EUCLID'S ELEMENTS OF Geometry.

In XV. Books:
With a ſupplement of divers *PROPOSITIONS* and *COROLLARIES.*
To which is added, a *Treatiſe* of REGULAR SOLIDS,
By *CAMPANE* and *FLUSSAS.*
LIKEWISE

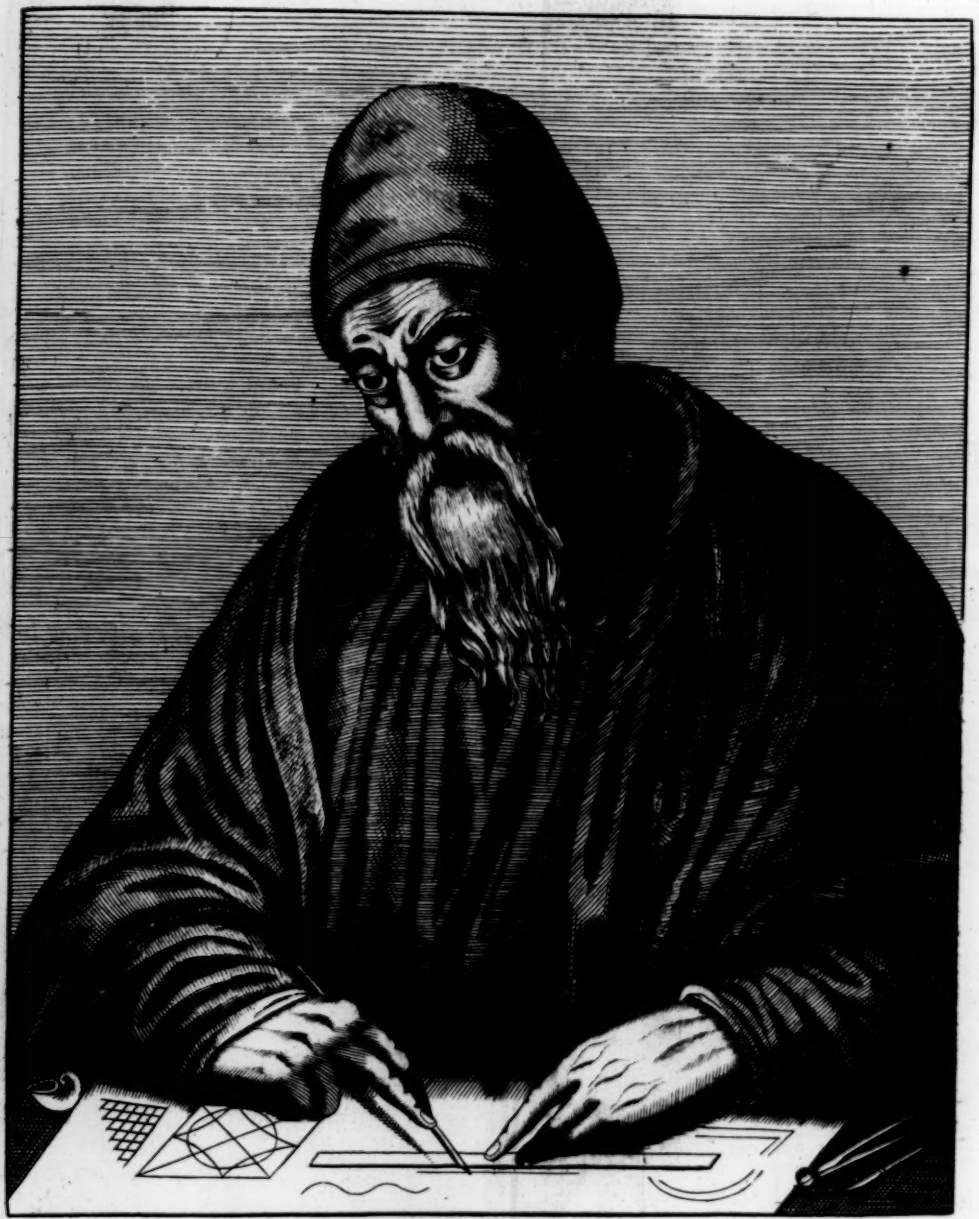
Euclid's DATA:

And MARINUS his Preface thereunto annexed.

Alſo a Treatiſe *of the* Diviſions *of* Superficies, *aſcribed to* Machomet Bagdedine, *but publiſhed by* Commandine, *at the requeſt of* John Dee *of* London; *whoſe* Preface *to the ſaid* Treatiſe *declares it to be the* Worke *of* EUCLIDE, *the* Author *of theſe* ELEMENTS.

Publiſhed by the Care and Induſtry of
JOHN LEEKE and GEORGE SERLE, Students in the *MATHEMATICKS.*

LONDON:
Printed, by R. & W. LEYBOURN, for GEORGE SAWBRIDGE at the *Bible* upon *Ludgate-hill,*
MDCLXI.



See the Bold-Shaddow of Vrania's Glory,
Immortall in His Race, no lesse in Story:
An Artist without Error, from whose Lyne,
Both Earth and Heav'ns, in sweet Proportions twine:
Behold Great EUCLID! But, behold Him well!
For 'tis in Him Divinity doth dwell. /

G. Wharton.

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TO THE

Duke of YORK.

May it pleaſe your HIGHNESSE:



BEfore a PRINCE of ſo approved Conduct, I ſhall not enumerate the many wayes that the MATHEMATICKS are Auxiliary in WAR; nor trouble you with ſuch Trite Stories to Court your belief thereof, as that of ARCHIMEDES; who ſitting in his Chair, with his *Nugæ Mathematicæ* (as he called them) threw multiplicity of Deaths amongſt the all-daring Legions of MARCELLUS: This Sir, is a taske needleſſe, and moſt injurious to your Highneſſe Merits; whoſe perfection in theſe

Sciences, could be only made queſtionable by ſuch exaggerations.

No, great Sir, The ſole reaſon for laying this Dedication: with ſo much preſumption, at your Highneſſe Feet, was our conſciouſneſs how particular an intereſt you have in the Prince of Mathematicians, whilſt all the world admires you, as the moſt abſolute Proficient both in *Mars*, and the *Muſes* Schools.

'Tis EUCLIDE, Sir, that implores your Princely Patrociny, the vouchſafe whereof, as it will procure him the favour of all; ſo it will be a Glorious Shield, to dazle the eyes of ſuch who would over-nicely cenſure the Lapſes of his Interpretors, and

SIR,

Your Highneſſes moſt humble
and moſt obliged Servants,

JOHN LEEKE,
GEORGE SERLE.

To the unfained lovers of Truth, and constant *Students* of Noble *Sciences*, JOHN DEE of *London*, heartly wisheth grace from Heaven, and most prosperous success in all their honest attempts and *exercises*.



DIvine PLATO, the great Master of many worthy Philosophers, and the constant avoucher, and pithy perswader of *Unum*, *Bonum*, and *Ens*: in his School and Academie, sundry times (besides his ordinary Scholers) was visited of a certain kind of men allured by the noble fame of *Plato*, and the great commendation of his profound and profitable doctrine. But when such Hearers, after long harkening to him, perceived, that the drift of his discourses issued out, to conclude, this *Unum*, *Bonum*, and *Ens*, to the Spiritual, Infinite, Eternal, Omnipotent, &c. Nothing being alledged or expressed, How; worldly goods: how, worldly dignity: how, health, strength, or lustinesse of body: nor yet the means, how a marvellous, sensible and bodily blisse and felicity hereafter, might be atteined: Straightway, the fantasies of those hearers were dampt, their opinion of *Plato* was clean changed: yea, his doctrine was by them despised; and his School, no more of them visited. Which thing, his Scholer, *Aristotle*, narrowly considering, found the cause thereof, to be, For that they had no fore-warning and information, in general, " whereto his Doctrine tended. For, so, might they have had occasion, either to have forborn his School haunting: (if they, then, had misliked his Scope and purpose) or constantly to have continued therein: to their full satisfaction: if such a final Scope and intent, had been to their desire. Wherefore, *Aristotle*, ever, after that, used in brief, to " fore-warn his own Scholers and hearers, both of what matter, and also to what end, he " took in hand to speak, or teach. While I consider the diverse trades of these two excellent Philosophers (and am most sure, both, that *Plato* right-well, otherwise could teach: and that *Aristotle* might boldly, with his hearers, have dealt in like sort as *Plato* did) I am in no little pang of perplexity: Because, that which I mislike, is most easie for me to perform (and to have *Plato* for my example.) And that which I know to be most commendable: and (in this first bringing, into common handling, the *Arts Mathematical*) to be most necessary: is full of great difficulty and sundry dangers. Yet, neither do I think it meet, for so strange matter (as now is meant to be published) and to so strange an audience, to be bluntly at first put forth, without a peculiar Preface: Nor (imitating *Aristotle*) well can I hope, that according to the amplenesse and dignity of the *State Mathematical*, I am able, either plainly to prescribe the material bounds: or precisely to expresse the chief purposes, and most wonderful applications thereof. And though I am sure, that such as did shrink from *Plato* his School, after they had perceived his final conclusion, would in these things have been his most diligent hearers (so infinitely might their desires, in fine and at length, by our *Art Mathematical*, be satisfied) yet, by this my Preface and fore-warning, Aswell all such, may (to their great behoof) the sooner, hither be allured: as also the

the *Pythagorical*, and *Platonical* perfect Scholer, and the constant profound Philosopher, with more ease and speed, may (like the Bee) gather, hereby, both wax and honey.

The intent of this Preface.

" Wherefore, seeing I find great occasion (for the causes alledged, and farther, in respect " of my *Art Mathematick general*) to use a certain fore-warning and Preface, whose " content shall be, that mighty, most pleasant, and fruitful *Mathematical Tree*, with his " chief arms and second (grafted) branches: " Both, what every one is and also, what com- " modity, in general, is to be looked for, aswell of grift as stock: And forasmuch as this " enterprize is so great, that, to this our time, it never was (to my knowledge) by any " atchieved: And also it is most hard, in these our drery dayes, to such rare and strange Arts, to win due and common credit: Nevertheless, if for my sincere endeavour, to satisfie your honest expectation, you will but lend me your thankful mind a while: and to such matter as, for this time, my pen (with speed) is able to deliver, apply your eye or ear attentively: perchance, at once, and for the first saluting, this Preface you will find a lesson long enough. And either you will, for a second (by this) be made much the apter: or shortly become, well able your selves, of the Lions claw, to conjecture his royal Symmetrie, and farther property. Now then, gentle, my friends and countrey-men, Turn your eyes, and bend your minds to that doctrine, which for our present purpose, my simple talent is able to yield you.

All things which are, and have being, are found under a triple diversity general. For, either, they are deemed Supernatural, Natural, or of a third being. Things Supernatural, are immaterial, simple, indivisible, incorruptible, and unchangable. Things Natural, are material, compounded, divisible, corruptible, and changable: Things Supernatural, are of the mind onely comprehended: Things Natural, of the sense exterior, are able to be perceived. In things Natural, probability and conjecture hath place: But in things Supernatural, chief demonstration and most sure Science is to be had. By which properties and comparisons of these two, more easily may be described, the state, condition, nature and property of those things, which, we before termed of a third being: which, by a peculiar name also, are called *Things Mathematical*. For, these, being (in a manner) middle, between things supernatural and natural: are not so absolute and excellent, as things supernatural; Nor yet so base and grosse, as things natural: But are things immaterial, and nevertheless, by material things able somewhat to be signified. And though their particular Images by Art, are aggregable and divisible: yet the general *Forms* notwithstanding, are constant, unchangeable, untransformable and incorruptible. Neither of the sense, can they, at any time, be perceived or judged. Nor yet, for all that in the royall mind of man, first conceived. But surmounting the imperfection of conjecture, weening and opinion; and comming short of high intellectual conception, are the Mercurial fruit of *Dianœtical* discourse, in perfect imagination subsisting. A marveilous newtrality have these things *Mathematical*, and also a strange participation between things supernatural, immortal, intellectual, simple and indivisible: and things natural, mortal, sensible, compounded and divisible. Probability and sensible proof, may well serve in things natural, and is commendable: In Mathematical reasonings, a probable Argument, is nothing regarded: nor yet the testimony of sense, any whit credited: But onely a perfect demonstration, of truths certain, necessary, and invincible: universally and necessarily concluded: is allowed as sufficient for an Argument exactly and purely Mathematical.

Number.

Note the word *Unite*, to express the Greek *Monas*, and not *Unity*, as we have all, commonly, till now, used.

Of *Mathematical* things, are two principal kinds, namely, *Number* and *Magnitude*. *Number*, we define to be a certain Mathematical Summe, of *Units*. And an *Unit* is that thing Mathematical, Indivisible, by participation of some likenesse of whose property, any thing, which is indeed, or is counted One, may reasonably be called One. We account an *Unit*, a thing *Mathematical*, though it be no *number*, and also indivisible because of it materially, Number doth consist: which principally, is a thing *Mathematical*. *Magnitude* is a thing *Mathematical*, by participation of some likeness of whose nature, any thing is judged long, broad, or thick. A thick *Magnitude* we call a *Solid*, or a *Body*. What *Magnitude* so ever is Solid or Thick, is also broad and long. A broad Magnitude we call a *Superficies* or a Plain. Every plain Magnitude, hath also length. A long Magnitude, we term a *Line*. A *Line* is neither thick nor broad, but onely long: Every certain Line, hath two ends: The ends of a Line, are *Points* called. A *Point*.

A Point.

Point is a thing *Mathematical*, indivisible, which may have a certain determined situation. If a Point move from a determined situation, the way wherein it moved, is also a *Line*, Mathematically produced;

produced: whereupon, of the ancient Mathematicians, a *Line* is called the race or courſe of a *Point*. A Point we define, by the name of a thing Mathematical: though it be no Magnitude, and indiviſible: becauſe it is the proper end, and bound of a *Line*: which is a true *Magnitude*. And *Magnitude* we may define to be that thing *Mathematical*, which is diviſible for ever, in parts diviſible, long, broad, or thick. Therefore though a Point be no *Magnitude*, yet *Terminatively* we reckon it a thing *Mathematical*, (as I ſaid) by reaſon it is properly the end and bonnd of a line. *A Line.* *Magnitude.*

Neither *Number* nor *Magnitude* have any Materiality. Firſt, we will conſider of *Number*, and of the Science *Mathematical*, to it appropriate, called *Arithmetick*, and afterward of *Magnitude*, and his Science called *Geometrie*. But that name contenteth me not: whereof a word or two hereafter ſhall be ſaid. How Immaterial, and free from all matter, *Number* is, who doth not perceive? yea, who doth not wonderfully wonder at it? For, neither pure *Element*, nor *Ariſtotles Quinta Eſſentia*, is able to ſerve for Number, as his proper matter. Nor yet the purity and ſimpleneſſe of Subſtance Spiritual or Angelical, will be found proper enough thereto. And therefore the great and godly Philoſopher *Anitius Boëtius*, ſaid: *Omnia quæcunque à primæva rerum natura conſtructa ſunt, Numerorum videntur ratione formata. Hoc enim fuit principale in animo Conditoris Exemplar.* That is: **All things (which from the very firſt original being of things, have been framed and made) do appear to be Formed by the reaſon of Numbers. For this was the principal Example or pattern in the mind of the Creator.** O comfortable allurement, O raviſhing perſwaſion, to deal with a Science, whoſe Subject is ſo ancient, ſo pure, ſo excellent, ſo ſurmounting all creatures, ſo uſed of the Almighty and incomprehenſible wiſedome of the Creator, in the diſtinct creation of all creatures: in all their diſtinct parts, properties, natures, and vertues, by order, and moſt abſolute number, brought from *Nothing*, to the *Formality* of their being and ſtate. By *Numbers* property therefore, of us, by all poſſible means, (to the perfection of the Science) learned, we may both wind and draw our ſelves into the inward and deep ſcarch and view, of all creatures diſtinct vertues, natures, properties, and *Forms*: And alſo farther, ariſe, clime, aſcend and mount up (with ſpeculative wings) in ſpirit, to behold in the Glaſſe of Creation, the *Form* of *Forms*, the *Exemplar Number* of all things *Numerable*: both viſible and inviſible: mortal and immortal, Corporal and Spiritual. Part of this profound and divine Science, had *Joachim* the Propheſier atteined unto: by *Numbers Formal*, *Natural*, and *Rational*, foreſeeing, concluding, and foreſhewing great particular events, long before their comming. His books yet remaining, hereof, are good proof. And the Noble Earl of *Mirandula*, (beſides that) a ſufficient witneſſe: that Joachim *in his propheſies, proceeded by no other way, then by Numbers Formal.* And this Earl himſelf, in *Rome*, * ſet up 900 Concluſions, in all kind of Sciences, openly to be diſputed of: and among the reſt, in his Concluſions *Mathematical* (in the eleventh Concluſion) hath in Latine this Engliſh ſentence. *By numbers, a way is had, to the ſearching out, and underſtanding of every thing, able to be known. For the verifying of which Concluſion, I promiſe to anſwer to the 74 Quaſtions, under written by the way of Numbers.* Which Concluſions, I omit here to rehearſe: aſwell avoiding ſuperfluous prolixitie: as, becauſe *Joannes Picus, Works*, are commonly had. But, in any caſe, I would wiſh that thoſe Concluſions were read diligently, and perceived of ſuch, as are earneſt Obſervers and Conſiderers of the conſtant law of Numbers: which is planted in things Natural and Supernatural: and is preſcribed to all Creatures, inviolably to be kept. For, ſo, beſides many other things, in thoſe Concluſions to be marked it would appear, how ſincerely, and within my bounds, I diſcloſe the wonderful myſteries by numbers, to be atteined unto. *Anno* 1488.

Of my former words, eaſie it is to be gathered, that *Number* hath a treble ſtate: One, in the Creator: another is every Creature (in reſpect of his compleat conſtitution:) and the third, in Spiritual, and Angelical Minds, and in the Soul of man. In the firſt and third ſtate, *Number*, is termed *Number Numbring*. But in all Creatures otherwiſe, *Number* is termed *Number Numbred*. And in our Soul, Number beareth ſuch a ſway, and had ſuch an affinity therewith; that ſome of the old Philoſophers taught, *Mans ſoul, to be a number moving it ſelf*. And indeed, in us, though it be a very Accident, yet ſuch an Accident it is, that before all Creatures it had perfect being, in the Creator, Sempiternally. *Number*

numbring therefore, is the diſcretion, diſcerning, and diſtincting, of things. But in God the Creator, This diſcretion, in the beginning, produced orderly and diſtinctly all things. For, his *numbring*, then, was his Creating of all things. And his continual *numbring* of all things, is the Conſervation of them in being. And, where and when he will lack an *Unit*: there and then, that particular thing ſhall be *Diſcreated*. Here I ſtay. But our Severalling, diſtincting, and *numbring*, createth nothing: but of Multitude conſidered, maketh certain and diſtinct determination. And albeit theſe things be waighty, and truths of great importance, yet (by the infinite goodneſs of the Almighty *Ternarie*,) Artificial Methods and eaſie wayes are made, by which the zealous Philoſopher may win neer this Riveriſh *Ida*, this Mountain of Contemplation: and more then Contemplation. And alſo, though *number* be a thing ſo immaterial, ſo divine, and eternal: yet by degrees, by little, and little, ſtretching forth, and applying ſome likeneſſe of it, as firſt, to things Spiritual: and then bringing it lower, to things ſenſibly perceived, as of a momentany ſound iterated: then to the leaſt things that may be ſeen, numerable: And at length (moſt groſly) to a multitude of any corporal things ſeen, or felt: and ſo of theſe groſſe and ſenſible things, we are trained to learn a certain Image, or likeneſſe of numbers, and to uſe Art in them to our pleaſure and profit. So groſſe is our converſation, and dull is our apprehenſion, while mortal Senſe, in us, ruleth the Common-wealth of our little world. Hereby we ſay, Three Lyons are three or a *Ternarie*. Which * *Ternaries*, are each, the *Union*, *knot* and *Uniformity* of three diſcreet and diſtinct *Units*. That is, we may in each *Ternary*, thrice ſeverally, point, and ſhew a part, *One*, *One*, and *One*. Where, in Numbring, we ſay One, Two, Three. But how far theſe viſible Ones, do differ from our Indiviſible Units (in pure *Arithmetick* principally conſidered) no man is ignorant. Yet from theſe groſſe and material things, may we be led upward, by degrees, ſo informing our rude Imagination, toward the conceiving of *numbers*, abſolutely: (Not ſuppoſing, nor admixting any thing created, Corporal or Spiritual, to ſupport, contein, or repreſent thoſe *numbers* imagined:) that at length, we may be able, to find the number of our own name, glorioufly exemplified and regiſtred in the book of the *Trinity*, moſt bleſſed and eternal.

But farther underſtand, that vulgar Practiſers, have Numbers, otherwiſe, in ſundry Conſiderations: and extend their name farther, then to Numbers, whoſe leaſt part is an *Unit*. For the common Logiſt, Reckenmaſter, or Arithmetician, in his uſing of Numbers: of an *Unit*, imagineth leſſe parts: and calleth them *Fractions*. As of an *Unit*, he maketh an half, and thus noteth it, $\frac{1}{2}$, and ſo of other (infinitely diverſe) parts of an *Unit*. Yea, and farther, hath *Fractions of Fractions*, *&c.* And foraſmuch as *Addition*, *Subſtraction*, *Multiplication*, *Diviſion*, and *Extraction of Roots*, are the chief, and ſufficient parts of *Arithmetick*: which is, the *Science that demonſtrateth the properties of numbers, and all operations, in numbers to be performed.* "How often therefore, theſe five ſundry "ſorts of Operations, do, for the moſt part of their execution, differ from the five opera- "tions of like general property and name, in our Whole numbers practiſable. So, often, "(for a more diſtinct doctrine) we, vulgarly account and name it, another kind of *Arith-metick*. And by this reaſon: the conſideration, doctrine, & working in whole numbers onely: where, of an *Unit*, is no leſſe part to be allowed: is named (as it were) an *Arithmetick* by it ſelf. And ſo of the *Arithmetick of Fractions*. In like ſort the neceſſary, wonder-ful, and Secret doctrine of Proportion, and proportionality hath purchaſed unto it ſelf a peculiar manner of handing and working: and ſo may ſeem another form of *Arithmetick*. Moreover, the *Aſtronomers*, for ſpeed, and more commodious calculation, have deviſed a peculiar manner of ordering numbers, about their circular motions, by Sexagenes and Sexageſines. By Signes, Degrees, Minutes, &c. which commonly is called the *Arithmetick* of *Aſtronomical* or *Phyſical Fractions*. That have I briefly noted by the name of *Arithmetick Circular*. Becauſe it is alſo uſed in Circles, not *Aſtronomical*, *&c.* Practiſe hath led *Numbers* farther, and hath framed them, to take upon them, the ſhew of *Magnitudes* property: Which is *Incommenſurability* and *Irrationality*. (For in pure *Arithmetick*, an *Unit*, is the common Meaſure of all Numbers.) And here, Numbers are become, as Lines, Plains, and Solids, ſometimes *Rational*, ſometimes *Irrational*. And have proper and peculiar characters, (as $\surd 3 . \surd \mathfrak{C}$. and ſo of other. Which is to ſignifie *Root Square*, *Root Cubick*: *and ſo forth*:) and proper and peculiar faſhions in the five principal parts: Wherefore the Practiſer eſteemed this a diverſe *Arithmetick* from the other.

Arithmetick Note.

1.

2.

3.

4.

other. Practiſe bringeth in, here, diverſe compounding of Numbers: as ſometime, two, three, four (or more) *Radical* Numbers diverſly knit by ſigns, of More and Leſſe: as thus $\surd 3\ 12 + \surd \mathfrak{C}\ 15$. Or thus $\surd 3\ 3\ 19 + \surd \mathfrak{C}\ 12 - \surd 3\ 2$, &c. And ſometime with whole numbers, or fractions of whole Number, among them: as $20 + \surd 3\ 24$. $\surd \mathfrak{C}\ 16 + 33 - \surd 3\ 10$. $\surd 3\ 3\ 44 + 12\frac{1}{4} + \surd \mathfrak{C}\ 9$. And ſo infinitely, may hap the varietie. After this; Both the one and the other hath fractions incident: and ſo is this *Arithmetick* greatly enlarged, by diverſe exhibiting, and uſe of Compoſitions and mixtings. Conſider how, I (being deſirous to deliver the Student from errour and Cavillation) do give to this *Practiſe*, the name of the *Arithmetick of Radical numbers*: Not of *Irrational* or *Surd numbers*: which other while, are Rational: though they have the Signe of a Root before them, which *Arithmetick* of whole Numbers moſt uſual, would ſay they had no ſuch Root, and ſo account them *Surd numbers*, which, generally ſpoken, is untrue: as *Euclid's* tenth Book may teach you. Therefore, to call them, generally *Radical numbers*, (by reaſon of the ſign $\surd$ prefixed) is a ſure way, and a ſufficient general diſtinction from all other ordering and uſing of Numbers: And yet (beſides all this) Conſider; the infinite deſire of knowledge, and incredible power of mans Search and Capacity: how, they, joyntly have waded farther (by mixing ſpeculation and practice) and have found out, and atteined, to the very chief perfection (almoſt) of *Numbers* practical uſe. Which thing is well to be perceived in that great Arithmetical Art of *Equation*: commonly called the *Rule of Coſs*, or *Algebra*. The Latines termed it, *Regulam Rei & Cenſus*, that is, the **Rule of the thing, and his value.** With an apt name, comprehending the firſt and laſt points of the Work. And the vulgar names, both in Italian, French, and Spaniſh depends (in naming it,) upon the ſignification of the Latin word *Res*: **A thing:** unleaſt they uſe the name of *Algebra*. And therein (commonly) is a double errour. The one of them which think it to be of *Geber*, his inventing: the other of ſuch as call it *Algebra*. For, firſt, though *Geber* for his great skill in Numbers, Geometry, Aſtronomy, and other marvellous Arts, might have ſeemed able to have firſt deviſed the ſaid Rule: and alſo the name carrieth with it a very neer likeneſſe of *Geber* his name: yet true it is, that a *Greek* Philoſopher and Mathematician, named *Diophantis*, before *Geber* his time, wrote 13 books thereof (of which ſix are yet extant: and I had them to * uſe, of the famous Mathematician, and my great frind, *Petrus Montaureus*:) *Anno* 1550. And ſecondly, the very name, is *Algiebar*, and not *Algebra*: as by the Arabian *Avicen*, may be proved: who hath theſe preciſe words in Latine, by *Andreas Alpagus* (moſt perfect in the Arabick tongue) ſo tranſlated. *Scientia faciendi Algiebar & Almachabel. i. Scientia inveniendi numerum ignotum, per additum numeri, & diviſionem, & equationem.* Which is to ſay: **The Science of working Algiebar, and Almachabel: that is: the Science of finding an unknown Number, by adding of a Number, and Diviſion, and Equation.** Here have you the name: and alſo the principal parts of the Rule, touched. To name it, *The rule, or art of Æquation*, doth ſignifie the middle part and the State of the Rule. This Rule hath his peculiar Characters, and the principal parts of *Arithmetick*, to it apperteining, do differ from the other *Arithmetical operations*. This *Arithmetick* hath Numbers Simple, Compound, Mixt, and Fractions accordingly. This Rule and *Arithmetick* of *Algiebar*, is ſo profound, ſo general, and ſo (in manner) conteineth the whole power of Numbers, Application practical: that mans wit, can deal with nothing more profitable about numbers: nor match, with a thing, more meet for the divine force of the Soul, (in humane ſtudies, affairs or exerciſes) to be tried in. Perchance you looked for, (long ere now) to have had ſome particular proof, or evident teſtimony of the uſe, profit, and Commodity of Arithmetick vulgar, in the common life and trade of men. Thereto, then I will now frame my ſelfe: But herein great care I have, leaſt length of ſundry proofs, might make you deem, that either I did diſdoubt your zealous mind to vertues ſchool, or elſe miſtruſt your able wits, by ſome, to gueſſe much more. A proof, then four, five, or ſix, ſuch, will I bring, as any reaſonable man, therewith may be perſwaded, to love and honour, yea learn and exerciſe the excellent Science of *Arithmetick*.

And firſt: Who, neerer at hand, can be a better witneſſe of the fruit received by *Arithmetick*, then all kind of Merchants? Though not alike, either need it, or uſe it. How could they forbear the uſe and help of the Rule, called the Golden Rule? Simple and

Compound: both forward and backward? How might they misse *Arithmetical* help in the Rules of Fellowship: either without time, or with time? and between the Merchant and his Factor? The Rules of Bartering in wares onely: or part in wares, and part in money, would they gladly want? Our Merchant Venturers, and Travellers over Sea, how could they order their doings justly and without losse, unlesse certain and general Rules for Exchange of money, and Rechange, were, for their use, devised? The Rule of Alligation, in how sundry cases, doth it conclude for them, such precise verities, as neither by natural wit, nor other experience they were able else to know? And (with the Merchant then to make an end) how ample and wonderful is the Rule of False positions? especially as it is now, by two excellent Mathematicians (of my familiar acquaintance in their life time) enlarged? I mean *Gemma Frisius* and *Simon Jacob*, Who can either in brief conclude, the general and Capital Rules? or who can Imagine the Myriades of sundry Cases, and particular examples in Act and earnest, continually wrought, tried and concluded by by the forenamed Rules only? How sundry other *Arithmetical practises*, are commonly in Merchants hands, and knowledge: They themselves can at large testifie.

The Mintmaster, and Goldsmith, in their mixture of Metals, either of divers kinds, or divers values; how are they, or may they, exactly be directed, and marvelously pleasured, if *Arithmetick* be their guide? And the honourable Physicians will gladly confesse themselves much beholding to the Science of *Arithmetick*, and that sundry wayes: But chiefly in their Art of Graduation, and compound Medicines. and though *Gallenus*, *Averrois*, *Arnoldus*, *Lullus*, and others have published their positions, aswel in the quantities of the Degrees above Temperament, as in the Rules, concluding the new *Form* resulting: yet a more precise, commodious, and easie *Method*, is extant: by a Countreyman of ours (above 200 years ago) invented. And forasmuch as I am uncertain, who hath the same: or when that little Latin treatise (as the Author writ it,) shall come to be printed: (Both to declare the desire I have to pleasure my Countrey, where in I may: and also, for very good proof of Numbers use, in this most subtile, and fruitful Philosophical Conclusion,) I intend in the mean while, most briefly, and with my farther help, to communicate the pith thereof unto you.

R. B.

First, describe a circle: whose Diameter let be an inch. Divide the circumference into four equal parts. From the Center, by those 4 sections, extend 4 right lines: each of 4 inches and a half long: or of as many as you list, about 4 without the circumference of the circle: So that they shall be of 4 inches long (at the least) without the Circle. Make good evident marks at every inches end. If you list, you may subdivide the inches again into 10 or 12 smaller parts, equal. At the ends of the lines, write the names of the 4 principal Elemental qualities. *Hot*, and *Cold*, one against the other. And likewise *Moist* and *Dry*, one against the other. And in the circle write *Temperate*. Which *Temperature* hath a good Latitude: as appeareth by the Complexion of man. And therefore we have allowed unto it, the foresaid Circle: and not a point Mathematical or Physical.

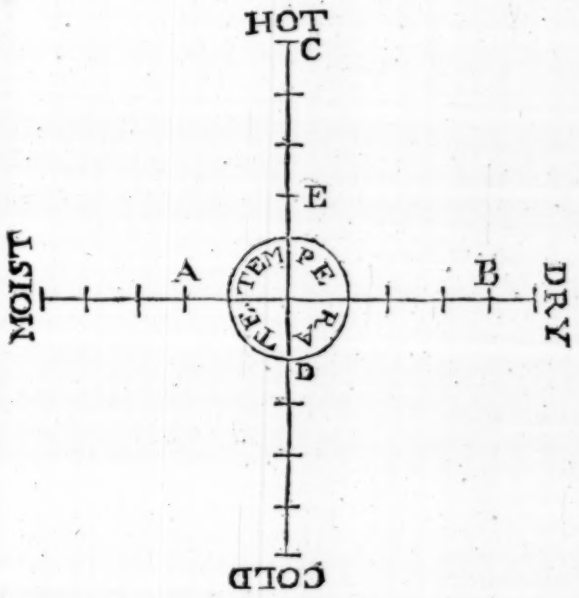
* Take some part of *Lullus* counsel in his Book *De Q. Essentia.*

Now, when you have two things Miscible, whose degrees are * truly known; Of necessity, either they are of one quantity and waight, or of diverse. If they be of one quantity and waight: whether their forms, be contrary qualities, or of one kind (but of divers intentions and degrees) or a *Temperate*, and a contrary, *The form resulting of their Mixture, is in the Middle between the degrees of the forms mixt.* As for example, let *A*, be *Moist* in the first degree: and *B*, *Dry* in the third degree. Adde 1 and 3, that maketh 4: the half or middle of 4 is 2. This 2 is the middle, equally distant from *A* and *B* (for the * *Temperament* is counted none. And for it, you must put a Cypher, if at any time, it be in mixture.) Counting them from *B*, 2 degrees, toward *A*: you find it to be *Dry* in the first degree: So is the *Form resulting* of the mixture of *A*, and *B*, in our Example. I will give you another example. Suppose, you have two things, as *C*, and *D*: and of *C*, the Heat to be in the 4th. degree: and of *D*, the Cold, to be remisse, even unto the *Temperament*. Now, for *C*, you take 4: and for *D*, you take a Cypher: which, added unto 4, yeildeth onely 4. The middle, or half, whereof, is 2. Wherefore the *Form resulting* of *C*, and *D*, is *Hot* in the second degree: for, 2 degrees, accounted from *C*, toward *D*, end just in the second degree of heat. Of the third manner, I will give also an example: which let be this: I have a liquid Medicine whose Qualitie of heat is in the fourth degree exalted: as was *C*, in the example foregoing: and an other liquid Medicine I have: whose Qualitie, is heat, in the first degree. Of each of these, I mixt alike quantitie: Subtract here,

* *Note.*

Note.

here, the leſſe from the more: and the reſidue divide into two equal parts: whereof the one part, either added to the leſſe, or ſubtracted from the higher degree, doth produce the



degree of the Form reſulting, by this mixture of *C*, and *E*. As, if from 4 you abate 1, there reſteth 3, the half of 3 is 1½: Add to 1 ½: you have 2 ½. Or ſubtract from 4 this 1½: you have likewiſe 2 ½ remaining. Which declareth, the *Form reſulting*, to be *Heat*, in the middle of the third degree.

The ſecond Rule.

" But if the quantities of two things Commixt, be divers, and the intenſions, (of their " Forms miſcible) be in divers degrees, and heigths. (Whether thoſe Forms be of one " kind, or of Contrary kinds, or of a Temperate and a Contrary, *What proportion is of " the leſſe quantity to the greater, the ſame ſhall be of the difference, which is between " the degree of the Form reſulting, and the degree of the greater quantity of the thing " miſcible to the difference, which is between the ſame degree of the Form reſulting, and " the degree of the leſs quantity.* As for example. Let two pound of Liquour be given, hot in the fourth degree: and one pound of Liquour be given, hot in the third degree. I would gladly know the Form reſulting, in the mixture of theſe two Liquours. Set down your numbers in order thus. Now by the Rule of Algiebar, have I deviſed a very eaſie, brief, and general manner of working in this caſe. Let us firſt ſuppoſe that *Middle Form reſulting* to be 1 ☌: as that Rule teacheth. And becauſe (by our Rule, here given) as the weight of 1, is to 2: So is the difference between 4, (the degree of the greater quantity) and 1 ☌: to the difference between 1 ☌ and 3: (the degree of the thing, in leſſe quantity. And withal, 1 ☌, being always in a certain middle, between the two heigths or degrees. For the firſt difference, I ſet 4—1 ☌: and for the ſecond, I ſet 1 ☌—3. And now again, I ſay, as 1 is to 2. ſo is 4—1 ☌ to 1 ☌—3. Wherefore, of theſe four proportional numbers, the firſt and the fourth multiplyed, one by the other, do make as much, as the ſecond and the third multiplied the one by the other. Let theſe multiplications be made accordingly. And of the firſt and the fourth, we have 1 ☌— 3, and of the ſecond and third, 8—2 ☌. Wherefore, your Æquation is between 1 ☌ — 3, and 8—2 ☌. Which may be reduced, according to the Art of Algiebar: as, here, adding 3, to each part, giveth the Æquation, thus, 1 ☌=11—2 ☌. And yet again, contracting or reducing it: Adde to each part, 2 ☌: Then have you 3 ☌ equal to 11, thus

lb. 2.	*Hot.* 4.
lb. 1.	*Hot.* 3.

thus represented 3 ☌ = 11. Wherefore, dividing 11, by 3: the quotient is 3 ⅔: the *Value* of our 1 ☌, *Coss*, or *Thing*, first supposed: and that is the heigth, or Intention of the *Form resulting*: which is *Heat*, in two thirds of the fourth degree: and here I set the shew of the work in conclusion thus. The proof hereof is easie, by substracting 3, from 3⅔, resteth ⅔. Substract the same heigth of the Form resulting, (which is 3 ⅔) from 4: then resteth ⅓. You see that ⅔ is double to ⅓: as 2 ℔. is double to 1 ℔. So should it be: by the Rule here given. Note. As you added to each part of the Æquation 3: so if you first added to each part 2 ☌, it would stand, 3 ☌ − 3 = 8: and now adding to each part 3: you have (as afore) 3 ☌ = 11.

℔. 2.	*Hot.* 4.	⅓	3 ⅔ *The Form resulting.*
℔. 1.	*Hot.* 3.	⅔	

And though, I, here, speak onely of two things misceible: and most commonly more then three, four, five or six, &c. are to be mixed: (and in one Compound to be reduced: and the Form resulting of the same, to serve the turn) yet these Rules are sufficient, duly repeated and iterated. In proceeding first, with any two: and then with the Form resulting and another, and so forth: For, the last work concludeth the Form resulting of them all: I need nothing to speak, of the mixture (here supposed) what it is. Common Philosophy hath defined it, saying, *Mixtio est miscibilium, alteratorum, per minima conjunctorum Unio.* Every word in the definition, is of great importance. I need not also spend any time, to shew, how, the other manner of distributing of degrees, doth agree to these Rules. Neither need I of the farther use belonging to the Crosse of Graduation (before described) in this place declare, unto such as are capable of that, which I have already said. Neither yet with examples, specifie the manifold varieties, by the foresaid two general Rules to be ordered. The witty and Studious, here, have sufficient: And they which are not able to attein to this, without lively teaching, and more in particular: would have larger discoursing, then is meet in this place to be dealt withall: And other (perchance) with a proud snuffe will disdain this little: and would be unthankful for much more. I, Therefore conclude; and with such as have modest and earnest Philosophical minds, to laud God highly for this: and to marvail that the profoundest and subtilest point, concerning *Mixture of Forms, and Qualities Natural*, is so matcht and married with the most simple, easie, and short way of the noble Rule of *Algiebar*. Who can remain, therefore, unperswaded, to love, allow, and honour the excellent Science of *Arithmetick*? For, here, you may perceive that the little finger of *Arithmetick* is of more might and contriving, then a hundred thousands mens wits, of the middle sort, are able to perform, or truly to conclude, without help thereof.

Note.

Now, will we farther, by the wise and valiant Captain, be certified, what help he hath, by the Rules of *Arithmetick*: in one of the arts to him apperteining: and of the Greeks "named Ταχτικὴ. That is, the skill of ordering Souldiers in Battle ray, after the best "manner to all purposes. This art so much dependeth upon Numbers use, and the Mathematicals, that *Ælianus* (the best writer thereof,) in his work to the Emperour *Hadrianus*, by his perfection, in the Mathematicals, (being greater then others before him had,) thinketh his book to passe all other the excellent work, written of that art, unto his dayes. For, of it, had written *Æneas*: *Cyneas of Thessaly*: *Pyrrhus Epirota*, and *Alexander* his sonne: *Clearchus*: *Pausanias*: *Evangelus*: *Polybius*, familiar friends to *Scipio*, *Empolemus*, *Iphicrates*, *Possidonius*, and very many other worthy Captains, Philosophers and Princes of immortal fame and memory. Whose faireſt flower of their garland (in this feat) was *Arithmetick*: and a little perceiverance, in *Geometrical* Figures. But in many other cases doth *Arithmetick* stand the Captain in great stead. As in proportioning of victuals for the army, either remaining at a stay; or suddenly to be encreased with a certain number of Souldiers: and for a certain time. Or by good arts to diminish his Company, to make the victuals, longer to serve the remanent, and for a certain determined time: If need so require. And so in sundry his other accounts, Reckonings, Measurings, and Proportionings, the wise, expert, and circumspect Captain will affirm the Science of *Arithmetick*, to be one of his chief Counsailors, directers and aiders. Which thing (by good means) was evident to the Noble, the Couragious, the Loyal, and Courteous *John*, late

Ταχτικὴ.

Earl

Earl of *Warwick*. Who was a young Gentleman, throughly known to very few. Albeit his lusty valiantneſſe, force, and Skill in Chivalrous feats and exerciſes: his humbleneſs, and friendlineſs to all men, were things, openly, of the world perceived. But what roots (otherwiſe,) vertue had faſtened in his breſt, what Rules of Godly and honourable life he had framed to himſelf: what vices, (in ſome then living) notable, he took great care to eſchew, what manly vertues, in other noble men, (flouriſhing before his eyes,) he Sythingly aſpired after: what proweſſes he purpoſed and meant to atchieve: with what feats of Arts, he began to furniſh and fraught himſelf, for the better ſervice of his King and Countrey, both in peace and war. Theſe (I ſay) his Heroical Meditations, fore-caſtings, and determinations, no twain, (I think) beſide my helf, can ſo perfectly, and truly report. And therefore, in Conſcience, I count it my part, for the honour, preferment, and procuring of vertue, (thus briefly) to have put his Name, in the Regiſter of *Fame Immortal*.

To our purpoſe. This *John* by one of his acts (beſides many other: both in *England* and *France*, by me, in him noted) did diſcloſe his hearty love, to vertuous Sciences: and his noble intent, to excell in Martial proweſſe: when we with humble requeſt, and inſtant Soliciting: got the beſt Rules (either in the time paſt, by Greek or Roman, or in our time uſed: and new Stratagems therein deviſed) for ordering of all Companies, ſummes, and Numbers of men, (many, or few) with one kind of weapon, or more appointed: with Artillery, or without: on horſback or on foot: to give or take on-ſet: to ſeem many, being few: to ſeem few, being many. To march in battail or journey: with many ſuch feats, to foughten field, Skirmiſh or Ambuſh appertaining: and of all theſe lively deſignments (moſt curiouſly) to be in velame parchment deſcribed: with Notes and peculiar marks, as the art requireth: and all theſe Rules and deſcriptions Arithmetical, incloſed in a rich Caſe of Gold, he uſed to wear about his neck, as his Jewel moſt precious, and Counſailour moſt truſty. Thus, *Arithmetick*, of him, was ſhrined in gold: Of *Numbers* fruit, he had good hope. Now Numbers therefore innumerable, in *Numbers* praiſe, his ſhrine ſhall finde.

This Noble Earle, died Anno 1552. ſcarce 24 years of age, having no iſſue by his wife, Daughter to the Duke *Somerſet*.

What need I, (for farther proof to you) of the Schoolmaſters of Juſtice, to require teſtimony: how needful, how fruitful, how skilful a thing *Arithmetick* is? I mean the Lawyers of all ſorts. Undoubtedly, the Civilians can marvellouſly declare: how, neither the Ancient Romane Laws, without good knowledg of *Numbers Art*, can be perceived: Nor (Juſtice infinite Caſes) without due proportion, (narrowly conſidered) is able to be executed. How juſtly, and with great knowledg of Art, did *Papinianus* inſtitute a law of partition, and allowance, between man and wife after a divorce? But how *Accurſius*, *Baldus*, *Bartolus*, *Jaſon*, *Alexander*, and finally, *Alciatus*, (being otherwiſe, notably well learned) do jumble, gueſſe and erre from the equity, art and intent of the Law-maker: *Arithmetick* can detect, and convince: and cleerly, make the truth to ſhine. Good *Bartolus* tired in the examining and proportioning of the matter, and with *Accurſius* Gloſſe much cumbred: burſt out, and ſaid: *Nulla eſt in toto libro, hac gloſſa difficilior: Cujus computationem nec Scholaſtici, nec Doctores intelligunt, &c.* That is: **In the whole book, there is no gloſſe harder then this: Whoſe account or reckoning, neither the Scholers nor the Doctours underſtand, &c.** What can they ſay of *Julianus* law, *Si ita Scriptum*, *&c.* Of the Teſtators will, juſtly performing, between the wife, ſonne, and daughter? How can they perceive the equity of *Aphricanus*, *Arithmetical* Reckoning, where he treateth of *Lex Falcidia*? How can they deliver him, from his Reprovers: and their maintainers: as *Joannes*, *Accurſius*, *Hypolitus*, and *Alciatus*? How juſtly and artificially, was *Africanus* reckoning made? Proportionating to the ſummes bequeathed, the Contributions of each part? Namely, for the hundred preſently receiued, $17\frac{1}{7}$. And for the hundred, received after ten moneths $12\frac{6}{7}$: which make the 30: which were to be contributed by the legataries to the heir. For, what proportion 100 hath to 75: the ſame hath $17\frac{1}{7}$ to $12\frac{6}{7}$: Which is *Seſquitertia*, that is, as 4, to 3, which make 7. Wonderful many places in the Civil law, require an expert *Arithmetician*, to underſtand the deep Judgment and juſt determination of the Ancient Roman Law-maker. But much more expert ought he to be, who ſhould be able to decide with equity, the infinite variety of Caſes, which doe, or may happen, under every one of thoſe laws and ordinances Civil. Hereby, eaſily, you may now conjecture: that in the Canon law: and in the laws of the Realm (which with us, bear the chief authority) Juſtice and equity might be greatly preferred, and skilfully executed through

due skill of Arithmetick, and proportions apperteining. The worthy Philosophers and prudent law-makers (who have written many books *De Republica:* How the best state of Common-wealths might be procured and mainteined,) have very well determined of Justice: (which, not onely is the Base and foundation of Common weales: but also the total perfection of all our works, words, and thoughts:) "defining it, to be that vertue, "by which, to every one, is rendered, that to him appertaineth. God challengeth this at our hands to be honoured as God: to be loved, as a Father, and to be feared, as a Lord and Master. Our neighbours proportion is also prescribed of the Almighty Law-maker: which is, to do to others, even as we would be done unto. These proportions, are in Justice necessary: in duty, commendable: and of Common-wealths, the life, strength, stay, and flourishing. *Aristotle* in his *Æthicks* (to fetch the seed of Justice, and light of direction, to use, and execute the same) was fain to fly to the perfection, and power of Numbers: for proportions Arithmetical and Geometrical. *Plato* in his book called *Epinomis*, (which book is the Treasure of all his doctrine) where his purpose is to seek a Science, which, when a man had it, perfectly: he might seem, and so be, indeed, *Wise*. He briefly of other Sciences discoursing, findeth them, not able to bring it to passe: But of the Science of Numbers, he saith. *Illa, quæ numerum mortalium generi dedit, id profectò efficiet. Deum autem aliquem, magis quàm fortunam, ad salutem nostram, hoc munus nobis arbitror contulisse, &c. Nam ipsum bonorum omnium Authorem, cur non maximi boni, Prudentiæ dico, causam arbitramur?* **That Science, verily, which hath taught mankind number, shall be able to bring it to passe. And, I think, a certain God, rather than Fortune, to have given us this gift, for our blisse. For, why should we not judg him who is the Authour of all good things, to be also the cause of the greatest good thing, namely, Wisedome?** There, at length, he proveth *Wisedome* to be atteined by good skill of *Numbers*. With which great Testimony, and the manifold proofs, and reasons, before expressed, you may be sufficiently and fully perswaded; of the perfect Science of *Arithmetick*, to make this account: That of all Sciences, next to *Theologie*, it is most divine, most pure, most ample, and general, most profound, most subtile, most commodious, and most necessary. Whose next Sister, is the absolute Science of *Magnitudes:* of which (by the direction and aid of him, whose *Magnitude* is infinite, and of us incomprehensible) I now intend so to write, that both with the *Multitude*, and also with the *Magnitude* of marvellous and fruitful verities, you (my frirnds and Countreymen) may be stir'd up and awaked, to behold what certain Arts and Sciences (to our unspeakable behoof) our heavenly Father, hath for us prepared, and revealed by sundry *Philosophers* and *Mathematicians*.

Justice.

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BOth, *Number* and *Magnitude*, have a certain Original seed, (as it were) of an incredible property: and of man, never able, fully to be declared. Of *Number*, an Unit, and of *Magnitude*, a Point, do seem to be much like Original causes: But the diversity nevertheless, is great. We defined an *Unit* to be a thing Mathematical indivisible: A Point, likewise, we said to be a Mathematical thing indivisible. And farther, that a Point may have certain determined Situation: that is, that we may assign, and prescribe a Point, to be here, there, yonder, &c. Herein (behold) our Unit is free, and can abide no bondage, or to be tyed to any place, or seat: divisible or indivisible. Again, by reason, a Point may have a situation limited to him: a certain motion, therefore, (to a place, and from a place) is to a Point incident, and appertaining. But an *Unit* cannot be imagined, to have any motion. A point, by his motion, produceth Mathematically, a line, (as we said before) which is the first kind of Magnitudes, and most simple: An *Unit*, cannot produce any number. A line, though it be produced of a Point moved, yet it doth not consist of points: Number, though it be not produced of an *Unit*, yet doth it consist of Units, as a material cause. But formally, Number, is the Union and Unity of Units. Which uniting and knitting, is the workmanship of our mind: which, of distinct and discreet Units maketh a Number: by uniformity, resulting of a certain multitude of Units. And so, every number may have his least part given: namely, an Unit: But not of a magnitude, (no, not of a Line,) the least part can be given: because, infinitely, division thereof, may be conceived. All Magnitude, is either a Line, a Plain, or a Solid. Which Line, Plain or Solid, of no Sense, can be perceived, nor exactly by hand (any way) represented: nor of

Number.

Nature

Nature produced: But, as (by degrees) Number did come to our perceiverance: So by visible forms, we are holpen to imagine, what our Line Mathematical is. What our Point, is. So preciſe, are our Magnitudes, that one Line is no broader than an other: for they have no breadth: Nor our Plains have any thickneſſe: Nor yet our Bodies, any weight: be they never ſo large of dimenſion. Our Bodies, we can have ſmaller, than either Art, or Nature can produce any: and greater alſo, then all the World can comprehend. Our leaſt Magnitudes, can be divided into ſo many parts, as the greateſt. As, a Line of an inch long, (with us) may be divided into as many parts, as may the diameter of the whole World, from Eaſt to Weſt: or any way extended: What priviledges, above all manual Art, and Natures might, have our two Sciences Mathematical? to exhibit, and to deal with things of ſuch power, liberty, ſimplicity, purity, and perfection? And in them, ſo certainly, ſo orderly, ſo preciſely, to proceed: as excellent, is that workman Mechanical Judged, who neereſt can approach to the repreſenting of works, Mathematically demonſtrated? And our two Sciences, remaining pure, and abſolute in their proper terms, and in their own matter, to have, and allow onely ſuch demonſtrations, as are plain, certain, univerſal, and of an eternal verity? This Science of *Magnitude*, his properties, conditions, and appurtenances: commonly, now is, and from the beginning, hath of all Philoſophers, been called *Geometry*. But verily, with a name too baſe and ſcant, for a Science of ſuch dignity and ampleneſſe. And, perchance, that name, by common and ſecret conſent, of all wiſe men, hitherto eath been ſuffered to remain: that it might carry with it a perpetual memory of the firſt and notableſt benefit, by that Science, to common people ſhewed: Which was, when Bounds and Meres of land and ground, were loſt, and confounded, (as in *Egypt*, yearly, with the over-flowing of *Nilus*, the greateſt and longeſt river in the World) or, that ground bequeathed, were to be aſſigned: or ground ſold, were to be laid out: or (when diſorder prevailed) that Commons were diſtributed into Severalties. For, where, upon theſe and ſuch like occaſions, Some by ignorance, ſome by negligence; Some by fraud, and ſome by violence, did wrongfully limit, meaſure, encroach, or challenge (by pretence of juſt content and meaſure) thoſe lands and grounds: great loſſe, diſquietneſſe, murther, and war did (full oft) enſue: Till, by Gods mercy, and mans induſtry, The perfect Science of Lines, Plains, and Solids (like a divine Juſticier,) gave unto every man his own. The people then, by this art pleaſured, and greatly relieved, in their lands juſt meaſuring: and other Philoſophers, writing rules for land meaſuring: between them both, thus, confirmed the name of *Geometria*, that is, (according to the very Etymologie of the word) Land-meaſuring. Wherein the people knew no farther of Magnitudes uſe, but in Plains: and the Philoſophers of them had no fit hearers, or Schollers: farther to diſcloſe unto, then of flat, plain *Geometry*. And though, theſe Philoſophers, knew of farther uſe, and beſt underſtood the Etymologie of the word, yet this name *Geometria*, was of them applyed generally to all ſorts of Magnitudes: unleſſe otherwhile, of *Plato* and *Pythagoras*. When they would preciſely declare their own doctrine. Then was* *Geometria*, with them, *Studium quod circa planum verſatur*. But, well you may perceive by *Euclid's Elements* that more ample is our Science, then to meaſure Plains: and nothing leſſe therein is taught (of purpoſe) then how to meaſure Land. Another name, therefore, muſt needs be had, for our Mathematical Science of Magnitudes: which regardeth neither clod nor turff: neither hill, nor dale: neither earth nor heaven: but is abſolute *Megethelogia*, not creeping on ground, and daſſling the eye with pole, perch, rod, "or line: but lifting the heart above the Heavens, by inviſible lines, and immortal beams: "meeteth with the reflexions of the light ineo mprehenſible, and ſo procureth joy, and per-"fection unſpeakable. Of which true uſe of our *Megethica*, or *Megethologia*, Divine *Plato* ſeemed to have good taſt and judgment: and (by the name of *Geometry*.) ſo noted it: and warned his Scholers thereof: as, in his ſeventh *Dialogue*, of the Commonwealth may evidently be ſeen. Where (in Latin) thus it is; right well tranſlated: *Profecto, nobis hoc non negabunt, Quicunque vel paululum quid Geometria guſtarunt, quin hæc Scientia, contrà, omnino ſe habeat, quàm de ea loquuntur, qui in ipſa verſantur.* In Engliſh, thus. **Verily (ſaith *Plato*) whoſoever hath (but even very little) taſted, of Geometry, will not deny unto us, this: but that this Science, is of another condition, quite contrary to that, which they that are exerciſed in it, do ſpeak of it.** And there it followeth of our *Geometry*. *Quòd quæritur cognoſcendi*

Geometry.

Plato 7. de Rep.

cognoſcendi illius gratia, quod ſemper eſt, non & ejus quod oritur quandóque & interit. Geometria, ejus quod eſt ſemper, Cognitio eſt. At tollet igitur (ô Generoſe Vir) ad Veritatem, animum: atque ita, ad Philoſophandum preparabit cogitationem, ut ad ſupera convertamus: quæ, nunc, contra quàm decet, ad inferiora dejicimus, &c. Quàm maximè igitur præcipiendum eſt, ut qui præclariſſimam hanc habitant Civitatem, nullo modo, Geometriam ſpernant. Nam & quæ præter ipſius propoſitum, quodam modo eſſe videntur, haud exigua ſunt, &c. It muſt needs be confeſſed (ſaith *Plato.*) That [Geometry] is learned, for the knowing of that which is ever: and not of that, which in time, both is bred and is brought to an end, &c. Geometry is the knowledge of that which is everlaſting. It will lift up therefore (O Gentle Sir) our mind to the verity: and by that means, it will prepare the Thought, to the Philoſophical love of wiſedome, that we may turn or convert, toward heavenly things, [both mind and thought] which now, otherwiſe than becommeth us, we caſt down on baſe or inferiour things, &c. chiefly, therefore Commandement muſt be given, that ſuch as doe inhabite this moſt honourable City, by no means, deſpiſe Geometry. For even thoſe things [done by it] which, in manner ſeem to be, beſide the purpoſe of Geometry: are of no ſmal importance, &c. And beſides the manifold uſes of *Geometry*, in matters apperteining to war, he addeth more, of ſecond unpurpoſed fruit, and commodity, ariſing by *Geometry*, ſaying: *Scimus quin etiam ad Deſciplinas omnes facilius per diſcendas intereſſe omnino, attigerit ne Geometriam aliquis, an non, &c. Hanc ergo Doctrinam, ſecundo loco diſcendam juvenibus ſtatuamus.* That is. But alſo, we know, that for the more eaſie learning of all Arts, it importeth much, whether one have any knowledg in Geometry, or no, &c. Let us therefore make an ordinance or decree, that this Science of young men ſhall be learned in the ſecond place. This was Divine *Plato* his judgment, both of the purpoſed, chief, and perfect uſe of *Geometry*: and of his ſecond, depending, derivative commodities. And for us Chriſtian men, a thouſand thouſand more occaſions are to have need of the help of * *Megethological* Contemplations: whereby to train our Imaginations and Minds by little and little to forſake and abandon, the groſſe and corruptible Objects, of our outward ſenſes: and to apprehend by ſure doctrine demonſtrative, Things Mathematical. And by them readily to be holpen and conducted, to conceive, diſcourſe, and conclude of things Intellectual, Spiritual, Eternal, and ſuch as concern our bliſſe everlaſting: which, otherwiſe (without ſpecial priviledg of Illumination, or Revelation from Heaven.) No mortal mans wit (naturally) is able to reach unto, or to compaſſe. And, verily, by my ſmall Talent (from above) I am able to prove and teſtifie that the literal Text, and order of our divine Law, Oracles and Myſteries, require more skill in Numbers and Magnitudes: then (commonly) the Expoſitors have uttered: but rather onely (at the moſt) ſo warned: and ſhewed their own want therein. (To name any, is needleſſe: and to note the places, is, here, no place; But if I be duly asked, my anſwer is ready.) And without the literal, Grammatical, Mathematical, or Natural verities of ſuch places, by good and certain Art, perceived no Spirituall ſenſe, proper to thoſe places, by Abſolute *Theologie*) will thereon depend. "No man, therefore, can doubt, but toward the atteining of knowledg incom- "parable, and Heavenly Wiſdome: Mathematical ſpeculations both of Numbers and "Magnitudes: are means, aids, and guides: ready, certain, and neceſſary. From henceforth, in this my Preface, will I frame my talk, to *Plato* his fugitive Scholers: or, rather, to ſuch, who well can, (and alſo will,) uſe their outward ſenſes, to the glory of God, the benefit of their Countrey, and their own ſecret contentation, or honeſt preferment on this earthly Scaffold. To them, I will orderly recite, deſcribe, and declare a great Number of Arts, from our two Mathematical fountains, derived into the fields of *Nature*. Whereby ſuch ſeeds, and roots, as lie deep hid in the ground of *Nature*, are refreſhed, quickned, and provoked to grow, ſhoot up, flour, and give fruit, infinite and incredible. And theſe Arts ſhall be ſuch, as upon magnitudes properties do depend more than upon Number. And by good reaſon we may call them Arts, and Arts Mathematical Derivative, for (at this time) I define An Art, to be a Methodical compleat Doctrine, having abun-

I. D. Herein I would gladly ſhake off, the earthly name of Geometry.

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An Art.

abundance of ſufficient and peculiar matter to deal with; by the allowance of the Metaphyſical Philoſopher: the knowledge whereof, to humane ſtate is neceſſary. And that I account an Art Mathematical derivative, which by Mathematical demonſtrative Method, in Numbers or Magnitudes, ordereth and confirmeth his doctrine, as much and as perfectly as the matter ſubiect will admit. And for that I intend, to uſe the name and propertie of a *Mechanician*, otherwiſe, then (hitherto it hath been uſed), I think it good (for diſtinction ſake) to give you alſo a brief deſcription, what I mean thereby. A Mechanician or a Mechanical workman, is he whoſe skill is without knowledg of Mathematical demonſtration, perfectly to work and finiſh any ſenſible work, by the Mathematician principal or derivative, demonſtrated or demonſtrable. Full well I know, that he which inventeth or maketh theſe demonſtrations, is generally called *A ſpeculative Mechanician*, which differeth nothing from a *Mechanical Mathematician*. So, in reſpect of divers actions, one man may have the name of ſundry Arts, as ſometime of a Logician, ſometimes, in the ſame manner otherwiſe handled) of Rethorician. Of theſe trifles, I make (as now, in reſpect of my Preface), ſmall account: to file them for the fine handling of ſubtile curious diſputers. In other places, they may command me, to give good reaſon: and here, I will not be unreaſonable.

Art Mathematical Derivative.

A Mechanician.

First, then, from the purity, abſoluteneſſe, and Immateriality of principal *Geometrie*, is that kind of *Geometrie* derived, which vulgarly is counted *Geometrie*, and is the Art of meaſuring ſenſible magnitudes, their juſt quantities and contents. This, teacheth to meaſure either at hand: and the Practiſer to be by the thing Meaſured: and ſo, by due applying of Compaſſe, Rule, Square, Yard, Ell, Perch, Pole, Line, Gauging rod, (or ſuch like Inſtrument) to the Length, Plain; or Solid meaſured, * to be certified, either of the length, perimetrie, or diſtance lineal: and this is called: *Mecometrie*. Or * to be certified of the content of any plain Superficies: whether it be in ground Surveyed, Board, or Glaſſe meaſured, or ſuch like thing: which meaſuring, is named, *Embadometrie*. * Or elſe to underſtand the Solidity and content of any bodily thing: as of Timber and Stone, or the content of Pits, Ponds, Wels, Veſſels, ſmall and great, of all faſhions. Where of Wine, Oil, Beer, or Ale Veſſels, &c. the Meaſuring commonly hath a peculiar name and is called *Gauging*. And the general name of theſe Solid meaſures is *Stereometry*. Or elſe, this *Vulgar Geometry*, hath conſideration to teach the Practiſer, how to meaſure things, with good diſtance between him and the thing meaſured: and to underſtand thereby, either * how farre, a thing ſeen (on land or water) is from the meaſurer: and this may be called *Apomecometrie*: Or, how high, or deep, above or under the level of the meaſurers ſtanding, any thing is, which is ſeen on land or water, called *Hypſometry*. * Or, it informeth the meaſurer, how broad any thing is, which is in the meaſurers view: ſo it be on land and water ſituated: and may be called *Platometry*. Though I uſe here to condition the thing meaſured, to be on land or water ſituated: yet, know for certain, that the ſundry height of Clouds, blazing Stars, and of the Moon, may (by theſe means) have their diſtances from the earth: and, of the blazing Stars and Moon, the ſolidity (as well as diſtances) to be meaſured: but becauſe, neither theſe things are vulgarly taught, nor of a common Practiſer ſo ready to be executed. I, rather let ſuch meaſures be reckoned incident to ſome of our other Arts, dealing with things on high, more purpoſely, than this vulgar Land meaſuring Geometry doth: as in *Perſpective* and *Aſtronomy*, &c.

I.

Geometry Vulgar.

1.

2.

3.

2.

1.

2.

3.

Note.

OF theſe Feats (farther applyed) is ſprung the Feat of *Geodeſie*, or Land meaſuring: more cunningly to meaſure and ſurveigh Land, Woods, and Waters, afar off. More cunningly, I ſay: But God knoweth (hitherto) in theſe Realms of *England* and *Ireland*, (whether through ignorance or fraud, I cannot tell in every particular) how great wrong and injury hath (in my time) been committed by untrue meaſuring and ſurveying of Land or wood, any way. And, this I am ſure: that the Value of the difference, between the truth and ſuch Surveyes, would have been able to have found (for ever) in each of our two Univerſities, and excellent Mathematical Reader: to each allowing (yearly) a hundred Marks of lawful money of this Realm: which indeed, would ſeem requiſite, here to be had (though by other wayes provided for) as well as the famous Univerſity of *Paris* hath two

Note.

Mathematical Readers, and each two hundred French Crowns yearly, of the French Kings magnificent liberality onely. Now, again, to our purpose, returning: Moreover, of the former knowledg Geometrical, are grown the skils of *Geography*, *Chorography*, *Hydrography*, and *Stratarithmetry*.

" GEOGRAPHIE teacheth ways, by which, in sundry forms, (as *Sphærike*, *Plain*, " or other) the Situation of Cities, Towns, Villages, Forts, Castles, Mountains, Woods, " Havens, Rivers, Creeks, and such other things, upon the out-face of the earthly Globe " (either in the whole, or in some principal member and portion thereof conteined) may " be described and designed, in commensurations Analogical to Nature and verity: and " most aptly to our view, may be represented. Of this Art how great pleasure, and how manifold commodities do come unto us daily and hourly: of most men is perceived. While some, to beautifie their Hals, Parlers, Chambers, Galeries, Studies, or Libraries with: other some, for things past, as battels fought, earth-quakes, heavenly firings, and such occurrents, in Histories mentioned: thereby lively, as it were, to view the place, the Region adjoyning, the distance from us: and such other circumstances. Some other presently to view the large dominion of the Turk: the wide Empire of the Moscovite: and the little morsel of ground, where Christendome (by profession) is certainly known. Little, I say, in respect of the rest, &c. Some either, for their own journeys directing into far lands: or to understand of other mens travails. To conclude, some for one purpose, and some for another, liketh, loveth, getteth, and useth Maps, Charts, and Geographical Globes. Of whose use, to speak sufficiently, would require a book peculiar.

CHOROGRAPHIE, seemeth to be an underling, and a twig of *Geographie*: " and yet nevertheless, is in practice manifold, and in use very ample. This teacheth " Analogically to describe a small portion or circuit of ground, with the contents: not " regarding what commensuration it hath to the whole, or any parcel, without it, con- " teined. But in the territory or parcel of ground, which it taketh in hand to make de- " scription of, it leaveth out (or undescribed) no notable, or odde thing, above the " ground visible. Yea, and sometimes, of things under ground, giveth some peculiar mark, " or warning: as of Metal-mines, Cole-pits, Stone quaries, &c. Thus a Dukedome, a Shire, " a Lordship, or lesse, may be described distinctly. But marvellous pleasant and profitable is it, in the exhibiting to our eye and commensuration, the plat of a City, Town, Fort, or Palace, in true Symmetry: not approaching to any of them: and out of Gun-shot, &c. Hereby the *Architect* may furnish himself, with store of what paterns he liketh; to his great instruction; even in those things which outwardly are proportioned: either simply in themselves, or respectively to Hils, Rivers, Havens, and Woods adjoyning. So also, term this particular description of places, *Topographie*.

" HYDROGRAPHIE, delivereth to our knowledg, on Globe or in Plain, the per- " fect Analogical description, of the Ocean, Sea-coasts, through the whole World, or " in the chief and principal parts thereof, with the Isles and chief particular places of " danger, conteined within the bounds and Sea-coasts described; as of Quick-sands, " Banks, Pits, Rocks, Races, Countertides, Whirle-pools, &c. This dealeth with the Element of the water chiefly: as *Geographie* did principally take the Element of the Earths description (with his appurtenances) to task. And besides this, *Hydrographie*, requireth a particular Register of certain Land-marks (where marks may be had) from the sea, well able to be skried, in what point of the Sea compasse they appear, and what apparent form, situation, and bigness they have, in respect of any dangerous place in the sea, or neer unto it, assigned: And in all Coasts, what Moon maketh full Sea, and what way, the Tides and Ebbs, come and go, the *Hydrographer* ought to record. The sounding likewise: and the Chanels wayes: their number, and depths ordinarily, at ebbe and floud, ought the *Hydrographer*, by observation and diligence of *Measuring*, to have certainly known, And many other points, are belonging to perfect *Hydrographie*, and for to make a *Rutter*, by: of which, I need not here speak: as of the describing, in any place, upon Globe or Plain, the 32 points of the Compasse, truly; (whereof, scarcely four, in *England*, have right knowledg, because, the lines thereof, are no strait lines, nor Circles.) Of making due projection of a Sphære in plain. Of the Variation of the Compasse, from true North. And such like matters (of great importance, all) I leave to speak of in this place, because, I may seem (already) to have enlarged the bounds and duty of an *Hydrographer*, much more than any man (to this day) hath noted or prescribed: yet am I well able to prove, all these

theſe things, to appertain, and alſo to be proper to the Hydrographer. The chief uſe and end of this Art, is the Art of Navigation, but it hath other divers uſes, even by them to be enjoyed, that never lack ſight of land.

STRATARITHMETRIE, is the skill (apperteining to the war,) by which a man can ſet in figure Analogical to any *Geometrical* figure appointed, any certain number or ſumme of men: of ſuch a figure capable, (by reaſon of the uſual ſpaces between Souldiers allowed: and for that, of men, can be made no Fractions: yet nevertheleſſe, he can order the given ſumme of men, for the greateſt ſuch figure, that of them can be ordered) and certifie of the overplus (if any be) and of the next certain ſumme, which, with the overplus, will admit a figure exactly proportional to the figure aſſigned. By which skill alſo, of any army or company of men; (the figures and ſides of whoſe orderly ſtanding, or array is known,) he is able to expreſſe the juſt number of men, within that figure conteined: or (orderly) able to be conteined. * And this figure, and ſides thereof, he is able to know: either by, and at hand, or a farre off. Thus farre ſtretcheth the deſcription " and property of *Stratarithmetrie*: ſufficient for this time and place. It differeth from " the feat *Tactical*, *De aciebus inſtruendis*, becauſe, there is neceſſary the wiſedome and " foreſight, to what purpoſe he ſo ordereth the men, and skilful ability, alſo, for any occa- " ſion, or purpoſe, to deviſe and uſe the apteſt and moſt neceſſary order, array and " figure of his Company and ſum of men. By figure, I mean, as either of a *Perfect Square*, " *Triangle*, *Circle*, *Ovale*, *long ſquare*, (of the Greeks it is called *Eteromekes*) *Rhombe*, *Rhomboid*, *Lunular*, *Ring*, *Serpentine*, and ſuch other Geometrical figures: Which in wars, have been, and are to be uſed for commodiouſneſſe, neceſſity and advantage, &c. And no ſmall skill ought he to have, that ſhould make true report, or neer the truth of the numbers and ſummes, of Footmen or Horſemen, in the Enemies ordering. A farre off, to make an eſtimate, between neer terms of More and Leſſe, is not a thing very rife, among thoſe that gladly would doe it. Great policy may be uſed of the Captains, (at times fit, and in places convenient) as to uſe figures, which make greateſt ſhew, of ſo many as he hath: and uſing the advantage of the three kinds of uſual ſpaces: (between Footmen and Horſmen) to take the largeſt: or when he would ſeem to have few, (being many:) contrariwiſe in figure and ſpace. The Herald, Purſuivant, Serjeant Royal, Captain, or whoſoever is careful to come neer the truth herein, beſides the judgment of his expert eye, his skill of ordering *Tractical*, the help of his Geometrical Inſtrument: Ring or Staffe Aſtronomical: (commodiouſly framed for carriage and uſe.) He may wonderfully help himſelf by perſpective Glaſſes. In which, (I truſt) our poſterity will prove more skilful and expert, and to greater purpoſes, than in theſe dayes, can (almoſt) be credited to be poſſible.

Thus have I lightly paſſed over the Artificial Feats, chiefly depending upon vulgar *Geometry*: and commonly, and generally reckoned under the name of *Geometry*. But there are other (very many) *Methodical Arts*, which declining from the purity, ſimplicity, and Immateriality, of our Principal Science of *Magnitudes*: do yet nevertheleſſe, uſe the great ayd, direction, and method of the ſaid principal Science, and have proper names, and diſtinct: both from the Science of *Geometry*, (from which they are derived) and one from the other. As PERSPECTIVE, ASTRONOMY, MUSICK, COSMOGRAPHIE, ASTROLOGY, STATIKE, ANTHROPOGRAPHIE, TROCHILIKE, HELICOSOPHIE, PNEUMATITHMIE, MENADRIE, HYPOGEIODIE, HYDRAGOGIE, HOROMETRIE, ZOGRAPHIE, ARCHITECTURE, NAVIGATION, THAUMATURGIKE, and ARCHIMASTRIE. I think it neceſſary, orderly, of theſe to give ſome peculiar deſcriptions: and withall, to touch ſome of their commodious uſes, and ſo to make this Preface, to be a little ſweet, pleaſant Noſegay for you, to comfort your Spirits, being almoſt out of courage, and in deſpair; (through brutiſh brute.) Weening that *Geometrie*, had but ſerved for building of an houſe, or a curious bridg, or the roof of Weſtminſter hall, or ſome witty pretty diviſe, or engin, appropriate to a Carpenter, or Joyner, &c. That the thing is far otherwiſe, than the world, (commonly) to this day, hath deemed, by word and work, good proof will be made.

Among theſe Arts, by good reaſon, PERSPECTIVE ought to be had, e're of *Aſtronomical Apparences*, perfect knowledge can be atteined. And becauſe of the prerogative of *Light*, being the firſt of *Gods Creatures*: and the eye, the light of our body, and

* *Note.* The difference between Stratarithmetry, and Tracticie.

J. D. Friend, You will find it hard, to perform my deſcription of this feat. But by Chorography, you may help your ſelf ſomewhat: where the figures known (in Sides and Angles) are not regular: And where, Reſolution, into Triangles can ſerve, &c. & yet you will finde it ſtrange to deal thus generally with Arithmetical figures: and, that for Battail ray. Their Contents differ ſo much from like Geometrical figures.

and his ſenſe moſt mighty, and his organ moſt artificial, and *Geometrical*. At *Perſpective*, we will begin therefore. **Perſpective is an Art Mathematical, which demonſtrateth the manner and properties of all Radiations, Direct, Broken, and Reflected.** This Deſcription, or Notation, is brief: but it reacheth ſo far, as the World is wide. It concerneth all Creatures, all Actions, and paſſions, by Emanation of beams performed. Beams or natural lines, (here) I mean, not of light onely, or of colour (though they, to eye, give ſhew, witneſſe, and proof, whereby to ground the art upon) but alſo of other *Forms*, both *Subſtantial* and *Accidental*, the certain and determined active Radial Emanations. By this Art (omitting to ſpeak of the higheſt points) we may uſe our eyes, and the light with greater pleaſure, and perfecter judgment: both, of things, in light ſeen, and of other: which by like order of lights Radiations, work and produce their effects. We may be aſhamed to be ignorant of the cauſe, why ſo ſundry ways our eye is deceived, & abuſed: as, while the eye weeneth a round Globe or Sphere (being far off) to be a flat and plain circle, and ſo likewiſe judgeth a plain Square to be a round: ſuppoſeth walls parallels, to approach, afar off: roof and floure parallels, the one to bend downward, the other to ariſe upward, at a little diſtance from you. Again, of things being in like ſwiftneſſe of moving to think the neerer to move faſter, and the farther much ſlower. Nay, of two things, whereof the one (incomparably) doth move ſwifter than the other, to deem the ſlower to move very ſwift, and the other to ſtand: What an errour is this, of our eye? Of the Rainbow, both of his Colours, of the order of the colours, of the bigneſs of it, the place and height of it, (&c.) to know the cauſes demonſtrative, is it not pleaſant, is it not neceſſary? of two or three Suns appearing: of blazing Stars: and ſuch like things: by natural cauſes, brought to paſſe, (and yet, nevertheleſſe, of farther matter ſignificative) it is not commodicus for man to know the very true cauſe and occcaſion Natural? yea, rather is it not, greatly, againſt the Soveraignty of mans nature, to be ſo over-ſhot and abuſed with things (at hand) before his eyes? as with a Peacocks tail, and a Doves neck: or a whole ore, in water holden, to ſeem broken. Things far off to ſeem neer, and neer, to ſeem far off. Small things to ſeem great, and great to ſeem ſmall. One man to ſeem an army. Or a man to be curſtly afraid of his own ſhadow. Yea, ſo much, to fear, that if you being (alone,) neer a certain Glaſſe, proffer with dagger or ſword, to foyne at the glaſſe, you ſhall ſuddenly be moved to give back (in manner) by reaſon of an Image appearing in the air, between you and the glaſſe, with like hand, ſword or dagger, & with like quickneſſe foyning at your very eye, likewiſe as you doe at the glaſſe. Strange, this is, to hear off, but more mervailous to behold, than theſe my words can ſignifie. And nevertheleſſe, by demonſtration Optical, the order and cauſe thereof, is certified: even ſo as the effect is conſequent. Yea, thus much more, dare I take upon me, toward the ſatisfying of the noble courage, that longeth ardently for the wiſedome of cauſes Natural: as to let him underſtand, that, in *London*, he may with his own eye, have proof of that, which I have ſaid herein. A Gentleman, (which, for his good ſervice, done to his Countrey, is famous and honourable: and for skill in the Mathematical Sciences, and Languages, is the odde man of this land, &c.) even he is able: and (I am ſure) will very willingly, let the glaſſe, and the proof be ſeen: and ſo I (here) requeſt him: for the encreaſe of wiſedome, in the honourable, and for the ſtopping of the mouths malicious; and repreſſing the arrogancy of the ignorant; ye may eaſily gueſſe, what I mean. This Art of *Perſpective* is of that excellency, and may be led to the certifying, and executing of ſuch things, as no man would eaſily believe: without Actual proof perceived. I ſpeak not hing of Natural *Philoſophy*, which, without *Perſpective*, cannot be fully underſtood, nor perfectly atteined unto. Nor of *Aſtronomy*: which without *Perſpective*, cannot well be grounded. Nor *Aſtrology*, naturally verified, and avouched. That part hereof, which dealeth with Glaſſes (which name Glaſſe, is a general name, in this Art, for any thing, from which a Beam reboundeth) is called *Catoptrike*, and hath ſo many uſes, both marvellous, and profitable: that, both, it would hold me too long, to note therein the principal concluſions, already known: And alſo (perchance) ſome things might lack due credit with you: And I, thereby to loſe my labour: and you, to ſlip into light judgment. * Before you have learned ſufficiently the power of Nature and Art.

A marvellous glaſſe.

S. W. P.

Now

NOw to proceed: ASTRONOMIE, **is an Art Mathematical, which demonstrateth the distance, Magnitudes, and all natural Motions, apparences, and passions proper to the Planets and fixed Stars, for any time past, present, and to come: in respect of a certain Horizon, or without respect of any Horizon.** By this Art we are certified of the distance of the starry Skie, and of each Planet, from the Center of the Earth, and of the greatnesse of any fixed Star seen, or *Planet*, in respect of the earths greatnesse. As, we are sure (by this Art) that the Solidity, Massinesse, and Body of the *Sun*, conteineth the quantity of the whole Earth, and Sea, a hundred threescore and two times, lesse by $\frac{1}{8}$ one eight part of the earth, but the Body of the whole earthly Globe and Sea, is bigger than the body of the Moon, three and fourty times lesse by $\frac{1}{8}$ of the *Moon*. Wherefore the *Sun* is bigger than the *Moon* 7000 times, lesse by $59\frac{32}{64}$ that is, precisely $6940\frac{25}{64}$ bigger than the *Moon*. And yet the unskilful man, would judg them a like big. Wherefore, of necessity, the one is much farther from us than the other. The *Sun* when he is farthest from the earth (which, now, in our age is, when he is in the 8 degree, of *Cancer*) is 1179 Semidiameters of the Earth, distant. And the *Moon* when she is farthest from the earth, is 68 Semidiameters of the earth and $\frac{1}{2}$. The neerest, that the *Moon* commeth to the Earth, is Semidiameters $52\frac{1}{4}$. The distance of the starry skie is from us, in Semidiameters of the Earth $20081\frac{1}{2}$. Twenty thousand fourscore, one, and almost a half. Subtract from this, the *Moons* neerest distance, from the Earth: and thereof remaineth Semidiameters of the earth $20029\frac{1}{4}$. Twenty thousand nine and twenty and a quarter. So thick is the heavenly Palace, that the *Planets* have all their exercise in, and most mervailously perform the Commandement and Charge to them given by the Omnipotent Majesty of the King of Kings. This is that, which in *Genesis* is called *Ha Rakia*. Consider it well. The Semidiameter of the Earth, conteineth of our common miles $3436\frac{4}{11}$ three thousand, four hundred thirty six and four eleventh parts of one mile. Such as the whole Earth and Sea, round about, is 21600, one and twenty thousand six hundred of our miles. Allowing for every degree of the greatest circle, threescore miles. Now if you weigh well with your self but this little parcel of fruit *Astronomical*, as concerning the bignesse, distances of *Sun*, *Moon*, *Starry skie*, and the huge massinesse of *Ha Rakia* will you not find your consciences moved, with the Kingly Prophet, to sing the confession of Gods Glory, and say, **The Heavens declare the glory of God, and the Firmament [Ha Rakia] sheweth forth the works of his hands.** And so forth, for those five first staves of that kingly Psalm. Well, well, It is time for some, to lay hold on wisedom, and to judg truly of things: and not so to expound the Holy Word, all by Allegories: as to neglect the Wisdom, Power, and Goodnesse of God, in, and by his Creatures, and Creation to be seen and learned. By Parables and Analogies of whose natures and properties, the course of the Holy Scripture, also, declareth to us very many Mysteries. The whole Frame of Gods Creatures, (which is the whole world) is to us, a bright glasse: from which, by reflexion, reboundeth to our knowledg and perceiverance, Beams and Radiations: representing the Image of his infinite Goodnesse, Omnipotency, and Wisedom. And we thereby are taught and perswaded to glorifie our Creator, as God: and be thankful therefore. Could the Heathenists find these uses, of these most pure, beautiful and mighty Corporal Creatures: and shall we, after that the true *Sonne* of right wiseneſſe is risen above the *Horizon* of our temporal *Hemisphere*, and hath so abundantly streamed into our hearts the direct beams of his goodnesse, mercy, and grace: Whose heat all Creatures feel: Spiritual and Corporal: Visible and Invisible: Shall we (I say) look upon the *Heaven*, *Stars*, and *Planets*, as an Oxe, and an Asse doth: no further, careful or inquisitive, what they are: why were they created: how doe they execute that they were created for? Seeing, all Creatures were for our sake created: and both we, and they, created chiefly to glorifie the Almighty Creator: & that by all means to us possible. *Nolite ignorare* (saith *Plato in Epinomis*,) *Astronomiam sapientissimum quiddam esse*. **Be ye not ignorant, Astronomie to be a thing of excellent Wisedome.** *Astronomy* was to us from the beginning commended, and in manner commanded by God himself. In asmuch as he made the *Sun* *Moon* and *Stars* to be to us for *Signes*, and knowledge of Seasons, and for distinctions of days and years. Many words need not. But I wish, every man should weigh this word *Signes*. And besides that, conferre it also with the tenth Chapter of *Jeremie*. And though

Note.

ſome think, that there they have found a rod: Yet modeſt reaſon, will be indifferent judg, who ought to be beaten therewith, in reſpect of our purpoſe. Leaving that: I pray you underſtand this: that without great diligence of Obſervation, Examination and Calculation, their periods and courſes (whereby Diſtinction of Seaſons, Years, and new Moons might preciſely be known) could not exactly be certified. Which thing to perform, is that Art, which we here have defined to be *Aſtronomie*. Whereby we may have the diſtinct courſe of Times, Days, Years & Ages: aſwel for conſideration of Sacred Prophecies, accompliſhed in due time, foretold: as for high Myſtical Solemnities holding: And for all other humane affairs, Conditions, and Covenants upon certain time, between man and man; with many other great uſes: Wherein, (verily,) would be great incertainty, confuſion, untruth, and brutiſh Barbarouſneſſe: without the wonderful diligence and skill of this Art: continually learning and determining Times, and periods of Time, by the Record of the heavenly book, wherein all times are written, and to be read with an *Aſtronomical Staffe*, inſtead of a feſcue.

MUSICK, of Motion, hath his Original cauſe. Therefore, after the Motions moſt ſwift, and moſt ſlow, which are in the Firmament, of Nature performed: and under the *Aſtronomers Conſideration*: now I will ſpeak of another kind of *Motion*, producing ſound, audible, and of man numerable. *Muſick* I call here that *Science*, which of the Greeks is called *Harmonice*. Not medling with the controverſie between the ancient *Harmoniſts* and *Canoniſts*. **Muſick is a Mathematical Science, which teacheth by ſenſe and reaſon, perfectly to judg, and order the diverſities of ſounds high and low.** *Aſtronomy* and *Muſick* are ſiſters, ſaith *Plato*. As for *Aſtronomy*, the eyes: ſo for *Harmonious Motion*, the ears were made. But as *Aſtronomy* hath a more divine contemplation and commodity, than mortal eye can perceive: ſo is *Muſick* to be conſidered, that the * Mind may be preferred before the ear. And from audible ſound, we ought to aſcend, to the examination: which numbers are *Harmonious*, and which not. And why, either, the one are: or the other are not. I could at large, in the heavenly * motions and diſtances, deſcribe a marvellous Harmony of *Pythagoras* Harp, with eight ſtrings. Alſo ſomewhat might be ſaid of * *Mercurius* two Harps, each of four ſtrings Elemental. And very ſtrange matter, might be alledged of the *Harmony*, to our * ſpiritual part appropriate. As in *Ptolomæus* third book, in the fourth and ſixth Chapters may appear. * And what is the cauſe of the apt bond, and friendly fellowſhip, of the Intellectual and Mental part of us, with our groſſe and corruptible body, but a certain Mean, and *Harmonious Spirituality, with both participating, and of both (in a manner) reſulting? In the * Tune of Mans voice, and alſo * the ſound of Inſtrument*, what might be ſaid of *Harmonie*: No common Muſician would lightly believe. But of the ſundry mixture (as I may term it) and concourſe, diverſe collation and application of theſe *Harmonies*: as of three, four, five or more: marvellous have the effects been: and yet may be found and produced the like: with ſome proportional conſideration for our time and being: in reſpect of the ſtate, of the things then: in which, and by which, the wondrous effects were wrought. *Democritus* and *Theophraſtus* affirmed that by *Muſick* griefs and diſeaſes of the mind and body might be cured or inferred. And we find in Record, that *Terpander*, *Arion*, *Iſmenias*, *Orpheus*, *Amphion*, *David*, *Pythagoras*, *Empedocles*, *Aſclepiades*, and *Timotheus* by *Harmonical* Conſonancy, have done and brought to paſſe, things more than marvellous to hear of. Of them then, making no farther diſcourſe in this place. Sure I am, that common *Muſick*, commonly uſed, is found to the *Muſicians* and Hearers, to be ſo commodious and pleaſant, That if I would ſay and diſpute but thus much: That it were to be otherwiſe uſed, then it is, I ſhould find more reprievers, then I could find privy or skilful of my meaning. In things therefore evident, and better known, then I can expreſs: & ſo allowed & liked of, (as I would wiſh ſome other things had the like hap) I will ſpare to enlarge my lines any farther, but conſequently follow my purpoſe.

1.

2.

3. 4.

5.

6.

7. 8.

7. D. Read in *Ariſtot.* his 8 book of Politicks the 5, 6, & 7 Chap. Where you ſhall have ſome occaſion farther to think of Muſick, then commonly is thought.

OF COSMOGRAPHIE, I appointed briefly in this place, to give you ſome intelligence. **Coſmographie is the whole and perfect deſcription of the heavenly, and alſo Elemental part of the World, and their homologal application, and mutual collation neceſſary.** This Art requireth *Aſtronomy*, *Geographie*, *Hydrographie*, and *Muſick*. Therefore, it is no ſmall Art, nor ſo ſimple, as in common practice, it is (ſlightly) conſidered. This matcheth Heaven, and

and the Earth in one frame, and aptly applyeth parts correſpondent: So, as, the Heavenly Globe, may (in practiſe) be duly deſcribed upon the Geographical and Hydrographical Globe. And there, for us to conſider an *Æquinoctial Circle*, an *Ecliptique line*, *Colours*, *Poles*, *Stars*, in their true Longitudes, Latitudes, Declinations, and Verticality: alſo Climes and Parallels: and by an *Horizon* annexed, and revolution of the Earthly Globe (as the Heaven, is by the *Primovant*, carried about in 24 equal hours) to learn the Riſings and Settings of Stars (of *Virgil* in his *Georgicks*, of *Heſiod*: of *Hippocrates* in his *Medicinal Sphære*, *to Perdicca* King of the *Macedonians*: of *Diocles*, to King *Antigonus*, and of other famous *Philoſophers* preſcribed) a thing neceſſary, for due manuring of the earth, for *Navigation*, for the Alteration of mans body: being whole, ſick, wounded, or bruiſed, By the Revolution, alſo, or moving of the Globe Coſmographical, the Riſing and Setting of the Sun: the lengths of days and nights: the Hours and times (both night and day) are known: with very many other pleaſant and neceſſary uſes: Whereof, ſome are known: but better remain, for ſuch to know and uſe: who of a ſpark of true fire, can make a wonderful bonfire, by applying of due matter, duly.

Of ASTROLOGIE, here I make an Art, ſeveral from *Aſtronomy*: not by new deviſe, but by good reaſon and authoritie: for, **Aſtrologie is an Art Mathematical, which reaſonably demonſtrateth the operations and effects, of the natural beams, of light, and ſecret influence: of the Stars and Planets: in every Element and elemental body: at all times in any Horizon aſſigned.** This Art is furniſhed with many other great Arts and experiences: As with perfect *Perſpective*, *Aſtronomy*, *Coſmographie*, *Natural Philoſophie*, of the four Elements, the Art of Graduation, and ſome good underſtanding in *Muſick*: and yet moreover, with another great Art, hereafter following, though I here ſet this before, for ſome conſiderations me moving. Sufficient (you ſee) is the ſtuffe, to make this rare and ſecret Art, of: & hard enough to frame to the concluſion Syllogiſtical: yet both the manifold and continual travails of the moſt ancient and wiſe Philoſophers, for the atteining of this art: and by examples of effects, to confirm the ſame; hath left unto us ſufficient proof and witneſs, and we alſo daily may perceive, That mans body, and all other Elemental bodies are altered, diſpoſed, ordered, pleaſured and diſpleaſured, by the influential working of the *Sun*, *Moon*, and the other *Stars* and *Planets*. And therefore, ſaith *Ariſtotle*, in the firſt of his *Meteorological* books, in the ſecond Chapter; *Eſt autem neceſſariò Mundus iſte ſupernis lationibus ferè continuus. Ut, inde, vis ejus Univerſa regatur. Ea ſiquidem cauſa prima putanda omnibus eſt, unde motus principium exiſtit.* That is: **This [Elemental] World is of neceſſity, almoſt, next adjoyning, to the Heavenly motions: That from thence all his vertue or force may be governed. For that is to be thought the firſt Cauſe unto all: from which, the beginning of motion, is.** And again, in the tenth Chapter. *Oportet igitur, & horum principia ſumamus, & cauſas omnium ſimiliter. Principium igitur ut movens præcipuumque & omnium primum, Circulus ille eſt, in quo manifeſte Solis latio, &c.* And ſo forth. His *Meteorological* books, are full of arguments, and effectual demonſtrations, of the vertue, operation, and power of the heavenly bodies, in and upon the four Elements, and other bodies of them (either perfectly or unperfectly) compoſed. And in his ſecond book, *De generatione & corruptione*: in the tenth Chapter. *Quo circa & prima latio, Ortus & Interitus cauſa non eſt: Sed obliqui Circuli latio: ea namque & continua eſt, & duobus motibus fit.* In Engliſh thus. **Wherefore the uppermoſt motion is not the cauſe of Generation and Corruption, but the motion of the Zodiack, for that both is continual, and is cauſed of two movings.** And in his ſecond book, and ſecond Chapter of his *Phyſicks*. *Homo namque generat hominem, atque Sol.* **For man** (ſaith he) **and the Sun, are cauſe of mans generation.** Authorities may be brought very many; both of 1000, 2000, yea and 3600 years Antiquity; of great *Philoſophers*. *Expert*, *Wiſe*, and godly men, for that Concluſion: which, daily and hourly, we men may diſcern and perceive by ſenſe and reaſon. All beaſts doe feel, and ſimply ſhew by their actions and paſſions, outward and inward. All Plants, Herbs, Trees, Flowers and Fruits, All finally, the Elements, and all things of the Elements compoſed, do give

Teſtimony (as *Ariſtotle* ſaith) that their **Whole diſpoſitions, vertues, and natural motions, depend of the Activity of the heavenly Motions and Influences. Whereby, beſides the ſpecial order and form, due to every ſeed: and beſide the Nature, proper to the individual Matrix, of the thing produced: What shall be the heavenly impreſſion, the perfect and circumſpect Astrologian hath to conclude?** Not onely (by *Apoteleſmes*) τὸ ὅτι, but by Natural and Mathematical demonſtration τὸ διότι. Whereunto, what Sciences are requiſite (without exception) I partly have here warned: and in my *Propædeumes* (beſides other matter there diſcloſed) I have Mathematically furniſhed up the whole Method. To this our age, not ſo carefully handled by any, that ever I ſaw, or heard of, I was (for * 21 years ago) by certain earneſt diſputations, of the Learned *Gerardus Mercator*, and *Antonius Gogava*, (and other) thereto ſo provoked: and (by my conſtant and invincible zeal to verity) in obſervations of heavenly Influences (to the minute of time,) than ſo diligent: and chiefly by the Supernatural influence, from the Star of *Jacob*, ſo directed: That any modeſt and ſober Student, carefully and diligently ſeeking for the Truth, will both find and confeſſe, therein, to be the Verity, of theſe my words: and alſo become a reaſonable Reformer, of three ſorts of people: about theſe influential operations, greatly erring from the truth. Whereof the one is **Light Believers**, the other **Light Deſpiſers**, and the third **Light Practiſers.** The

* *An.* 1548 *and* 1549 *in Lovain.*

Note.

1. firſt, and moſt common ſort, think the Heavens and Stars, to be anſwerable to any their

2. doubts or deſires: which is not ſo: and, indeed, they, too much over-reach, The ſecond ſort think no influential vertue (from the heavenly Bodies) to bear any ſway in Generation and Corruption, in this Elemental World. And to the *Sun*, *Moon* and *Stars* (being ſo many, ſo pure, ſo bright, ſo wonderful big, ſo far in diſtance, ſo manifold in their motions, ſo conſtant in their periods, &c.) they aſſign a ſlight, ſimple office or two, and ſo allow unto them (according to their capacities) as much vertue, and power influential, as to the Signe of the *Sun*, *Moon*, and ſeven *Stars*, hanged up (for Signs) in *London*, for diſtinction of houſes, and ſuch groſſe helps, in our wordly affairs, and they underſtand not (or will not underſtand) of the other workings, and vertues of the Heavenly *Sun*, *Moon* and *Stars*: not ſo much, as the Mariner or Husbandman: no, not ſo much, as the *Elephant* doth, as the *Cynocephalus*, as the *Porpentine* doth: nor will allow theſe perfect and incorruptible mighty bodies, ſo much virtual Radiation and Force, as they ſee in a little piece of a *Magnes ſtone*: which, at great diſtance, ſheweth his operation. And perchance they think, the Sea and Rivers (as the Thames) to be ſome quick thing, and ſo to ebbe and flow, run in and out, of themſelves, at their own fantaſies. God help, God help. Surely theſe men come too ſhort: and either are to dull: or wilfully blind: or, perhaps too ma-

3. licious. The third man is the common and vulgar *Aſtrologian*, or Practiſer, who being not duly, artificially and perfectly furniſhed: yet, either for vain glory, or gain: or like a ſimple Dolt, and blind Bayard, both in matter and manner erreth: to the diſcredit of the *Wary* and modeſt *Aſtrologian*: and to the robbing of thoſe moſt noble corporal Creatures, of their Natural Vertue: being moſt mighty, moſt beneficial to all elemental Generation, Corruption and the appurtenances: and moſt Harmonious in their Monarchie: For which things being known, and modeſtly uſed: we might highly and continually glorifie God, with the princely Prophet, ſaying. **The Heavens declare the Glory of God: who made the Heavens in his wiſedome: who made the Sun for to have dominion of the day: the Moon and Stars to have dominion of the night: whereby day to day uttereth talk, and night to night declareth knowledge. Praiſe him, all ye Stars and Light.** Amen.

In order, now followeth, of STATIKE, ſomewhat to ſay what we mean by that name: and what commodity, doth, on ſuch Art, depend. **Statike is an Art Mathematical, which demonſtrated the cauſes of heavineſſe of all things; and of motions and properties, to heavineſſe and lightneſſe belonging.** And foraſmuch as, by the Bilanx, or Balance (as the chief ſinſible Inſtrument) Experience of theſe demonſtrations may be had: we call this Art *Statike*: that is, *the Experiments of the Balance.* Oh, that men wiſt, what profit (all manner of ways) by this

Art

Art might grow, to the able examiner, and diligent practiſer. " Thou onely, knoweſt all " things preciſely (ô God) who haſt made Weight and Balance, thy Judgment; who " haſt created all things in *Number*, *Weight* and *Meaſure*: and haſt weighed the " Mountains and Hils in a Balance: who haſt peyſed in thy hand, both Heaven and " Earth. We therefore warned by the Sacred Word, to conſider thy Creatures: and by " that conſideration, to win a glimps (as it were,) or ſhadow of perceiverance, that thy " wiſedome, might, and goodneſſe is infinite, and unſpeakable, in thy Creatures declared: " and being farther advertiſed, by thy merciful goodneſſe, that, three principal ways, " were of thee, uſed in Creation of all thy Creatures, namely, *Number*, *Weight*, and " *Meaſure*. And for as much as, of *Number* and *Meaſure*, the two Arts (ancient, fa- " mous, and to humane uſes moſt neceſſary) are, already, ſufficiently known and ex- " tant: This third key, we beſeech thee (through thy accuſtomed goodneſſe) that it may " come to the needful and ſufficient knowledge, of ſuch thy Servants, as in the work- " manſhip, would gladly find, thy true occaſions (purpoſely of thee uſed) whereby we " glorifie thy name, and ſhew forth (to the weaklings in faith) thy wondrous Wiſedome " and Goodneſs. *Amen.*

Marvel nothing at this pang (godly friend, you gentle and zealous Student.) Another day, perchance you will perceive, what occaſion moved me. Here, as now, I will give you ſome ground, and withall ſome ſhew, of certain commodities, by this Art ariſing. And becauſe this Art is rare, my words and practiſes might be too dark: unleſſe you had ſome light holden before the matter: and that beſt will be, in giving you, out of *Archimedes* demonſtrations, a few principal concluſions, as followeth.

I. The ſuperficies of every Liquor, by it ſelf conſiſting, and in quiet, is Spherical: the center whereof, is the ſame, which is the center of the Earth.

II. If Solid Magnitudes, being of the ſame bigneſs, or quantity, that any Liquor is, and having alſo the ſame Weight: be let down into the ſame Liquor, they will ſettle downward, ſo, that no part of them, ſhall be above the ſuperficies of the Liquor: and yet, nevertheleſſe, they will not ſink utterly down, or drown.

III. If any Solid Magnitude being Lighter than a Liquor, be let down into the ſame Liquor, it will ſettle down, ſo far into the ſame Liquor, that ſo great a quantity of that Liquor, as is the part of the Solid Magnitude, ſetled down into the ſame Liquor: is in Weight, equal, to the Weight of the whole Solid Magnitude.

IV. Any Solid Magnitude, Lighter then a Liquor, forced down into the ſame Liquor, will move upward, with ſo great a power, by how much, the Liquor having equal quantity to the whole Magnitude, is heavier than the ſame Magnitude.

V. Any Solid Magnitude, heavier than a Liquor, being let down into the ſame Liquor, will ſinke down utterly: and will be in that Liquor, Lighter by ſo much, as is the weight or heavineſs of the Liquor, having bigneſſe or quantity equal to the Solid Magnitude.

VI. If any Solid Magnitude, Lighter than a Liquor, be let down into the ſame Liquor, the weight of the ſame Magnitude, will be, to the Weight of the Liquor. (Which is equal in quantity to the whole Magnitude,) in that proportion, that the part, of the Magnitude ſetled down is to the whole Magnitude.

I. D. The cutting of a Sphere according to any proportion aſſigned, may by this propoſitiõ be done Mechanically by tẽpering liquor to a certain weight in reſpect of the weight of the Sphere therein ſwimming. A Paradox. A common errour noted.

BY theſe verities, great errours may be reformed in opinion of the Natural Motion of things Light, and Heavy. Which errours are in Natural Philoſophy (almoſt) of all men allowed: to much truſting to authority, and falſe Suppoſitions. As, **Of any two Bodies, the Heavier to move downward faſter than the Lighter.** This errour is not firſt by me, Noted: but by one *John Baptiſt de Benedictis*. The chief of his Propoſitions, is this: which ſeemeth a Paradox.

If there be two Bodies of one form, and of one kind, equal in quantity or unequal, they will move by equal ſpace, in equal time: So that both their movings be in air, or both in water: or in any one middle.

Hereupon, in the feat of GUNNING, certain good diſcourſes (otherwiſe) may receive great amendment and furtherance, In the intended purpoſe, alſo, allowing ſomewhat to the imperfection of Nature: not anſwerable to the preciſeneſſe of demonſtration. Moreover, by the foreſaid propoſitions (wiſely uſed.) The Air, the Water, the Earth, the Fire, may be neerly known, how light, or heavy they are (Naturally) in their aſſigned parts: or in the whole. And then to things Elemental, turning your practiſe: you may deal

N. T. The wonderful uſe of theſe Propoſitions.

deal for the proportion of the Elements, in the things Compounded. Then to the proportions of the Humours in man: their weights, and weight of his Bones, and Fleſh, &c. Then, by weight, to have conſideration of the force of man, any manner of way: in whole, or in parc. Then may you of Ships water drawing, diverſly in the Sea and in freſh water, have pleaſant conſideration: and of weighing up of any thing, ſunken in Sea, or in freſh water, &c. And (to lift up your head aloft:) by weight, you may as preciſely, as by any Inſtrument elſe, meaſure the Diameters of *Sun* and *Moon*, &c. Friend, I pray you weigh theſe things with the juſt Balance of Reaſon. And you will find marvels upon marvels: and eſteem one Drop of Truth (yea, in natural Philoſophy) more worth than whole Libraries of Opinions undemonſtrated: or not anſwering to Natures Law, and your experience. Leaving theſe things, thus: I will give you two or three light practiſes to great purpoſe: and ſo finiſh my Annotation *Statical*. In Mathematical matters, by the Mechanicians aid, we will behold here the Commodity of weight. Make a Cube of any one Uniform: and through like heavy ſtuffe: of the ſame ſtuffe make a Sphere or Globe preciſely, of a Diameter equal to the Radical ſide of the Cube. Your ſtuffe, may be Wood, Copper, Tinne, Lead, Silver, &c. (being as I ſaid of like nature, condition, and like weight throughout.) And you may by Say Ballance have prepared a great number of the ſmaleſt weights: which by thoſe Balance can be diſcerned or tried: and ſo have proceeded to make you a perfect Pyle, company and number of weights: to the weight of ſix, eight, or twelve pound weight, moſt diligently tried, all and of every one, the Content known, in your leaſt weight that is weighable. [They that cannot have theſe weights of of preciſeneſſe: may by Sand, uniform, and well duſted, make them a number of weights, ſomewhat neer preciſeneſſe: by halfing ever the Sand: they ſhall, at length, come to a leaſt common weight. Therein, I leave the farther matter, to their diſcretion, whom need ſhall pinch.] The *Venetians* conſideration of weight may ſeem preciſely enough: by eight deſcents progreſſional * halfing, from a grain: your Cube, Sphere, apt Balance, and convenient weights being ready: fall to work. * Firſt, weigh your Cube. Note the number of the weight. Weigh, after that, your Sphere. Note likewiſe, the Number of the weight. If you now find the weight of your Cube, to be to the weight of the Sphere, as 21 is to 11: Then you ſee, how the Machanician and *Experimenter*, without Geometry and Demonſtration, are (as neerly in effect) taught the proportion of the Cube to the Sphere: as I have demonſtrated in the end of the twelfth book of *Euclid*. Often try with the ſame Cube and Sphere. Then, change your Sphere and Cube to another matter: or to another bigneſs: till you have made a perfect univerſal Experience of it. Poſſible it is, that you ſhall winne to neerer termes, in the proportion.

When you have found this one certain Drop of Natural verity, proceed on, to inferre, and duly to make aſſay, of matter depending. As becauſe it is well demonſtrated, that a Cylinder, whoſe height, and Diameter of his baſe, is equal to the Diameter of the Sphere, is *Seſquialter* to the ſame Sphere, (that is, as 3. to 2.) To the number of the weight of the Sphere, adde half ſo much, as it is: and ſo have you the number of the weight of that Cylinder. Which is alſo comprehended of our former Cube: ſo, that the baſe of that Cylinder, is a Circle deſcribed in the ſquare, which is the baſe of our Cube. But the Cube and Cylinder, being both of one height, have their Baſes in the ſame proportion, in the which, they are one to another in their maſſineſſe or ſolidity. But before, we have two numbers, expreſſing their maſſineſſe, ſolidities, and quantities, by weight: wherefore, we have * the proportion of the ſquare, to the Circle, inſcribed in the ſame ſquare. And ſo are we fallen into the knowledge ſenſible and experimental of *Archimedes* great ſecret, of him, by great travail of mind, ſought and found. Wherefore to any Circle given, you can give a ſquare equal: * as I have taught in my Annotation, upon the firſt propoſition of the twelfth book: and likewiſe to any ſquare given, you may give a Circle equal: * If you deſcribe a Circle, which ſhall be in that proportion to your Circle inſcribed, as the ſquare is to the ſame Circle: This, you may do, by my Annotations, upon the ſecond propoſition of the twelfth book of *Euclide*, in my third Problem there. Your diligence may come to a proportion of the ſquare to the Circle inſcribed neerer the truth, than is the proportion of 14 to 11: and conſider, that you may begin at the Circle and Square, and ſo come to conclude of the Sphere and the Cube, what their proportion is: as now, you came from the Sphere to the Circle. For of Silver or Gold, or Latton Lamyns, or plates, (thorough one hole drawn, as the manner is,) if you make a ſquare figure, and weigh it: and then, deſcribing thereon, the

The practiſe Statical, to know the proportion between the Cube and the Square.

7. D.

* For, ſo, have you 256 parts of a grain.

* The proportion of the Square, to the Circle inſcribed.

* The Squaring of the Circle Mechanically.

* To any ſquare given, to give a Circle equal.

the Circle inſcribed: and cut off, and file away, preciſely (to the Circle) the overplus of the Square: you ſhall then, weighing your Circle, ſee, whether the weight of the Square, be to your Circle, as 14 to 11, as I have noted, in the beginning of *Euclids* twelfth book, &c. after this reſort to my laſt propoſitions, upon the laſt of the twelfth: and there, help your ſelf, to the end, and, here, Note this, by the way. That we may ſquare the Circle, without having knowledge of the proportion of the Circumference to the Diameter: as you have here perceived. And otherwiſe alſo, I can demonſtrate it. So that many have cumbered themſelves ſuperfluouſly, by travailing in that point firſt, which was not of neceſſity, firſt: and alſo very intricate. And eaſily you may, (and that diverſly) come to the knowledge of the circumference: the Circles quantity, being firſt known. Which thing I leave to your conſideration: making haſte to diſpatch another Magiſtral Problem: and to bring it neerer to your knowledge, and readier dealing with, than the world (before this day,) had it for you, that I can tell off. And that is, *A Mechanical doubling of the Cube, &c.* Which may thus be done: **Make of Copper-plates, or Tin-plates a four ſquare upright Pyramis or a Cone: perfectly faſhioned in the hollow, within. Wherein, let great diligence be uſed, to approach as neer as may be) to the Mathematical perfection of thoſe figures. At their baſes: let them be all open: every where, elſe, moſt cloſe, and juſt to. From the verter, to the Circumference of the baſe of the Cone: and to the ſides of the baſe of the Pyramis: Let four ſtraight lines be drawn, in the inſide of the Cone and Pyramis: making at their fall, on the perimeters of the baſes, equal angles, on both ſides themſelves, with the ſaid perimeters. Then four lines (in the Pyramis, and as many in the Cone) divide one, in 12 equal parts; and another in 24, another in 60, and another in 100, (reckoning up from the verter.) Or uſe other numbers of diviſion, as experience ſhall teach you. Then * ſet your Cone or Pyramis with the verter downward, perpendicularly, in reſpect of the baſe, (though it be otherwiſe, it hindereth nothing.) So let them moſt ſtedily be ſtayed.** Now if there be a Cube, which you would have doubled. Make you a prety Cube of Copper, Silver, Lead, Tinne, Wood, Stone, or Bone. Or elſe make a hollow Cube or Cubick coffen, of Copper, Silver, Tinne, or Wood, &c. Theſe you may ſo proportion in reſpect of your Pyramis or Cone, that the Pyramis or Cone, will be able to contein the weight of them, in water three or four times at the leaſt: what ſtuffe ſo ever they be made of. Let not your ſolid angle at the vertex, be too ſharp: but that the water may come with eaſe, to the very vertex, of your hollow Cone or Pyramis. Put one of your ſolid Cubes in a Balance apt: take the weight thereof exactly in water. Poure that water, (without loſſe) into the hollow Pyramis or Cone quietly. Mark in your lines, what numbers the water cutteth: Take the weight of the ſame Cube again: in the ſame kind of water, which you had before: put that * alſo, into the Pyramis or Cone, where you did put the firſt. Mark now again, in what number or place of the lines, the water cutteth them. Two wayes you may conclude your purpoſe: it is to wit, either by numbers or lines. By numbers: as, if you divide the ſide of your Fundamental Cubes into ſo many equal parts, as it is capable of, conveniently, with your eaſe, and preciſeneſſe of the diviſion. For, as the number of your firſt and leſs line (in your hollow Pyramis or Cone) is to the ſecond or greater (both being counted from the vertex) ſo ſhall the number of the ſide of your Fundamental Cube, be to the number belonging to the Radical ſide of the Cube, double to your Fundamental Cube: Which being multiplyed Cubick wiſe, will ſoon ſhew it ſelf, whether it be double or no, to the Cubick number of your Fundamental Cube. By lines, thus: As your leſſe and firſt line (in your hollow Pyramis or Cone) is to the ſecond or greater, ſo let the Radical ſide of your Fundamental Cube, be to a fourth proportional line, by the 12 propoſition of the ſixth book of *Euclid*. Which fourth line, ſhall be the Root Cubick, or Radical ſide of the Cube, double to your Fundamental Cube: which is the thing we deſired. For this, may I (with joy) ſay, ΕΥΡΗΚΑ, ΕΥΡΗΚΑ, ΕΥΡΗΚΑ: thanking the Holy and glorious Trinity, having greater cauſe thereto, then * *Archimedes* had (for finding the fraud uſed in the Kings Crown of Gold:) as all men may eaſily judge: by

Note. Squaring of the Circle, without knowing of the proportion between circumference and Diameter.

To double the Cube readily, by Art Mechanical, depending upon demonſtration Mathematical.

J. D. The 4 ſides of this Pyramis muſt be 4 Iſoſceles triangles alike & equal.

J. D. * In all workings with this Pyramis or Cone, let their Situations be in all points & conditions, alike, or all one: while you are about one work. Elſe you will erre.

J. D. * Conſider well when you muſt put your waters together: & when you muſt empty your firſt water, out of your Pyramis, or Cone. Elſe you will erre.

* *Vitruvius* lib. 9. cap. 3.

God be thanked for this invention and the fruit ensuing.

by the diversity of the fruit following of the one and of the other. Where I speak before of a hollow Cubick Coffin: the like use is of it, and without weight. Thus. Fill it with water precisely full, and pour that water into your Pyramis or Cone: and here note the lines cutting in your Pyramis or Cone. Again, fill your Coffin like as you did before. Put that water, also to the first. Mark the second cutting of your lines. Now, as you proceeded before, so must you here proceed. * And if the Cube, which you double, be never so great: you have, thus, the proportion (in small) between your two little Cubes: And then, the side of that great Cube (to be doubled) being the third, will have the fourth, found, to it proportional: by the 12 of the 6 of *Euclide*.

Note.

* Note, as concerning the Spherical superficies of the water.

Note, that all this while, I forget not my first Proposition Statical, he rehearsed: that, the Superficies of the water, is Spherical. Wherein use your discretion to the first line, adding a small hair breadth, more: and to the second half a hair breadth more, to his length. For you will easily perceive, that the difference can be no greater, in any Pyramis or Cone, of you to be handled. Which you shall thus try. *For finding the swelling of the water above level.* " Square the Semidiameter, from the Center of the Earth, to your " first waters superficies. Square then, half the subtendent of that watry superficies: " (which subtendent must have the equal parts of his measure, all one, with those of the " Semidiameter of the Earth, to your watry superficies:) subtract this square from the " first: Of the residue, take the Root square. That Root, subtract from your first Semi- " diameter of the Earth to your watry superficies: that which remaineth, is the height of " the water, in the middle, above the Level. Which you will find, to be a thing insensible: and though it were greatly sensible,* yet, by help of my sixth Theoreme upon the last Proposition of *Euclides* twelfth book, noted: you may reduce all to a true Level. But farther diligence of you is to be used, against accidental causes of the waters swelling: as by having (somewhat) with a moist sponge, before, made moist your hollow Pyramis or Cone, will prevent an accidental cause of swelling, &c. Experience will teach you abundantly: with great ease, pleasure, and commodity.

* *Note.*

Note this Abridgment of doubling the Cube, &c.

Thus, may you double the Cube Mechanically, Treble it, and so forth, in any proportion. Now will I abridg your pain, cost, and care herein. Without all preparing of your Fundamental Cubes: you may (alike) work this conclusion. For that was rather a kind of Experimental Demonstration, than the shortest way; and all upon one Mathematical Demonstration depending. Take water (as much as conveniently will serve your turn, as I warned before of your Fundamental Cubes bigness.) Weigh it precisely. Put that water into your Pyramis or Cone. Of the same kind of water, then take again, the same weight you had before; put that likewise into the Pyramis or Cone; for in each time your marking of the lines how the water doth cut them, shall give you the proportion between the Radical sides, of any two Cubes, whereof the one is double to the other, working as before I have taught you: * saving that for your Fundamental Cube his Radical side: here, you may take a right line, at pleasure.

Note *

To give Cubes one to the other in any Proportion Rational or Irrational.

Yet farther proceeding with our Drop of Natural truth: **You may now give Cubes one to the other in any proportion given, Rational or Irrational.** On this manner. Make a hollow Parallelipipedon of Copper or Tinne: with one Base wanting, or open, as in our Cubick Coffin. From the bottom of that Parallelipipedon, raise up, many perpendiculars, in every of his four sides. Now if any proportion be assigned you, in right lines: " Cut one of your perpendiculars (or a line equal to it, or lesse " than it) likewise: by the 10 of the 6 of *Euclide*. And those two parts, set in two sundry " lines of those perpendiculars (or you may set them both, in one line) making their be- " ginnings to be, at the Base: and so their lengths to extend upward. Now, set your " hollow Parellelipipedon, upright, perpendicularly, steady. Pour in water handsomely, " to the height of your shorter line. Pour that water, into the hollow Pyramis or Cone. " Mark the place of the rising. Settle your hollow Parallelipipedon again. Pour water " into it: unto the height of the second line, exactly. Pour that water * duly into the " hollow Pyramis or Cone: mark now again, where the water cutteth the same line, " which you marked before. For, there, as the first marked line, is to the second: So " shall the two Radical sides be, one to the other, of any two Cubes: which in their so- " lidity, shall have the same proportion which was at the first assigned: were it Rational " or Irrational.

* Emptying, the first.

Thus

Thus, in ſundry wayes you may furniſh your ſelf with ſuch ſtrange and profitable matter: which long hath been wiſhed for. And though it be Naturally done, and Mechanically: yet hath it a good Demonſtration Mathematical. Which is this: Alwayes, you have two like Pyramids: or two like Cones, in the proportions aſſigned: and like Pyramids or Cones, are in proportion, one to the other, in the proportion of their Homologal ſides (or lines) tripled. Wherefore, if to the firſt, and ſecond lines, found in your hollow Pyramis or Cone, you joyn a third and a fourth in continual proportion, that fourth line ſhall be to the firſt, as the greater Pyramis or Cone is to the leſſe: by the 33 of the eleventh of *Euclid.* If Pyramis to Pyramis, or Cone to Cone, be double, then ſhall * Line to Line, be alſo double, &c. But as our firſt line, is to the ſecond, ſo is the Radical ſide of our Fundamental Cube, to the Radical ſide of the Cube to be made, or to be doubled: and therefore, to thoſe twain alſo, a third and a fourth line, in continual proportion, joyned: will give the fourth line in that proportion to the firſt, as our fourth Pyramidal, or Conick line was to his firſt: but that was double or treble, &c. as the Pyramids or Cones were, one to another (as we have proved) therefore, this fourth, ſhall be alſo double or treble to the firſt, as the Pyramids or Cones were one to another: But our made Cube, is deſcribed of the ſecond in proportion of the four proportional lines: therefore * as the fourth line is to the firſt, ſo is that Cube to the firſt Cube: and we have proved the fourth line to be the firſt, as the Pyramis or Cone is to the Pyramis or Cone: Wherefore the Cube is to the Cube, as Pyramis to Pyramis, or Cone is to Cone. But we * ſuppoſe Pyramis to Pyramis, or Cone to Cone, to be double or treble, &c. Therefore Cube is to Cube, double or treble, &c. Which was to be demonſtrated. And of the Parallelipipedon, it is evident, that the water ſolid Parallelipipedons, are one to another, as their heights are, ſeeing they have one baſe. Wherefore the Pyramids or Cones made of thoſe water Parallelipipedons, are one to the other, as the lines are (one to the other) between which, our proportion was aſſigned. But the Cubes made of lines, after the proportion of the Pyramidal or Conick *Homologal* lines are one to the other, as the Pyramids or Cones are, one to the other (as we before did prove) therefore, the Cubes made, ſhall be one to the other, as the lines aſſigned, are one to the other: which was to be demonſtrated. Note. * This my demonſtration is more general, than onely in Square Pyramis or Cone: Conſider well. Thus, have I, both Mathematically, and Mechanically, been very long in words, yet (I truſt) nothing tedious to them, who, to theſe things, are well affected. And verily I am forced (avoiding prolixity) to omit ſundry ſuch things, eaſie to be practiſed: which to the Mathematician, would be a great Treaſure: and to the Mechanician no ſmall gain. * Now may you,

Between two lines given, find two middle proportionals in continual proportion: by the hollow Parallelipipedon, and the hollow Pyramis, or Cone. Now any Parallelipipedon rectangle being given: three right lines may be found proportional in any proportion aſſigned, of which ſhall be produced a Parallelipipedon, equal to the parallelipipedon given. Hereof, I noted ſomewhat upon the 36 propoſition, of the 11 book of *Euclide.* Now, all thoſe things, which *Vitruvius* in his Architecture, ſpecified, able to be done, by doubling of the Cube. Or, by finding of two middle proportional lines, between two lines given, may eaſily be performed. Now, that Probleme, which I noted unto you, in the end of my Addition, upon the 34 of the 11 book of *Euclide*, is proved poſſible. Now may any regular body be transformed into another, &c. Now, any regular body, any Sphere, yea any mixt Solid: and (that more is) irregular Solids, may be made (in any proportion aſſigned) like unto the body firſt given. Thus, of a *Manneken*, (as the *Dutch* Painters term it) in the ſame *Symmetry*, may a Giant be made: and that, with any geſture, by the *Manneken* uſed: and contrariwiſe. Now, may you, of any Mould or Model of a Ship, make one, of the ſame Mould (in any aſſigned proportion) bigger or leſſer. Now, may you, of any * Gun, or little piece of Ordnance, make another, with the ſame *Symmetrie* (in all points) as great, and as little, as you will. Mark that, and think on it. Infinitely, **may you apply this, ſo long ſought for, and now ſo eaſily concluded: and withall, ſo willingly and frankly communicated to ſuch, as faithfully deal with vertuous ſtudies.** Thus can the Mathematical mind, deal Speculatively in his own Art: and by good means, mount above the clouds and ſtars: And thirdly, he can by order, deſcend, to frame Natural things, to wonderful uſes: and when he liſt, retire

The Demonſtrations of this doubling of the Cube, and of the reſt.

J. D.
* Hereby help your ſelf to become a preciſe practiſer. And ſo conſider how nothing at all, you are hindred (ſenſible) by the convexity of the water.
* By the 33 of the eleventh Book of *Eucli.*

J. D.
* And your diligence in practice can ſo (in weight of water) perform it: Therefore, now, you are able to give good reaſon of your whole doing.
* Note this Corallary.
* The great Commodities following of theſe new Inventions.



Such is the fruit of the Mathematical Sciences and Arts.

home into his own Center: and there, prepare more means, to Aſcend or Deſcend by: and all to the glory of God, and our honeſt delectation in Earth.

Although, the Printer, hath looked for this Preface, a day or two, yet could I not bring my pen from the paper, before I had given you comfortable warning and brief inſtructions, of ſome of the commodities by *Staticke*, able to be reaped: In the reſt, I will therefore, be as brief, as it is poſſible: and withall deſcribing them ſomewhat accordingly. And that, you ſhall perceive, by this, which in order commeth next. For whereas it is ſo ample and wonderful, that an whole year long, one might find fruitful matter therein, to ſpeak of and alſo in practice is a Treaſure endleſſe: yet will I glanſe over it, with words very few.

This do I call ANTHROPOGRAPHIE. Which is an Art reſtored, and of my preferment to your ſervice. I pray you think of it, as of one of the chief points of Humane knowledg. Although it be but now firſt confirmed, with this new name: yet the matter, hath from the beginning, been in conſideration of all perfect Philoſophers. **Anthropographie is the deſcription of the Number, Meaſure, Weight, Figure, Situation and Colour of every diverſe thing, conteined in the perfect body of MAN with certain knowledge of the Symmetrie, Figure, Weight, Characterization, and due local motion of any parcel of the ſaid Body, aſſigned: and of numbers, to the ſaid parcel apperteining.** This is one part of the Definition, meet for this place: Sufficient to notifie, the particularity, and excellency of the Art: and why it is, here, aſcribed to the Mathematicals. If the deſcription of the heavenly part of the World had a peculiar Art, called *Aſtronomy*. If the deſcription of the earthly Globe, hath his peculiar Art, called *Geographie*. If the matching of both hath his peculiar Art, called *Coſmographie*. Which is the deſcription of the whole and univerſal frame of the World: Why ſhould not the deſcription of him, who is the leſſe World and from the beginning called *Microcoſmus*, (that is, **The leſſe World**: And for whoſe ſake and ſervice, all bodily creatures elſe, were created: Who alſo participateth with Spirits and Angels, and is made to the Image and ſimilitude of *God*, have his peculiar Art? and be called the *Art of Arts*: rather than either to want a name, or to have too baſe and improper a name? You muſt of ſundry profeſſions, borrow or challenge home, peculiar parts hereof: and farther proceed: as God, Nature, Reaſon and Experience ſhall inform you. The *Anatomiſts* will reſtore to you, ſome part: *Phyſiognomiſts* ſome: The *Chyromantiſts* ſome. The *Metapoſcopiſts* ſome. The Excellent *Albert Durer*, a good part: the Art of Perſpective, will ſomewhat, for the eye help forward: *Pythagoras*, *Hypocrates*, *Plato*, *Galenus*, *Meletius*, and many other (in certain things) will be Contributaries. And farther, the Heaven, the Earth, and all other Creatures, will each ſhew, and offer their Harmonious ſervice, to fill up, that, which wanteth hereof: and with your own Experience, concluding: you may Methodically regiſter the Whole, for the poſterity: Whereby, good proof will be had, of our Harmonious, and Microcoſmical conſtitution. The outward Image and view hereof, to the Art of *Zographie*, and painting, to Sculpture, and Architecture, (for Church, Houſe, Fort, or Ship) is moſt neceſſary and profitable: for that, it is the chief baſe and foundation of them, Look in * *Vitruvius*, whether I deal ſincerely, for your behoof, or no. Look in *Albertus Durerus de Symmetria humani Corporis*. Look in the 27 and 28 chapters, of the 2 Book, *De occulta Philoſophia*. Conſider the *Ark* of *Noe*. And by that, wade farther. Remember the *Delphical Oracle*, *NOSCE TE IPSUM* (**know thy ſelf**) ſo long ago pronounced: of ſo many a Philoſopher repeated: and of the *Wiſeſt* attempted: And then you will perceive how long ago, you have been called to the School, where this Art might be learned. Well. I am nothing affraid, of the diſdain of ſome ſuch, as think Sciences and Arts, to be but ſeven, perhaps, thoſe ſuch, may, with ignorance, and ſhame enough, come ſhort of them ſeven alſo: and yet nevertheleſſe, they cannot preſcribe a certain number of Arts: and in each certain unpaſſible bounds to God, Nature, and mans Induſtry. New Arts daily riſe up: and there was no ſuch order taken, that, all Arts, ſhould in one age, or in one land, or of one man, be made known to the world. Let us embrace the gifts of God, and ways to wiſedome, in this time of grace, from above, continually beſtowed on them, who thankfully will receive them: *Et bonis omnia Co-operabuntur in bonum.*

MAN is the leſſe World.

☞ * Microcoſmus.

* *Lib.3.cap.1.*

☞

TRO-

TROCHILIKE, is **that Art Mathematical, which demonstrateth the properties of all Circular Motions, Simple and Compound.** And because the fruit hereof vulgarly received, is the Wheels, it hath the name of *Trochilike:* as a man would say *Wheel Art.* By this Art, a Wheel may be given, which shall move once about, in any time assigned. Two Wheels may be given, whose turnings about in one and the same time (or equal times) shall have one to the other, any proportion appointed. By Wheels, may a streight line be described: Likewise, a Spiral line in plain, Conical Section lines, and other Irregular lines at pleasure may be drawn. These and such like are principal Conclusions of this Art: and help forward many pleasant and profitable Mechanical works: As Mils, to saw great and very long Deal-boards, no man being by. Such have I seen in *Germany*, and in the City of *Prague*, in the kingdome of *Bohemia:* Coyning Mils, Hand Mils for Corn grinding: And all manner of Mils and Wheel work: By Wind, Smoak, Water, Weight, Spring, Man or Beast moved. Take in your hand *Agricola de re Metallica:* and then shall you (in all Mines) perceive how great need is, of Wheel work. By Wheels, strange works and incredible are done: as will, in other Arts hereafter appear. A wonderful example of farther possibility, and present Commodity was seen in my time, in a certain Instrument: which by the Inventer and Artificer (before) was sold for twenty Talents of Gold: and then had (by misfortune) received some injury and hurt. And one *Janellus* of *Cremona* did mend the same, and presented it unto the Emperour *Charls* the Fifth. *Hieronymus Cardanus*, can be my witnesse, that therein, was one Wheel, which moved, and that in such rate, that, in 7000 years onely, his own period should be finished. A thing almost incredible: But how farre, I keep me within my bounds: very many men (yet alive) can tell.

Saw Mils.

HELICOSOPHIE, is neer Sister to *Trochilike*, and is **An Art Mathematical which demonstrateth the designing of all Spiral lines in Plain, on Cylinder, Cone, Sphere, Conoid, and Sphearoid, and their properties appertaining.** The use hereof in *Architecture*, and diverse Instruments and Engines, is most necessary. For, in many things, the Scrue worketh the feat, which else, could not be performed. By help hereof, it is * recorded, that, where all the power of the City of *Syracusa*, was not able to move a certain Ship (being on ground) mighty *Archimedes*, setting to his Scruish Engine, caused *Hiero* the King, by himself, at ease, to remove her, as he would. Whereat the King wondring: Ἀπὸ ταύτης τῆς ἡμέρας, περὶ παντὸς, Ἀρχιμήδει λόγον πιστεύειν. *From this day, forward*, (said the King) *Credit ought to be given to* Archimedes, *whatsoever he saith.*

* *Athenæus* lib. 5. cap. 8.

Proclus pag. 18

PNEUMATITHMIE, **demonstrateth by close hollow Geometrical Figures (regular and irregular) the strange properties (in motion or stay) of the Water, Air, Smoak, and Fire in their continuity, and as they are joyned to the Elements next them.** This Art, to the Natural Philosopher, is very profitable, to prove that *Vacuum*, or *Emptinesse*, is not in the world. And that, all Nature, abhorreth it so much, that, contrary to ordinary law, the Elements will move, or stand. As, water to ascend rather: than between him and Air, Space or place should be left, more than (naturally) that quantity of Air requireth, or can fill. Again, water to hang, and to descend: rather than by descending, to leave emptinesse at his back. The like is of Fire and Air, they will descend: when, either their Continuity should be dissolved, or their next Element forced from them. And as they will not be extended to discontinuity: So will they not, nor yet of mans force, can be prest or pent in space, not sufficient and answerable to their bodily substance. Great force and violence will they use, to enjoy their natural right and liberty. Hereupon, two or three men together, by keeping Air under a great Cauldron, and forcing the same down, orderly, may without harm descend to the Sea bottome: and continue there a time, &c. Where, Note, how the thicker Element (as the water) giveth place to the thinner (as is the Air:) and receiveth violence of the thinner, in manner, &c. Pumps and all manner of Bellows, have their ground of this art: and many other strange devises: as *Hydraulica*, Organes going by water, &c. Of this Feat, called commonly *Pneumatica*) goodly works are extant, both in Greek and Latine. With old and learned Schoolmen, it is called *Scientia de pleno & vacuo.*

To go to the bottom of the Sea without danger.

MENADRIE, is an Art Mathematical, which demonstrateth, how, above Natures vertue, and power simple: vertue, and force may be multiplyed: and so, to direct, to lift, to pull to, and to put or cast fro, any multiplied or simple, determined vertue, Weight or Force: naturally, not, so, directible or moveable. Very much is this Art furthered, by other Arts, as in some points by *Perspective*, in some, by *Statick*, in some, by *Trochilike*, and in other by *Helicosophie*, and *Pneumatithmie*. By this Art, all Cranes, Gibbets, and Engines to lift up, or to force any thing any manner of way, are ordered, and the certain cause of their force is known. As, the force, which one man hath with the *Dutch* Waggen Rock: therewith, to set up again, a mighty waggen laden, being overthrown. The force of the Crossebow Rack, is certainly here demonstrated. The reason, why one man doth with a leaver, lift that, which six men, with their hands onely, could not so easily do. By this Art, in our common Cranes in *London*, where power is to crane up the weight of 2000 pound: by two wheeles more (by good order added) Art concludeth, that there may be craned up 200000 pound weight, &c. So well knew *Archimedes* this Art: that he alone with his devices and engines (twice or thrice) spoyled and discomfited, the whole Army and Host of the *Romans*, besieging *Syracusa*, *Marcus Marcellus the Consul*, being their General Captain. Such huge stones, so many with such force, and so far, did he with his engines hail among them out of the City. And by Sea likewise: though there Ships might come to the wals of *Syracusa*, yet he utterly confounded the *Roman* Navy: what with his mighty Stones hurling: what with pikes * of 18 foot long, made like shafts: which he forced almost a quarter of a mile, what, with his catching hold of their ships, and hoysing them up above the water, and suddenly letting them fall into the Sea again: what with his * Burning Glasses: by which he fired their other ships a far off: what with his other policies, devices, and engines, he so manfully acquitted himself: that all the force, courage, and policy of the *Romans* (for a great season) could nothing prevail, for the winning of *Syracusa*: whereupon, the *Romans* named *Archimedes*, *Briareus*, and *Centimanus*. *Zonaras* maketh mention of one *Proclus* who so well had perceived *Archimedes* Art of *Menadrie*, and had so well invented of his own, that with his Burning Glasses, being placed upon the walls of *Bysance*, he multiplyed so the heat of the Sun, and directed the beams of the same against his enemies Navy with such force, and so suddenly (like lightening) that he burned and destroyed both man and ship. And *Dion* specifieth of *Priscus* a *Geometrician* in *Bysance*, who invented and used sundry Engines, of force multiplyed: which was cause, that the Emperour *Severus* pardoned him his life after he had wonne *Bysance*. Because he honoured the Arr, wit, and rare industry of *Priscus*. But nothing inferiour to the invention of these Engines of Force, was the invention of Guns. Which, from an *English-man* had the occasion and order of first inventing: though in another land, and by other men, it was first executed. " And they that should see the record, where the occasion and order general of Gunning, " is first discoursed of, would think: that small things slight and common: comming to " wise mens consideration, and industrious mens handling, may grow to be of force in- " credible.

Plutarchus in Marco Marcello.
Syneſius in Epistola.
Polybius.
Plinius.
Quintilianus.
T. Livius.
* *Atheneus.*
* *Galenus.*
Athemius.

Burning Glass.

Guns.

HYPOGEIODIE, is an Art Mathematical, demonstrating, how under the Spherical Superficies of the Earth, at any depth, to any perpendicular line assigned (whose distance from the perpendicular of the entrance: and the Azimuth likewise, in respect of the said entrance is known) certain way may be prescribed and gone: And how any way above the Superficies of the Earth designed, may under Earth, at any depth limited, be kept: going always perpendicularly, under the way, on Earth designed: and contrariwise, any way, (straight or crooked,) under the Earth, be given: upon the outface, or superficies of the Earth, to Line out the same: so, as, from the Center of the Earth, perpendiculars drawn to the Spherical superficies of the Earth, shall precisely fall in the correspondent points of those two wayes. This, with all other cases and circum-

circumſtances herein, and appurtenances, this Art demonſtrateth. This Art is very ample in variety of Concluſions, and very profitable ſundry wayes to the Common-wealth. The occaſion of my inventing this Art, was at the requeſt of two Gentlemen, who had a certain work (of gain) under ground and their grounds did joyn over the work: and by the reaſon of the crookedneſſe, divers depths, and heights of the way under ground, they were in doubt, and at controverſie, under whoſe ground, as then, the work was. The name onely (before this) was of me publiſhed, *De Itinere Subterraneo.* The reſt be at Gods will. For Pioners, Miners, Diggers for Metals, Stone, Cole, and for ſecret paſſages under ground, between place and place (as this land hath diverſe) and for other purpoſes, any man may eaſily perceive, both the great fruit of this Art, and alſo in this Art. the great aid of Geometry.

HYDRAGOGIE, **demonſtrateth the poſſible leading of Water by natures law, and by artificial help, from any head (being a ſpring ſtanding or running Water) to any other place aſſigned.** Long hath this Art been in uſe: and much thereof written: and very marvellous works therein performed: as may yet appear in *Italy*, by the Ruines remaining, of the *Aqueducts.* In other places of Rivers, leading through the Main land Navigable many a Mile: and in other places, of the marvellous forcings of water to aſcend: which all declare the great skill to be required of him, who ſhould in this Art be perfect, for all occaſions of waters poſſible leading. To ſpeak of the allowance of the Fall, for every hundred foot: or of the Ventils (if the waters labour be far and great) I need not: ſeeing, at hand (about us) many expert men can ſufficiently teſtifie, in effect, the order: though the Demonſtration of the Neceſſity thereof, they know not: Not yet, if they ſhould be led, up and down, and about Mountains, from the head of the Spring: and then a place being aſſigned: and of them to be demanded, how low or high, that laſt place is, in reſpect of the head, from which (ſo crookedly, and up and down, they be come: Perhaps, they would not, or could not) very readily or neerly reſolve that Queſtion. *Geometry* therefore, is neceſſary to *Hydragogie.* Of the ſundry ways to force water to aſcend, either by *Tympane*, *Kettelmils*, *Scrue*, *Cteſibike*, or ſuch like, in *Vitruvius*, *Agricola*, (and other,) fully, the manner may appear. And ſo, thereby, alſo be moſt evident, how the Arts of *Pneumatithmie*, *Helicoſophie*, *Statike*, *Trochelike*, and *Menadrie*, come to the furniture of this in ſpeculation, and to the Commodity of the Common-wealth in practice.

HOROMETRIE, **is an Art Mathematical, which demonſtrateth how at all times appointed, the preciſe uſual denomination of time, may be known, for any place aſſigned.** Theſe words are ſmooth and plain eaſie *Engliſh*, but the reach of their meaning is farther than you would ligh ly imagine. Some part of this Art, was called in old time *Gnomonice*: and of late, *Horologiographia*: and in *Engliſh*, may be termed *Dialling.* Ancient is the uſe, and more ancient is the Invention. The uſe, doth well appear to have been (at the leaſt) above two thouſand and three hundred years ago: in * King *Achaz* Dial, then, by the Sun, ſhewing the diſtinction of time. By Sun, Moon, and Stars, this Dialling may be performed, and the preciſe time of day, or night known. But the demonſtrative delineation of theſe Dials, of all ſorts, requireth good skill both of *Aſtrononie* and *Geometry* Elemental, Spherical, Phenomenal, and Conical. Then to uſe the grounds of the Art, for any regular Superficies, in any place offered: and (in any poſſible apt poſition thereof) thereon to deſcribe (all manner of ways) how, uſual hours, may be (by the *Suns* ſhadow) truly determined: will be found no ſleight Painters work. So to paint and preſcribe the Suns Motion, to the breadth of a hair. In this Feat (in my youth) I invented a way, **How in any Horizontal, Mural; or Equinoctial, Dial, &c. at all hours (the Sun ſhining) the ſign and degree aſcendent, may be known.** Which is a thing very neceſſary, for the riſing of thoſe fixed ſtars: whoſe Operation in the air, is of great might evidently. I ſpeak no further, of the uſe hereof. But foraſmuch as, mans affairs, require knowledge of Times and Moments, when neither Sun, Moon, or Star, can be ſeen: Therefore, by Induſtrie Mechanical, was invented firſt, how by Water, running orderly, the Time and Hours might be known: whereof, the famous *Cteſibius*, was Inventor: a man of *Vitruvius* to the skie (juſtly) extolled. Then, after that, by ſand running, were hours meaſured:

* 2 *Kings* 20.

Then

Then, by *Trochilike* with weight, and of late time, by *Trochilike* with Spring: without weight. All these by Sun or Stars direction, (in certain time) require oversight and reformation, according to the heavenly Æquinoctial Motion: besides the inequality of their own Operation. There remaineth (without parabolical meaning herein) among the Philosophers, a more excellent, more commodious, and more marvellous way, than all these: of having the motion of the Primovant (or first Æquinoctial motion) by Nature and Art, imitated: which you shall (by further search in weightier studies) hereafter, understand more of. And so it is time to finish this Annotation, of Times distinction, used in our common, and private affairs: the commodity whereof no man would want, that can tell, how to bestow his time.

A perpetual Motion.

ZOGRAPHIE, **is an Art Mathematical, which teacheth and demonstrateth, how, the intersection of all visual Pyramids, made by any plain assigned, (the Center, distance, and lights, being determined) may be, by lines, and due proper colours represented.** A notable Art, is this, and would require a whole Volume, to declare the propertie thereof: and the Commodities ensuing. Great skill of *Geometrie*, *Arithmetick*, *Perspective*, and *Anthropographie*, with many other particular Arts, had the Zographer, need of, for his perfection. For, the most excellent Painter, (who is but the proper Mechanician, and Imitator sensible, of the Zographer) hath atteined, to such perfection, that sense of man and beast, have judged things painted, to be things natural and not artificial: alive and not dead. This Mechanical Zographer (commonly called the Painter) is marvellous in his skill: and seemeth to have a certain divine power: as of friends absent, to make a friendly, present comfort, yea, and of friends dead, to give a continual, silent presence: not onely with us, but with our posterity for many ages. And so proceeding, consider, how in Winter, he can shew you, the lively view of Sommers joy, and riches: and in Sommer, exhibite the countenance of Winters doleful state and nakednesse. Cities, Towns, Forts, Woods, Armies, yea, whole Kingdomes (be they never so far, or great) can he with ease, bring with him, home (to any mans judgment) as patterns lively of the things rehearsed. In one little house, can he, enclose (with great pleasure of the beholders) the portraiture lively, of all visible Creatures, either on earth or in the earth, living: or in the waters lying, creeping, sliding or swimming: or of any fowl or fly, in the Air flying. Nay, in respect of the Stars, the Skie, the Clouds: yea, in the shew of the very light it self, that divine creature, can he match our eyes judgment, most neerly. What a thing is this? things not yet being, he can represent, so, as, at their being: the picture shall seeme (in manner) to have created them. To what Artificer, is not picture, a great pleasure and commodity? which of them all, will refuse the direction and aid of Picture? The Architect, the Goldsmith, and the Arras weaver: of Picture, make great account. Our lively Herbals, our portraitures of birds, beasts, and fishes: and our curious Anatomies, which way, are they most perfectly made, or with most pleasure, of us beholders? Is it not by Picture onely? and if Picture, by the industry of the Painter, be thus commodious and marvellous: what shall be thought of *Zographie*, the Schoolmaster of Picture, and chief Governour? Though I mention not *Sculpture*, in my Table of Arts Mathematical: yet may all men perceive, How, that *Picture* and *Sculpture*, are Sisters Germane: and both, right profitable, in a Commonwealth and of *Sculpture*, aswel as of *Picture*, excellent artificers have written great books in commendation. Witnesse I take, of *George Vasari*, *Pittore Aretino*: of *Pomponius Gauricus*: and others. To these two Arts, (with other,) is a certain odde art, called *Athalmasat*, much beholding: more, than the common *Sculptor*, *Entayler*, *Carver*, *Cutter*, *Graver*, *Founder*, or *Painter*, (*&c.*) know their art to be commodious.

An Objection.

ARCHITECTURE, to many may seem not worthy, or not meet, to be reckoned among the *Arts Mathematical*. To whom, I think good, to give some account of my so doing. Not worthy, (will they say,) because it is but for building of a House, Palace, Church, Fort, or such like grosse works: and you also, defined the *Arts Mathematical*, to be such as dealed with no Material or corruptible thing: and also did demonstratively proceed in their Faculty, by Number or Magnitude. First, you see, that I count here, *Architecture*, among those *Arts Mathematical*, which are derived from the principals: and you know, that such may deal with natural things, and sensible matter. Of which some

The Answer.

draw

draw neerer, to the ſimple and abſolute Mathematical Speculation, then others do. And " though the *Architect* procureth, informeth, and directeth the *Mechanician*, to hand- " work, and the building actual, of Houſe, Caſtle, or Palace, and is chief Judge of the " ſame: yet with himſelf (as chief *Maſter* and *Architect*) remaineth the Demon- " ſtrative reaſon and cauſe of the *Mechanicians* work, in Line, Plain, and Solid: by " *Geometrical*, *Arithmetical*, *Optical*, *Muſical*, *Aſtronomical*, *Coſmographical*, (and " to be brief) by all the former derived *Arts Mathematical*, and other Natural arts " able to be confirmed and ſtabliſhed. If this be ſo, then, may you think, that *Architecture* hath good and due allowance, in this honeſt company of *Arts Mathematical* Derivative. I will herein crave Judgment of two moſt perfect *Architects*, the one being *Vitruvius* the Roman: who did write the Books thereof, to the Emperour *Auguſtus*, (in whoſe days our Heavenly Arch-maſter was born) & the other *Leo Baptiſta Albertus*, a Florentine, who alſo publiſhed ten books thereof. *Architectura* (ſaith *Vitruvius*) *eſt Scientia pluribus diſciplinis, & variis eruditionibus ornata: cujus judicio probantur omnia, quæ ab cæteris Artificibus perficiuntur opera.* That is: Architecture is a Science garniſhed with many doctrines and divers inſtructions: by whoſe Judgement, all works, by other workmen finiſhed, are judged. It followeth. *Ea naſcitur ex Fabrica, & Ratiocinatione, &c. Ratiocinatio autem eſt, quæ, res frabricatas, ſolertia ac ratione proportionis, demonſtrare atque explicare poteſt.* Architecture groweth of Framing, and Reaſoning, &c. Reaſoning is that, which of things framed with fore-caſt, and proportion: can make demonſtration, and manifeſt declaration. Again. *Cùm, in omnibus enim rebus, tùm maximè etiam in Architectura, hæc duo inſunt: quod ſignificatur, & quod ſignificat. Significatur propoſita res, de qua dicitur: hanc autem ſignificat Demonſtratio, rationibus doctrinarum explicata.* Foraſmuch, as in all things: therefore chiefly in Architecture, theſe two things are: the thing ſignified, and that which ſignifieth. The thing propounded, whereof we ſpeak, is the thing ſignified. But Demonſtration, expreſſed with the reaſons of divers doctrines, doth ſignifie the ſame thing. After that. *Ut literatus, ſit peritus Graphidos, eruditus Geometriæ, & Optices non ignarus: inſtructus Arithmetica: hiſtorias complures noverit, Philoſophos diligenter audiverit: Muſicam ſciverit: Medicinæ non ſit ignarus, reſponſa Juriſperitorum noverit: Aſtrologiam, Cœlique rationes cognitas habeat.* An Architect (ſaith he) ought to underſtand Languages, to be skilful of Painting, well inſtructed in Geometry, not ignorant of *Perſpective*, furniſhed with *Arithmetick*, have knowledge of many Hiſtories, and diligently have heard Philoſophers, have skill of Muſick, not ignorant of Phyſick, know the anſwers of Lawyers, and have Aſtronomie, and the courſes Cœleſtial, in good knowledge. He giveth reaſon, orderly, wherefore, all theſe Arts, Doctrines, and Inſtructions, are requiſite in an excellent Architect. And (for brevity) omitting the Latin text, thus he hath. Secondly, it is behooful for an Architect to have the knowledge of Painting: that he may the more eaſily faſhion out, in patterns painted, the form of what work he liketh: and Geometrie giveth to Architecture many helps: and firſt teacheth the uſe of the Rule, and the Compaſſe: whereby (chiefly and eaſily) the deſcriptions of Buildings, are diſpatched in Ground-plats: and the directions of Squares, Levels, and Lines. Likewiſe, by Perſpective, the Lights of the Heaven, are well led, in the Buildings: from certain quarters of the World. By Arithmetick, the charges of Buildings are ſummed together: the meaſures are expreſſed, and the hard queſtions of Symmetries, are by Geometrical means and methods diſcourſed on, &c. Beſides this, of the Nature of things (which in Greek is called φυσιολογία) Philoſophy doth make declaration. Which it is neceſſary for an Architect, with diligence to have learned: becauſe it hath many and divers Natural queſtions: is ſpecially in Aqueducts. For in their courſes, leadings about, in the level ground, and in the mountings, the natural ſpirits or breaths are ingendred divers wayes: the hindrances, which they cauſe, no man can help, but he, which out of Philoſophy, hath learned the original cauſes of things. Likewiſe, whoſoever ſhall read *Cteſibius*, or *Archimedes* books, (and of others, who have written ſuch Rules) cannot think, as they doe: unleſſe he ſhall have received of Philoſophers, inſtructions in theſe things: and Muſick he muſt needs know: that he may have underſtanding, both of Regular and Mathematical Muſick: that he may temper well his Baliſts, Catapults, and Scorpions, &c. Moreover, the Braſen Veſſels, which in Theatres, are placed by Mathematical order, in ambries, under the ſteps: and the diverſities of the ſounds (which the Grecians call ἠχοι) are ordered according to Muſical Symphonies and Harmonies: being diſtributed in

in the Circuits, by *Diateſſaron*, *Diapente*, and *Diapaſon*. That the convenient voice, of the players ſound, when it came to theſe preparations, made in order, there being increaſed: with that increaſing, might come more clear and pleaſant, to the ears of the lookers on, &c. And of Aſtronomy, is known the Eaſt, Weſt, South, and North. The faſhion of the Heaven, the Æquinox, the Solſticie, and the courſe of the Stars. Which things, unleſſe one know: he cannot perceive, any thing at all, the reaſon of Horologies. Seeing therefore, this ample Science, is garniſhed, beautified, and ſtored, with ſo many and ſundry skils and knowledges: I think, that none can juſtly account themſelves Architects of the ſodain. But they onely, who from their childs years aſcending by theſe degrees of knowledges, being foſtered up with the atteining of many Languages and Arts, have won to the high Tabernacle of Architecture, &c. And to whom Nature hath given ſuch quick Circumſpection, ſharpneſſe of Wit, and Memory, that they may be very abſolutely skilfull in Geometry, Aſtronomy, Muſick, and the reſt of the Arts Mathematical: Such ſurmount and paſſe the calling, and ſtate of Architects: and are become Mathematicians, &c. And they are found ſeldom: as in times paſt was, *Ariſtarchus Samius*, *Philolaus*, and *Archytas Tarentynes*: *Apollonius Pergeus*, *Eratoſthenes Cyreneus*: *Archimedes*, and *Scopas*, *Syracuſians*. Who alſo left to their poſterity, many Engines and Gnomonical works: by numbers, and natural means, invented and declared.

A Mathematician.

Thus much, and the ſame words (in ſenſe) in one onely Chapter of this incomparable *Architect Vitruvius*, ſhall you find. And if you ſhould, but take his book in your hand, and ſlightly look through it, you would ſay ſtraightway: This is *Geometry*, *Arithmetick*, *Aſtronomy*, *Muſick*, *Anthropographie*, *Hydragogie*, *Horometrie*, *&c.* and (to conclude) the ſtore-houſe of all workmanſhip. Now, let us liſten to our other Judg, our Florentine, *Leo Baptiſta*, and narrowly conſider, how he doth determine of *Architecture*. *Sed anté que ultra progrediar*, *&c.* But before I proceed any further (ſaith he) I think, that I ought to expreſſe, what man I would have to be allowed an Architect. For, I will not bring in place a Carpenter: as though you might compare him to the Chief Maſters of "others arts. For the hand of the Carpenter, is the Architects Inſtrument: But I will appoint "the Architect to be that man, who hath the skill, (by a certain and marvellous means "and way) both in mind and Imagination to determine: and alſo in work to finiſh: "what works ſo ever, by motion of weight and coupling and framing together of bodies, "may moſt aptly be commodious for the worthieſt uſes of Man. And that he may be able to perform theſe things, he had need of atteining and knowledge of the beſt and moſt worthy things, &c. The whole Feat of Architecture in building, conſiſteth in Lineaments and in framing. And the whole power and skill of Lineaments, tendeth to this: that the right and abſolute way may be had, of Coapting and joyning Lines and angles: by which, the face of the building, or frame may be comprehended and concluded. And it is the property of Lineaments, to preſcribe unto buildings, and every part of them, an apt place, and certain number: a worthy manner and a ſeemly order: that ſo the whole form and figure of the building, may reſt in the very Lineaments, &c. And we may preſcribe in mind and imagination the whole forms, * all material ſtuffe being ſecluded. Which point we ſhall attein, by noting and fore-pointing the angles, and lines, by a ſure and certain direction and connexion. Seeing then, theſe things are thus: Lineament ſhall be the certain and conſtant preſcribing, conceived in mind: made in lines and angles: and finiſhed with a learned mind and wit. We thank you Maſter *Baptiſt*, that you have ſo aptly brought your art and phraſe, thereof, to have ſome Mathematical perfection: by certain order, number, form, figure, and *Symmetrie* mental: all natural and ſenſible ſtuffe ſet apart. Now then it is evident, (Gentle Reader) how aptly and worthily I have preferred *Architecture*, to be bred and foſtered up in the Dominion of the peerleſſe *Princeſſe*, *Mathemaca*, and to be a natural Subject of hers. And the name of *Architecture*, is of the principality, which this Science hath, above all other Arts. And *Plato* affirmeth the *Architect* to be *Maſter* over all, that make any work. Whereupon, he is neither Smith nor Builder: nor, ſeparately, any Artificer: but the Head, the Provoſt, the Director, and Judge of all artificial works, and all Artificers. For, the true *Architect*, is able to teach, demonſtrate, diſtribute, deſcribe, and judge all works wrought. And he, onely ſearcheth out the cauſes and reaſons of all Artificial things. Thus excellent, is *Architecture*: though few (in our days) attein thereto: yet may not the art, be otherwiſe thought on, than in very deed it is worthy. Nor we may not of ancient Arts, make new and imperfect Definitions in our days: for

Vitruvius.

Who is an Architect.

* The Immaterialitie of perfect Architecture.

What Lineament is.

Note.

for ſcarcity of Artificers: No more than we may pinch in, the Definitions of *Wiſedome*, or *Honeſty*, or *Friendſhip*, or *Juſtice*. No more will I conſent, to diminiſh any whit of the perfection and dignity, (by juſt cauſe) allowed to abſolute *Architecture*. Under the Direction of this Art, are three principal, neceſſary *Mechanical Arts*. Namely, *Houſing*, *Fortification*, and *Naupegie*. *Houſing*, I underſtand, both for Divine Service, and Mans common uſage: publick and private. Of *Fortification* and *Naupegie*, ſtrange matter might be told you: But perchance, ſome will be tyred, with this Bed-rol, already rehearſed: and other ſome, will nicely nip my groſſe and homely diſcourſing with you: made in poſt haſte: for fear you ſhould want this true and friendly warning, and taſt giving, of the *Power Mathematical*. Life is ſhort, and uncertein: Times are perilous, &c. And ſtill the Printer awayting, for my pen ſtaying: all theſe things, with farther matter of Ingratefulneſſe, give me occaſion to paſſe away, to the other Arts remaining, with all ſpeed poſſible.

The Art of NAVIGATION, demonſtrateth how, by the ſhorteſt good way, by the apteſt direction, and in the ſhorteſt time, a ſufficient Ship, between any two places (in paſſage Navigable) aſſigned: may be conducted: and in all ſtorms and natural diſturbances chancing, how to uſe the beſt poſſible means, whereby to recover the place firſt aſſigned. What need the *Maſter Pilot* had of other Arts, here before recited, it is eaſie to know: as, of *Hydrographie*, *Aſtronomy*, *Aſtrologie*, and *Horometrie*. Preſuppoſing continually, the common Baſe, and Foundation of all: namely, *Arithmeticke*, and *Geometry*. So that he be able to underſtand and judg his own neceſſary Inſtruments, and furniture neceſſary. Whether they be perfectly made or no: and alſo can, (if need be) make them himſelf. As Quadrants, the Aſtronomers Ring, the Aſtronomers Staffe, the Aſtrolabe Uumverſal. An Hydrographical Globe. Charts Hydrographical, true, (not with parallel Meridians.) The common Sea Compaſſe: The Compaſſe of variation: The Proportional and Paradoxal Compaſſes (of me invented, for our two Muſcovy Maſter Pilots, at the requeſt of the Company) Clocks with ſpring: hour, half hour, and three hour Sand-glaſſes: and ſundry other Inſtruments: and alſo be able on Globe, or Plain to deſcribe the Paradoxal Compaſſe: and duly to uſe the ſame, to all manner of purpoſes, whereto it was invented. And alſo, be able to Calculate the Planets places for all times. *An.* 1599.

Moreover, with Sun, Moon or Star (or without) be able to define the Longitude and Latitude of the place, which he is in: So that the Longitude and Latitude of the place, from which he ſayled, be given: or by him, be known: whereto appertaineth expert means, to be certified ever, of the Ships way, &c. and by fore-ſeeing the Riſing, Setting, Noonſteding, or Midnighting of certain tempeſtuous fixed Stars: or their Conjunctions, and Anglings with the Planets, &c. he ought to have expert conjecture of ſtorms, tempeſts and ſpouts: and ſuch like Meteorological effects, dangerous on Sea. For (as *Plato* ſaith,) *Mutationes opportunitateſque temporum preſentire, non minus rei militari, quàm Agriculture, Navigationique convenit.* **To foreſee the alterations and opportunities of times is convenient, no leſſe to the Art of War, than to huſbandry and Navigation.** And beſides ſuch cunning means, more evident tokens in Sun and Moon, ought of him to be known: ſuch (as the Philoſophical Poet) *Virgilius* teacheth in his *Georgicks*, where he ſaith.

Sol quoque & exoriens & quum ſe condet in undas,
Signa dabit, Solem certiſſima ſigna ſequuntur, &c.

——— Nam ſæpè videmus,
Ipſius in vultu varios errare colores. Georgic. 1.
Cærulens, pluviam denunciat, igneos Euros,
Sin macula incipient rutilo immiſcerier igni,
Omnia tum pariter vento, nimbiſque videbis
Fervere: non illa quiſquam me nocte per altum
Ire, neque à terra moveat convellere funem, &c.
Sol tibi ſigna dabit. Solem quis dicere falſum
Audeat? ——— &c.

And so of Moon, Stars, Water, Air, Fire, Wood, Stones, Birds, and Beasts and of many things else, a certain Sympathical forewarning may be had: sometimes to great pleasure and profit, both on Sea and Land. Sufficiently, for my present purpose, it doth appear, by the premisses, how *Mathematical* the *Art* of *Navigation* is, and how it needeth and also useth other *Mathematical Arts:* And now if I would go about to speak of the manifold Commodities, comming to this Land, and others, by ships and *Navigation*, you might think, that I catch at occasions, to use many words, where no need is.

Yet this one thing may I, (justly) say. In *Navigation*, none ought to have greater care, to be skilful, than our *English* Pilots. And perchance, some, would more attempt: and other some, more willingly would be aiding, if they wist certainly, what priviledge, God hath endued this Island with, by reason of Situation, most commodious for *Navigation*, to places most Famous and Rich. And though (of * late) a young Gentleman, a couragious Captain, was in great readiness, with good hope, and great causes of perswasion, to have ventured for a **Discovery**, (either *Westerly*, by *Cape de Paramantia:* or *Easterly*, above *Nova Zemla*, and the *Cyrimisses*) and was at the very neer time of attempting, called and imployed otherwise (both then, and since,) in great good service to his Country, as the Irish Rebels * have tasted: Yet, I say, (though the same Gentleman, do not hereafter deal therewith) some one or other should listen to the matter: and by good advice, and discreet circumspection, by little and little, winne to the sufficient knowledge of that **Trade and Voyage**: which now I would be sorry, (through carelesness, want of skill and courage) should remain unknown; or unheard of. Seeing, also, we are, herein, half challenged by the learned, by half request published. Thereof, verily might grow commodity, to this Land chiefly, and to the rest of the Christian Common-wealth far passing all riches and worldly Treasure.

* *An.* 1567 *S. H. G.*

* *An.* 1569.

THAUMATURGIKE, is that Art Mathematical, which giveth certain order, to make strange works, of the sense to be perceived, and of men greatly to be wondred at. By sundry means, this *Wonder-work* is wrought. Some by *Pneumatithmie:* as the works of *Ctesibius* and *Hero:* some by weight, whereof *Timæus* speaketh: some by Strings strained, or Springs, therewith imitating lively Motions: some by other means, as the Images of *Mercury:* and the brazen head, made by *Albertus Magnus* which did seem to speak. *Boethius* was excellent in these feats. To whom *Cassiodorus* writing, saith. **Your purpose is to know profound things, and to shew marvels. By the disposition of your Art, Metals do low: Diomedes of brasse, doth blow a Trumpet loud, a brasen Serpent hisseth: Birds made sing sweetly. Small things we rehearse of you, who can imitate the heaven, &c.** Of the strange self-moving, which at Saint *Denis*, by *Paris*, * I saw once or twice (*Orontius* being then with me, in company) it were to strange to tell. But some have written it: and yet (I hope) it is there, to be seen. And by *Perspective* also strange things, are done: as partly (before) I gave you to understand in *Perspective:* as, to see in the air aloft, the lively image of another man, either waking to and fro: or standing still. Likewise, to come into an house, and there to see the lively shew of Gold, Silver or precious Stones: and comming to take them in our hand, to find nought but Air. Hereby, have some men (in all other matters counted wise) fouly over-shot themselves: misdeeming of the means. Therefore, said *Claudius Cælestinus*, *Hodie magna literatura viros, & magna reputationis videmus, opera quædam quasi miranda, supra Naturam putare, de quibus in Perspectiva doctus causam faciliter reddidisset.* That is. **Now a dayes, we see some men, yea at great learning and reputation, to judge certain works as marvellous above the power of nature: of which works, one that were skilful in Perspective, might easily have given the cause.** Of *Archimedes Sphere*, *Cicero* witnesseth. Which is very strange to think on. **For when Archimedes (saith he) did fasten in a Sphere, the movings of the Sun, Moon, and of the five other Planets, he did, as the God, which (in Timæus of Plato) did make the World. That one turning should rule Motions most unlike in slowness and swiftness.** But a greater cause of marvelling we have by *Claudianus* report hereof. Who affirmeth this,

* *An.* 1551.

De his quæ Mundo mirabiliter eveniunt, cap. 8.

Archi-

Archimedes work, to have been of Glasse, and discourseth of it more at large: which I omit. The Dove of wood, which the *Mathematician Archytas* did make to flie, is by *Agellius* spoken of. Of *Dædalus* strange Images, *Plato* reporteth. *Homer* of *Vulcans Self-movers*, (by secret wheels,) leaveth in writing. *Aristotle* in his *Politicks* of both, maketh mention. Marvellous was the workmanship of late dayes, performed by good skill of *Trochilike*, &c. For in *Noremberg*, a Flie of Iron, being let out of the Artificers hand, did (as it were) flie about by the guests at the Table, and at length, as though it were wearie, return to his Masters hand again. Moreover, an artificial Eagle, was ordered, to flie out of the same Town, a mightie way, and that aloft in the air, toward the Emperour coming thither: and followed him, being come to the gate of the Town.* Thus you see what Art Mathematical can perform, when skill, will, industrie, and abilitie, are duly applied to proof.

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A Digression Apologetical.

AND for these, and such like marvellous Acts and Feats, Naturally, Mathematically, and Mechannically, wrought and contrived: ought any honest Student, and Modest Christian Philosopher, be counted, and called a **Conjurer**? Shall the folly of Ideots, and the Malice of the Scornful, so much prevail, that He, who seeketh no worldly gain or glory at their hands: But onely, of God, the treasure of heavenly wisdome, and knowledge of pure verity: Shall he (I say) in the mean space, be robbed and spoiled of his honest name and fame? He that seeketh (by S. *Pauls* advertisement) in the Creatures Properties, and wonderful vertues, to finde just cause, to glorifie the Æternal, and Almighty Creator by: Shall that man be (in hugger mugger) condemned, as a Companion of the Hel-hounds, and a Caller, and Conjurer of wicked and damned Spirits? He that bewaileth his great want of time, sufficiently (to his contentation) for learning of Godly wisdome, and Godly Verities in: and onely therein setteth all his delight: Will that man leese and abuse his time, in dealing with the Chief enemy of Christ our Redeemer: the deadly foe of all mankind: the subtile and impudent perverter of Godly Verity: the Hypocritical Crocodile: the Envious Basilisk, continually desirous, in the twinkling of an eie, to destroy all Mankind, both in Body, and Soul, eternally? Surely (for my part, somewhat to say herein) I have not learned to make so brutish, and so wicked a Bargain. Should I, for my xx, or xxv years Study: for two or three thousand Marks spending: seven or eight thousand Miles going and travelling, onely for good learning sake: And that, in all manner of weathers: in all manner of wayes and passages: both early and late: in danger of violence by man: in danger of destruction by wilde beasts: in hunger: and thirst: in perillous heats by day, with toil on foot: in dangerous damps of cold, by night, almost bereaving life: (as God knoweth): with lodgings, oft times, to small ease: and sometime to lesse security. And for much more (than all this) done and suffered, for Learning and atteining of Wisdome: Should I (I pray you) for all this, no otherwise, nor more warily: or (by Gods mercifulnesse) no more luckily, have fished, with so large, and costly, a Net, so long time in drawing and that with the help and advice of Lady Philosophy, and Queen Theology): but at length, to have catched, and drawn up, * a Frog? Nay, a Devil? For, so, doth the Common peevish Pratler Imagine and Jangle: And, so, doth the Malicious scorner, secretly, and bravely and boldly face down, behinde my back. Ah, what a miserable thing, is this kinde of Men? How great is the blindnesse and boldnesse, of the Multitude, in things above their Capacity? What a Land: what a People: what Manners: what Times are these? Are they become Devils, themselves: and by false witnesse bearing against their Neighbour, would they also, become Murderers? Doth God, so long give them respite, to reclaim themselves in, from this horrible slandering of the guiltlesse: contrary to their own Consciences: and yet will they not cease? Doth the Innocent, forbear the calling of them, Juridically to answer him, according to the rigour of the Laws: and will they despise his Charitable patience? As they, against him, by name, do forge, fable, rage, and raise slander, by Word and Print: Will they provoke him by word and Print, likewise, to Note their Names to the World: with their particular devices, fables, beastly Imaginations, and unchristian-like slanders? Well: Well, O (you such) my unkinde Countreymen! O unnatural Countreymen! O unthankful Countreymen! O Brainsick, Rash, Spiteful and Disdainful Countreymen. Why oppresse you me thus violently with your slandering of me: Contrary to Verity: and contrary to your own Consciences? And I, to this hour, neither by word, deed, or thought, have been, any way, hurtful, damageable,

* A Proverb, Fair fisht and caught a Frog.

damageable, or injurious to you or yours? Have I, ſo long, ſo dearly, ſo far, ſo carefully, ſo painfully, ſo dangerouſly ſought and travelled for the learning of Wiſdome, and atteining of Vertue: And in the end (in your judgement) am I become, worſe, than when I began? Worſe than a Mad man? A dangerous Member in the Common-wealth: and no Member of the Church of Chriſt? Call you this to be Learned? Call you this, to be a Philoſopher? and a lover of Wiſdome? To forſake the ſtraight heavenly way: and to wallow in broad way of damnation? To forſake the light of heavenly Wiſdome: and to lurk in the dungeon of the Prince of darkneſſe? To forſake the Verity of God, and his Creatures, and to fawn upon the impudent, craftie, obſtinate Liar, and continual diſgracer of Gods Veritie, to the uttermoſt of his power? To forſake the Liſe and Bliſſe Eternal: and to cleave unto the Author of Death everlaſting? that murderous Tyrant, moſt greedily awaiting the Prey of Mans ſoul? Well: I thank God, and our Lord Jeſus Chriſt, for the comfort, which I have by the Examples of other men, before my time: To whom, neither in godlineſſe of life, nor in perfection of learning, I am worthy to be compared: and yet they ſuſteined the verie like injuries that I do: or rather greater. Patient *Socrates* his *Apologie* will teſtifie: *Apuleius* his *Apologies*, will declare the brutiſhneſſe of the Multitude: *Joannes Picus*, Earl of *Mirandula*, his *Apologie* will teach you, of the raging ſlander of the malicious Ignorant againſt him. *Joannes Trithemius* his *Apologie* will ſpecifie, how he had occaſion to make publike Proteſtation: as wel by reaſon of the Rude Simple: as alſo in reſpect of ſuch, as were counted to be of the wiſeſt ſort of men. "Many could I "recite: But I defer the preciſe and determined handling of this matter: being loth to de- * ☞ "tect the Folly and Malice of my Native Countreymen. * Who, ſo hardly, can diſgeſt "or like any extraordinarie courſe of Philoſophical ſtudies: not falling within the Com- "paſſe of their Capacitie: or where they are not made privie of the true and ſecret cauſe, "of ſuch wonderful Philoſophical Feats. Theſe men, are of four ſorts, chiefly. The firſt, I may name, *Vain pratling buſie-bodies.* The ſecond, *Fond Friends.* The third, *Imperfectly Zealous*: and the fourth, *Malicious Ignorant.* To each of theſe (briefly, and in cha-

1. ritie) I will ſay a word or two, and ſo return to my Preface. *Vain pratling buſie-bodies*, uſe your idle aſſemblies, and conferences, otherwiſe, than in talk of matter, either above

2. your capacities, for hardneſſe: or contrarie to your conſciences in veritie. *Fond Friends*, leave off, ſo to commend your unacquainted friend upon blind affection. As, becauſe he knoweth more, than the common Student: that, therefore he muſt needs be skilful, and a doer, in ſuch matter and manner, as you term *Conjuring*. Weening thereby, you advance his fame: and that you make other men, great marvels of your hap, to have ſuch a learned friend. Ceaſe to aſcribe Impietie, where you pretend Amitie. For, if your tongues were true, then were that your friend *Untrue*, both to God and his Sovereign. Such *Friends*

3. and *Fondlings*, I ſhake off, and renounce you: Shake you off, your Folly. *Imperfectly Zealous*, to you, do I ſay: that (perhaps) well, do you mean: but far you miſſe the Mark: If a Lamb you will kill, to feed the flock with his bloud. Sheep, with Lambs bloud, have no natural ſuſtenance: No more, is Chriſts flock, with horrible ſlanders, duly edified. Nor your fair pretenſe, by ſuch raſh ragged Rhetorique, any whit, wel graced. But ſuch, as ſo uſe me, will finde a foul crack in their credit. Speak that you know: And know, as you ought: Know not, but Hear-ſay, when life lieth in danger. Search to the quick, and

4. let charity be your guide. *Malicious Ignorant*, what ſhall I ſay to thee? *Prohibe linguam tuam à malo. A Detractione pareitur lingua.* **Cauſe thy tongue to refrain from evil. Refrain your tongue from ſlander.** Though your tongues be ſharpened Serpent-like, and Adders poiſon lie in your lips: yet take heed, and think, betimes, with your ſelf, *Pſal. 140.* *Vir linguoſus non ſtabilietur in terra. Virum violentum venabitur malum, donec præcipietur.* For, ſure I am, *Quia faciet Dominus Judicium afflicti, & vindictam pauperum.*

Thus, I require you my aſſured friends and Countreymen (you Mathematicians, Mechanicians, and Philoſophers, charitable and diſcreet) to deal in my behalf, with the light and untrue tongued, my envious Adverſaries or Fond friends. And farther, I would wiſh, that at leaſure, you would conſider, how *Baſilius Magnus* layeth *Moſes* and *Daniel* before the eyes of thoſe, which count all ſuch ſtudies Philoſophical (as mine hath been) to be *Act. 7. 6.* ungodly or unprofitable. Weigh well Saint *Stephen* his witneſſe of *Moſes*. *Eruditus eſt Moſes omni Sapientia Ægyptiorum: & erat potens in verbis & operibus ſuis.*

Moſes

Moſes was inſtructed in all manner of wiſdome of the Egyptians, and he was of power both in his words and works. You ſee this Philoſophical Power and Wiſdome which *Moſes* had, to be nothing miſliked of the Holy Ghoſt. Yet *Plinius* hath recorded *Moſes* to be a wicked *Magician*. And that (of force) muſt be, either for this Philoſophical wiſdome, learned, before his calling to the leading of the Children of *Iſrael*: or for thoſe his wonders wrought before King *Pharaoh*, after he had the conducting of the *Iſraelites*. As concerning the firſt, you perceive, how Saint *Stephen*, at his Martyrdome (being full of the Holy Ghoſt) in his Recapitulation of the Old Teſtament, hath made mention of *Moſes* Philoſophy: with good liking of it: And *Baſilius Magnus*, alſo avoucheth it, to have been to *Moſes* profitable (and therefore I ſay, to the Church of God neceſſarie.) But as concerning *Moſes* wonders done before King *Pharaoh*: God himſelf ſaid: *Vide ut omnia oſtenta, quæ poſui in manu tua, facias coram Pharaone.* **See that thou do all thoſe wonders before Pharaoh, which I have put in thy hand.** Thus, you evidently perceive, how raſhly, *Plinius* hath ſlandered *Moſes*, of vain fraudulent *Magick*, ſaying: *Eſt & alia Magices Factio, à Moſe, Jamne, & Jotape, Judæis pendens: ſed multis millibus annorum poſt Zoroaſtrem, &c.* Let all ſuch, therefore, who, in Judgement and Skill of Philoſophie, are far inferiour to *Pliny*, take good heed, leſt they over-ſhoot themſelves raſhly, in judging of *Philoſophers ſtrange acts*: and the means how they are done. But much more ought they to beware of forging, deviſing, and imagining monſtrous feats, and wonderful works, when and where no ſuch were done: no, not any ſpark or likelihood, of ſuch, as they, without all ſhame, do report. And (to conclude) moſt of all, let them be aſhamed of Man, and afraid of the dreadful and juſt Judge: both fooliſhly or maliciouſly to deviſe: and then divelliſhly to father their new fond Monſters on me: Innocent in hand and heart: for treſpaſſing either againſt the law of God or man, in any my Studies, or Exerciſes, Philoſophical, or Mathematical: As in due time, I hope, will be more manifeſt.

Lib. 30. c. 10.

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NOw end I, with ARCHEMASTRIE. Which name, is not ſo new, as this Art is rare. For another Art, under this, a degree (for skill and power) hath been endued with this Engliſh name before. And yet this may ſerve for our purpoſe, ſufficiently, at this preſent. **This art, teacheth to bring to actual experience, ſenſible, all worthy concluſions by all the Arts Mathematical purpoſed, and by true Natural Philoſophy concluded: and both addeth to them a farther ſcope, in the terms of the ſame Arts, and alſo by his proper Method, and in peculiar terms, proceedeth, with help of the foreſaid Arts, to the performance of compleat Experiences, which of no perticular Art, are able (Formally) to be challenged.** If you remember, how we conſidered *Architecture*, in reſpect of all common handworks: ſome light may you have, thereby, to underſtand the Sovereigntie and propertie of this Science. *Science* I may call it, rather, than an Art: for the excellencie and Maſterſhip it hath, over ſo many, and ſo mightie Arts and Sciences. And becauſe it proceedeth by *Experiences*, and ſearcheth forth the cauſes of Concluſions, by *Experiences*: and alſo putteth the Concluſions themſelves, in *Experience*, it is named of ſome *Scientia Experimentalis*. The **Experimental Science**, *Nicolaus Cuſanus* termeth it ſo, in his *Experiments Statical*, and another *Philoſopher* of this Land Native (the flower of whoſe worthy fame can never die nor wither) did write thereof largely, at the requeſt of *Clement the Sixth*. The Art carrieth with it, a wonderful credit: by reaſon, it certifieth, ſenſibly, fully, and compleatly to the utmoſt power of Nature and Art. This Art certifieth by *Experiences* compleat and abſolute: and other Arts, with their arguments and demonſtrations, perſwade: and in words prove very well their Concluſions. * But words and arguments, are no ſenſible certifying: nor the full and final fruit of Sciences practicable. And though ſome Arts have in them *Experiences*, yet they are not compleat, and brought to the uttermoſt, they may be ſtretched unto, and applied ſenſibly. As for example: the natural Philoſopher diſputeth and maketh goodly ſhew of reaſon: And the Aſtronomer, and the Optical Mechanician put ſome things in *Experience*: but neither, all, that they may: nor yet ſufficiently, and to the utmoſt, thoſe, which they do. There, then, the *Arch-maſter*, ſteppeth in,

R. B.

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in, and leadeth forth on, the *Experiences*, by order of his doctrine *Experimental*, to the chief and final power of Natural and Mathematical Arts. Of two or three men, in whom this Description of *Archimystry* was *Experimentally* verified, I have read and heard: and good record, is of their such perfection. So that this Art is no fantastical imagination: as some Sophister, might, *Cum suis Insolubilibus*, make a flourish: and dazle your imagination: and dash your honest desire and courage, from believing these things, so unheard of, so marvellous, and of such importance. Well: as you will. I have forewarned you. I have done the part of a friend: I have discharged my Dutie toward God: for my small Talent, at his most merciful hands received. To this Science, doth the *Science Aliran-giat*, great Service. Muse nothing of this name. I change not the name, so used, and in Print, published by others: being a name proper to the Science. Under this, commeth *Ars Sintrillia*, by *Artephius*, briefly written. But the chief Science, of the Archimystry, (in this world) as yet known, is another (as it were) OPTICAL Science: whereof, the name shall be told (God willing) when I shall have some, (more just) occasion, thereof, to discourse.

Here I must end, thus abruptly (Gentle friend, and unfeigned lover of honest and necessary verities.) For, they, how have (for your sake, and vertues cause) requested me, (an old foreworn Mathematician) to take pen in hand: (through the confidence they reposed in my long experience: and tryed sincerity) for the declaring and reporting somewhat, of the fruit and commodity, by the **Arts Mathematical to be atteined unto**: even they, sore against their wils are forced, for sundry causes, to satisfie the workmans request, in ending forthwith: He, so feareth this, so new an attempt, and so costly: and in matter so slenderly (hitherto) among the Common Sort of Students, considered or esteemed.

And where I was willed some what to alledge, why, in our vulgar speech, this part of the principal Science of *Geometry*, called *Euclids Geometrical Elements*, is published, to your handling: being unlatined people, and not Universitie Scholers: Verily, I think it needlesse.

1. For, the Honour, and Estimation of the **Universities and Graduates**, is, hereby nothing diminished. Seeing, from, and by their Nurse Children, you receive all this Benefit, how great soever it be.

2. Neither are their studies any whit hindred. No more than the Italian *Universities*, as *Academia Bononiensis*, *Ferrariensis*, *Florentina*, *Mediolanensis*, *Patavina*, *Papiensis*, *Perusina*, *Pisana*, *Romana*, *Senensis* or any one of them find themselves, any deal disgraced, or their Studies any thing hindred, by *Frater Lucas de Burgo*, or by *Nicolaus Tartalea*, who in vulgar Italian language, have published, not onely *Euclids Geometry*, but of *Archimedes* somewhat: and in Arithmetick, and Practical Geometry, very large Volumes, all in their vulgar speech. Nor in *Germany*, have the famous *Universities*, any thing been discontent with *Albertus Durerus*, his Geometrical Institutions in Dutch: or with *Guilielmus Xylander*, his learned translation of the first six books of *Euclide*, out of the Greek, into the High Dutch. Nor with *Gualterus H. Riffius*, his Geometrical Volume: very diligently translated into the High Dutch tongue, and published. Nor yet the *Universities* of *Spain* or *Portugal*, think their reputation to be decayed: or suppose any their studies to be hindred by the Excellent *P. Nonnius*, his Mathematical works, in vulgar speech by him put forth. Have you not, likewise in the French tongue, the whole Mathematical Quadrivie? and yet neither *Paris*, *Orleans*, or any of the other Universities of *France* at any time, with the Translators, or Publishers offended: or any mans studie thereby hin dred?

3. And surely, the common and vulgar scholer (much more the Grammarian) before his comming to the *Universty*, shall (or may) be, now (according to *Plato* his Counsel) sufficiently instructed in *Arithmetick* and *Geometry*, for the better and easier learning of all manner of *Philosophy Academical* or *Peripatetical*. And by that means, go more cheerfully, more skilfully, and speedily forward in his studies, there to be learned. And, so, in lesse time, profit more, than (otherwise) he should or could do.

4. Also many good and pregnant *English* wits, of young Gentlemen, & of other, who never intend to meddle with the profound search and studie of Philosophie (in the *Universities* to be learned) may nevertheless, now, with more ease and liberty, have good occasion, vertuously to occupier the sharpnesse of their wits: where else (perchance) otherwise, they would

would in found exercises, spent (or rather lose) their time: neither serving God: nor furthering the weal, common or private.

And great Comfort, with good hope, may the *Universities* have, by reason of this *English* **Geometrie and Mathematical Preface**, that they (hereafter) shall be the more regarded, esteemed, and resorted unto. For when it shall be known and reported, that of the *Mathematical Sciences* onely, such great Commodities are ensuing (as I have specified:) and that in deed, some of you unlatined students, can be good witnesse, of such rare fruit by you enjoyed (thereby:) as either, before this, was not heard of: or else not so fully credited: " Well, may all men conjecture, that far greater aid and better furniture, " to win to the perfection of all Philosophy, may in the Universities be had: being the " Store-house and Treasury of all Sciences and all Arts, necessary for the best and most " noble State of Common-wealths. 5. *Universities.*

Besides this, how many a common Artificer, is there in these Realms of *England* and *Ireland*, that dealeth with Numbers, Rule, and Compasse: Who, with their own skill and experience, already had, will be able (by these good helps and informations) to find out, and devise, new Works, strange Engines, and Instruments: for sundry purposes in the Commonwealth? or for private pleasure? and for the better mainteining of their own estate? I will not (therefore) fight against mine own shadow. For no man (I am sure) will open his mouth against this enterprize. No man (I say) who either hath charity toward his brother, (& would be glad of his furtherance in vertuous knowledge:) or that hath any care and zeal for the bettering of the common state of this Realm. Neither any, that make accompt, what the wiser sort of men (*Sage* and *Stayed*) do think of them. To none (therefore) will I make any *Apologie*, for a vertuous act doing: and for commending, or setting forth, profitable Arts, to *English* men in their English tongue. " But, unto God our Creator, let us all be " thankful: for that, **As he, of his goodnesse, by his power, in his " wisedome, hath created all things in Number, Weight and Mea- " sure.** So, to us, of his great mercy, he hath revealed means, whereby to attein the " sufficient and necessary knowledge of the foresaid his three principal Instruments: " Which means, I have abundantly proved unto you, to be the *Sciences* and *Arts Ma-* " *thematical.* 6.

And though I have been pinched with straightnesse of time: that, no way, I could so pen down the matter (in my mind) as I determined: hoping of convenient leasure: Yet, if vertuous zeal, and honest intent, provoke and bring you to the reading of this Compendious Treatise, I doe not doubt, but as the verity thereof (according to our purpose) will be evident unto you: So the pith and force thereof, will perswade you: and the wonderful fruit thereof: highly pleasure you. And that you may the easier perceive and better remember, the principal points, whereof my Preface treateth, I will give you the **Ground-plat** of my whole discourse, in a Table annexed: from the first to the last, somewhat Methodically contrived. *The ground Plat of this Preface in a Table.*

If haste, hath caused my poor pen, any where to stumble: You will (I am sure) in part of recompence, (for my earnest and sincere good will to pleasure you) consider the rockish huge mountains, and the perilous unbeaten ways, which (both night and day, for the while) it hath toyled and laboured through, to bring you this good News, and Comfortable proof of Vertues fruit.

So I commit you unto Gods merciful direction, for the rest: heartily beseeching him to prosper your Studies, and honest Intents: to his Glory, and the Commodity of our Country. *Amen.*

Written at my poor House,
At Mortlake

Anno 1570 *Febr.* 9.

Here followeth the GROUND-PLAT of the Mathematical Preface of Maſter JOHN DEE. By which a brief view of what hath been before at large delivered doth appear.

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TO THE READER.

ALthough it be usual for Authors or Commentators to write somewhat in praise of the Subject of which they treat: Yet we suppose it neither necessary, or (we hope) will it be expected that we speak much or little in praise of the Mathematicks, or of this Book, of the Elements of Geometry, the ground and foundation of all Mathematical productions; the general use that hath been, and is still made thereof, almost throughout the World, as well in respect of its Original Greek, as the Translations thereof into all Languages, doth sufficiently declare the excellency and worth of the Art, the Book, and the famous Author and Philosopher EUCLIDE: An account of whose Original, and the time wherein he flourished, we have thought good to insert; as also that full and learned Preface of the famous Mathematician John Dee, then which nothing of that nature can be more ample or satisfactory.

We shall only advertise the Reader what order and method hath been used and observed by us in this Book (which we now present to his favourable acceptance) which hath not been done by any other that have published these Elements in the English Tongue.

And firſt, to ſpeak of the Schemes, he is to take notice that throughout the Book, the greater black lines ſtand for, and denote the Data, *or things given, the leſſer black lines the things required, and the pricked lines ſerve for the Conſtruction and Demonſtration; the like method is alſo obſerved in the Circular lines given and required.*

Secondly, he is to take notice that the explications of the things given and required, are comprehended in the words of the Propoſition, by meanes of the correſpondent letters relating to the ſeveral parts of the Schemes, in ſuch ſort, as the Propoſitions may be read with them, or without them, and ſo by this meanes the Demonſtrations are much abreviated, yet our endeavours have been to render all things as plain, eaſie, and intelligible to the Reader as may be, and we hope they will be ſo found.

Thirdly, Whereas moſt Authors that have publiſhed the Elements of EUCLIDE, *do end at the fifth Propoſition of the Fifteenth Book, we have in this Book added the reſt of the Propoſitions belonging to the Fifteenth Book, as alſo the whole Sixteenth, according to* Fluſſas, *and alſo the Treatiſe of Regular Solids, which Propoſitions will be found of excellent uſe in the Geometrie of Solids: Thoſe Propoſitions alſo we have reduced to the ſame method as the precedent, and ſo conſequently they are much abreviated.*

Fourthly, He is to take notice, that at the end of the Second Book, we have added the Ten firſt Propoſitions of that Book according to Barlaam *a famous Greek Author, to ſhew the agreement and correſpondency that is between Lines and Numbers, Geometry and Arithmetick, the ſaid Propoſitions being proved true by both, they having the ſelf ſame proprieties and paſſions in Numbers, which* EVCLIDE *in this Second Book demonſtrates by magnitudes.*

Fifthly, We have added EUCLIDES Data, *a Book of excellent uſe in things relating to Algebra, as alſo a learned Preface thereon, written by the* Philoſopher Marinus, *fully unfolding the ſignification of the Term, and the profit and utility of the Treatiſe; the ſame method of bringing the letters of the Scheme into the* Propotſition, *being obſerved here as in the other Bookes.*

Laſtly, We have added two excellent Treatiſes of the Diviſions

visions of Superficies, one of them written by our EUCLIDE, *though ascribed to* Machomet Bagdedine, *the other by* Frederick Commandine *of* Urbin, *touching the same matter: These two Treatises, together with* EUCLIDES Data, *were never before published in the English Tongue, nor is there any thing extant in English of that nature so full and ample, in all respects, as those are.*

There are some other Treatises ascribed to our Author, viz. *his* Opticks, Catoptricks, Phænomena, *and* Musick, *which we purposely omitted, because they are not Elementary, as also because those Subjects have been more fully treated on by other later Authors. And moreover it is questioned, by many whether they were the Workes of this our* EUCLIDE, *or some other.*

Thus as exactly as we can, we have given you a short view of what we thought fit to be premised concerning this Work, which if diligently read, will undoubtedly be attended with as much profit as delight: The greatest Errors (as it is almost impossible but some will passe, in a Work of this Nature) are denoted at the end of the Book: As for the lesse mistakes, we leave them to your goodnesse, either to correct them, or excuse them.

FAREWEL.

An Account of the Author according to Ancient Phylosophers.

THe famous *EUCLIDE* Instituter of these Mathematical Elements (to premise something concerning the Author, and something also of this his Noble Science) is variously reported of, both for his Person, and the Age and Time in which he flourished. Several Writers are of opinion, as *Campanus* and *Theon*, in their Edition of these Elements, have published that he was borne at *Megara*, a Town adjacent to *Isthmus*, a Hearer of *Socrates*, who denominated a Sect of Philosophers, his Disciples, from that place, which said Sect was otherwise called the *Dialectick*; for that their manner of Writing was by way of Question and Answer, the Property of Logick onely. Of this *EUCLIDE* *Diogenes Laertius* hath writ much, in his *Lives* of the Philosophers; as also *Cicero* in his Third Book of Academick Questions, where he saith that *EUCLIDE* was of *Megara*, a Disciple of *Socrates*, whose Scholars affirmed that to be the only good, which was *One*, *Alike*, the *Same*, and *Perpetual*. This relation of *EUCLIDE* is backt by *Valer. Max.* who in his Eighth Book saith, that *Plato* sent his Scholars to informe themselves of a certain Question, to *EUCLIDE* the Geometrician: But if credit may be given to *Proclus* a Noble Writer, and other Ancient Writers, This our *EUCLIDE* was younger then him of *Megara*, and flourished in the Reign of *Ptolomy* the First, who governed *Ægypt* after the death of *Alexander* the Great, in the 115th. Olympiad, and before the Birth of Christ 309 Yeares; which is the most probable opinion, from this Reason: For *Diogenes Laertius*, diligently enumerating the Works of *EUCLIDE* of *Megara*, makes no mention of this most renowned Volume, of the Geomerical Elements, by which this our *EUCLIDE* hath decreed to himself a never dying immortal Fame and Glory. Neither is it to be thought, that *Diogenes* a Person so well versed in the Moniments of Philosophers, would have wittingly past over this most eximious piece, or not take notice that it was made by his familiar *EUCLIDE*. Therefore certainly (though it be uncertaine from whence he had his Birth) this our *EUCLIDE* was younger then the other: A Philosopher he was, of great praise at first for his Learning in the Academicks, from which he addrest himself wholly to the Study of the Mathematicks, in which he so farre excelled, that by universal consent, and judgment, he is deservedly stiled the Prince of Ma-

Mathematicians. He writ ſeveral Books concerning that Science, in which his accurateneſſe, and admirable Doctrine, is very conſpicuous; ſuch are his *Opticks*, *Catoptricks*, *Elementary Inſtitutions*, for the Attainment of Muſick, *Phœnomena*, and a Book entituled *Data*, *A Treatiſe of Diviſions*, which though aſcribed to *Machomet Bagdedine*, is ſuſpected to be his, being a very exact piece, and newly publiſhed by the endeavours of *John Dee* of *London*, and *Frederick Commandine* of *Urbin*. He writ alſo (as *Proclus* ſaith) the *Conick Elements*, which yet never ſaw the light, and ſome ſuch other Tracts. But more eſpecially he wrote this Book of *Geometrical Elements*, never enough (by univerſal vogue) to be commended, being wrought with ſuch admirable method, and ſuch immenſe Learning, that no man ever that yet writ of the ſame, came ſo much as neer him. In which as he ſhewed his acute and piercing brain, ſo did he not publiſh all things belonging to that Science, but onely the chiefe and moſt neceſſary, ſupported by the moſt firme and ſolid Arguments. And what the profit of theſe his Elements are to the ſtudious, hath by what we have already ſaid, and by what we ſhall now ſay, plainly appear. They are called the Elements of Geometry, for this cauſe, for that no Mathematical Operation can be undertaken, nor no advantage be made, without them. For all Mathematical Writers, as *Archimedes*, *Appollonius*, *Theodoſius*, and others, in their Demonſtrations uſe theſe Elements, as Principles long ſince known, approved, and demonſtrated. Therefore as he, who would learn to read, muſt firſt learn his A B C, and daily uſe them repeated in the forming and expreſſing of words, ſo he that would render the Science of the Mathematicks eaſie to him, muſt perfectly know theſe Geometrical Elements: For from theſe, as from a full Fountaine, all dimenſion and diviſion of Latitude, Longitude, Altitude, Profundity, as of Fields, Mountains, Iſlands, take their riſe and beginning; from hence the obſervation (by Inſtruments) of the Stars in Heaven, the making of all Scioterick Horologies, The force of Engines, Judgment of Weights, all diverſity of various apparences (ſuch as are ſeen in Glaſſes, in Pictures, in Water, and in the Aire variouſly radiated) have their Original. By theſe Principles (further) the Medium and Centre of this whole Fabrick of the World was found out, and the Poles or Hinges on which it is roled; and finally, the certain figure and quantity of it. By the ſtrength of this alone, the riſing, ſetting, depreſſion, return, aſcenſion, deſcenſion, and culmination of all the perpetual motions in the Heaven, the diverſity of day and night, in all climates, and Latitudes throughout the Year, are infallibly known. The divers Conjunctions alſo, and Oppoſitions and Aſpects of the Planers, are ſo eaſily known, that their places in the Heaven, the Eclipſes and defects of the Sun and Moon are ſo certainly demonſtrated, that they can be, and are predicted by the Mathematicks to all after Ages. To ſumme up all, this vaſt Work of God and Nature is preſented to the eye of our underſtanding, by the benefit and help of Geometry. Adde hereto, that this Science renders things incredible, and which ſurpaſſe all beliefe, moſt perſpicuous and evident to us. Such a Story is that which Hiſtorians teſtifie of *Archimedes* of *Syracuſa*, For King *Hieron* having built a Ship (which he purpoſed to ſend to *Ptolomy* King of *Ægypt*) of ſuch an immenſe bulk, that all the Syracuſians with their uttermoſt ſtrength, could not remove, *Archimedes* being a moſt skilful Geometrician, promiſed to launch it, which with little labour accordingly in the ſight of them all he performed, to ſuch an amazement

of

of the King, that he expreſt with aſtoniſhment, *That what ever* Archimedes *ſhould from thenceforth ſay, ought indubitably to be believed.* Not much unlike this, was his other famous Operation before the ſame King, who had commanded a Crown to be made of Gold, *Archimedes* without melting it down, told the King the reſpective Weight of Gold and Silver, which the fraud and cunning of the Artificer had mixed and incorporated together. Nor may we paſſe by in ſilence that uſual Saying of his, *That (truſting to the Force and Efficacy of Geometrical Demonſtrations) if there were another World whereon he could ſet his foot, he would remove this whereon we are, from its place.* And by the ſame reaſon, with never ſo little ſtrength, remove never ſo ponderous and maſſie weights. Divers like things have been performed by him, and other induſtrious Geometricians, which are recorded at large. But ſuch a fame did this Science purchaſe to *Archimedes*, that *Marcellus* the Roman General, againſt whom he alone had long defended his Native City of *Syracuſa*, by certain Engines invented by Geometrical Demonſtrations, having taken the City, and given the plunder and ſpoil, and ſack of it to his Souldiers, forbidding any quarter to be given to the Citizens, did yet by his Proclamation command the ſaving of the life of *Archimedes*, whom when againſt that his command, he knew to be killed by the hands of a common Souldier, he did moſt paſſionately grieve, and beſtowed that Honour upon him being dead, which he could not confer upon him living. And *Cicero* highly pleaſes himſelf, and much glories in it, that when he was Quæſtor in *Sicily*, he had found out and ſeen his Monument and Sepulchre. Therefore none may wonder that Geometry was in ſuch eſteem among the Grecians. And this alſo much advances the excellency and utility of Geometry, becauſe its Demonſtrations are ſo clear, that no man can without it be a perfect Methodiſt; which *Galen* that famous Philoſopher and Prince of Phyſicians, in his Books ingeniouſly acknowledgeth: Who being every way accompliſhed in the Dialectick Learning; and having run through all the Schools both of the Stoicks and Peripateticks of his time, and with much intentneſſe of mind, conſidered and ſtudied their Precepts and Rules, declares that they contein nothing in them, which might conduce to the knowledge of Demonſtration; but rather moſt of them were yet in controverſie amongſt themſelves, others quite repugnant to natural reaſon; ſo that he was almoſt brought to the opinion of the *Pyrrhonian* Philoſophers, to doubt of every thing, and to determine nothing; but that by the knowledge of Geometry and Arithmetick (in which he was trained up by his Grand-father) he was recovered, and farther reclaimed from that errour. His counſel therefore is, that men apply their minds to ſtudy the Arithmetick Characters, and the Demonſtration of Lines. So alſo *Plato* is of this opinion; *Becauſe* (ſaith he) Geometry *conduceth to the acquiring and right underſtanding of all other Arts and Sciences.* To conclude, this is the chiefe Glory of Geometry, that it loyters not, or employes it ſelf about theſe inferiour Machines, from whence it had its Original, but hath ſoared up into Heaven, and reſetled humane minds, (groveling before in the duſt) in Cœleſtial Seats, and hath capacitated us to the underſtanding both of this whole Fabrick of the World, and the Adminiſtration and Government thereof.

THE

THE FIRST ELEMENT OF EUCLIDE.

THE ARGUMENT.

THis Firſt Book comprehends the properties of the firſt and chiefeſt of right lined figures, *viz.* of *Triangles* and *Parallelograms*, as touching the equality, or inequality of their Area, and of their ſides and angles; wherein firſt is declared the original and properties of Triangles compared among themſelves: Then intermixing the properties of parallels, the nature of *Parallelograms* and the affections that are in them are demonſtrated; after which is ſhewn the relation of Triangles and Parallelograms, and after what manner a Parallelogram may be made equal to a Triangle; Laſtly, it is demonſtrated that in a right angled Triangle, the Square that is made of the ſide ſubtending the right angle, is equal to the Squares that are made of the ſides comprehending the right angle. And that all theſe things may the better be accompliſhed, here is alſo taught the diviſion of a right lined angle, & of a right line into two equal parts, the conſtruction of a perpendicular line, and after what manner a right lined angle may be made equal to a right lined angle, and other things of the like nature.

A DE-

DEFINITIONS.

1 *A point is that which hath no part.* *As* A

IN Greek it is read σημεῖον, that is to ſay, a Sign; For ſeeing that it is void of all Magnitude, that which is externally made, is the ſigne of that which is conceived in the Minde; and it is (in a manner) the ſame as a Unite in Number, an Inſtant in Time, and a Sound in Muſick.

2 *A Line is a Length without breadth.* *As* A—B.

THere is no ſuch Line in any material thing, but as the point, ſo the line which we draw, is the ſign of that which we conceive in Mind: For if the point which we conceive be moved, and leaveth an imaginary tract, that ſhall be a line, long by reaſon of the motion, but not broad, becauſe the point from whence it proceedeth is void of all extention.

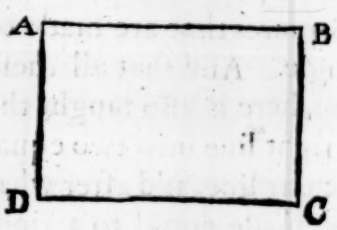
3 *The Extreams or Ends of a Line are points.*

THat is, the beginning and end of a Length, as it is a Length, is a point: becauſe a Mathematician doth not conſider Magnitude, but as it is finite: Therefore when *Euclide* ſpeakes of an infinite line, he underſtandeth an indeterminate line, or a line of any Magnitude.



4 *A Right Line is that which lyeth equally between its points.*

OR whoſe extreams doth ſhadow all the middle parts, as ſaith *Plato*: or the leaſt of all thoſe which have the ſame extreams, as *Archimedes* will have it: For ſeeing that a line is conceived to be made by the flux of a point, if it floweth equally between its points, or by the ſhorteſt Space, it is called a right line, as A B; If a point be carryed with an uniform motion and diſtance from a certain point, it is called a circular line as C D; If it moves unevenly, in ſome place lower, and in others higher, and the extreams do not ſhadow all the middle parts, it is called a mixt line, as E F: From hence *Ariſtotle* ſaith ingeniouſly *Lib.* 1. *de Cœlo*, that according to this threefold diſtinction of a line, there can be only three motions, two ſimple, the right and circular, and the third mixt of both.



6 *A Superficies is that which hath only length and breadth.*

AS a line the firſt Species of continued quantity, is produced by the flux of a point, ſo a Superficies, the ſecond Species, is conceived to be produced by the flux of a line tranſverſly, or ſide-wayes, as if the line A D be ſuppoſed to move ſide-wayes to B C, it ſhalll produce the Superficies A B C D, which may be divided in length as a line, and likewiſe

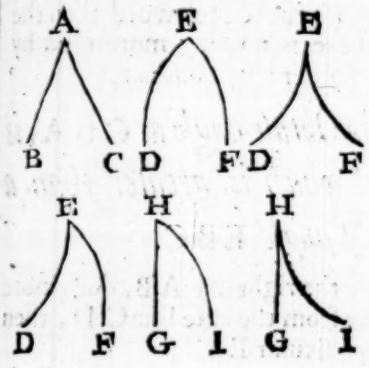
wise in breadth. Conceive a shadow, saith *Proclus*, and thou shalt conceive a Superficies long and broad, yet having no manner of thickness.

6 *The Extreams or Ends of a Superficies are lines.*

THis Definition is only to be understood of a plain or a mixt Superficies, and not of a circular, for the extream thereof is only a line, and not lines.

7 *A Plain Superficies is that which lyeth equally between his lines.*

WHat I have said of a right line, the same is to be understood of a Plain Superficies.

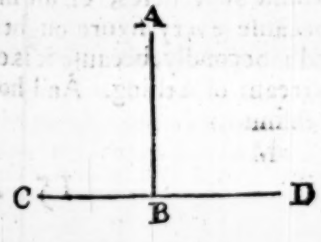


8 *A Plain angle* BAC, *is an inclination or bowing of two lines* AB *and* AC *in a Plain, the one to the other, the one touching the other, and not lying streight forth at length.*

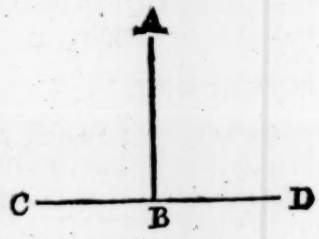
HEre the Causes of an angle are explained, the Material are two lines which touch one another: The Formal is an Inclination of the one to the other. From whence it followeth, First, that those two lines ought not so to touch, that they may lye joyned in a direct line the one to the other, that is, so as they make one right line, but the one ought to incline to the other: Secondly, it followeth that the quantity of an angle consists in a greater or lesser inclination of the lines, and not in the length of the lines: Thirdly, it followeth that it is not necessary that the two lines being produced, should intersect one another after the contact, as *Pelletarius* would have it, for that is only true in right lined angles, but it sufficeth that they touch one another, and be inclined to one another. Lastly, if the angle be in a Plain Superficies, it is called a Plain angle. And in every figure although we call every angle by three letters, yet the middlemost is alwayes to be understood for the Angle.

9 *And when the lines which contein the angle are right lines, it is called a right lined angle.*

AND if both of them be curved as DEF, it is a curve lined angle, and if one be curved, and the other right as GHI, it is a mixt angle.



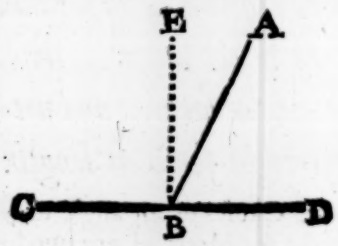
10 *When a right line as* AB, *standing upon a right line as* CD, *maketh the angles on either side,* ABC *and* ABD *equal, then*



either of thoſe angles is a right angle, and the right line A B, *that ſtandeth erected, is called a perpendicular line to that upon which it ſtandeth* C D.

THen either angle is ſaid to be equal, when the right line A B inclineth not more to C then to D.

That which the Greeks call καθετος, is rendred in Latine *Perpendicularis*, but the Mathematicians do more frequently uſe the Greek word then the Latine, chiefly in the Opticks, where there is nothing more uſed by them then πρὸς καθετον, alſo in Latine they render it *ad Cathetum*.



11 *An Obtuſe angle as* C B A, *is that which is greater than a right angle* E B C.

BEcauſe the right line A B, doth more recline from the baſe line C D, then the perpendicular E B.

12 *An Acute angle, as* A B D, *is that which is leſſer than a right angle* E B D.

13 *A Term is that which is the extream of any thing.*

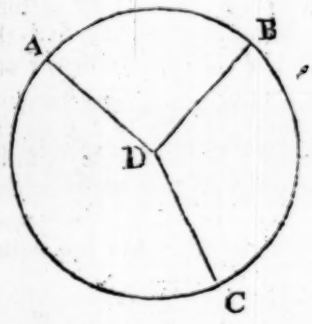
SUch are a point, a line, a ſuperficies; that is to ſay, a point is the Term of a line, a line of a ſuperficies, and a ſuperficies of a Body.

14 *A Figure is that which is comprehended by one Term, or many.*

IT is ſaid by one Term, becauſe one Term comprehendeth a Circle and an Ellipſis, that is to ſay, a Circular, or an Elliptical line: but there are alwayes more Terms required to comprehend right lined figures.

Moreover it is to be obſerved, the Terms ought to encompaſſe and comprehend the quantity which is called a Figure, and not only terminate it. From whence it followeth, firſt, that a line properly is no Figure, ſeeing that points do not encompaſſe a line, but only terminate it: It followeth, ſecondly, that there can be no figure of an infinite Superficies, or an infinite body, if any ſuch may be given; firſt becauſe every figure ought to encompaſſe and comprehend the thing figured: Secondly, becauſe it is encompaſſed with Terms, but a Term is the extream of a thing. And how can that which hath an end and extreams be infinite?

15 *A*



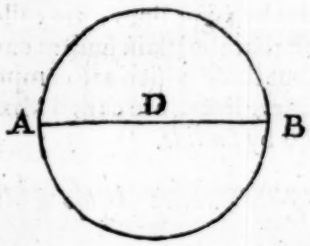
15 *A Circle is a plain figure, comprehended by one line* A B C, *which is called a Circumference: unto which all right lines drawn from one point within the figure, as* D A, D B, D C, *and falling upon the Circumference thereof, are equal the one to the other.*

16 *And that point is called the center of the Circle.*

Theodosius hath the same in his first Book of *Sphericks*, (concerning a Sphere) Definition the first, and second, but in the fifth Definition he thus describeth a Pole.

The Pole of a Circle in a Sphere, is a point in the Superficies of the Sphere, from whence all right lines drawn to the Circumference of the Circle are equal the one to the other.

From which it may be gathered that there is only this difference between the Center and the Pole of a Circle on the Sphere, because the Center is conceived to be within the figure: But the Pole is in the Superficies of the Sphere.

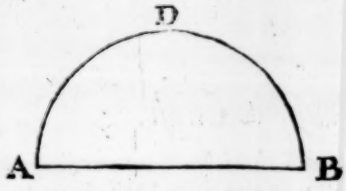


17 *The Diameter of a Circle is a certain right line, as* A B, *drawn by the center* D, *and being terminated by the circumferenee on either side, as* A *and* B, *divides the circle into two equal parts.*

HEre you shall observe three things, first, that all the Diameters of the same Circle are equal to one another, seeing that their halves are equal by the 15th. Definition, the second which followeth from the first is, that although there may be infinite right lines drawn in a Circle not passing by the Center, yet only those right lines which are drawn by the Center and terminated in the Circumference, are called Diameters, because the Diameter only can measure the Circle: seeing that they only be all equal the one to the other, and of a determinate length; but the others are alwayes unequal and uncertain: For *Ptolomie* saith in his *Analemma* that the measure of any thing ought to be certain, and determinate, not indefinite; Wherefore let not beginners wonder if it be put in the Feminine Gender by the Mathematicians, For Diameter is the same as the dimetient line, or a line dividing into two equal parts.

The

The third is that the Diameter doth divide the Circle into two equal parts, which *Thales* doth thus demonstrate in *Proclus*; Conceive in mind the portion of a Semicircle, so to be fitted to another portion, so as the Diameter may be the Base of both of them: If the Circumference of one do altogether agree to the Circumference of the other, it is manifest that those two portions made by the Diameter are equal the one to the other; seeing that neither of them doth exceed the other: But if one Circumference do not agree with the other, but falleth either without or within it, or partly without, and partly within, then the right lines drawn from the Center to the Circumference shall be equal, and not equal, which is absurd.



18 *A Semicircle is a figure which is conteined by the Diameter* A B, *and by that line* A D B. *which is taken away from the circumference of the circle.*

19 Right lined Figures, *are those which are conteined under right lines.*

After the Definition of the Circle, *Euclide* intending to give the Description of divers figures, teacheth first of all what figures are called Right lined Figures; for of those he principally treats in the following Books.

Therefore all plain figures enclosed on all sides by right lines, are called Right lined Figures; from whence it is manifest that the plain figures environed by Curve lines, are called Curvilined: but those which are comprehended partly by Right lines, and partly by Curve lines, are called Mixt: Now divers kinds of Figures are here described by *Euclide*.

20 Trilateral, *or three sided figures, are those which are conteined under three right lines.*

When *Euclide* here saith, that Right lined Trilateral figures are such as are environed by three right lines, he shews us cleerly how a Triangle ought to be defined: For seeing that in Right lined figures there are as many angles as there are sides, or of right lines of which they are constituted, the Triangle shall be a figure conteined under three right lines whose Species shall be hereafter declared.

21 Quadrilateral, *or four sided figures, are those which are conteined under four right lines.*

In like manner, the Quadrangle, or Quadrilateral figure, is a figure conteined of four right lines, whose divers Species and Kinds, shall be hereafter declared.

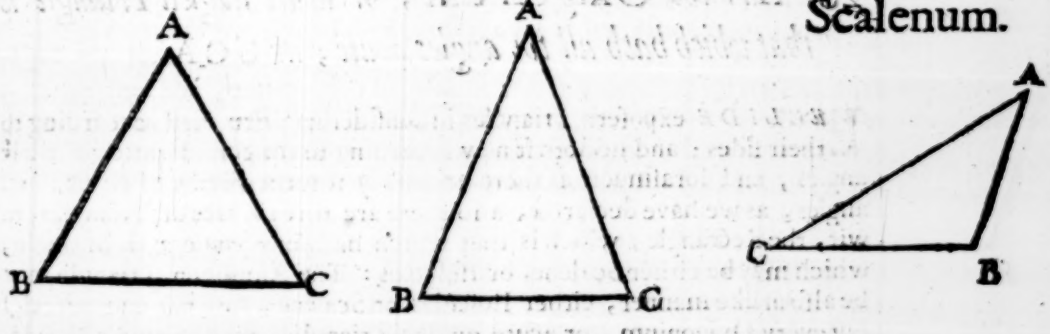
22 Mul-

22 Multilateral, *or many ſided figures, are thoſe which are conteined under more than four right lines.*

FOraſmuch as the Species or Kinds of Right lined figures are innumerable, by reaſon of the infinite progreſſion of numbers: For three right lines environing a figure, do conſtitute the firſt Kind, under which are conteined all Triangles, four right lines conſtitute the ſecond Kind, which compriſeth all quadrangular or quadrilateral figures, five right lines compound the third Kind, and ſix the fourth, and ſo *ad infinitum*: Therefore ſeeing that right lined figures cannot be infinitely defined, *Euilde* calleth all other right lined figures, comprehended by more than four right lines, by this General Name [*Multilateral*] contenting himſelf with the Denomination of the Trilateral and Quadrilateral, it may be for this reaſon, becauſe he treateth principally of thoſe in his Books, and by the knowledg of thoſe the Definition of the reſt may be eaſily underſtood.

23 *Now of* Trilateral *or three ſided figures, that which hath three equal ſides is called an* Equilateral *Triangle; as* ABC.

Equilateral. Iſoſceles. Scalenum.



24 *But that which hath only two ſides equal, is called an* Iſoſceles *Triangle; as* CAB.

25 *And that which hath the three ſides unequal, is called a* Scalenum, *as* BAC.

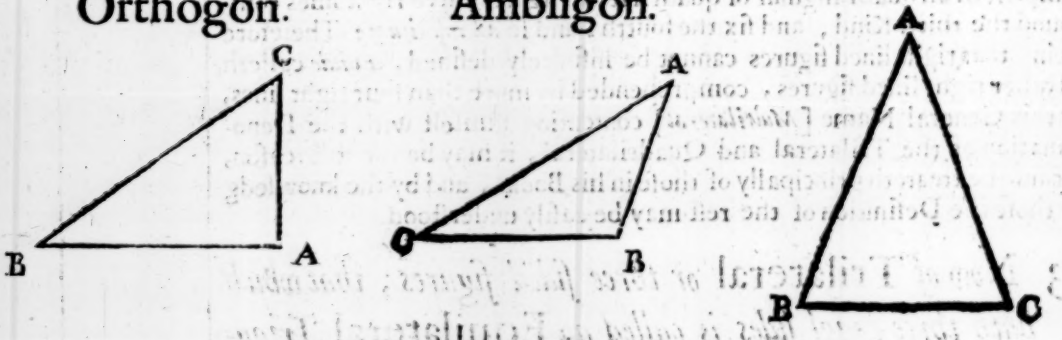
BY theſe three Definitions he expoſeth the three Species or Kinds of Triangles, the firſt and moſt ſimple of which is the *Equilateral* Triangle, or Triangle of three equal ſides, the ſecond is the *Iſoſceles* Triangle, which hath onely two ſides equal, and the third is the *Scalenum*, which hath all the three ſides unequal, the Equilateral Triangles are alwayes uniform, foraſmuch as their angles are alwayes equal: But the Iſoſceles and Scalene Triangles are diverſified in infinite manners.

26 *More-*

26 *Moreover, of three ſided, or* Trilateral *figures, the* Orthogonal, *or Rectangled Triaugle, is that which hath one right angle, as* B A C.

Oxigon.

Orthogon. Ambligon.



27 *The* Ambligonium, *or obtuſe angled Triangle, is that which hath one angle obtuſe, as* A B C.

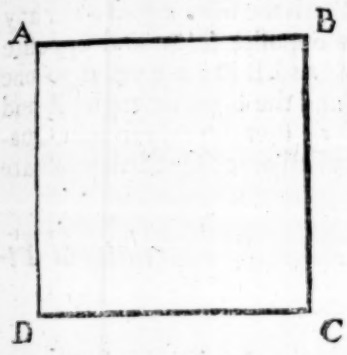
28 *And the* Oxigonium, *or acute angled Triangle is that which hath all the angles acute; as* B C A.

EUCLIDE expoſeth Triangles in conſidering them; firſt, according to their ſides; and he doth it now according to the conſideration of their angles, and foraſmuch as there are onely three varieties of Right lined angles, as we have declared, alſo there are three Kinds of Triangles, to wit, the Rectangle, which is that which hath one angle a right angle, which may be either Scalene, or Iſoſceles: The Ambligon Triangle may be alſo in like manner, either Iſoſceles or Scalene, (but not equilateral;) but every Oxigonium, or acute angled Triangle, may be either Equilateral, Iſoſceles, or Scalene.

¶ Here it is to be noted, that in every Triangle, two of the lines being taken for two ſides, the third remaining ſide is uſually called by the Mathematicians the [Baſe] of the Triangle, whether it be the loweſt ſide of the Triangle or not, in ſuch ſort, as each of the lines which encloſeth the Triangle, may be taken for the Baſe.

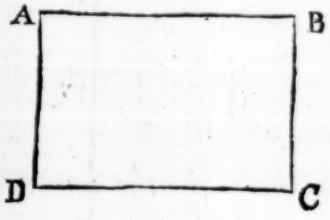
29 *But of* Quadrilateral, *or four ſided figures, the Square is that which is equilateral, and right angled; that is to ſay, which hath the four ſides equal, and the angles right.*

After



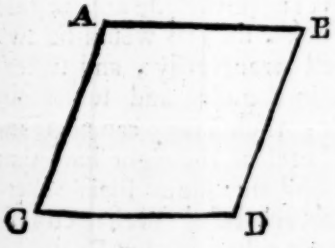
After the Trilateral and three sided figures he expoſeth the Quadrilateral and four ſided figures, ſpeaking only of five ſorts, of which the firſt four are Regular, but the fifth and laſt Irregular: Therefore the firſt figure is called a Square, of which all the ſides are equal to one another, and all the angles right angles, as is the Figure A B C D; and therefore the Quadrangle which is equilateral, and not right angled, or contrarily rectangled, and not equilateral, ſhall not be called a Square.

30 *An* Oblong, *or Long Square is a figure rectangled, but not equilateral.*

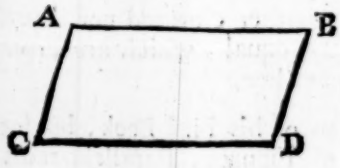


THe ſecond Quadrilateral Figure is called by the Greeks ετερόμηκες, that is to ſay, longer of one part, which is that which hath the four angles right angles, but hath not the four ſides equal to one another, although the oppoſite ſides are equal to one another, as appears by this Figure A B C D, where the ſides A B and D C oppoſite to one another, are equal, and in like manner, the ſides A D and B C.

31 *A* Rhombus, *is a figure equilateral, but not right angled.*



THis Figure which is the third kind of Quadrilateral, or four ſided figures, hath the four ſides equal, but the four angles are not right angles, although that the oppoſite angles are equal to one another, two to two, the angles A and D being equal to one another, but not right angles; and in like ſort, the angles B and C.



32 *A* Rhomboides *is a Figure which hath the oppoſite ſides, and angles equal, but is neither equilateral, nor right angled.*

THis Figure called Rhomboides, neither hath the ſides equal, nor any angle a right angle, but only hath the oppoſite ſides and oppoſite angles equal to one another, as the ſides A C and B D, are equal to one another, and in like manner, A B and D C, and the oppoſite angles A and D, and likewiſe B and C equal to one another; therefore theſe four Quadrilateral figures may be termed regular, but all other Quadrilaterals are irregular.

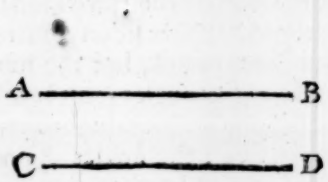
33 *But beſides theſe figures, all other Quadrilateral Figures are called* Trapeziums.



ALL other Quadrilateral Figures, which differ from thoſe before mentioned; that is to ſay, which are neither Squares, nor long Squares, Rhombs, nor Rhomboides, are called Trapeziums, and are Irregulars, conſidering that they may be of divers manners and formes.

34 *Parallels are right lines, which being in one and the ſame Plain, and prolonged infinitely on both parts, neither meet on the one ſide, nor on the other, as the lines* AB *and* CD.

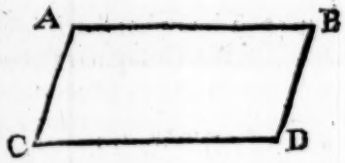
TO the end the right lines be parallel, or equi-diſtant, it ſufficeth not, that being prolonged infinitely on both parts, that they meet not at a point, or never interſect, but it is neceſſary that they be alſo in one and the ſame Plain Superficie: For divers right lines being not in one and the ſame Plain Superficie, prolonged infinitely, will never interſect at one point; yet nevertheleſſe are not ſaid to be Parallels, ſuch as would be two lines poſited tranſverſly, and underſtood to be in the air, and touch not one another; thoſe I ſay, can never interſect; Therefore the right lines that are in one and the ſame Plain Superficie, and which being prolonged infinitely on both ſides, (as for Example, the lines A B and C D,) will never interſect, thoſe ſhall be termed Parallels; or lines equally diſtant, *Poſidonius* thus defineth Parallels.



Parallels, (ſaith he) are ſuch lines as are neither elevated nor depreſſed in a Plain, but have all the perpendiculars equal, which are drawn from all the points taken in the one or the other line.

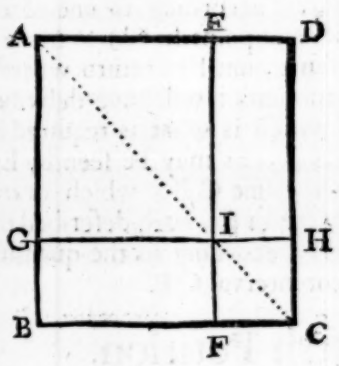
¶ *EUCLIDE* here endeth the Definitions of his Firſt Book, but forasmuch as mention is made therein of Figures, Parallelograms, and of their Complements, we have judged it convenient to explain the two following Definitions, to render the Demonſtrations more intelligible.

35 *A* Parallelogram *is a Quadrilateral, or four sided figure, whose opposite sides are parallel, or equidistant.*



AS if in the Figure A B C D, the side A B be parallel to the side opposite C D, and A C parallel to B D, it shall be called Parallelogram; that is to say, a Figure made of lines which are parallels, and of these there are four sorts; to wit, the Square, the long Square, which are also called rectangled Parallelograms; the Rhombe and the Rhomboides, Parallelograms not rectangled, from whence may be collected, that there are only four sorts of Parallelograms.

36 *But when in a Parallelogram there is drawn a* Diameter *or* Diagonal, *and two right lines parallel to the sides, cutting the Diameter in one and the same point, in such sort as the Parallelogram be divided by those parallel lines, into four* Parallelograms, *those two through which the Diameter doth not passe, are called Complements, but the two others, through which the Diameter doth passe, are said to be about the Diameter.*



LEt A B C D, be a Parallelogram, in which having drawn the Diameter A C, and the line E F, cutting the same in the point I, and in like manner, H G cutting the same Diameter in the same point I, and parallel to A D and BC, it is manifest, that the whole Parallelogram is divided into four Parallelograms, two of which, to wit, D I and I B, through which the Diameter doth not passe, are by Geometricians called Complements, or Supplements of the Parallelograms; the remaining Parallelograms E G and F H, are said to be about the Diameter, forasmuch as the Diameter doth passe through them.

Petitions or Requests.

1 *It is required that it be granted, that a right line may be drawn from any point to any point.*

THis first Petition will be manifest, if you consider what hath been heretofore said of a line; For seeing that a line is an imaginary flowing of a point, and by this means a right line is a flowing, proceeding from the direct way, it will happen that if some point be understood to move to another point, a right line will be drawn from one point to another, which *EUCLIDE* requireth by this first Petition; as you may see by the line AB, drawn from the point A, to the point B.

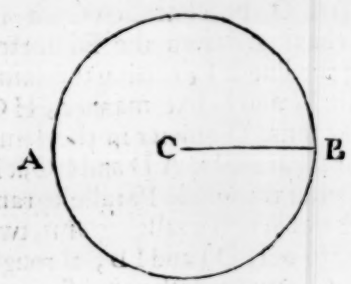
A———B

2 *And that a terminated right line may be produced streight forth continually.*

AS if it were understood that this point were moved yet further forth directly, the terminated right line shall be continued; and this production may be continued *ad infinitum*, seeing that we may imagine this point to move infinitely; as if the right line AB were continued to the point C, and then to the point D, &c.

A——B——C . D

3 *And that upon any Center, and at any distance, a Circle may be described.*



NOw if we conceive in the understanding, some terminated right line, applyed according to one of the extremities at a point fixed, to move about the same, until it return where it began its motion, it will have described a Circle, which is what is required by this Petition, as may be seen by Example in the line CB, which drawn about the Center C, hath described the Circle BA, acording to the quantity, distance, or interval CB.

Others add this fourth Petition.

4 *That any Magnitude being given, it is possible to take another greater or lesser.*

FOr all continued quantity may be augmented by addition, and by division may be infinitely diminished: Therefore there cannot be given a continued quantity so great, but a greater may yet be given; nor one so little,

little, but a leſſer may be given; This holds as true in Numbers, by addition, for each number may be augmented infinitely, by the continual addition of Unity; although in the diminiſhing thereof, you arrive to Unity indiviſible.

Common Sentences, or Axioms.

1 *Things that are equal to one and the ſame, are alſo equal to one another.*

FRom this firſt Axiom it is evident, that a thing greater, or leſſe than one of the equal things, ſhall be alſo greater or leſſe than the other; and if one of the equal things be greater or leſſe than ſome Magnitude, the other ſhall be in like manner greater or leſſe than the ſame Magnitude.

2 *And if to equal things, equal things be added, the whole ſhall be equal.*

3 *And if from equal things, equal things be taken, the remainders ſhall be equal.*

4 *And if to unequal things, equal things be added, the whole ſhall be unequal.*

IN like manner, it is manifeſt, that if to unequal things, things unequal be added, the greateſt to the greateſt, and the leaſt to the leaſt, that the whole ſhall be unequal, the one greater, the other leſſe.

5 *And if from unequal things, equal things be taken, the remainders ſhall be unequal.*

IT follows from hence, that if from unequal things, unequal things be taken; to wit, from the greateſt, leſſe, and from the leaſt more, the remainders ſhall be unequal; that is to ſay, this ſhall be greater, and that other leſſe.

Now in all theſe Axioms, excepting only the firſt, by theſe words, *equal quantities*, you ought alſo to underſtand one and the ſame, common to divers: for if to equal things there be added one common thing, the whole ſhall be equal; and if from equal things, a common thing be taken, the remainders ſhall be alſo equal: and if to unequal things, there be added one and the ſame common thing, or to one and the ſame common thing, there be added unequal things, the whole ſhall be unequal, and if from unequal things there be taken one and the ſame common thing, or from one and the ſame common thing, there be taken unequal things, the remainders ſhall be unequal.

6 *And the things that are double to one and the ſame thing, are alſo equal to one another.*

Whence

WHence it followes, that, that which is double to one of the equal things, is alſo double to the other: Likewiſe thoſe things which are triple, quadruple, quintuple, &c. to one and the ſame thing, are equal to one another, as is manifeſt.

7 *And the things which are the one half of one and the ſame thing, are alſo equal to one another.*

COntrariwiſe it follows, that thoſe things which are equal to one another, are the halves of one and the ſame; likewiſe the things which are the third, fourth, or fifth parts, &c. are alſo equal to one another.

In thoſe two Maximes, by one and the ſame quantity, ought alſo to be underſtood, equal quantities, for things double, triple, &c. of things which are equal, are in like manner equal to one another. Alſo the things which are the halves, third, or fourth parts, &c. of equal things, are alſo equal to one another.

8 *And the things which agree to one another, are alſo equal to one another.*

THat is to ſay, that two quantities, the one being poſited on the other, the one exceedeth not the other, but both agree together, thoſe are equal; as two right lines ſhall be ſaid to be equal to one another, when the one being poſited on the other, that which is placed above, totally agrees with the other, in ſuch ſort, as that it neither exceedeth the other, nor is exceeded by it: The ſame alſo of two right lined angles, equal to one another, when the one being poſited on the other, the uppermoſt exceedeth not the other, nor is exceeded by it, but the lines which conſtitute the one, do totally fall upon thoſe which conſtitute the other; for that being ſo, the inclination of the lines are equal; although, nevertheleſſe, thoſe lines are unequal to one another: but it is to be underſtood, that the Magnitudes which agree to one another, are equal, according to that only in which they agree. Now length agrees only to length, a Superficie to a Superficie, and the inclination of lines, to the inclination of lines, &c.

Clavius willing to convert this Axiom, ſaith thus, [and contrariwiſe the things which are equal to one another, agree with one another, if the one be poſited on the other;] which nevertheleſſe, cannot be, for if a ſquare be deſcribed, and on the Diameter of that Square there be deſcribed another ſquare, the firſt ſquare ſhall be equal to the half of the laſt; that is to ſay, to an Iſoſceles Triangle, and nevertheleſſe, will not agree, being poſited the one on the other: There may be alſo made a Curvilined angle, equal to a right angle, an obtuſe angle, or acute angle, as *Proclus* hath demonſtrated, which nevertheleſſe will not agree, being poſited on one another. Laſtly, a right lined Triangle may be made equal to another right lined Triangle, which may not agree, being poſited on one another, as is manifeſt by the 37 and 41 of the firſt Book: Therefore this Axiom cannot be converted univerſally, but thus it may. Equal right lines agree with one another, being poſited the one on the other: in like manner, equal right lined angles, as alſo plain Superficies, alike and equal, &c.

9 *And*

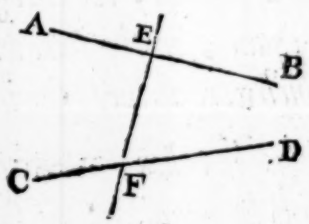
9 *And the whole is greater than its part.*

10 *All right Angles are equal to one another.*

THis Axiom is manifeſt by the 10 Definition, *Pappus* ſhews on this place that this Axiom cannot be converted; for every angle equal to a right angle is not a right angle, as we have ſhewn in the 8th. Common Sentence; but only, that every right lined angle equal to a right angle, is alſo a right angle; and this may be taken for an Axiom.

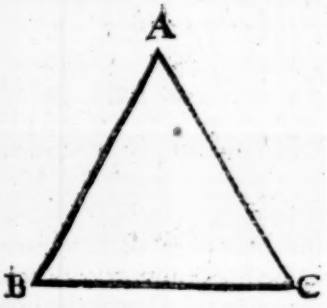
11 *And if on two right lines there fall another right line, making the interior angles on the ſame part leſſe than two right angles; thoſe two right liues being infinitely prolonged, will interſect and cut one another, on that part where the two angles are leſſe than two right angles.*

AS if a right line E F, falling on the two right lines A B and C D, maketh the two interior angles B E F, and D E F on the ſame part B D, leſſe than two right angles; *EUCLIDE* ſaith, that thoſe lines being prolonged on the part B D, where the angles are made leſſe than two right angles, that finally they ſhall interſect at a point, which is manifeſt, for when the two interior angles on the ſame part are equal to two right angles; then the two lines cannot interſect in any part, but are alwayes equally diſtant the one from the other, as ſhall be demonſtrated in the 28th. Prop. of this Book; wherefore if the interior angles on the ſame part be leſſe than two right angles, it is neceſſary that the Space between the lines ſtreighten more and more, and grow leſſe and leſſe, and on the other part that they open and enlarge themſelves more and more: Therefore thoſe lines at laſt will interſect at a point.



12 *Two right lines contein no Space.*

THis Axiom hath no difficulty in it, for if two lines make an angle at a point, they ſpread and dilate themſelves ſtill more and more both wayes, if they be prolonged, as is manifeſt by this Figure, for having drawn a right line from A to B, from the ſame point A, cannot be drawn another right line to the point B, by another way, in ſuch ſort as that between AB and the other line, there ſhould remain a Space incloſed; ſo as that another right line drawn from the point A,



as

as A C, ſhall make an angle with A B, in ſuch ſort as that another line ought yet to be drawn, as B C, to encloſe the Space: but a right line and a Curviline, or elſe two curved lines, may well contein or encloſe a Space, as is manifeſt.

After theſe Axioms of EUCLIDE, *Clavius* and other Interpreters, have yet added theſe following Axioms.

1 *Two right lines, have not one and the ſame Common Section.*

2 *Two right lines meeting at a point, if they be prolonged, they will neceſſarily interſect.*

There are yet added theſe ſix.

1 *If to equal things, unequal things be added, the exceſſe or difference of the whole ſhall be equal, or the ſame with the difference of the things added.*

2 *If to unequal things, equal things be added, the exceſſe, or difference of the whole ſhall be equal, or the ſame with the exceſſe or difference of thoſe which were at the beginning.*

3 *If from equal things, unequal things be taken, the exceſſe or difference of the remainders ſhall be equal, or the ſame with the exceſſe or difference of the things taken away.*

4 *If from unequal things, equal things be taken, the exceſſe or difference of the remainders ſhall be the ſame with the exceſſe or difference of the whole.*

5 *The whole is equal to all its parts taken together.*

6 *If the whole be double to the whole, and the part cut off, double to the part cut off, the reſt ſhall be double to the reſt.*

NOw theſe ſix pretended Axioms are alſo hereafter demonſtrated by *EUCLIDE*, as ſhall (God aſſiſting) be made appear. Wherefore ſeeing that they are proved elſwhere, following the Demonſtrations of

of *EUCLIDE*, they ought not to be taken for Axioms: We ſhall demonſtrate according to *EUCLIDE* the five firſt, at the Second Propoſition of this Book, and the Sixth, at the fifth Propoſition of the Fifth Book.

From what hath been before ſaid, we may gather, with *Proclus* and *Geminus*, this difference between Requeſts, and Maximes or Axioms; that ſeeing both the one and the other, are known of themſelves, and indemonſtrable, that Requeſts are of the nature of Problemes; foraſmuch as by them there is required ſome thing to be done; but Axioms reſemble Theoremes; ſeeing that by them it is not required that any thing ſhould be done, but only there is propoſed ſome Sentence well known. But the Requeſt or Petition differeth from the Problem, in that the Conſtruction of the Requeſt ſtandeth in need of no Demonſtration, but no one will conſent to the conſtruction of a Problem, without Demonſtration, for that ſome difficult thing is propoſed to be conſtrued. In like manner, there is difference between Axiom and Theorem, for the Axiom ought not to be demonſtrated; but the Theorem ought in no wiſe to be conſented to, if not demonſtrated: For no one will require the proof or Demonſtration of this Propoſition, [Things which are equal to one and the ſame thing, are likewiſe equal to one another;] but of this there will be required Demonſtration, [Of every Triangle, the interior angles are equal to two right angles;] and the ſame is to be underſtood of the other Axioms and Theorems, as alſo of Petitions and Problems.

It is likewiſe very certain, that of Requeſts or Petitions, ſome of them are more proper to Geometry, as thoſe three which *EUCLIDE* hath propoſed to us: other ſome are common to Geometry and Arithmetick, as is this, [A quantity may be infinitely augmented;] for as well the number as the Magnitude, may be augmented by Addition, ſo as that no end can be found in this augmentation; the ſame may be ſaid of the Axioms for the Eighth, Tenth, Eleventh, and are proper to Geometry only, but all the others are applyed to the Demonſtrations both Geometrical and Arithmetical, for even ſo as equal Magnitudes taken from equal Magnitudes, do leave equal Magnitudes, be the Magnitudes conſiſting of Lines, Superficies, or of Solid Bodies; So likewiſe equal numbers, taken from equal numbers, leave equal numbers; and having ſaid this of theſe three Kinds of Principles, let us now come to the Demonſtrations, by which the nature of theſe Principles will be more perfectly known; for there are divers principles among Mathematicians, and ſuch as are not eaſily underſtood, before their uſe be known in their Demonſtrations, which will be found by experience.

¶ To the end the courſe of the Demonſtrations be not interupted, we have cited the Principles and Propoſitions of *EUCLIDE* in the Margent, with a Letter of the Alphabet; the ſame Letter being alſo put in the Diſcourſe, there where the Propoſition ought to be cited, written in the Margent with the ſame Letter, and may thus be underſtood;

1 Def. The first Definition, and so of the rest.
1 Pet. The First Petition, or Request.
1 c.s. or 1. ax. The first Common Sentence, or Axiom.
1. 1. The first Proposition of the first Book.
1. 2. The first Proposition of the second Book, and so of the rest, in this manner.
Color. 32. 1. The Corollary of the 32 Proposition of the first Book.

What a Proposition is.

THe principles thus placed and ended, now follow the Propositions, which are Sentences set forth to be proved by reasoning and Demonstrations; and therefore they are again repeated in the end of the Demonstration: For the Proposition is ever the conclusion, and that which ought to be proved.

Propositions are of two sorts, the one is called a Problem, the other a Theorem.

What a Problem is.

A Problem is a Proposition which requireth some action or doing, as the making of some figure, or to divide a figure or line, to apply figure to figure, to adde figures together, or to subtrah one from another, to describe, to inscribe, to circumscribe one figure within or without another, and such like: As of the first Proposition of the first Book is a Problem, which is thus, *Upon a given terminated right line, to make a Equilateral triangle.* For in it, besides the Demostration, and Comtemplation of the mind, is required somewhat to be done: namely, to make an Equilateral triangle upon a line given. And in the end of every Problem, after the Demonstration, is concluded after this manner: *Therefore, &c. Which was to be done.*

What a Theorem is.

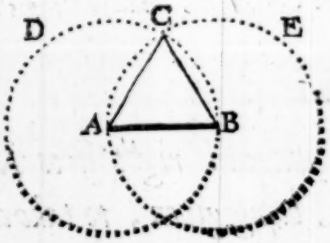
A Theorem is a Proposition which requireth the searching out and demonstration of some property or passion of some figure: Wherein is only Speculation and Contemplation of Mind, without doing or working of any thing, As the seventeenth Proposition of the first Book, which is thus, *Of every triangle two angles are lesse than two right angles, after what manner soever they be taken*; is a Theorem. For in it is required only to be proved and made plain by reason and Demonstration, that two angles are lesse than two right angles, &c. without further working or doing. And in the end of every Theorem, after the Demonstration, is concluded after this manner: *Therefore, &c. Which was to be demonstrated.*

PRO-

PROPOSITIONS,

PROBLEMES, & THEOREMES.

PROPOSITION 1. PROBLEM 1.



Upon a given terminated right line A B, *to make an Equilateral Triangle* A B C.

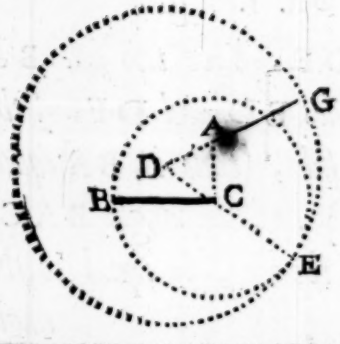
Construction ON the Center A, at the distance A B, [a] deſcribe the Circle B C D; and on the Center B, at the ſame diſtance B A, [b] deſcribe the Circle A C E, and from the point C, where the Circles interſect one another, [c] draw the two right lines C A and C B: Then I ſay, that the Triangle A B C is Equilateral.

a) 3. pet. 1.
b) 3. pet. 1.
c) 1. pet. 1.

Demonſtration FOr the right line A C is equal to the right line A B [d], and the right line C B is equal to the ſame right line A B [e]; Therefore [f] the right lines A C and B C, are alſo equal the one to the other: And the three right lines C A, A B, and B C, being equal, [g] the Triangle A B C is equilateral: Wherefore upon a given terminated right line A B, is made an Equilateral triangle A B C; Which was required to be done.

d) 15. d. 1.
e) 15. d. 1.
f) 1. ax. 1.
g) 23. d. 1.

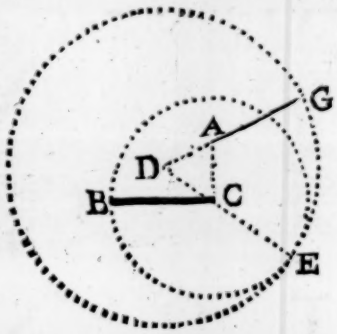
PROP. 2. PROBL 2.



From a point given A, *to draw a right line* A G, *equal to a right line given* B C.

Construction DRaw [a] from A to C, the right line A C, upon which make [b] the equilateral triangle C A D, then on the Center C, at the diſtance B C, deſcribe [c] the Circle B E,

a) 1. pet. 1.
b) 1. 1.
c) 3. pet. 1.



d) 2. pet. 1. e) 3. pet. 1. and [d] produce the side D C to E, and on the Center D, at the distance D E, [e] describe the Circle E G. and produce the side D A to G. Then I say, that the right line A G, (drawn from the given point A) is equal to (the given line) B C.

Demonstration FOr the right lines D E and D G are [f] equal: Therefore taking away the equal right lines D A and D C, there shall remaine the equal right lines [g] A G and C E: But C E is equal to C B: Therefore also A G is equal to C B.

f) 15. d. 1. g) 3. ax. 1.

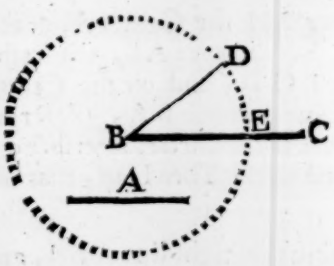
Wherefore from a point given A, is drawn a right line A G, equal to a right line given B C; Which was required to be done.

The position of the point A, in or without the given line B C doth varie the Case, but yet there is alwayes the same Construction and Demonstration.

SCHOLIUM.

The line A G *may be taken equal to* B C *with the Compasses, but in so doing, it answers to no Petition, as* Proclus *well observes.*

PROP. 3. PROBL. 3.



Two unequal right lines A, *and* B C, *being given, to take away from the greater* B C, *a right line* B E, *equal to the lesser* A.

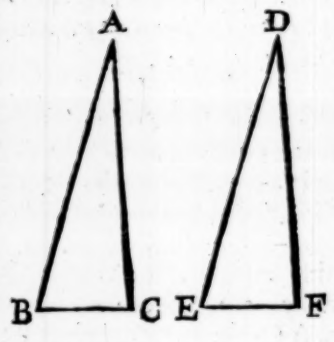
a) 2. 1. b) 3. pet. 1. *Construction* FRom the given point B, [a] draw the right line B D, equal to the given right line A, and on the Center B, [b] at the distance B D describe the Circle D E: Then I say, that the part cut off is equal to A.

c) 15. d. 1. d) 1. ax. 1. *Demonstration* FOr the right line B E is equal to the right line B D [c], and B D is equal to A by Construction; Therefore, the part cut off B E, is equal to the given line A [d]: Which was required to be done.

PROP. 4. THEOR. 1.

If two triangles B A C *and* E D F, *have two sides* B A *and* A C *of the one, equal to two sides* E D *and* D F, *of the other, each side to his correspondent side, (that is,* B A *equal to* E D, *and* A C *equal to* D F,) *and the angle* B A C, *conteined by the equal right line of the one, be equal to the angle*

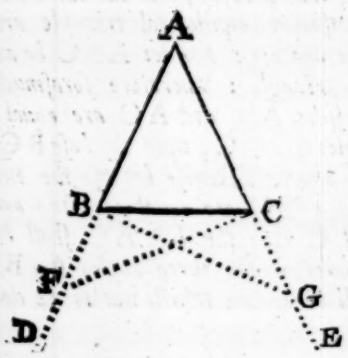
angle E D F, *conteined by the equal right lines of the other: The base also* B C *of the one, shall be equal to the base* E F *of the other; and the one triangle* B A C, *shall be equal to the other triangle* E D F, *and the other angles remaining, shall be equal to the other angles remaining, the one to the other; that is, the angle* A B C *to the angle* D E F, *and the angle* A C B *to the angle* D F E, *by which equal sides are subtended.*



Demonstration. FOr the triangle A B C being put upon the triangle D E F, and the point A upon the point D, and the right line A B upon the right line D E the point B shall fall upon the point E, because the right line A B, is equal to the right line D E: Seeing therefore that the right line A B doth agree with the right line D E, and the right line A C with the right line D F; because the angle B A C is equal to the angl E D F: Therefore, also the point C shall agree with the point F, wherefore also the base B C shall agree with the base E F. For if (B agreeing with E, and C with F,) the base B C doth not agree with the base E F: Then two right lines shall include a Superficies, which is absurd. Therefore the base B C, agrees with the base E F, and is equal to it: Wherefore the whole triangle A B C, shall agree with the whole triangle D E F, (for all the angles and sides do agree) and is equal to it, and the remaining angles of the one, shall agree with the remaining angles of the other, and shall be equal the one to the other, *viz.* the angle A B C to the angle D E F, subtended by the equal sides A C and D F, and the angle A C B to the angle D F E, subtended by the equal sides A B and D E.

PROP. 5. THEOR. 2.

The angles A B C *and* A C B, *which are at the base of* Isosceles *triangles* A B C, *are equal the one to the other, and the equal right lines* A B *and* A C, *being produced, (as to* D *and* E,) *the angles* C B D *and* B C E, *that are under the base, are equal the one to the other.*

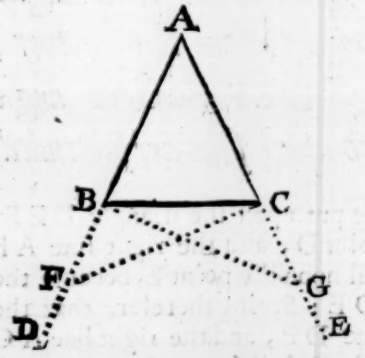


Con-

Construction LEt any point as F be taken in the line B D, and from the greater line A E, [a] let the lesser A G, be taken equal to the line A F, and [b] draw the right lines F C and G B.

a) 3. 1.
b) 1. pet. 1.

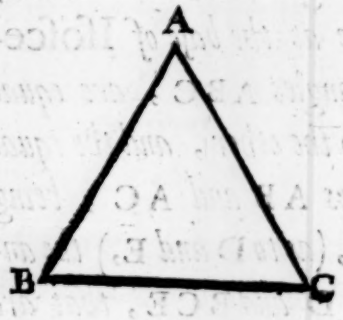
Demonstration SEeing that A F is equal [c] to A G, and A B to A C [d]: Therefore these two F A and A G are equal to these two G A and A B, the one to the other, and they contein a common angle F A G: Therefore [e] the base F C is equal to the base G B, and the triangle A F C is equal to the triangle A G B, and the remaining angles shall be equal to the remaining angles, the one to the other, by which equal sides are subtended, that is, A C F to A B G, and A F C to A G B, *and because* A F *is equal to* A G, *and* A B *to* A C. *Therefore*, B F is equal [f] to C G, and it is proved that F C is equal to G B: Therefore these two B F and F C are equal to these two C G and G B, the one to the other; and the angle B F C is equal to the angle C G B, and they have one common base B C: Therefore [g] the triangle B F C shall be equal to the triangle C G B, and the remaining angles of the one equal to the remaining angles of the other, the one to the other, by which equal sides are subtended: Therefore the angle F B C is equal to the angle G C B, and the angle B C F to the angle C B G: And because the whole angle A B G is equal to the whole angle A C F, of which the angle C B G is equal to B C F; the remaining angle A B C shall be equal to the remaining angle [h] A C B, and they are at the base of the triangle, and it is proved that the angles under the base F B C and G C B are equal: Therefore the angles A B C and A C B, at the base of Isosceles Triangles are equal, &c. Which was to be demonstrated.



c) by Con.
d) by Supp.
e) 4. 1.
f) 3. ax. 1.
g) 4. 1.
h) 3. pet. 1.

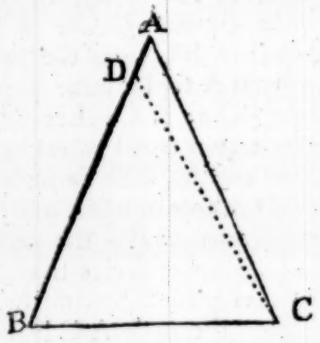
COROLLARIE.

From this Fifth Proposition it also followeth, that every equilateral triangle is equiangled; that is to say, that the three angles of whatsoever equilateral triangle are equal to one another: For let A B C *be an equilateral triangle: Therefore foraſmuch as the two sides* A B *and* A C *are equal, the two angles* B *and* C, *upon the base* B C, *shall be also equal. Likewise because the two sides* A B *and* B C *are equal, the two angles* A *and* C *upon the base* A C *shall be equal: Therefore the three angles* A, B, *and* C *shall be equal: Which was to be demonstrated.*



PROP.

PROP. 6. THEOR. 3.



If two angles A B C *and* A C B, *of a triangle* A B C, *be equal to one another, the ſides* A B *and* A C *ſubtending thoſe equal angles ſhall be alſo equal to one another.*

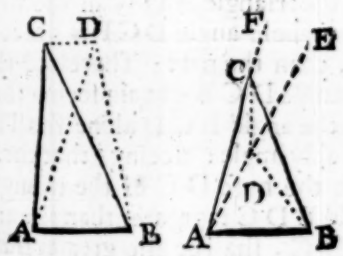
Conſtruction FOr if they be unequal, one of them ſhall be greater than the other; Let A B therefore be the greater, (if poſſible,) and let D B [a] be cut off, equal to A C, and let D C be drawn. a) 3. 1.

Demonſtration FOraſmuch as D B is equal to A C, and B C common, the two ſides D B and B C of the triangle D B C, ſhall be equal to the two ſides A C and C B, of the Triangle A C B, each ſide to his correſpondent ſide, and the angle A B C is equal to the angle A C B by Suppoſition; Therefore [b] the baſe A B ſhall be equal to the baſe D C, and the whole triangle D B C, equal to the whole triangle A C B; that is, the part equal to its whole; which is abſurd; therefore A B ſhall not be unequal to A C: Therefore if two angles, &c. Which was to be demonſtrated. b) 4. 1.

COROLLARIE.

It follows from this Propoſition, that every equiangled triangle (that is to ſay, all whoſe angles are equal to one another,) is equilateral; for ſeeing it hath been demonſtrated, that the angles B *and* C *being equal, the ſubtending ſides* A B *and* A C *ſhall be alſo equal, it may be concluded in the ſame manner, that ſeeing the angles* A *and* B *are equal, the ſubtending ſides* A C *and* C B *ſhall be alſo equal, and ſo of the reſt: Therefore the three ſides ſhall be equal, and therefore the triangle is equilateral.* As in the preceding Corollary of the Fifth Propoſition.

PROP. 7. THEOR. 4.

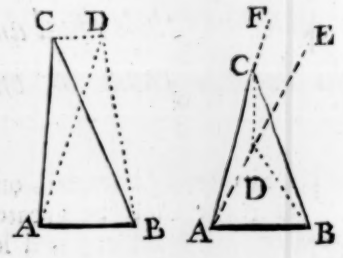


If two right lines A C *and* B C, *drawn from the extremities* A *and* B, *of one and the ſame right line* A B, *do meet in a point* C, *there cannot be drawn from the ſame extremities* A *and* B, *to another point, and on the ſame ſide, two other right lines equal to the firſt, each line to his correſpondent line.*

It

IT is most certain, that from the point A, towards C, to another point as D, a line may be drawn equal to AC, but the line drawn from B to the same point D, shall not be equal to BC: Likewise from B there may be drawn a line equal to BC, on the same side to some other point: but the line drawn from A to the same point, shall not be equal to AC: Let therefore [a] AD be drawn equal to AC, on the same side, and let BD be joyned: I say, that BD cannot be equal to BC; for having drawn CD, the figure ACD shall be an Isosceles triangle, AC and AD being equal by construction [b]: Therefore [c] the angle ACD shall be equal to the angle CDA, on the base CD; But [d] the angle BCD is lesse than the angle ACD, the part then its whole; it [e] shall be therefore also lesse than CDA, which is shewn to be equal to ACD; and therefore yet much lesse than the whole CDB: But if the line BD were also equal to the line BC, the triangle BCD would be an Isosceles triangle; and therefore [f] the angle BCD should be equal to the angle CDB, on the base CD. But BCD is shewn to be much lesse than CDB, it should be then equal, and much lesse, which is impossible: Therefore BD shall not be equal to BC. Which was to be demonstrated.

a) 2. 1.



b) 5. 1.
c) 5. 1.
d) 9. c. s.
e) 9. c. s.
f) 5. 1.

And if from the point B there be drawn a line equal to BC, to some other point then C, and on the same side, it will be shewn by the same reason, that the line drawn from the point A, to the same point, shall not be equal to AC.

Otherwise, (if it be possible,) from the extremities A and B, let there be drawn AD and BD, to another point D, towards C, equal to the first AC and BC, each to his correspondent, to wit AD, equal to AC, having the same extremity A, and BD equal to BC, having the same extremity B, and let DC be joyned; and if the point D be within the triangle, (as in the second figure;) Let AC and AD be prolonged directly towards E and F, as much as shall be expedient.

Forasmuch as the line AD is put equal to AC, the angle EDC alone, shall be equal to DCB and BCF, taken for one only angle, [g] seeing they are both of them under the base DC, of the triangle ADC in the first figure, or as in the second figure: But [h] the whole angle DCF is greater than the angle DCB alone, [h] the whole, than the part: Therefore the angle EDC shall be also greater than the angle DCB; again seeing that the line BC is put equal to the line BD, the angle BCD alone shall be equal to EDC and EDB, taken for one only angle: Seeing [i] that these angles BCD, EDC, and EDB, are on the base DC of the triangle BDC: but we have shewn that the angle EDC is greater than the angle BCD; Therefore the same angle EDC, shall be also greater than the angle EDC and EDB, the part than the whole, which is impossible: Therefore, If two, &c. Which was to be demonstrated.

g) 5. 1.
h) 9. c. s.
h) 9. c. s.
i) 5. 1.

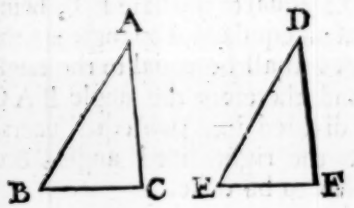
COROLLARIE.

Hence it follows, that having drawn two lines from the extremities of a right line

to

to one and the same point, as A C *and* B C, *if from the same extremities there be drawn two others, equal to the two first, each to his correspondent, on the same side, they will meet together in one and the same point, being inclined the one to the other, is manifest by the aforegoing figures.*

PROP. 8. THEOR. 5.



If two triangles A B C, *and* D E F, *have two sides* A B *and* A C, *equal to two sides* D E *and* D F, *each to his correspondent side, and the base* B C *be equal to the base* E F, *they shall also have the angle* A, *conteined of those sides* A B *and* A C, *equal to the angle* D, *conteined of the sides* D E *and* D F.

Demonstration FOr if it be understood that the triangle A B C be put on the triangle D E F; to wit, the point B on the point E, and the base B C on the base E F, the point C will agree with the point F: [a] Seeing that B C and E F are equal, and the two right lines B A and C A will agree on the two others E D and F D, [b] being equal to them, each to his correspondent, and the point A on the point D; and therefore the angle A shall be equal to the angle D, otherwise they would meet together in another point, which is impossible; [c] Seeing that if two right lines drawn from the extremities of another right line, do meet in one point, &c. Therefore A shall be equal to D: Therefore, If two triangles, &c. Which was to be demonstrated.

a) 8. c. f.
b) 8. c. f.
c) 7. 1.

COROLLARIE.

It may be gathered from this Proposition that the other angles are also equal to the other angles, each to his correspondent, and the whole triangle, to the whole triangle; for seeing that the angle A, *is shewn to be equal to the angle* D, *the sides* A B *and* A C *being equal to* D E *and* D F, *each to his correspondent, it follows* [d] *that the triangle is equal to the triangle, and the other angles equal to the other angles, each to his correspondent.*

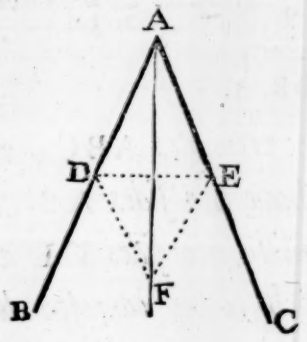
d) 4. 1.

PROP. 9. PROBL 4.

To divide a given right lined angle B A C, *into two equal parts.*

Construction LEt there be taken in the line A B, a point at pleasure D; and from the greatest line A C prolonged infinitely, let [a] there be cut off A E equal to A D, and let D E be drawn, on which [b] let there be described the equilateral triangle D F E, and let A F be drawn; I say that the angle B A C is divided into two equal angles.

a) 2. 1.
b) 1. 1.

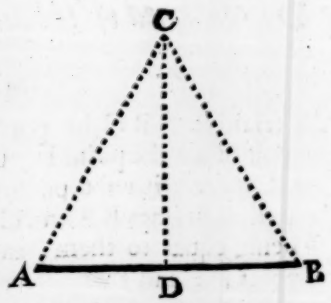


c) 8. 1.

Demonſtration FOraſmuch as E A hath been taken equal to A D and A F is common, the two ſides D A and A F are equal to the two ſides E A and A F, each to his correſpondent, and the baſe D F equal to the baſe E F; being the ſides of an equilateral triangle; c the angle D A F ſhall be equal to the angle E A F: and therefore the angle B A C ſhall be divided into two equal parts: Therefore the right lined angle, &c. Which was to be done.

PROP. 10. PROBL. 5.

To divide a terminated right line, given A B, *into two equal parts.*



a) 1. 1.
b) 9. 1.

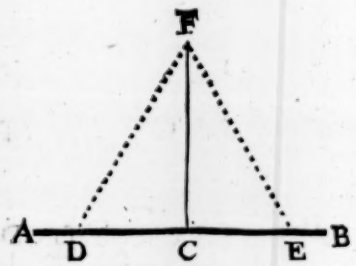
Conſtruction ON A B a let there be deſcribed the equilateral triangle A B C, and then b let the angle A C B be divided in two equal parts by the right line C D: I ſay, that A B is divided into two equal parts in the point D.

Demonſtration FOraſmuch as A C is equal to C B, (they being the ſides of an Equilateral triangle,) and C D is common, the two ſides A C and C D are equal to the two ſides B C and C D, each to his correſpondent, and the angle A C D equal to the angle B C D, by conſtruction; wherefore the baſe A D c ſhall be equal to the baſe B D: Therefore A B ſhall be divided in two equal parts. Which was to be done.

c) 4. 1.

PROP. 11. PROBL. 6.

On a given right line A B, *and from a given point therein* C, *to draw a right line* C F, *at right angles.*



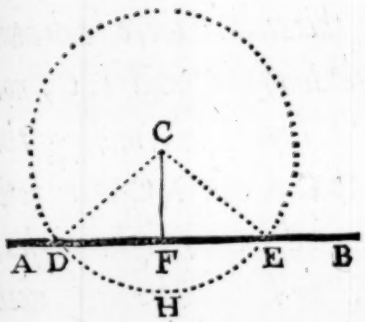
a) 2, 3. 1.
b) 5. 1.

Conſtruction IN the part A C let there be taken any point, as D, then a let C E be taken equal to C D, then on D E let there be deſcribed the equilateral triangle D F E; and let C F be drawn; b I ſay that the ſame C F is at right angles to the line A B.

De-

Demonſtration FOraſmuch as C D is equal to C E, and C F common, the triangles D C F and E C F ſhall have the two ſides D C and C F equal to the two ſides E C and C F, each to his correſpondent, and the baſes D F and E F equal, being the ſides of an equilateral triangle, the angles at the point C ſhall be equal; therefore both the one and the other, are right angles [c]: Therefore C F ſhall be at right angles to A B: Therefore on a right line, &c. Which was to be done. c) 10. def.

PROP. 12. PROBL. 7.

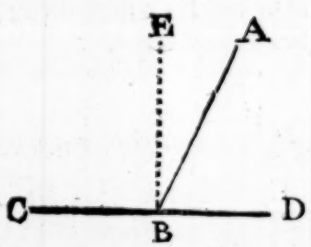


On an infinite right line given A B, *and from a given point* C, *which is not in it, to draw a perpendicular line* C F.

Conſtruction LEt the contingent point H be taken on the other ſide of A B, and from the point C as a Center at the diſtance C H, let the circle D E be deſcribed, cutting A B in the points D and E; then [a] let D E be divided into two equal parts in the point F, and let C F, C D, and C E be drawn; I ſay that C F is a perpendicular to A B. a) 10. 1.

Demonſtration FOraſmuch as D F is equal to E F, by Conſtruction, and F C common, the two ſides D F and F C of the triangle D F C, are equal to the two ſides E F and F C of the triangle E F C, each to his correſpondent, and the baſe C D equal to the baſe C E, they being both drawn from the center C to the circumference; [b] the angles on both ſides at the point F ſhall be equal; and [c] therefore both the one and the other, are right angles: Therefore C F is a perpendicular drawn to A B: Which was to be done. b) 8. 1. c) 10. def.

PROP. 13. THEOR. 6.

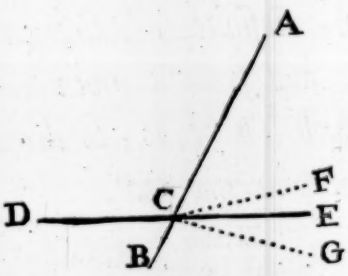


When a right line A B. *falling on a right line* C D, *maketh angles* A B D *and* A B C, *either it makes two right angles, or two angles equal to two right angles.*

Demonſtration FOr if the angle A B D be equal to A B C; they [a] ſhall be both right angles; if it be unequal, [b] let B E be drawn at right angles to C D; therefore E B D and E B C ſhall be right angles; Now foraſmuch as the two angles D B A and A B E [c] are equal to the right angle E B D, all the parts taken together, to their whole, if [d] the common angle E B C be added; then, the three angles D B A, A B E, a) 10. def. b) 11. 1. c) 9. c. ſ. d) 2. c. ſ.

and E B C ſhall be equal to the two right angles D B E and E B C: Again, foraſmuch as the angle A B C is equal to the two angles A B E and E B C, the whole, to all his parts taken together, if you add the common D B A, the two angles D B A and A B C ſhall be equal to the three angles D B A, A B E, and E B C, but thoſe three have been ſhewn to be equal to two right angles; therefore D B A and A B C ſhall be alſo equal to two right angles: Therefore when a right line, &c. Which was to be demonſtrated.

PROP. 14. THEOR. 7.



If to a right line A B, *and at a point therein* C, *there be drawn two right lines* D C *and* E C, *not from the ſame ſide, making the angles* D C A *and* A C E, *on both ſides, equal to two right angles, thoſe right lines* D C *and* C E, *will directly meet with one another.*

Demonſtration. FOr otherwiſe D C being prolonged towards E, would fall either above or below C E; Let it fall firſt of all above C E, (if poſſible,) as in the point F, in ſuch ſort as D C F may be a right line.

Foraſmuch as the right line A C, falling on the right line D C F, makes the two angles D C A and A C F, [a] the ſame angles ſhall be right angles, or equal to two right angles. But the two angles D C A and A C E are equal to two right angles, by Suppoſition; and [b] all the right angles are equal to one another; therefore the two angles D C A and A C F are equal to the two angles D C A and A C E, taking away therefore the common angle D C A, [c] there will remain A C F equal to A C E, the part to its whole [d], which is impoſſible. The ſame abſurdity will happen, if it ſhall be ſaid that D C being prolonged, doth fall below C E, as in the point G; therefore D C and E C ſhall meet in a direct line. Therefore, &c. Which was to be demonſtrated.

a) 13. 1.
b) 10. c. ſ.
c) 3. c. ſ.
d) 9. c. ſ.

COROLLARIE I.

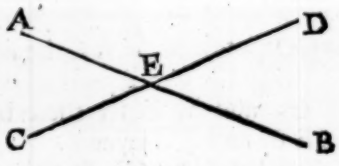
It follows from this Propoſition, that two right lines indirectly meeting with one another, have not one and the ſame common ſegment; that is to ſay, one and the ſame part common as F C *and* E C, *which meet indirectly in the point* C, *the line* D C *cannot be ſaid to be common to* C E *and* C F; *that is to ſay, that* D C *and* C E *making a right line* D C *and* C F *will not alſo make a right line, for otherwiſe, if as well* D C E, *as* D C F, *were a right line; (as we even now demonſtrated,) the angle* H C F *ſhould be equal to the angle* A C E, *the part to the whole, which is impoſſible.*

COROLLARIE II.

It follows yet that two right lines meeting with one another indirectly in a certain point

point, if they be both prolonged, they must of neceſsity cut one another, they being prolonged from the ſame part where they meet, as if the lines EC *and* FC, *which meet in the point* C, *be prolonged, to wit* EC, *it will fall directly in the point* D, *as hath been ſhewn; but if* FC *be alſo prolonged from the part of* C, *it will cut the line* EC, *prolonged at* D; *that is to ſay, it will fall under* CD: *for otherwiſe* FC *ſhould fall on* CD; *and therefore* EC *and* FC *ſhould have one and the ſame common ſegment, which is ſhewn to be impoſſible; and if it ſhould be ſaid that* FC *being prolonged, would fall above* CD, *the ſame abſurdity as before would follow; to wit, that the part ſhould be equal to its whole, which is impoſſible.*

PROP. 15. THEOR. 8.



If two right lines AB *and* CD, *do cut one another in* E, *they ſhall make the oppoſite angles at the head equal to one another,* AED *to* CEB, *and* AEC *to* BED.

Demonſtration. FOraſmuch as DE falls on AB, the two angles AED and DEB ſhall be equal to two right angles; Again, foraſmuch as BE falls on CD, the two angles CEB and BED, [a] ſhall be equal to two right angles, and all the right angles are equal to one another: therefore the two angles AED and DEB, [b] ſhall be equal to the two angles CEB and BED; therefore if the common angle DEB be taken away; [c] there will remaine the angle AED equal to CEB: in like manner it may be ſhewn that the angle AEC is equal to the angle DEB: Therefore, If two lines, &c. Which was to be demonſtrated.

a) 13. 1.
b) 10. c. ſ.
c) 3. c. ſ.

COROLLARIE I.

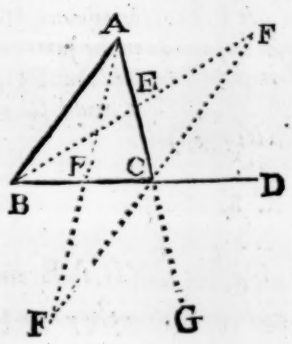
EUCLIDE *gathers from the Demonſtration of this Theorem, (according to the opinion of* Proclus) *that two right lines cutting one another, do make four angles equal to four right angles; for it hath been demonſtrated, that as well the two angles* AED *and* DEB, *as the two angles* AEC *and* CEB, *are equal to two right angles, by the* 13th. *Therefore the angles conſtituted at the point* E, *are equivalent to twice two right angles; and therefore are equal to four right angles.*

COROLLARIE II.

By the ſame reaſon we may gather that all the angles conſtituted at one and the ſame point, are only equal to four right angles: For if from the point E *in the aforegoing Figure, there be drawn as many right lines as you pleaſe, the four angles conſtituted in the point* E, *ſhall be divided into divers parts; the which are equal to their whole, [d] taken together; therefore thoſe four being equal to four right angles, (by the firſt Corollary;) all their parts taken together, ſhall be likewiſe equal to four right angles; from whence it is manifeſt that every Space, conſtituted in a Plain, about ſome point, is equivalent to four right angles, as is manifeſt: foraſmuch as all the angles which may be conſtituted about that point, are equal to four right angles: In like manner, it is manifeſt that any number of lines at pleaſure, mutually cutting one another, will make the angles at the point of Section, equal to four right angles.*

d) Cor. 3. 1.

PROP. 16. THEOR. 9.



Of every triangle, as A B C, *one ſide* B C *being prolonged, as to* D, *the exterior angle* A C D, *is greater than either of the interior and oppoſite angles* C A B *or* C B A.

Demonſtration FOr let the ſide A C be divided in two equal parts [a] in the point E, and by E let there be drawn B E F; and let [b] E F be made equal to E B, and F C joyned.

a) 10. 1.

b) 2. 1.

Foraſmuch as A E is equal to E C, and E F to E B, by Conſtruction, the two ſides A E and E B of the triangle A E B, ſhall be equal to the two ſides C E and E F of the triangle F E C, each to his correſpondent, and the angles at the head E, conteined by thoſe equal ſides, [c] are equal; the baſe A B ſhall be equal to the baſe [d] F C, and the whole triangle to the whole triangle, and the other angles equal to the other angles, each to his correſpondent, conteined by the ſame equal ſides; to wit, E C F equal to E A B: But the whole [e] A C D is greater than E C F, it ſhall be therefore alſo greater than its equal E A B: In like manner, having divided B C into two equal parts in the point E, and drawn A E F, in ſuch ſort as E F may be equal to E A and C F joyned, and the ſide A C prolonged to the point G, it will appear that the exteriour angle B C G is greater than the interiour and oppoſite angle A B C: and therefore A C D which is [f] equal to B C G, ſhall be alſo greater than A B C: Therefore, Of every triangle one ſide, &c. Which was to be demonſtrated.

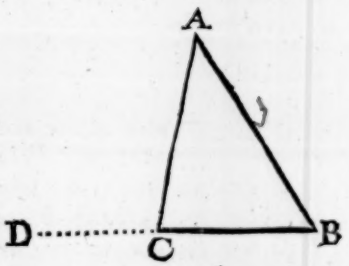
c) 15. 1.

d) 4. 1.

e) 9. c. ſ.

f) 15. pet.

PROP. 17. THEOR. 10.



Of every triangle as A B C, *two angles are leſſe than two right angles, after what manner ſo ever they be taken.*

Demonſtration FOr firſt of all the ſide B C being prolonged to the point D, it will appear that the two angles A B C and A C B, are leſſe than two right angles.

a) 16. pet.

Foraſmuch as the exteriour angle A C D is greater [a] than the oppoſite and interiour angle B, if the common angle A C B be added, [b] the two angles A C D and A C B, will be greater than B and A C B; but A C B and A C D are equal [c] to two right angles: Therefore B and A C B are leſſe than two right angles: In like manner, the ſide C A being prolonged to

b) 4. c. ſ.

c) 13. pet.

to the point E, it will appear that C A B and A B C are leſſe than two right angles, &c. Therefore, Of every triangle, &c. Which was to be demonſtrated.

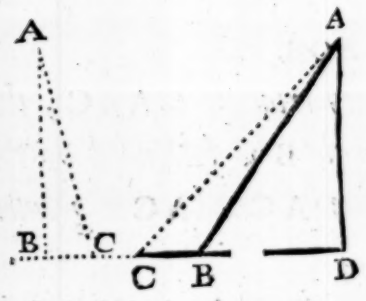
COROLLARIE I.

From theſe Demonſtrations, it follows alſo that every triangle that hath one angle either right or obtuſe, hath the other angles acute: for ſeeing it is apparent that any two angles of every triangle are leſſe than two right angles; it is neceſſary that if one angle be right or obtuſe, that all the other angles be acute, otherwiſe two angles of a triangle, ſhould be right angles, or greater than two right angles.

COROLLARIE II.

It follows alſo from this Propoſition, that if a right line make two unequal angles with another right line, the one obtuſe, the other acute, that the perpendicular line drawn from any point thereof on the other line, will fall on that ſide where the acute angle is.

Let A B with C D make two unequal angles; to wit, A B D acute, and A B C obtuſe, and from any point, as from A, let A D be drawn perpendicular to C B: I ſay that A D will fall on the ſide of the acute angle A B D; Otherwiſe, Let it fall (if poſſible) on the ſide of the obtuſe angle A B C, and let it be A C, therefore the two angles, to wit, the obtuſe A B C, and the right angle A C B of the triangle A B C, ſhall be greater than two right angles, which is abſurd, being they are [d] *leſſe than two right angles; therefore the perpendicular drawn from the point A, on A B, ſhall not fall on the ſide of the obtuſe angle; therefore it ſhall fall on the ſide of the acute angle.*



d) 17. 1.

COROLLARIE III.

Likewiſe it is manifeſt from this Propoſition, that all the angles of an Equilateral triangle, and the two angles on the baſe of an Iſoſceles triangle are acute: For ſeeing that [c] *any two angles of an Equilateral triangle, and the two upper angles on the baſe of an Iſoſceles triangle are equal to one another; and as well the firſt two* [f] *as the laſt two, are leſſe than two right angles; each of them ſhall be leſſe than a right angle that is to ſay, it ſhall be acute, for if one of them were a right or obtuſe angle, the two ſhould be equal to two right angles, or greater than two right angles.*

c) 5. 1.

f) 17. 1.

PROP. 18. THEOR. 11.

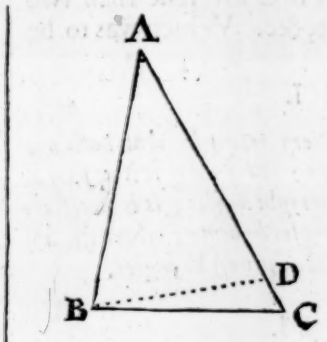
Of every triangle, as A B C, the greateſt ſide A C, ſubtendeth the greateſt angle A B C.

Demonſtration FOr ſeeing that A C is greater than A B, let [a] A D be cut off equal to A B, and let B D be joyned; foraſmuch as A B and A D are equal, the two angles [b] A B D and A D B ſhall be equal; but the exterior angle A D B is greater [c] than the interior and oppoſite angle

a) 3. 1.

b) 5. 1.

c) 16. 1.



angle C: Therefore its equal ABD, ſhall be alſo greater than C: and therefore the whole ABC yet much greater than the angle C. By the ſame reaſon it will appear that the angle C is greater than the angle A: Therefore, Of every triangle, &c. Which was to be demonſtrated.

COROLLARIE.

It follows from this Propoſition, that the three angles of a Scalenum triangle are unequal: For let the triangle ABC *be a Scalenum, whoſe greateſt ſide let be* AC, *the leaſt* BC, *and the mean* AB; *I ſay, that all the angles of that triangle are unequal: For ſeeing that the ſide* AC *is put greater than the ſide* AB, *by this Propoſition the angle* B *ſhall be greater than the angle* C, *by the ſame reaſon, the angle* C *ſhall be greater than the angle* A; *foraſmuch as the ſide* AB, *is put greater than the ſide* BC: *Therefore the three angles are unequal:* B *the greateſt,* C *the mean, and* A *the leaſt.*

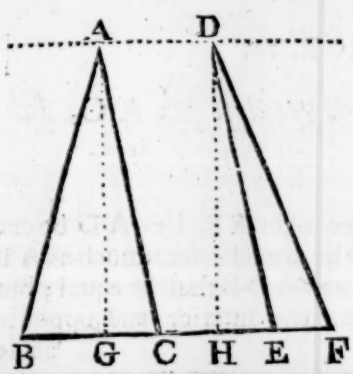
PROP. 19. THEOR. 12.

Of every triangle as ABC, *the greateſt angle* B *is ſubtended by the greateſt ſide* AC, (AC *is greater than* AB.)



Demonſtration FOr if it be denied, it ſhall be equal, or leſſe: if equal, a) 5. 1. the angle B [a] ſhall be equal to the angle C, which is abſurd; for B is put greater than C, if the ſide AC be ſaid to be leſſe than AB; the angle B ſubtended of the leaſt ſide AC, ſhall be leſſe than b) 18. 1. the angle C: ſubtended of the greateſt ſide AB [b], which is yet more abſurd: for the angle B is greater than the angle C, by ſuppoſition: Therefore the ſide AC ſhall be greater than AB: Likewiſe it will appear, that the ſide AC is greater than the ſide BC, putting the angle B greater than the angle A: Therefore, Of every triangle, &c. Which was to be demonſtrated.

COROLLARIE.



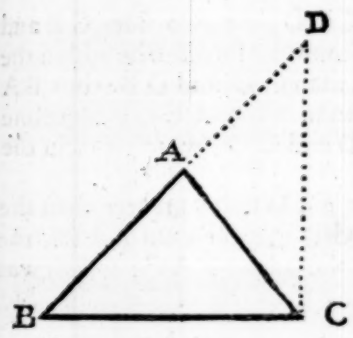
From this Propoſition it follows, that of all the right lines drawn from one and the ſame point, on a right line, that line which is perpendicular, is the leaſt.

For, Let the lines AB, AG, *and* AC, *in the firſt triangle, be drawn from the point* A, *on the line* BC, *of which let* AG *be perpendicular to* BC: *I ſay, that* AG *is the leaſt: For ſeeing that in the triangle* ABG, *the two angles* AGB *and* ABG *are* c) 17. 1. [c] *leſſe than two right angles, and* AGB *is a right angle,*

ABG

A B G *ſhall be leſſe than a right angle; that is to ſay acute; therefore* [d] A B *ſhall be greater than* A G, *and in this manner, it might be demonſtrated that each of the lines in the other triangle; namely, the triangle* D E F, *is greater than the perpendicular* D H: *Therefore the perpendicular* A G *ſhall be the leaſt.* d) 19. 1.

PROP. 20. THEOR. 13.

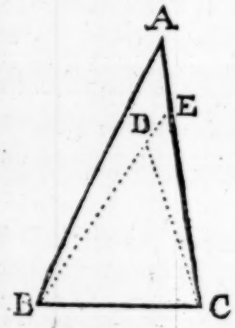


Of every triangle as A B C, *two ſides are greater than the other; after what manner ſoever they be taken (that is,* B A *and* A C *taken together, are greater than* B C; *likewiſe* A C *and* C B *are greater than* A B; *and laſtly,* A B *and* B C *are greater than* A C.)

Demonſtration LEt the ſide B A be prolonged towards D, [a] in ſuch ſort as A D may be put equal to A C; and let C D be drawn: Foraſmuch as the two ſides A D and A C are made equal, the two angles [b] on the baſe C D; to wit D, and A C D, ſhall be equal: but the angle B C D [c] is greater than the angle A C D, it ſhall be therefore alſo greater than D; therefore in the triangle B C D the ſide B D oppoſite [d] to the greateſt angle B C D, ſhall be greater than the ſide B C, oppoſite to the leaſt angle D; but the ſide B D is equal to the two ſides A C and A B: (for ſeeing that A D is equal to A C, if the common ſide [e] A B be added, the whole B A D ſhall be equal to A C and A B:) therefore the two ſides A B and A C together, ſhall be greater than the other ſide C B: In like manner, it may be demonſtrated, that two ſides (which you pleaſe,) of any triangle, are greater than the other ſide: Therefore, Of every triangle, &c. Which was to be demonſtrated.

a) 2. 1.
b) 5. 1.
c) 9. c. ſ.
d) 19. 1.
e) 2. c. ſ.

PROP. 21. THEOR. 14.



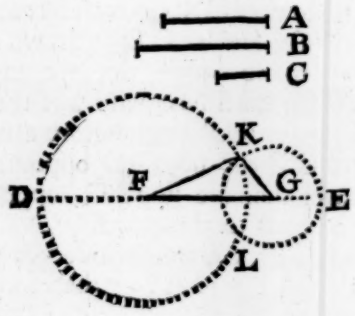
If on one of the ſides B C *of a triangle* A B C, *and from the extremities* B *and* C, *are drawn two right lines* B D *and* C D *within, meeting in a point* D, *thoſe lines* B D *and* C D *ſhall be leſſe than the two other ſides* B A *and* C A *of the triangle* B A C: *But they ſhall contein a greater angle, (the angle* B D C *is greater than the angle* B A C.

Demonſtration FOr let B D be prolonged to the point E of the ſide A C: foraſmuch as in the triangle B A E, the two ſides B A and
a) 20. 1. A E, [a] are greater than the other B E, if the common ſide C E be ad-
b) 4. c. ſ. ded, [b] the ſides B A and A C ſhall be greater than the ſides B E and E C.

Again, foraſmuch as, in the triangle C E D, the two ſides C E and
c) 20. 1. E D are [c] greater than the other C D, if the common ſide D B be added, the
d) 4. c. ſ. two ſides C E and E B, [d] ſhall be greater than C D and D B; but B A and A C have been ſhewn to be greater than B E and E C; therefore B A and A C ſhall be yet greater than B D and C D, which was in the firſt place propoſed.

c) 16. 1. Secondly, foraſmuch as the exterior angle B D C is [c] greater than the interior angle D E C, and the ſame D E C is greater than B A C, the angle B D C ſhall be much greater than the angle B A C: which was in the ſecond place propoſed: Therefore, If on one of the ſides, &c. Which was to be demonſtrated.

PROP. 22. PROBL. 8.



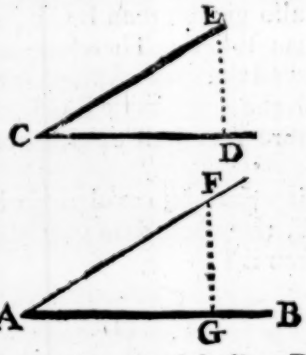
Of three right lines F K, G K, *and* F G, *equal to three given right lines,* A, B, *and* C; *to conſtitute a triangle* F G K: *But two of them, after what manner ſoever they are taken, ought to be greater than the other; foraſmuch as of every triangle, two ſides after what manner ſoever they be taken, are greater than the other ſide.*

Conſtruction LEt there be taken ſome right line, as D E, prolonged infinitely towards E, and of the ſaid line D E. Let there be ta-
a) 3. 1. ken D F equal to [a] A and F G equal to B; and laſtly G E equal to C, and from the point F as a center at the ſpace F D. Let the circle D K be deſcribed againe from the center G, and with the ſpace or diſtance G E, let the circle E K be deſcribed; and let F K and G K be drawn: I ſay, that the triangle F G K is made of three lines equal to the three given lines A, B, and C.

Demonſtration FOraſmuch as the point F is the center of the circle D K,
b) 15. def. 1. the line [b] F K ſhall be equal to F D, by the conſtruction;
c) 1. c. ſ. therefore F K and A ſhall be [c] equal to one another: Again, foraſmuch as G is the center of the circle E K, the line G K ſhall be equal to G E, by the ſame reaſon: But the line C is equal to G E by the conſtruction: therefore G K and C ſhall be equal, and F G is taken equal to B: Therefore the three right lines F K, F G, and G K, are equal to the three given right lines A, B, and C, &c. Which was to be done.

PROP.

PROP. 23. PROBL. 9.

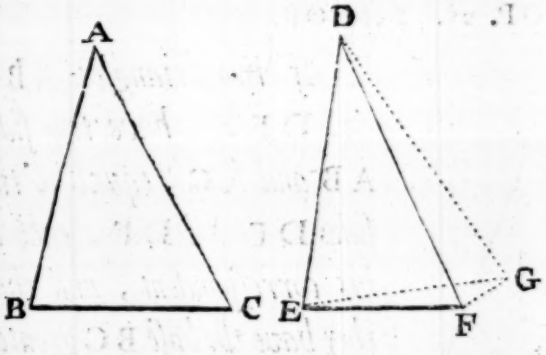


To a given right line A B, *and at a point therein* A, *to conſtitute a right lined angle* F A G, *equal to a given right lined angle* D C E.

Conſtruction LEt the contingent points C and E be taken in both the one and the other lines D C and D E; and let C E be drawn; then of three lines equal to the right lines D C, C E, and E D, [a] let there be conſtituted the triangle A F G; in ſuch ſort, as A F be put equal to D C, F G to D E, and A G to C E. a) 22. 1.

Demonſtration FOraſmuch then as the two ſides A F and A G of the triangle A F G, are equal to the two ſides C D and C E, of the triangle C D E, each to his correſpondent, and the baſe F G equal to the baſe E D; the angle F A G ſhall be [b] equal to the angle C D E: Therefore to a given right line, &c. Which was to be done. b) 8. 1.

PROP. 24. THEOR. 15.

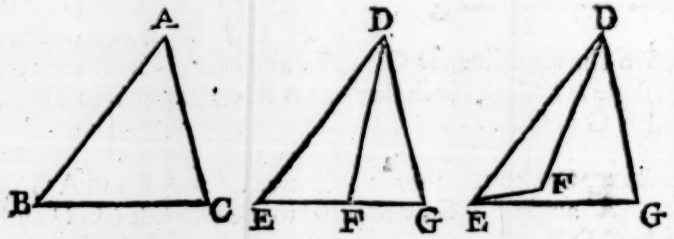


If two triangles A B C, *and* D E F, *have two ſides* A B *and* A C *equal to two ſides* D E *and* D F, *each to his correſpondent, and the angle* A *conteined of thoſe equal ſides* A B *and* A C, *greater than the angle* E D F, *they ſhall have the baſe* B C *greater than the baſe* E F.

Demonſtration FOr ſeeing that the angle A is greater than the angle E D F; at the line E D, and at a point therein D, let there be conſtituted the angle [a] E D G, equal to the angle A; and let D G be taken, equal to one of the two A C or D F; and let E G and F G be drawn: Foraſmuch then as the triangles A B C and E D G have the two ſides A B and A C equal to the two ſides D E and D G each to his correſpondent, and the angle A, equal to the angle E D G, conteined of the ſame equal ſides, [b] the baſe B C ſhall be equal to the baſe E G: Again, a) 23. 1. b) 5. 1.

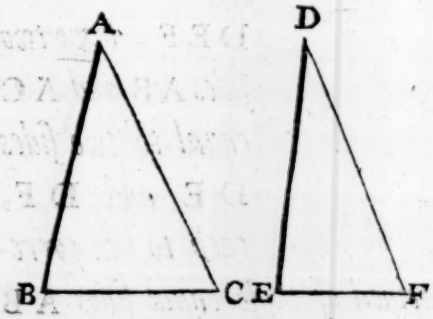
foraſmuch as I D G is equal to D F, the triangle D G F is an Iſoſceles tri-
c) 9. c. ſ. angle: and therefore c the two angles D G F and D F G, on the baſe,
d) 19. 1. ſhall be equal: But the angle D G F is d greater than the angle E G F: therefore D F G equal thereto, ſhall be alſo greater than E G F, and therefore the whole E F G, yet greater than E G F: Therefore in the triangle E F G the ſide E G which doth ſubtend the greateſt angle E F G, ſhall be greater than E F, which ſubtendeth the leſſer angle E G F: But B C is ſhewn to be equal to E G: Therefore B C ſhall be alſo greater than E F.

e) 4. 1. Secondly, Let the baſe E G fall on E F, it e ſhall be equal to the baſe B C, as before; but E G is greater than F E, the whole than part thereof:
f) 9. c. ſ. Therefore B C f ſhall be alſo greater than E F.



g) 21. 1. Laſtly, let E G fall under E F; foraſmuch as the two ſides D G and and E G are g greater than the two ſides D F and F E, and D F is made
h) 5. c. ſ. equal to D G, h taking away the equal ſides D G and D F, there will remaine E F leſſe than E G or B C, which we have ſhewn to be equal to E G: Therefore, If two triangles, &c. Which was to be demonſtrated.

PROP. 25. THEOR. 16.

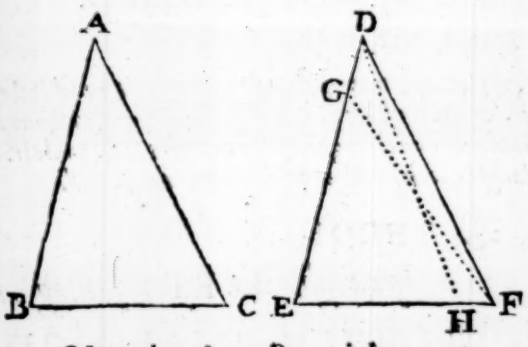


If two triangles A B C *and* D E F, *have two ſides* A B *and* A C, *equal to two ſides* D E *and* D F, *each to his correſpondent, and that they have the baſe* B C *greater than the baſe* E F, *they will have the angle* A, *conteined by the ſame equal right lines* A B *and* A C, *greater than the angle* D, *conteined by the equal right lines* D E *and* D F.

Demonſtration FOr if A were not greater, it ſhould be equal or leſſe, if equal, the ſides A B and A C being equal to the two ſides
a) 4. 1. D E and D F, each to his correſpondent; a the baſe B C ſhould be equal to the baſe E F, which is abſurd, for B C is put greater than E F: If leſſe, A B and A C being equal to D E and E F, each to his correſpondent;
b) 24. 1. B C ſhould be b leſſe than E F, which is more abſurd, for it is greater by

by ſuppoſition: Therefore ſeeing that it cannot be either equal, or leſſe, it ſhall be greater. Therefore, If two triangles, &c. Which was to be demonſtrated.

PROP. 26. THEOR. 17.



If two triangles ABC, *and* DEF, *have two angles* B *and* C, *equal to two angles* E, *and* DFE, *each to his correſpondent, and a ſide equal to a ſide; that is to ſay, either* BC *equal to* EF, *that which is adjacent to thoſe equal angles* B *and* C, *or elſe* AB *and* DE, *that which ſubtendeth one of thoſe equal angles* C *and* DFE, *they will have the other ſides* AC *and* CB, *equal to the other ſides* DF *and* FE, *each to his correſpondent, and the other angle* A, *equal to the other angle* D.

Demonſtration FOr if AB be not equal to DE, one of them ſhall be greater, which let be DE, (if poſſible,) from which let GE be cut off [a] equal to AB, and let GF be drawn; Foraſmuch as, the two ſides AB and BC of the triangle ABC, are equal to the two ſides GE and EF of the triangle GEF, each to his correſpondent, and the angle B equal to the angle E, by ſuppoſition, the baſe [b] GF ſhall be equal to the baſe AC, and all the triangle GEF, equal to all the triangle ABC, and the angle GFE equal to the angle C; but the angle DFE, is equal to the angle C by ſuppoſition; therefore the angle GFE ſhall be in like manner equal to the angle DFE, the part to its whole, which is abſurd: therefore the ſide AB is not equal to DE: Therefore ſeeing that the two ſides AB and BC are equal to the two ſides DE and EF, each to his correſpondent, and the angles B and E conteined of thoſe ſides, alſo equal, the baſe [c] AC ſhall be equal to the baſe DF, and the other angles A and alſo equal, which was propoſed.

a) 3. 1.
b) 4. 1.
c) 4. 1.

Secondly, Let the ſides AB and DE, ſubtending the equal angles C and DFE, be equal; I ſay again, that the other ſides AC and CB are equal to the other ſides DF and FE, each to his correſpondent, BC to EF, and AC to DF, and the other angle A equal to the other angle D; For if BC be not equal to EF: Let one of them, to wit EF, be the greater, from [d] which let EH be taken, equal to BC, and let DH be drawn.

d) 3. 1.

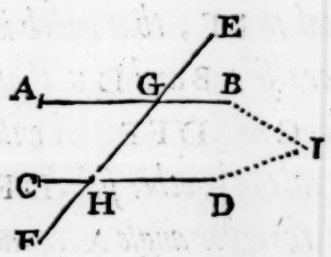
Foraſmuch then as the two ſides AB and BC, are equal to the two ſides DE and EH, and the angles B and E conteined of them, being put equal,

c) 4. 1. equal, the baſe is equal to the baſe, and the angle DHE [c] equal to the angle C: therefore the angle DHE ſhall be equal to the ſame angle DFE, the exterior angle to its interior and oppoſite angle, which is f) 16. 1. [f] abſurd. Therefore BC is not unequal to EF: Therefore by the fourth Propoſition of this Book it may be gathered as before, that the two other ſides, are equal to the two other, and the other angle to the other angle: Therefore, If two triangles, &c. Which was to be demonſtrated.

COROLLARIE.

It is eaſie to gather from this Propoſition, that the whole triangle is equal to the whole triangle, for by conſequence, as the two ſides AB *and* BC *are equal to the two* DE *and* EF, *each to his correſpondent, containing thoſe equal angles, the whole triangle is alſo equal to the whole triangle; which hath been already demonſtrated.*

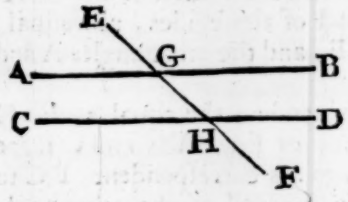
PROP. 27. THEOR. 18.



If a right line EF, *falling on two other right lines* AB *and* CD, *makes the alternate angles* AGH *and* GHD, *equal to one another; thoſe right lines* AB *and* CD, *ſhall be parallel to one another.*

Demonſtration FOr if they be not parallel, they being infinitely prolonged on both parts, they will meet either on the part of AC or of BD, otherwiſe they ſhall be parallels; Let them meet then (if poſſible,) on the part of BD, as in the point I; and ſo GIH ſhall be a triangle, a) 16. 1. whoſe exterior angle [a] AGH ſhall be greater than the interior and oppoſite GHD, which is abſurd; for AGH is equal to GHD, by Suppoſition: Therefore AB and CD ſhall not meet, and b) 35. def. [b] therefore they ſhall be parallels: In like manner, it might be demonſtrated alſo that they can never meet on the part of AC: Therefore, If a right line, &c. Which was to be demonſtrated.

PROP. 28. THEOR. 19.



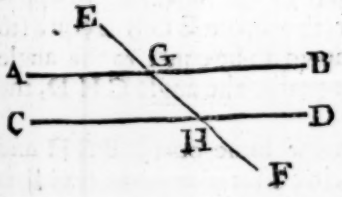
If a right line EF *falling on two right lines* AB *and* CD, *doth make the exterior angle* AGE, *equal to the interior and oppoſite angle* GHC, *and on the ſame part, or the interior angles on the ſame part,* AGH *and* CHG, *equal to two right angles; thoſe right lines* AB *and* CD *ſhall be parallel to one another.*

Demon-

Demonſtration FOraſmuch as the angle G H C is equal to the angle E G A by ſuppoſition; and that the angle B G H [a] is equal to the ſame angle E G A, being both oppoſites at the head G, the two angles B G H and G H C, which are alternate and oppoſite angles, [b] ſhall be alſo equal to one another: Therefore [c] A B and C D ſhall be parallel to one another. a) 15. 1. b) 1. c. ſ. c) 27. 1.

Secondly, If E F falling on A B and C D doe make the interior angles, and on the ſame part; to wit A G H and C H G, equal to two right angles: I ſay, that A B and C D are alſo parallel: For ſeeing that A G E and A G H [d] are equal to two right angles, and that the interior angles A G H and C H G are put equal to two right angles, the angles A G E and A G H ſhall be equal to the two angles A G H and C H G, taking away therefore the common angle A G H, there will remaine the exterior angle, equal to the interior and oppoſite angle on the ſame part G H C: Therefore A B and C D ſhall be parallels by the firſt part of this Propoſition: Therefore, If a right line, &c. Which was to be demonſtrated. d) 13. 1. 3. c. ſ.

PROP. 29. THEOR. 20.



If a right line E F *doth fall on two parallel right lines* A B *and* C D, *it will make the alternate angles* A G H *and* D H G *equal to one another, and the exterior angle* E G B, *equal to its interior and oppoſite angle* G H D, *and on the ſame part; and the two interior angles* B G H *and* G H D, *and on the ſame part, equal to two right angles.*

Demonſtration FOr if they be not equal, let the one of them, to wit A G H, be the greater, foraſmuch then as the angle A G H is greater than the angle D H G; if the common angle B G H be added, [a] the two angles A G H and B G H, ſhall be greater than the two angles B G H and D H G; but the two angles A G H and B G H [b] are equal to two right angles: Therefore B G H and D H G ſhall be leſſe than two right angles; therefore A B and C D ſhall not be parallels, [c] which is abſurd: for that they are put parallels: Therefore the angle A G H is not greater than the angle D H G, and therefore is equal thereto; if it were ſaid to be leſſe, the ſame abſurdity would happen. a) 4. c. ſ. b) 13. 1. c) 11. c. ſ.

Secondly, I ſay that the exterior angle E G B is equal to its interior and oppoſite angle, and on the ſame part G H D: Foraſmuch as the angle G H D is ſhewn to be equal to the angle A G H, and that the angle E G B is [d] equal to the ſame angle A G H, the angles E G B and G H D ſhall [e] be equal to one another; and ſo it may be ſhewn that E G A is equal to G H C. d) 15. 1. e) 1. c. ſ.

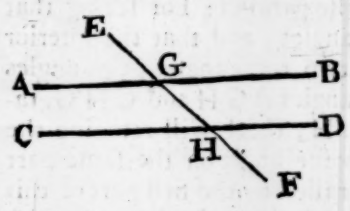
Thirdly, I ſay that the two interior angles, and on the ſame part, B G H and G H D, are equal to two right angles.

For

For ſeeing that the exterior angle E G B is ſhewn to be equal to G H D, if the common angle B G H be added; f the two angles B G H and G H D ſhall be equal to the two angles E G B and B G H; g but E G B and B G H are equal to two right angles: Therefore B G H and G H D ſhall be alſo equal to two right angles, by the ſame reaſon, the two angles A G H and G H C will appear to be equal to two right angles: Therefore, &c. Which was to be demonſtrated.

f) 3. c. ſ.
g) 13. per.

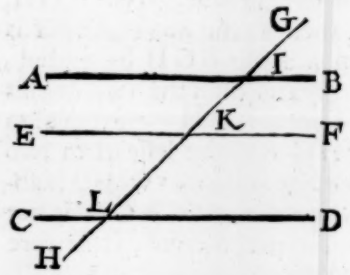
Otherwiſe, Let A B and C D be parallels: I ſay that the alternate angles A G H and H G D are equal to one another: If it be not ſo, the one of them ſhall be the greater: Let then A G H be the greater (if poſſible.) Wherefore adding the angle H G B, the two angles H G B and D H G ſhall be leſſe than the two angles H G B and A G H; but H G B and A G H are equivalent to two right angles; therefore H G B and D H G ſhall be leſſe than two right angles; therefore A B and C D ſhall not be parallel, which is contrary to Suppoſition: Therefore the angle A G H is equal to the angle G H D.



Secondly, The exterior angle E G B is equal to the interior and oppoſite angle, and of the ſame part G H D; for the angle E G B is equal to the angle A G H, and the angle G H D is ſhewn to be equal to the angle A G H: Therefore the angle E G B ſhall be equal to the angle G H D, the exterior to the interior angle.

Laſtly, The two interior angles, and on the ſame part, B G H and G H D are equal to two right angles: For ſeeing that the angle E G B is ſhewn to be equal to the angle G H D, if the angle B G H be added to each of them; the two angles G H D and B G D ſhall be equal to the two angles E G B and B G H: But the angles E G B and B G H are equal to two right angles; therefore the angles B G H and D H G ſhall be alſo equal to two right angles: Which was to be demonſtrated.

PROP. 30. THEOR. 21.



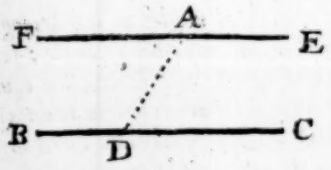
The right lines A B *and* C D, *parallel to one and the ſame right line* E F, *are alſo parallel to one another.*

Demonſtration LEt G H be drawn, cutting the ſame right lines in the points I, K, and L; foraſmuch as the right line G H doth fall on the parallels A B and E F, the angle A I K ſhall be equal a to the alternate angle F K I; Again, foraſmuch as G H falls on the parallels C D and E F, the angle D L K ſhall be equal to the angle I K F, the interior, to the exterior angle, or the exterior to the interior and oppoſite on the ſame part But A I K is equal to the ſame I K F; therefore A I K b and D L K ſhall be equal to one another, which being alternate angles, c the lines A B and C D ſhall be parallel to one another: Therefore, The right lines, &c. Which was to be demonſtrated.

a) 29. 1.
b) 1. c. ſ.
c) 27. 1.

PROP.

PROP. 31. PROBL 10.

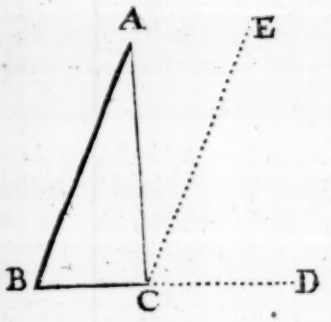


From a given point A, *to draw a right line parallel to a given right line* BC.

Construction LEt the contingent point D be taken in the given line BC; and let AD be drawn, making any angle at pleasure with BC, as the angle ADC, and in the point A of the given right line AD, let there be constituted the [a] right lined angle FAD, equal to the given right lined angle ADC, and let FA be prolonged directly towards E. [a] 23. 1.

Demonstration FOrasmuch as the two alternate angles FAD and CDA are equal by construction, the line AF [b] shall be parallel to BC, and drawn from the given point A. Which was to be done. [b] 27. 1.

PROP. 32. THEOR. 22.



Of every triangle, as ABC, *one of the sides* BC *being prolonged, the exterior angle* ACD *is equal to the two interior and opposite angles* A *and* B, *and the three interior angles* A, B, *and* ACB, *of any triangle, are equal to two right angles.*

Construction FOr from the point C let CE be drawn parallel to AB.

Demonstration FOrasmuch as AC doth fall on the parallels AB and CE, the alternate angles [a] A and ACE are equal; Again, forasmuch as BD falls on the Parallels AB and CE, the angles B and ECD shall be [b] equal, the exterior angle to the interior and opposite angle on the same part: Therefore the two angles ACE and ECD; that is to say, the whole ACD shall be equal to the two angles A and B, the exterior angle to the two interior and opposite angles; Which was proposed. [a] 29. 1. [b] 29. 1.

Secondly, I say that the three interior angles of the same triangle ABC; to wit, A, B, and ACB, are equal to two right angles.

Forasmuch as A and B are equal to the angle ACD, as hath been shewn, if the common angle ACB be added, the three angles [c] A, B, and ACB, shall be equal to the two angles ACB and ACD; but the two angles ACB and ACD are equal to two [d] right angles; therefore [c] 2. c. 5. [d] 13. 1.

the three angles A, B, and A C B are equal to two right angles: Therefore of every triangle, &c. Which was to be demonſtrated.

COROLLARIE I.

From this 32 Propoſition may be gathered that the three angles of any triangle taken together, are equal to three angles taken together, of any other triangle: foraſmuch as the three angles of the one, as the three angles of the other, are equal c) 32. 1. *to two right angles. Therefore if two angles of one triangle are equal to two angles of another triangle, the third angle of the one ſhall be alſo equal to the third angle of the other. Therefore if two angles of one triangle are equal to two angles of another triangle, each to his correſpondent, the other angle of the one ſhall be equal to the other angle of the other, and thoſe triangles ſhall be equiangled.*

COROLLARIE II.

It is alſo evident, that in every Iſoſceles triangle, whoſe angle conteined of the equal ſides is a right angle, that each of the other angles which are on the baſe is the f) 32. 1. *half of a right angle, for both of them together do conſtitute a right angle:* f *Seeing the three angles are equal to two right angles, and that the third angle is put a right angle; therefore ſeeing that the two remaining angles are equal to one another, each of them ſhall be the half of a right angle: But if the angle conteined of the equal ſides be obtuſe, each of the other angles ſhall be leſſe than half of a right angle, and the two angles together, leſſe than a right angle; If laſtly, the above mentioned angle be acute, each of the other angles ſhall be greater than half a right angle: foraſmuch as the two together, are greater than a right angle.*

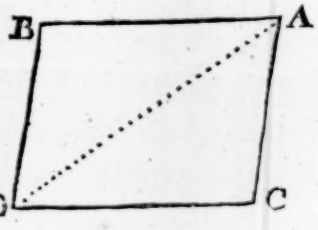
COROLLARIE. III.

It is likewiſe manifeſt, that each angle of an equilateral triangle, is the two g) 32. 1. *third parts of a right angle, for the two right angles,* g *to which the three angles of an equilateral triangle are equal, being divided into three angles, do make each angle the two third parts of a right angle.*

COROLLARIE IV.

It is alſo evident, that if a perpendicular line be drawn from one of the angles of an equilateral triangle on the oppoſite ſide there will be conſtituted two Scalene triangles, each of which ſhall have the angle adjacent to the perpendicular a right angle, another ſhall be the two thirds of a right angle; to wit, that which is alſo an angle of the equilateral triangle: Laſtly, the other angle ſhall be the third part of a right angle.

PROP. 33. THEOR. 23.



The right lines B D *and* A C, *which joyn together two right lines* B A *and* D C *equal and parallel, and on the ſame part, the ſame right lines* B D *and* A C *are in like manner equal and parallel.*

De-

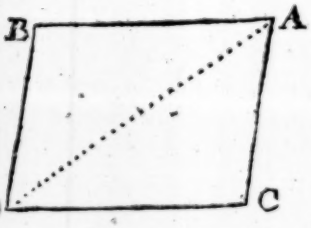
Demonſtration FOr let A D be drawn, foraſmuch as A D falls on the parallels B A and C D, [a] the alternate angles B A D and A D C ſhall be equal to one another, and foraſmuch as B A is equal to D C by ſupooſition, and A D common, the two ſides B A and A D of the triangle A B D ſhall be equal to the two ſides D C and D A of the triangle D C A, each to his correſpondent, and the angle B A D equal to the angle C D A conteined of thoſe equal ſides, the baſe [b] B D ſhall be equal to the baſe A C, and the whole triangle A B D ſhall be equal to the wholle triangle D C A, and the other angles equal to the other angles, each to its correſpondent; to wit, C equal to B, and D A C equal to B D A, which being alternate angles, [c] the lines B D and A C ſhall be parallel: But they are alſo ſhewn to be equal: Therefore they ſhall be equal and parallel: Therefore, &c. Which was to be demonſtrated.

a) 29. 1.

b) 4. 1.

c) 27. 1.

PROP. 34. THEOR. 24.



In Parallelograms the oppoſite ſides A B *and* C D, *and the oppoſite angles* D B A *and* A C B, *are equal to one another, and their Diameter* A D *doth divide them into two equal parts.*

Demonſtration FOr ſeeing that B A and D C are parallels, and that A D falleth on them, [a] the alternate angles B A D and A D C are equal to one another; Again, ſeeing that B D and A C are parallels, the alternate angles B D A and C A D are alſo equal: Therefore ſeeing that the two angles B A D and B D A of the triangle A B D, are equal to the two angles C A D and C D A of the triangle A C D, each to his correſpondent, and the ſide adjacent to thoſe equal angles, common to both the triangles, [b] the two ſides B D and B A, ſhall be equal to the two other ſides D C and A C, each to his correſpondent: to wit, B A equal to its oppoſite D C, and B D equal to its oppoſite A C; and the angle B equal to the angle C: And foraſmuch as the two angles at the point A are ſhewn to be equal to the two angles at the point D, the whole angle A ſhall be equal to its oppoſite and whole angle D: Therefore the ſides and the angles oppoſite to one another, are equal.

a) 29. 1.

b) 26. 1.

But foraſmuch as in the triangle A B D and D C A, the two ſides B A and B D are equal to the two ſides C D and C A, each to its correſpondent, and the angles B and C, conteined of thoſe ſides, are equal, as hath been ſhewn, [c] the whole triangle B A D ſhall be equal to the whole triangle A C D: Therefore the Diameter doth divide the Parallelogram into two equal parts: Therefore the ſides and the angles, &c. Which was to be demonſtrated.

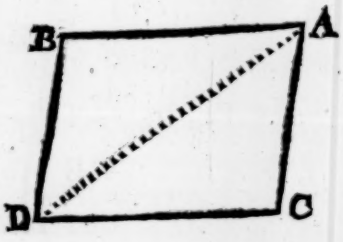
c) 4. 1.

COROLLARIE. I.

From this Propoſition may be in the firſt place gathered, that every quadrilateral or four ſided figure, which hath the oppoſite ſides equal, is a Parallelogram; for the two ſides B D *and* B A *being equal to* A C *and* C D, *each to his correſpondent, and the baſe* A D *common, the angles* B *and* C *ſhall be equal, and the two other angles alſo ſhall be equal to the two other angles, each to his correſpondent; ſo all the alternate angles which are at the points* A *and* D, *ſhall be equal to one another: Therefore the figure ſhall be a Parallelogram, all the ſides being Parallels.*

COROLLARIE II.

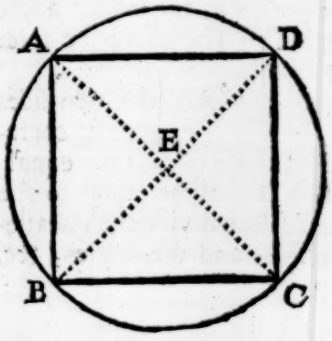
It follows alſo, that every four ſided figure which hath the oppoſite angles equal, is a Parallelogram, as in the former figure, the oppoſite angles being equal to wit, A *to* D, *and* B *to* C: *And by what hath been ſhewn in the* 32 *Propoſition, the four angles being equal to four right angles, the two angles* B *and* B A C *ſhall be equal to two right angles: In like manner* B *and* B D C *ſhall be alſo equal to two right angles: therefore as well* B A *and* C D *as* B D *and* A C *ſhall be parallels; and therefore the figure propoſed is a Parallelogram, and the ſame would happen if all the angles were right angles.*



We ſhall alſo ſay, that although the Diagonal line divideth every Parallelogram into two equal triangles, it doth not follow, that every figure of four ſides which divideth it ſelf into two equal triangles by the Diagonal, is a Parallelogram, as is eaſie to be underſtood in drawing from the point B *towards* A, *a line equal to* A C, *and from the point* D *a line equal to* B A, *on the ſame part, which will meet with one another at another point, as is manifeſt by what hath been heretofore demonſtrated; the facility whereof requireth no figure.*

COROLLARIE III.

It follows alſo that in the long Square and Rhomboides (for there are but four ſorts of Parallelograms, as we have already ſaid) the Diagonal line doth cut the angles unequally, becauſe of the inequality of the ſides in each triangle. See the former figure.



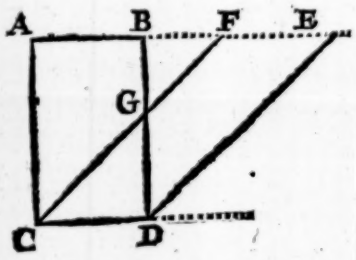
But in the Square and the Rhombe, the ſaid Diagonal line doth cut the angles into two equal parts; foraſmuch as the two ſides of one of the triangles are alſo equal to the two ſides of the other triangle, each to his correſpondent, and the angle conteined of them equal to the angle, and as appears by this ſquare A B C D, *in which the Diagonal lines* A C *and* B D *are alſo equal, and do cut one the other into two equal parts in the point* E, *becauſe of the equality of all the angles which are in the points* A, B, C, D, *and in the point* E.

But if the figure A B C D *were a Rhombe,*

the

the two angles oppoſite ſhould be obtuſe, and the two others acute: Wherefore the Diagonal lines would be unequal; and yet nevertheleſſe would cut each other in two equal parts, as is eaſie to be underſtood by what hath already been demonſtrated, and alſo in the long Square and Rhomboides, and all figures of four ſides, in which the Diagonal lines do cut one another into two equal parts, is a Parallelogram; All which things are ſo eaſily demonſtrated, if the Propoſitions and Demonſtrations before mentioned be underſtood, as there was ſmall need of making any mention of them here.

PROP. 35. THEOR. 25.

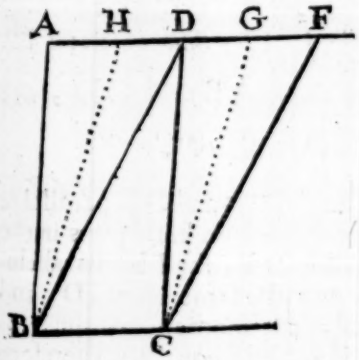


The Parallelograms A C D B *and* F C D E, *conſtituted on one and the ſame baſe* C D, *and between the ſame parallels* A B *and* C D *are equal to one another.*

Demonſtration FOraſmuch as A C D B is a parallelogram, A B is [a] equal to C D, by the ſame reaſon F E is equal to C D: [b] therefore A B and F E are equal to one another; therefore if the common line B F be added, [c] the whole A F ſhall be equal to the whole B E; [d] but A C is alſo equal to B D; therefore the two ſides F A and A C of the triangle F A C ſhall be equal to the two ſides E B and B D of the triangle B E D, each to his correſpondent, and the angles A and B conteined of thoſe ſides, [e] are equal, the exterior angle to the interior angle, and contrarily: therefore the baſe [f] F C ſhall be equal to the baſe E D, and the whole triangle F A C to the whole triangle E B D, from which the common triangle B G F being taken away, [g] the trapezium A B G C will remain equal to the Trapezium E F G D; to which Trapeziums if you again adde the common triangle C D G, the whole Parallelogram A C D B ſhall be equal to the whole Parallelogram E D C F: Therefore the Parallelograms conſtituted, &c. Which was to be demonſtrated.

a) 34. 1.
b) 1. c. ſ.
c) 2. c. ſ.
d) 34. 1.
e) 29. 1.
f) 4. 1.
g) 2. c. ſ.

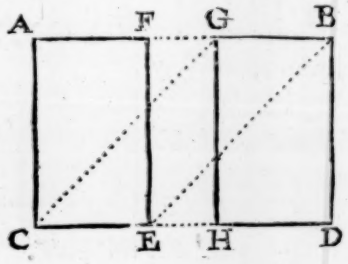
But if the Parallelograms conſtituted on one and the ſame baſe B C, were A B C D and D B C F, as in this ſecond figure; as well the ſide A D as D F, ſhould be equal to B C, as aforeſaid, and A B to D C, and the exterior angle F D C, to the interior and oppoſite angle A: Wherefore the two ſides F D and D C being equal to D A and A B, each to his correſpondent, and the angle F D C equal to the angle A, the whole triangle F D C ſhall be equal to the whole triangle D A B. Therefore if to both, there be added the common triangle B C D, the Parallelogram A B C D ſhall be equal to the Parallelogram D B C F.



And laſtly, if the Parallelograms conſtituted on the baſe B C, were A B C D and H B C G, as well A D as H G would yet be equal to B C, and

and therefore equal to one another: Therefore taking away the common line H D, there would remaine D G, equal to A H, and as before, the triangle G D C ſhall be equal to the triangle H A B: Therefore if to each of them there be added the common Trapezium H B C D, the whole Parallelogram A B C D ſhall be equal to the whole Parallelogram H B C G. Which was to be demonſtrated.

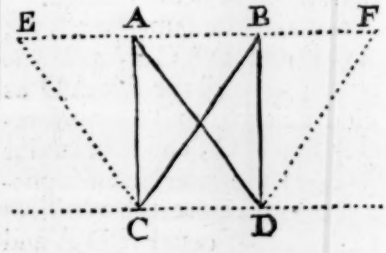
PROP. 36. THEOR. 26.



The Parallelograms A C E F *and* B G H D, *conſtituted on equal baſes* C E *and* H D, *and between the ſame parallels* A B *and* C D *are equal to one another.*

Demonſtration FOr let the right lines C G and E B be drawn; foraſmuch
a) 34. 1. as G B is [a] equal to H D, and C E is put equal to the ſame
b) 1. c. ſ. H D the two ſides, [b] G B and C E ſhall be equal to one another, the
which being parallels by ſuppoſition, the lines C G and E B which joyn
c) 33. 1. them, [c] ſhall be alſo equal and parallel, and B G and C E ſhall be a Parallelogram: Therefore the Parallelogram A C E F ſhall be equal to the
d) 35. 1. Parallelogram G C E B, being [d] on the ſame baſe C E, and between the ſame Parallels, by the ſame reaſons, the Parallelogram G H D B ſhall be equal to the ſame Parallelogram G C E B, being on the ſame baſe G B, and between the ſame Parallels: Therefore the Parallelograms A C E F,
e) 1. c. ſ. and D B G H [e] are equal to one another: Therefore the Parallelograms, &c. Which was to be demonſtrated.

PROP. 37. THEOR. 27.

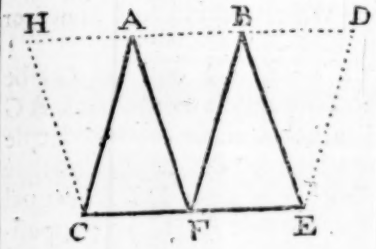


The triangles A C D *and* B C D, *conſtituted on one and the ſame baſe* C D, *and between the ſame parallels* A B *and* C D, *are equal to one another.*

a) 13. 1. *Demonſtration* LEt C E be drawn parallel to A D, [a] and D F parallel to C B, meeting with A B, prolonged on both parts in the points E and F, the figures A D C E and B C D F, ſhall be Parallelograms, the which being conſtituted on one and the ſame baſe C D, and
b) 35. 1. between the ſame Parallels, [b] ſhall be equal to one another; but the Diameters A C and B D doth divide them into [c] two equally: Therefore
c) 34. 1. their halves (which are the triangles propoſed A C D and B C D) [d] ſhall
d) 7. c. ſ. be alſo equal to one another: Therefore the triangles, &c. Which was to be demonſtrated.

PROP.

PROP. 38. THEOR. 28.

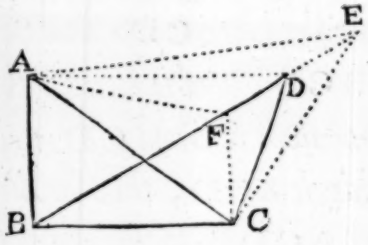


The triangles A C F *and* B E F, *conſtituted on equal baſes* C F *and* F E, *and between the ſame Parallels* A B *and* C E *are equal to one another.*

Demonſtration. FOr let C H and E D be drawn [a] parallel to F A and F B, meeting A B prolonged on both parts in the points H and D, A F C H and B F E D ſhall be Parallelograms, and [b] equal to one another, of which the triangles propoſed A C F and B F E are [c] the halves: Therefore thoſe triangles ſhall [d] be alſo equal to one another: Therefore the triangles, &c. Which was to be demonſtrated.

a) 31. 1.
b) 36. 1.
c) 34. 2.
d) 7. c. ſ.

PROP. 39. THEOR. 29.



The equal triangles A B C *and* D B C, *conſtituted on one and the ſame baſe* B C, *and on the ſame part* A D, *are alſo between the ſame parallels; that is to ſay, that the line drawn from* A *to* D, *is parallel to* B C.

Demonſtration. FOr if A D were not parallel to B C, there might be drawn from the point A a parallel to the ſame B C, which would paſſe above A D, or under it: Let therefore [a] A F be drawn as a parallel to B C, if A D be not a parallel; and let C F be joyned: Foraſmuch as A F and B C are parallels, the triangle F B C ſhall be equal to the triangle [b] A B C; but the triangle D C B is equal to the ſame triangle A B C by Suppoſition: therefore the triangles [c] F B C and D C B ſhall be equal to one another, the part to the whole, which is abſurd.

If it be ſaid that the line parallel to B C drawn from the point A towards D, ſhall paſſe above A D, the ſame abſurdity will happen; therefore A D is parallel to B C: Therefore the equal triangles, &c. Which was to be demonſtrated.

a) 31. 1.
b) 37. 1.
c) 1. c. ſ.

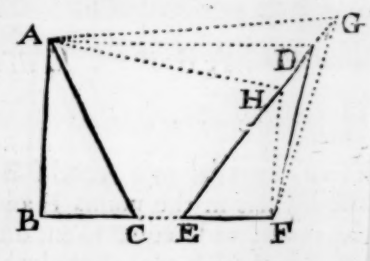
PROP. 40. THEOR. 30.

The equal triangles A B C *and* D E F, *conſtituted on equal baſes* B C *and* E F, *and on the ſame part* A D, *are alſo between the ſame parallels; that is to ſay, that the line drawn from the point* A *to* D, *is parallel to* B F.

Demon-

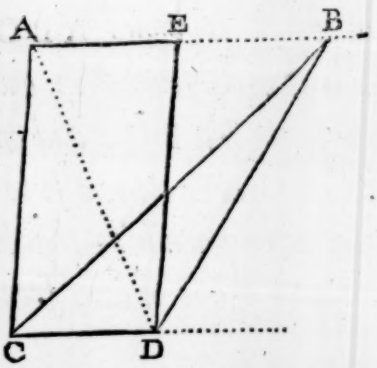
Demonſtration FOr otherwiſe there might be drawn from the point A
a) 31. 1. a line parallel to [a] BF; which ſhould fall above or under AD.

Let AG be drawn parallel to BF, (if poſſible,) and let GF be joyned: Foraſmuch as the lines AG and BF are parallels, the triangle GEF ſhall be equal to the triangle ABC, but the triangle DFE is equal to the ſame triangle ABC by Suppoſition: Therefore the triangles DFE and [b] GEF ſhall be equal to one another, the part to the whole, which is abſurd if the parallel be ſaid to paſſe below AD, the ſame abſurdity will happen: Therefore AD is parallel to BF: Therefore, The equal triangles, &c. Which was to be demonſtrated.



b) 1. c. ſ.

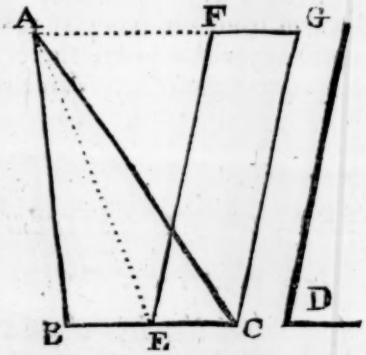
PROP. 41. THEOR. 31.



If a Parallelogram ACDE *hath the ſame baſe* CD, *as a triangle* BCD, *and is between the ſame parallels* AB *and* CD, *as is the triangle* BCD, *the Parallelogram* ACDE *ſhall be double to the triangle* BCD.

Demonſtration FOr having drawn the Diameter AD, the triangle ACD
a) 37. 1. [a] ſhall be equal to the triangle BCD: But the Parallelo-
b) 34. 1. gram ACDE is double to the triangle ACD: foraſmuch [b] as the triangles ACD and AED are equal: Therefore the ſame Parallelogram ACDE is alſo double to the triangle BCD: Therefore, If a Parallelogram, &c. Which was to be demonſtrated.

PROP. 42. PROBL. 11.



To conſtitute a Parallelogram EFCG *equal to a given triangle* ABC, *in a given right lined angle* D.

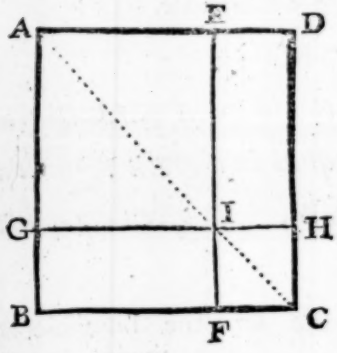
THat is to ſay, that hath one angle equal to a given right lined angle.

Conſtruction LEt one of the ſides of the triangle ABC, (which
a) 10. 1. you pleaſe) be divided; (to wit BC,) into two [a] equal parts in the point E,

E, and let A E be joyned; and let the angle C E F be made equal to the given angle D; then let [b] A F be drawn parallel to B C, meeting E F in the point F; alſo let C G be drawn from the point C, parallel to E F, meeting A F prolonged in the point G, ſo ſhall the Parallelogram F E C G be conſtituted in the angle C E F, which is equal to the given angle D, which ſaid Parallelogram I ſay is equal to the given triangle A B C. b) 31. 1.

Demonſtration FOraſmuch as the triangles A B E and A E C are equal; [c] being conſtituted on the equal baſes B E and E C, and between the ſame parallels A F and B C, the whole triangle A B C ſhall be double to the triangle A E C: But the Parallelogram F E C G is [d] alſo double to the ſame triangle A E C: Therefore the triangle A B C [e] and the Parallelogram F C ſhall be equal to one another, and the parallelogram F C ſhall have the angle C E F equal to the given angle D, by conſtruction: Therefore we have conſtituted, &c. Which was to be done. c) 38. 1. d) 41. 1. e) 6. c. ſ.

PROP. 43. THEOR. 32.



Of every parallelogram, as A B C D, *the complements* D I *and* I B *of the parallelograms* G E *and* F H, *which are about the Diameter* A C, *are equal to one another.*

Demonſtration FOr ſeeing that the triangles A B C and A D C [a] are equal to one another (for the Diameter A C divideth the parallelogram into two equal parts, and that the two triangles A G I and A E I are alſo equal to one another, by the ſame reaſon, if thoſe two later be cut off from the two firſt, [b] the Trapeziums G B F C I and E D H C I, ſhall remain equal to one another; But the triangles F I C and H I C are [c] alſo equal to one another; therefore if theſe triangles be taken from the Trapeziums, the complements [d] D I and I B, will remain equal: Therefore, Of every, &c. Which was to be demonſtrated. a) 34. 1. b) 3. c. ſ. c) 34. 1. d) 3. c. ſ.

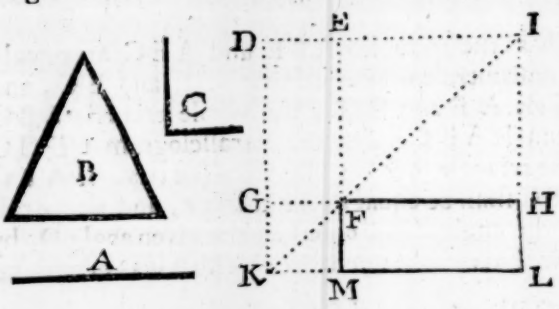
PROP. 44. PROBL. 12.

On a given right line A, *to apply a parallelogram,* L M F H, *equal to a given triangle* B, *in a given right lined angle* C.

Conſtruction LEt the parallelogram D E F G be conſtituted, [a] equal to the given triangle B, having the angle G F E, equal to C; then let G F be prolonged to the point H, in ſuch ſort as F H may be equal to A, and by H let there be drawn H I, [b] parallel to F E, meeting D E prolonged in the point I; after that let the Diameter I F be drawn, by the point F, meeting D G prolonged in K, and by K let K L be drawn, a) 42. 1. b) 31. 1.

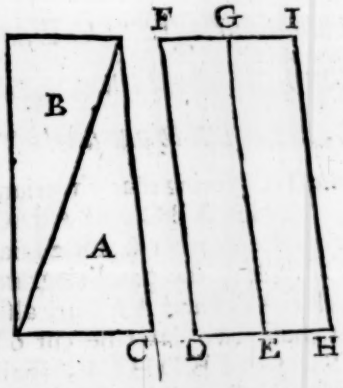
c) 31. 1. parallel c to G H, meeting I H prolonged in L, and let E F be prolonged to the point M: I ſay, that the Parallelogram L M F H is the Parallelogram required.

d) 15. 1. *Demonſtration* FOr it hath the ſide F H equal to the given line A, by the conſtruction, and the angle H F M is d equal to the angle E F G, the which is made equal to the angle C: Therefore H F M ſhall be alſo equal to C, and laſtly, the parallelogram F L is equal to the parallelogram G E, the complement equal to the complement, which ſaid parallelogram G E is made equal to the triangle B: Therefore F L ſhall be alſo equal to B: Therefore on a given, &c. Which was to be done.



PROP. 45. PROBL. 13.

To conſtitute a parallelogram D I, *equal to a given right lined figure* A B, *in a given right lined angle* C.



a) 42. 1. *Conſtruction* LEt the Parallelogram D G be made a equal to the triangle A, having the angle F D E equal to the given right lined angle C; then to the line G E, let the parallelogram E I be applyed, equal b to the triangle B, having the angle G E H equal to the angle C; and ſo what was required is done.

b) 44. 1.

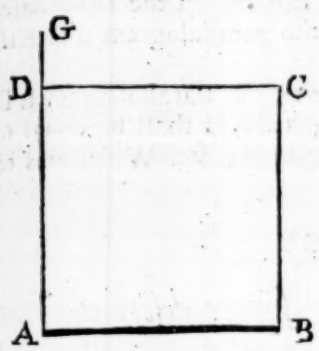
c) 1. c. ſ. d) 2. c. ſ. *Demonſtration* FOraſmuch as each of the angles F D E and G E H, is made equal to the angle C, c they ſhall be equal to one another; therefore if the angle G E D be added; d the two angles G E D and G E H ſhall be equal to the two angles G E D and F D E; but F D E and G E D are equal to two right angles; therefore G E D and G E H ſhall be alſo equal to two right angles; Therefore e D E H ſhall be a right line: By the ſame reaſons it may be proved that F G and G I do make a right line. Now ſeeing that each of theſe lines D F and H I are equal and parallel f to E G, they ſhall be equal and parallel to one another, and the right lines F I and D H which joyn them together, g ſhall be alſo equal and parallel: Therefore D I ſhall be a parallelogram, made of the two parallelograms D G and E I, equal to the two triangles A and B: Therefore the parallelogram D I is equal to the right lined figure A B, having the angle D equal to the given right lined angle C: Which was to be done.

e) 14. 1. f) 34. 1. g) 30. 1.

PROP.

PROP. 46. PROBL. 14.

On a given right line AB, *to deſcribe a Square* ABCD.



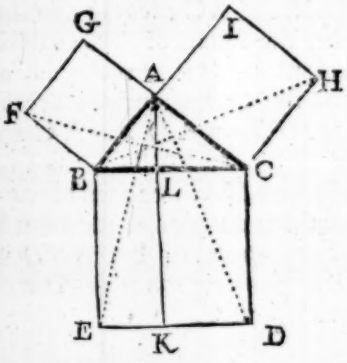
Conſtruction LEt the right line AG be drawn [a] at right angles to AB, and from the point D let [b] DC be drawn parallel to AB, likewiſe let BC be drawn parallel to AD, meeting DC in the point C, the figure ABCD ſhall be a Parallelogram, and therefore the ſide DC [c] is equal to AB, and AD equal to BC: But AD is put equal to AB; therefore BC [d] ſhall be alſo equal to AB; therefore ABCD ſhall be equilateral: I ſay it is alſo a Rectangle.

a) 11. 1.
b) 31. 1.
c) 34. 1.
d) 1. c. ſ.

Demonſtration FOraſmuch as AD and BC are parallel, the [e] angles A and B are equal to two right angles, but A is made a right angle, therefore B ſhall be alſo a right angle, and the two oppoſite angles D and C [f] are equal to them, and therefore right angles: Therefore the Parallelogram AC is rectangled, and equilateral, and therefore ſhall be a Square: Therefore, On a right line, &c. Which was to be done.

e) 28. 1.
f) 34. 1.

PROP. 47. THEOR. 33.



In right angled triangles as ABC, *the ſquare* BD *of the ſide* BC, *which ſubtendeth the right angle* BAC, *is equal to the ſquares* AF *and* CI, *of the ſides* AB *and* AC, *which do contein the ſame right angle* BAC.

Demonſtration LEt AK be drawn [a] parallel to BE or to CD, cutting BC in the point L, and let the right lines AD, AE, CF, and BH be joyned: Foraſmuch as the angles BAC and BAG are right angles, [b] GA and AC ſhall make one only right line, by the ſame reaſon BA and AI ſhall alſo make one right line.

a) 31. 1.
b) 14. 1.

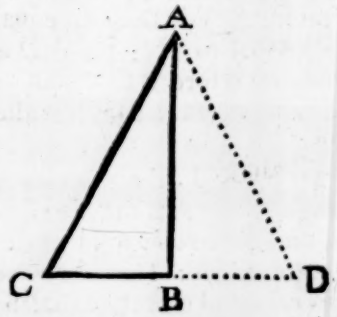
Again, Foraſmuch as the angles ABF and CBE are equal (as being right angles, if the common angle ABC be added, the whole ABE ſhall be [c] equal to the whole CBF; likewiſe the whole ACD ſhall be equal to the whole BCH: Foraſmuch therefore as the two ſides AB and BE of the triangle ABE, are equal to the two ſides CB and BF of the triangle CBF, each to his correſpondent, (as appears by the Definition of Squares,) and the angles ABE and CBF conteined by thoſe ſides are ſhewn to be equal: [d] Thoſe triangles ABE and CBF ſhall be equal; but the Parallelogram AF is [e] double to the triangle CBF,

c) 2. c. ſ.
d) 4. 1.
e) 41. 1.

CBF, being on the ſame baſe, and between the ſame parallels: It ſhall be therefore alſo double to the triangle ABC, and the Parallelogram BK is double to the ſame triangle ABC, it being on the ſame baſe, and between the ſame parallels: Therefore the parallelogram BK ſhall be equal to the ſquare AF.

By the ſame diſcourſe it might be ſhewn that the Parallelogram LD is equal to the ſquare CI: Therefore the ſquare EC ſhall be equal to the two ſquares AF and CI: Therefore, In triangles, &c. Which was to be demonſtrated.

PROP. 48. THEOR. 34.



If the ſquare deſcribed of one of the ſides AC *of a triangle* ABC *be equal to the ſquares of the two other ſides* AB *and* BC *of the ſame triangle* ABC, *the angle* ABC *is a right angle.*

Demonſtration LEt BD be drawn equal to BC, and at [a] right angles to AB, and let AD be joyned: Seeing that ABD is a right angle, the ſquare of AD [b] is equal to the two ſquares of AB and BD: but the ſquare of BD is equal to the ſquare of BC, ſeeing that the lines BD and BC are made equal, wherefore the ſquare of AD ſhall be equal to the ſquares of BA and BC: Therefore ſeeing that the ſquare of AC is equal to the ſame ſquares of BA and BC, [c] the ſame ſquares of AC and AD ſhall be equal; and therefore the lines AD and AC are equal: Foraſmuch then, as in the triangles ABD and ABC, the two ſides BD and BA are equal to the two ſides BA and BC, each to his correſpondent, and the baſes AD and AC equal, the angles at the point B conteined of thoſe [d] ſides ſhall be equal to one another: But ABD is made a right angle: Therefore ABC ſhall be alſo a right angle: Therefore the ſquare, &c. Which was to be demonſtrated.

a) 11. 1.
b) 47. 1.
c) 1. c. ſ.
d) 8. 1.

The End of the Firſt Element of EUCLIDE.

THE

THE SECOND ELEMENT OF EUCLIDE.

THE ARGUMENT.

EUCLIDE treats in this ſecond Book of the power of right lines, ſeeking the quantity of the ſquares of the parts of every right line divided, and of rectangled Parallelograms conteined under the parts of the ſame divided line; compared as well to one another, as with the ſquare of the whole, &c. and for that cauſe he here unfoldeth in the firſt place, by theſe two Definitions two things which are neceſſary for the well underſtanding of what is to be demonſtrated hereafter.

DEFINITIONS.

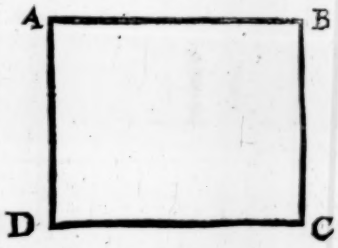
1 *Every rectangled Parallelogram is ſaid to be conteined under two righe lines, which do contein the right angle.*

IN this firſt he declares under what lines the rectangled parallelogram is ſaid to be conteined, and what is to be underſtood by a parallelogram to be conteined under two right lines; which for the more eaſie underſtanding, you muſt firſt know that every rectangled parallelogram is that which hath all his angles right angles, and of thoſe there are two ſorts, to wit, the ſquare, and the oblong or long ſquare, longer on one part then on the other: for in thoſe all the angles are right angles, as we have

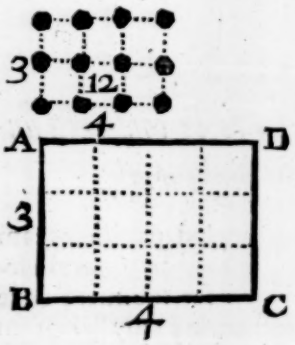
have already ſaid in the Definitions: And in every parallelogram if one angle be given a right angle, the three others will of neceſſity be right angles; For example, in the parallelogram A B C D, Let the angle A be a right angle, I ſay that the three others B, C, and D, are alſo right angles; for ſeeing that A B and D C are parallels, a the two interior angles A and B, are equal to two right angles: But A is a right angle by ſuppoſition; therefore B ſhall be alſo a right angle: But b forasmuch as each one of them is equal to his oppoſite, as the angle A to the angle C, and the angle B to the angle D, the angles D and C ſhall be alſo right angles: Therefore *EUCLIDE* ſaith that every parallelogram right angled is ſaid to be conteined under two right lines, which contein a right angle, as the Rectangled parallelogram A B C D, is ſaid to be conteined under A B and A D, or under A D and D C, or laſtly, under A B and B C: forasmuch as each two of thoſe lines ſo taken, do expreſſe the magnitude or ſpace of the whole parallelogram, to wit, one of them, as A B or D C its length, and the other as A D or B C its breadth: Therefore theſe two lines of the rectangled parallelogram, which contein the right angle being expreſſed, we conceive immediately the whole quantity thereof, and alſo its length and breadth is underſtood: It happens alſo that by the imaginary motion of one of the lines, according to the other, the whole parallelogram is conſtituted; for if it be conceived in the underſtanding, that the right line A B be moved tranſverſly according to A D, in ſuch ſort as that it may alwayes conſtitute a right angle with A D, until the point A arrive to the point D, and the point B to the point C, the whole parallelogram A D C B ſhall be deſcribed: the ſame will happen if A D be propoſed to move tranſverſly according to A B; therefore by good reaſon, a right angled parallelogram is ſaid to be conteined under two ſuch right lines.

a) 29. 1.

b) 34. 1.



Now this hath a great affinity and neerneſſe with the multiplication of one number by another, for even ſo, as by the multiplication of 3 by 4, 12 is produced, which is conſtituted in form of a parallelogram, from whence it is ſaid to be conteined under 3 and 4: So the parallelogram A B C D, conteined under the two right lines A B and B C, (of which A B conteins 3 Palmes, or other meaſures, and B C 4) conteins 12 ſquare Palmes, or other meaſures, which is made by the imaginary paſſing or motion of the line A B, of 3 Palmes in the line, or according to the line B C of 4 Palmes, as is repreſented by the figure; and is well known to Arithmeticians and Geometricians, and is demonſtrated by *Regiomontanus* in his firſt Book of triangles, Prop. 16. Whence it comes, that ſome ſay, that the rectangled parallelogram



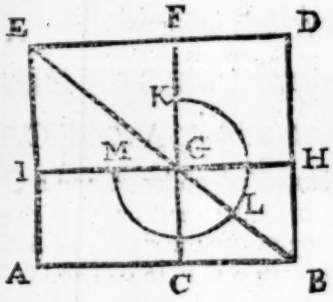
is

is constituted, or made of the product of two lines which are about the right angle, the one being multiplyed by the other; as the fore-mentioned Parallelogram is produced by the multiplication of the line A B, in the line B C, or (which is the same thing,) of the line B C in the line A B, for the same parallelogram is produced, whether the lesser line be applyed to the greater, or the greater to the lesser. In like manner, also the same number is produced, if the lesser number be multiplyed by the greater, or the greater by the lesser, as it is demonstrated by *EUCLIDE* in the seventh Book, Proposition 16. For that 12 is as well produced by 3 multiplyed in 4, as by 4 multiplyed in 3.

¶ The Reader is here advertised that in this second and the other following Books, the rectangled parallelogram is termed by *EUCLIDE* simply Rectangle, which other Geometricians do also observe, so that by Rectangle is to be understood alwayes Rectangled Parallelogram: Again, to the end the same letters be not so often repeated, Geometricians are accustomed to expresse the Parallelogram as well Rectangled, as not Rectangled, by two letters only; to wit, those which are diametrically opposite, as to denote the Parallelogram before mentioned, we say the Parallelogram A C, or else B D.

2 *Of every Parallelogram, one of the Parallelograms (which you please,) described about the Diameter, together with the two complements, is called* Gnomon.

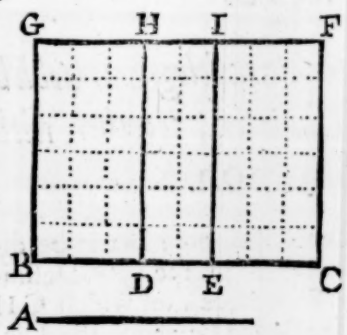
IN the Parallelogram A B D E, be it rectangled, or not: Let it be divided into four Parallelograms, as was shewn in the 37th. Definition of the First Book, of which C H and I F, are said to be about the Diameter, and the two others A G and G D are said to be complements; the figure compounded of which you please of the Parallelograms which are about the Diameter, as of C H, with the two complements A G and G D, such as is the figure M L K is called Gnomon: by the same reason, the figure H D E I C G, compounded of the Parallelogram I F, which is about the Diameter, and the two complements A G and G D shall be called Gnomon.



PRO-

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.



If there be two right lines A *and* BC, *and that one of them* BC, *be divided into as many parts as you pleaſe* BD, DE, *and* EC, *the Rectangle conteined under thoſe two right lines* A *and* BC, *is equal to the Rectangles conteined under the undivided line* A, *and under each of the parts of the divided line* BC, *to wit, under* A *and* BD, *under* A *and* DE, *and under* A *and* EC.

Demonſtration LEt the Rectangle be BF, compriſed under A and BC; that is to ſay, that BG be put equal to A, which may thus be done, draw BG and GF [a] at right angles, to the right line BC, and make each equal to A, and joyn the right line GF, ſo BG and CF ſhall be [b] parallels, becauſe of the right angles B and C, and alſo equal to one another; ſeeing [c] that each of them is put equal to A: therefore the right lines GF and BC which joyn them, [d] ſhall be alſo equal and parallel, therefore BF ſhall be a Rectangle, and conteined under BC and A; that is to ſay, his equal BG, according to the firſt Definition of this Book: Then from the points D and E, draw DH and EI, [e] parallel to BG, and therefore parallel [f] to one another: And ſeeing that by the ſame conſtruction BH and BI are Parallelograms, the right lines DH and EI ſhall [g] be equal to BG, that is to ſay to A: Seeing then that BG is equal to A, the Rectangle BH ſhall be conteined under the undivided line A, and the ſegment BD; and the Rectangle DI, ſhall be conteined under the undivided line A, and the ſegment DE: In like manner EF ſhall be

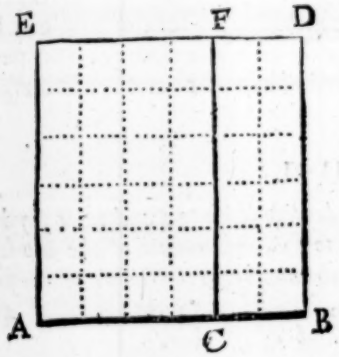
a) 11. 1.
b) 28. 1.
c) 1. c. ſ.
d) 33. 1.
e) 31. 1.
f) 30. 1.
g) 34. 1.

be conteined under A, and the ſegment E C: Now ſeeing that all thoſe parts taken together, are equal to their whole B F, it is evident that the Rectangle compriſed of the two lines A and B C, ſhall be equal to each of the Rectangles compriſed of A, and each of the ſegments of B C: Therefore if there be two lines, &c. Which was to be demonſtrated.

SCHOLIUM.

We have divided theſe figures into equal parts, to render the Demonſtrations more eaſie, and to fit them to numbers, (See the precedent figure:) Let the line B G *equal to* A, *be* 6 *feet, or other meaſures, and the line* B C 8, *by the firſt Definition of this Book, the Rectangle* B F *ſhall be* 48 *Superficial feet, and by the ſame reaſon, the Rectangle* B H *ſhall be* 18, D I 12, *and* E F *alſo* 18, *which three Rectangles together do make alſo* 48, *to wit, all the parts taken together, equal to their whole.*

PROP. 2. THEOR. 2.



If a right line A B *be divided at pleaſure in* C, *the Rectangles* C D *and* A F, *conteined under the whole* A B, *and each of the parts* A C *and* C B, *are equal to the ſquare of the whole* A B.

Demonſtration DEſcribe [a] A D the ſquare of the whole line A B, and by C [b] draw C F parallel to A E or B D, it ſhall be [c] equal to A E; that is to ſay, to A B, to which A E is equal, by the Definition of a ſquare: Foraſmuch then as A E is equal to A B, the Rectangle A F ſhall be conteined under the whole A B, and the ſegment A C: In like manner, foraſmuch as C F is equal to A B, the Rectangle C D is conteined under the whole A B, and the other ſegment C B: Therefore ſeeing the Rectangles C D and A F are equal to the ſquare A D, it is manifeſt that the Rectangles conteined under the whole A B, and each of the ſegments A C and C B are equal to the ſquare of the whole A B: Therefore, If a line, &c. Which was to be demonſtrated.

a) 46. 1.
b) 31. 1.
c) 34. 1.

SCHOLIUM.

This may alſo be ſeen by numbers, by dividing 6 *in* 4 *and* 2, *as appeares by the figure, applying it to numbers,* A F *ſhall be* 24, *and* C D 12, *according to the firſt Definition, which make* 36, *and ſo much is the ſquare of* A D 6.

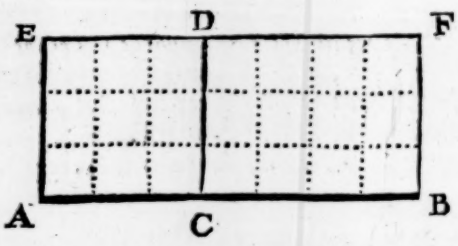
PROP. 3. THEOR. 3.

If a right line A B *be divided at pleaſure in* C, *the Rectangle* A F, *conteined under the whole* A B, *and one of the*

parts A C, *is equal to the Rectangle* C F, *conteined under thoſe parts, and* A D *the ſquare of the part* A C *firſt taken.*

Demonſtration DEſcribe A D the ſquare of the part A C, and by the point B draw B F [a] parallel to A E, meeting E D prolonged at F.

a) 31. 1.

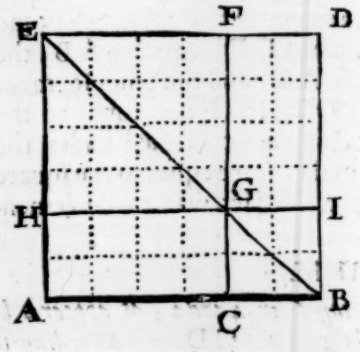


Foraſmuch then as the right line A E is equal to the ſegment A C, by the Definition of a ſquare, the Rectangle A F ſhall be conteined under the whole A B and the part A C: Again, foraſmuch as C D is equal to C A, by the ſame reaſon, the Rectangle C F ſhall be conteined under the parts A C and C B; therefore ſeeing that the Rectangle A F is equal to the ſquare A D, and to the Rectangle C F: It is mauifeſt that the Rectangle conteined under the whole A B, and the ſegment A C is equal to the Rectangle conteined under the part A C and C B, and to the ſquare of A C firſt taken: Therefore, If a right line, &c. Which was to be demonſtrated.

SCHOLIUM.

To fit this Theorem to numbers: Let A B 7 *be divided into* 4 *and* 3: *I ſay that the number* 21, *the product of* 3 *by* 7, *is equal to* 12, *the product of the two ſegments* 4 *and* 3 *multiplyed by one another, and to the number* 9 *the ſquare of the firſt part* 3: *In like manner* 23 *the product of* 7 *by* 4, *is equal to* 12 *the product of* 4 *and* 3, *and to the number* 16, *the ſquare of* 4 *the number firſt taken.*

PROP. 4. THEOR. 4.



If a right line A B *be divided at pleaſure in* C, *the ſquare* A D *of the whole* A B, *is equal to the ſquares* H F *and* C I, *of the parts* A C *and* C B, *and to twice the Rectangle* A G *or* G D, *conteined under the ſame parts* A C *and* C B.

Demonſtration DEſcribe the ſquare of the line A B, to wit, A D, and having drawn the Diameter B E, draw through the point C the right line C F, parallel to A E or B D, cutting the Diameter in the point G, and by G draw I G H parallel to A B, or to D E, and ſo the ſquare of A B ſhall be divided into four Parallelograms.

Foraſmuch as in the triangle A B E, the ſides A B and A E are equal, the two angles A B E and A E B [a] on the baſe E B are equal: But the

a) 5. 1.

angle

angle in the ſquare, and the three angles A, A E B, and A B E [b] are equal to two right angles, and therefore the two angles A B E and A E B, are each half a right angle, being equal to one another, and foraſmuch as H I and A B are parallels, the exterior angle G H E [c] ſhall be equal to its interior and oppoſite angle, and therefore a right angle: But the three angles of the triangle [d] E G H are equal to two right angles, and G E H is ſhewn to be half of a right angle, and E H G a right angle: Therefore the other H G E ſhall be alſo half a right angle, therefore in the triangle E G H, the two angles G E H and E G A ſhall be equal, being each of them half a right angle; therefore the two ſides H E and [e] H G ſhall be equal: But the two oppoſite ſides E F and F G [f] are equal unto them, each to its correſpondent: Therefore the Parallelogram H F ſhall be a ſquare, having all the ſides equal, and the angles right angles: For one of the angles, to wit E H G, being a right angle, all the four ſhall be right angles, as is ſhewn by the firſt Definition of this Book, and by the ſame reaſons it may be ſhewn that C I is a ſquare: Therefore F H and C I, [g] are the ſquares of the parts A C and C B; ſeeing that H G is equal to A C.

b) 32. 1.
c) 29. 1.
d) 32. 1.
e) 6. 1.
f) 34. 1.
g) 34. 1.

The Rectangles A G and G D ſhall be in like manner conteined under the ſegments A C and C B: foraſmuch as C G and G I are equal to C B, becauſe of the ſquare C I, and F G is equal to G H, by reaſon of the ſquare F H; that is to ſay [h] to A C: Therefore ſeeing the ſquare A D is equal to the two ſquares H E and C I, and to the two Rectangles A G and G D, it is manifeſt that A D the ſquare of the whole A B, is equal to the ſquares of the parts A C and C D, and to twice the Rectangle conteined under the ſame part A C and C B: Therefore, If a right line, &c. Which was to be demonſtrated.

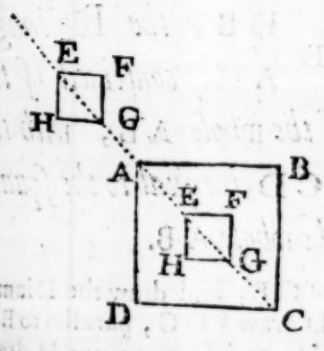
h) 34. 1.

COROLLARIE I.

By this Demonſtration it is manifeſt that the Parallelograms deſcribed about the Diameter of a ſquare are ſquare.

Which is apparent by the Demonſtration of this Theorem, where it is demonſtrated that the Rectangles C I *and* F H, *which are about the Diameter* B E *are ſquares: for the ſame Demonſtration ſerves in all other ſquares.*

Nevertheleſſe, this Corollary ought to be underſtood of Parallelograms, which are about the Diameteter of the ſquare, and which have ſome angle common with the whole ſquare, as are the Parallelograms C I *and* F H, *for the one hath the angle* A B D, *and the other the angle* A E D, *common with the ſquare.*



It is alſo true of any Parallelograms which are about the Diameter of the ſquare, being prolonged, although it have no angle common with the ſquare, provided their ſides be parallel to the ſides of the ſquare.

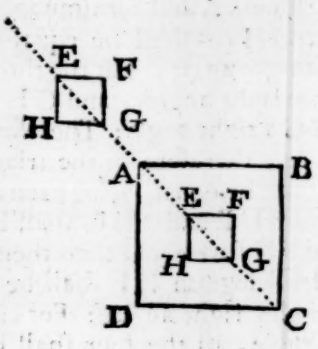
For about the Diameter A C *of the ſquare* B D, *deſcribe the Parallelogram* F H, *be it without or within the ſquare, which notwithſtanding muſt have the ſides parallel to the ſides of the ſquare: I ſay that* F H *is a ſquare.*

For ſeeing that A B *and* E F *are parallels* [i] *the angles* B A C *and* F E G *ſhall be equal, the exterior to the interior, and by the ſame reaſon the angles* B C A *and* F G E *ſhall be equal alſo: But the angles* B A C *and*

i) 29. 1.

BCA *are the halves of right angles, as hath been already demonstrated: Therefore the angles* FEG *and* FGE *shall be also the halves of right angles,* k *and therefore the sides* EF *and* FG *opposite to them, shall be equal, and* l *the angle* F *shall be a right angle: Therefore seeing that the sides* EF *and* FG m *are equal to their opposite sides* GH *and* HE, *the Parallelogram* FH *shall be equilateral; but it is also rectangled, as hath been shewn by the first Definition of this Book: Foraſmuch as one of the angles* F *is demonstrated to be a right angle: Therefore* FH *shall be a ſquare: Which was propoſed.*

k) 6. 1.
l) 32. 1.
m) 34. 1.



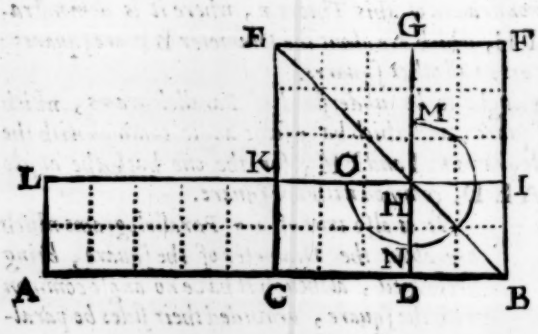
COROLLARIE. II.

It follows alſo from the Demonſtrateon of this Propoſition that the Diameter of any ſquare doth divide its angle into two equal parts, for it hath been demonſtrated that the angles AEB *and* DEB *in the fore-going Propoſition, are the halves of right angles, which is alſo demonſtrated by the Eighth, and Thirty fourth Propoſition of the firſt Book.*

SCHOLIUM.

This fourth Theorem ſhall be alſo ſhewn by numbers; (See the figure belonging to this Propoſition) let the line AB *of 6 feet be divided into* AC *and* CB, *4 and 2, the ſquare of* AB *ſhall be 36 Superficial feet, and the ſquares of* AC *and* CB, *to wit,* HF *and* CI *ſhall be 16 and 4, and the two rectangles under* AC *and* CB, *which are* AG *and* GD, *are each 8, all which parts together make 36; to wit, as much as the ſquare of the line* AB, *which is* ABDE, *as is manifeſt.*

PROP. 5. THEOR. 5.



If a right line AB *be divided into two equal parts* AC *and* CB, *and into two unequal parts* AD *and* DB, *the Rectangle* AH, *conteined of the unequal parts* AD *and* DB, *of the whole* AB, *with the ſquare* KG, *of the intermediate part* CD, *is equal to the ſquare* CF, *deſcribed of* CB, *the half of the whole* AB.

Demonſtration DEſcribe CF the ſquare a of CB, and draw the Diameter BE, and by the point D draw b DG, parallel to BF, or CE, cutting the Diameter in the point H, and by the point H draw KI, parallel to AB, and from the point A draw AL, parallel to CE, meeting

a) 46. 1.
b) 33. 1.

meeting with I K prolonged in the point L; The Parallelograms K G and D I, about the Diameter B E shall be [c] squares, and therefore D H equal to D B and K H to C D: therefore the Rectangle A H shall be conteined under A D and D B, and K G shall be the square of C D, the which A H with K G, is equal to the square C F: For seeing that [d] the complements C H and H F are equal, if the common square D I be added, the Parallelogram D F shall be equal to the Parallelogram C I: But the Parallelogram A K is [e] equal to the Parallelogram C I, being constituted on equal bases A C and C B, and between the same parallels A B and L I: Therefore A K shall be equal to D F, to which if you adde the common C H, the Gnomon M N O shall be equal to the Rectangle A H: Wherefore seeing that the Gnomon M N O and the square K G, are equal to the square C F, the Rectangle A H with the square K G, shall be equal to the same square C F: Therefore, If a line, &c. Which was to be demonstrated.

c) C. 1. 4, 2.

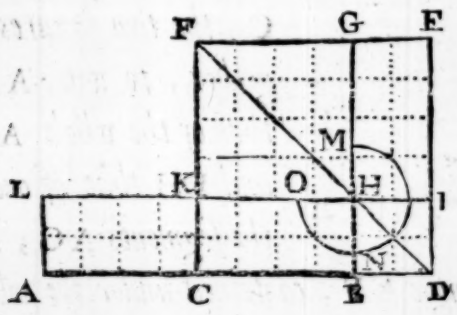
d) 43. 1.

e) 36. 1.

SCHOLIUM.

The same is also manifest in applying it to numbers: Let the line A B *be* 10; *then as well* A C *as* C B *shall be* 5, *and let* C D *be* 3, A D *shall be* 8, *and* D B 2: *Therefore the Rectangle* A H *shall be* 16, *and the square* K G 9, *and the Gnomon also* 16, *which with the square* K G, *is* 25, *which is the square* C F, *equivalent to the Rectangle* A H 16, *and* K G 9, *which together make* 25, *as is manifest.*

PROP. 6. THEOR. 6.



If a right line A B, *be divided into two equal parts in* C, *and there be added to it another right line* B D, *directly, the Rectangle* A I, *conteined under the whole* A B, *with the added line* B D, *to wit* A D, *and the added line* B D, *with the square* K G, *of the half* C B, *is equal to the square* C E, *described of the line* C D, *compounded of the half* C B, *and the added line* B D, *as of one line.*

Demonstration. ON C D describe the square C E, and having drawn the Diameter D F by B, draw B G parallel to D E or C F, dividing the Diameter in the point H; and by H draw I K, parallel to C D, and by A draw A L parallel to C F, meeting I K prolonged in the point L: Therefore B I and K H shall be squares, and D I shall be equal to B D, [a] and K H is equal to C B: Wherefore the Rectangle A I shall be conteined under A D and D B; and K G shall be the square of C B, which Rectangle A I and square K G, are equal to the square C E: for seeing that

a) 34. 1.

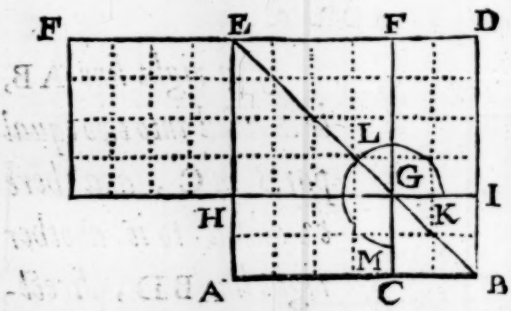
b) 36. 1. that the Parallelogram A K is equal to the Parallelogram C H; [b] Seeing that the baſes A C and C B are equal, and the Parallelogram H E is equal to the ſame C H, the complement to the complement, A K and H E ſhall be equal to one another: Therefore if you add to them the common C I, the Gnomon M N O ſhall be equal to the Rectangle A I: Therefore ſeeing the Gnomon M N O, with the ſquare K G, is equal to the ſquare C E, the Rectangle A I with the ſquare K G, ſhall be alſo equal to the ſquare C E: Therefore, If, &c. Which was to be demonſtrated.

This is alſo eaſie to be underſtood by the figure, where the Rectangle A K and H E are each 8 equal parts, to each of which if you add the Rectangle C I 12, the Rectangle A I will be equivalent to the Gnomon, to wit 20: Therefore A I with K G ſhall be equal to the ſquare C E.

SCHOLIUM.

To apply this Theorem to Numbers: Let 8 be divided into two equal parts, to wit into 4 and 4; and let 2 be added, it will be ſeen how the number 20, the product of the whole compound number 10, multiplied by the added 2 with 16, the ſquare of the half 4, is equal to 36, the ſquare of 6 compounded of the half 4, and of the added number 2, as may be alſo underſtood by the figure it ſelf.

PROP. 7. THEOR. 7.



If a right line A B, *be divided at pleaſure in* C, *the two ſquares together, to wit,* A D, *that of the whole* A B, *and* F H *that of one of the ſegments* A C, *are equal to twice the Rectangle* A F, *conteined under the whole* A B, *and the ſaid ſegment* A C, *and to* C I *the ſquare of the other ſegment* C B.

Demonſtration DEſcribe the ſquare of the line A B, to wit A D, and having drawn the Diameter B E by C, draw C F parallel to A E or B D, cutting the Diameter in the point G, by which point draw H I parallel to AB, by the firſt Corollary of the fourth Propoſition, C I and
a) 34. 1. H F ſhall be ſquares: And [a] foraſmuch as G H is equal to A C, and the figure H F ſhall be the ſquare of the ſegment A C: Again, foraſmuch as A E is equal to A B, the Rectangle A F ſhall be conteined under the whole AB, & the ſegment AC, by the ſame reaſon, the Rectangle H D ſhall be conteined under the ſame A B and A C: Foraſmuch as D E and E H are equal to A B and A C, by reaſon of the ſqnares A D and H F.

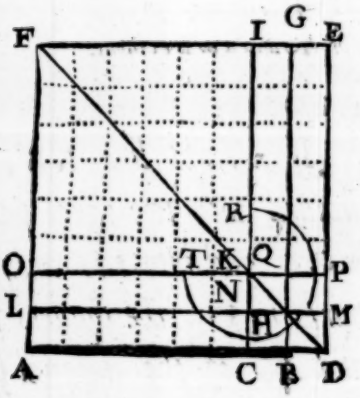
Foraſmuch then as the ſquare A D is equal to the Rectangles A F and F B; that is to ſay, to the Gnomon K L M, and to the ſquare C I, if you add

add the common ſquare H F, the ſquares A D and H F ſhall be equal to the Rectangles A F and D H, (each of which is conteined under the whole A B and the ſegment A C,) and to C I the ſquare of the other part C B: Therefore, If a right line, &c. Which was to be demonſtrated.

SCHOLIUM.

This is manifeſt alſo if it be applyed to numbers, for A B *being divided into* 6 *parts,* A C *ſhall be* 4, *and* C B 2, *and* A F *ſhall be* 24, *and* G D *with* H F *alſo* 24, *and* C I 4, *which parts together, make* 52 *equal parts: And the two ſquares together* A D *and* H F *do alſo make* 52 *ſuch parts.*

PROP. 8. THEOR. 8.



If a right line A B, *be divided at pleaſure in* C, *four times the Rectangle* A H, *conteined under the whole* A B, *and one of the ſegments* C B, *with the ſquare* O I *of the other ſegment* A C, *is equal to the ſquare* A E, *deſcribed of the whole* A E, *and of the ſaid ſegment* C B, *as of one line* C D.

Demonſtration PRolong A B towards D, and put B D equal to B C, and on the whole A D deſcribe the ſquare A E, and having drawn the Diameter D F, draw C I and B G parallels to D E, cutting the Diameter in the points H and K, through which points draw L M and O P, parallel to A D, which doth divide the firſt Parallels in the points N and Q.

In the firſt place the Parallelograms O I, N Q, B M, L G, and C P, which are about the Diameter D F ſhall be ſquares, by the firſt Corollary of the fourth Propoſition: And foraſmuch as [a] Q K is equal to A C; O I ſhall be the ſquare of the ſegment A C: Again, ſeeing that N H is equal [b] to C B, N Q ſhall be the ſquare of the ſegment C B: and therefore equal to the ſquare B M; ſeeing that C B and B D are equal, therefore each of the lines B H and H Q is equal to the ſegment C B, and ſo the two Rectangles A H and L Q, ſhall be compriſed under the whole A B, and the ſegment C B: [c] Seeing that L H is equal to A B, by the ſame reaſon, N G and H E ſhall be compriſed under A B and C B: Seeing that the right lines N H and H M are equal to C B and B D; and G H and E M equal to F L, that is to ſay to L H, or to his equal A B: And foraſmuch as the ſquares N Q and B M are equal; if you add the common rectangle K G, B M and K G together, ſhall be equal to the Rectangle N G: Wherefore the five rectangles A H, L Q, H E, B M, and K G, which conſtitute the Gnomon R H T, are equal to four times the Rectangle conteined under the whole A B, and the ſegment C B:

a) 34. 1.

b) 34. 1.

c) 34. 1.

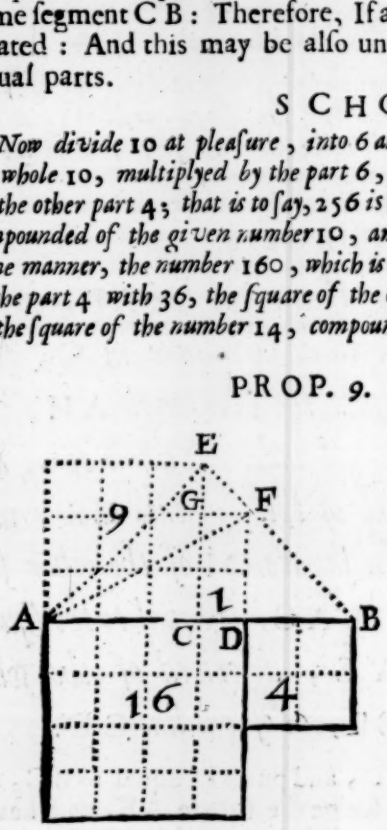
Now

Now seeing that the Gnomon R S T, and the square O I, are equal to the square A E; four times the Rectangle conteined under the whole A B, and the segment C B, with the square of the other segment A C, is equal to the square of the line A D, compounded of A B and of the same segment C B: Therefore, If a line, &c. Which was to be demonstrated: And this may be also understood by the figure divided into equal parts.

SCHOLIUM.

Now divide 10 *at pleasure, into* 6 *and* 4, *the number* 240, *which is made of the whole* 10, *multiplyed by the part* 6, 4 *times, with the number* 16 *the square of the other part* 4; *that is to say,* 256 *is equal to the square number of* 16, *which is compounded of the given number* 10, *and of the said part* 6, *as is manifest: In the same manner, the number* 160, *which is made of the whole* 10, 4 *times multiplyed by the part* 4 *with* 36, *the square of the other part* 6; *that is to say* 196, *is equal to the square of the number* 14, *compounded of* 10 *and* 4 *as is evident.*

PROP. 9. THEOR. 9.



If a right line A B, *be divided into two equal parts in* C, *and into two unequal parts in* D, *the squares* 16 *and* 4, *of the unequal segments* A D *and* D B, *of the whole* A B, *are double to the square* A E, *of the half* A C, *and the square* 1 *of the intermediate Section* C D.

Demonstration FRom the point C draw C E at right angles to A B, and equal to A C, and joyn A E and E B together; and from D draw also D F at right angles to A B, cutting E B in the point F, by which point draw F G, parallel to A B, dividing C E in the point G; and lastly joyn A F; Forasmuch as in the triangle A C E, the two sides A C and C E are equal, the two angles C A E and C E A shall be
a) 32. 1. equal: but A C E is a right angle; therefore the two [a] others are equivalent to a right angle: and therefore the angle A E C shall be the half of a right angle: By the same reason, the angle B E C shall be shewn to be half a right angle, and by consequence the whole A E B shall be a angle.

b) 29. 1. Again, forasmuch as in the triangle F G E, the angle E G F [b] is equal
e) 32. 1. to the right angle E C B, the exterior angle to the interior, the two others [c] shall be equal to a right angle: But it is demonstrated that the
d) 6. 1. angle F E G is half a right angle; Therefore F E G shall be also half a right angle, and therefore are both equal: Therefore [d] the sides E G and G F shall be equal to one another: In the same manner it shall be shewn that the two lines F D and D B are equal to one another.

Now forasmuch as in the triangle A C E, the angle C is a right angle, the

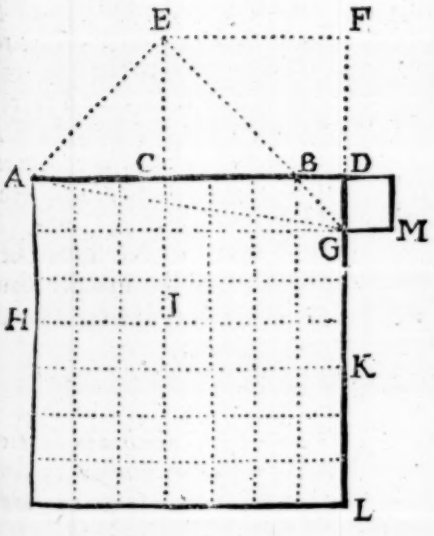
e the ſquare of the ſide A E ſhall be equal to the two ſquares of A C and C E, the which are equal to one another, being the lines A C and C E are equal: Therefore the ſquare of A E ſhall be double to the ſquare of A C: Again, foraſmuch as in the triangle E G F, the angle G is a right angle, and the two ſides G E and G F equal f the ſquare of E F ſhall be double to the ſquare of G F; that is to ſay, of the ſquare of C D, which is g equal to G F: Therefore the two ſquares of A E and E F, are double to the two ſquares of A C and C D: But theſe ſaid ſquares of A E and E F h are equal to the ſquare of A F: Therefore the ſquare of A F ſhall be double to the two ſquares of A C and C D: and the ſquare of A F is equal to the two ſquares of A D and D F: Therefore the two ſquares of A D and D F are double to the two ſquares of A C and C D: But the ſquare of D F is equal to the ſquare of D B: for thoſe lines are ſhewn to be equal: In like manner, therefore the ſquares of the ſegments A D and D B ſhall be alſo double to the ſquares of A C and C D, the half of the line A B, and of the middle Section: Therefore, If a line, &c. Which was to be demonſtrated,

e) 47. 1.
f) 47. 1.
g) 34. 1.
h) 34. 1.

SCHOLIUM.

Let 10 be divided equally into 5 and 5, and unequally into 7 and 3, the middle Section ſhall be 2, as by the fifth Propoſition, the ſquares of the unequal parts 7 and 3, which are 49 and 9, are double to 25 and 4, the ſquares of the half and of the middle Section, as is manifeſt..

PROP. 10. THEOR. 10.

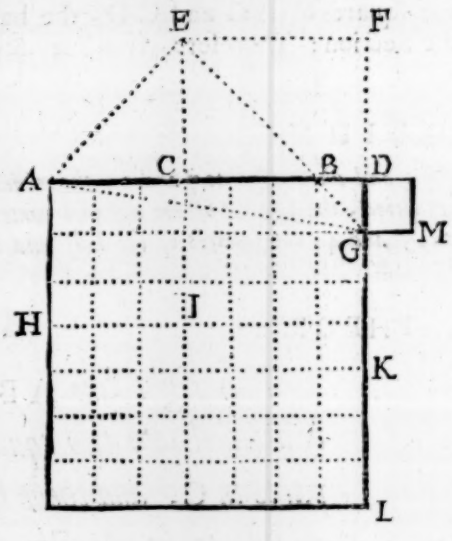


If a right line A B, *be divided into two equal parts in* C, *and there be added to it directly a right line* B D, *the two ſquares* A L *and* B G *together, of the whole* A B, *with the added line* B D, *are double to the ſquare* H C, *deſcribed of the half* A C, *and to the ſquare* C K, *compounded of the half* C B, *and of the added line* B D, *as of one line.*

Demonſtration DRaw C E at right angles to the line A B, and make it equal to the half A C, and joyn the right lines A E and E B, and by D draw F D G, parallel to

CE, meeting with E B prolonged in the point G, and by E draw E
parallel to C D, meeting D F in the point F, and let the right line A
be joyned, we ſhall now ſhew that the angle A E B is a right angl
as in the foregoing Propoſition, and C E B half a right angle;
a) 29. 1. a and therefore its alternate angle E G F in like manner half a righ
b) 34. 1. angle: But the angle b F is a right angle being oppoſite to the righ
c) 34. 1. angle C in the Parallelogram C F; c therefore the other angle F E G ſha
be alſo half a right angle, and therefore equal to E G F: Wherefore E
d) 6. 1. and F G d oppoſite to the angles F E G and E G F; are in like manner equa
by the ſame diſcourſe it ſhall be ſhewn that B D and D G are equal;
foraſmuch as the angle B D G is a right angle, and B G D half a
right angle, &c.

c) 47. 1. Foraſmuch therefore, as the ſquare of A E is e equal to the equi
ſquares of the equal right line
A C and C E, the ſame ſquare
of A E ſhall be double to the
ſquare of A C: Again, for-
aſmuch as the ſquare of E
f) 47. 1. f is equal to the equal ſquare
of the equal right lines E
and F G, the ſame ſquare o
E G ſhall be in like manner
double to the ſquare of E F, o
g) 34. 1. of C D its equal; for g C D i
equal to E F: therefore the two
ſquares of A E and E G are
double to the ſquares of A C
and C D; But the ſquare of
h) 47. 1. A G is equal to the ſquares h of
A E and E G, and therefore
double to the ſquares of A C
and C D, and the ſquares of
A D and D G are equal to the ſquare of A G: Therefore the ſquares o
A D and D G are double to the ſquares deſcribed of A C and C D:
Therefore ſeeing that the ſquare of D G is equal to the ſquare of it
equal B D, the ſquares of A D and D B ſhall be in like manner double
to the ſquares deſcribed of A C and C D: Therefore, If a right line, &c.
Which was to be demonſtrated.



SCHOLIUM.

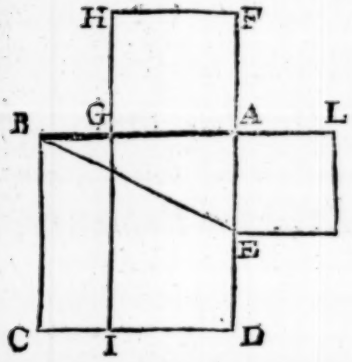
Divide the number 10 *into two equal parts* 5 *and* 5, *to which add ſome other number as* 3, *in ſuch ſort as the whole compound number may make* 13, *whoſe ſquares* 169 *and* 9, *of the numbers* 13 *and* 3, *are double to the ſquares* 25 *and* 64 *which are produced of the numbers* 5 *and* 8, *as is manifeſt.*

PROP. 11. PROBL. 1.

To divide a given right line A B, *in ſuch ſort as that the Rectangle* C G *conteined under the whole* A B, *and one of the ſegments* B G, *may be equal to the ſquare* A H *of the other ſegment* A G.

Demon-

Conſtruction DEſcribe the ſquare A C of the right line A B, and having divided the ſide D A into two equal parts in the point E, draw E B, and prolong D A to the point F, in ſuch ſort as E F be put equal to E B, and from A B cut off A G equal to A F, for A B is greater than A F: Seeing [a] that E A and A B are greater than E B, or E F its equal: Therefore taking away the common part E A, there will reſt A B greater than A F: I ſay that A B is divided in the point G, in ſuch ſort as the Rectangle conteined under A B and B G, is equal to the ſquare of A G, in ſuch ſort as that B G is the leſſer ſegment, and A G the greater. a) 26. 1.



Demonſtration FOr deſcribe A H the ſquare of A G, and prolong the ſide H G towards I, in ſuch ſort as that H I may divide D C in the point I, which [b] ſhall be parallel to F D and B C; therefore C G and D H ſhall be rectangled Parallelograms, and C G ſhall be conteined under the whole A B, and the ſegment B G; for C B is equal to A B; which ſaid Rectangle is equal to the ſquare G F of the other ſegment A G. b) 34. 1.

For ſeeing that D A being divided into two equal parts in E, and A F added thereto, [c] the Rectangle conteined under D F and F A; that is to ſay, the Rectangle D H, (for F H is equal to F A,) with the ſquare of the half A E, is equal to the ſquare of E F; that is to ſay, to the ſquare of E B, which is equal to E F: But the ſquare of E B is equal [d] to the ſquares of E A and A B: Wherefore the Rectangle D H, with the ſquare of A E ſhall be alſo equal to the ſquares of A E and A B; taking away therefore the common ſquare of A E, there will remain the Rectangle D H, equal to the ſquare of A B, which is the ſquare A C: Therefore if the common Rectangle D G be taken away, there will remain the Rectangle G C, equal to the ſquare G F, which was propoſed: Therefore we have divided, &c. Which was to be done. c) 6. 2. d) 47. 1.

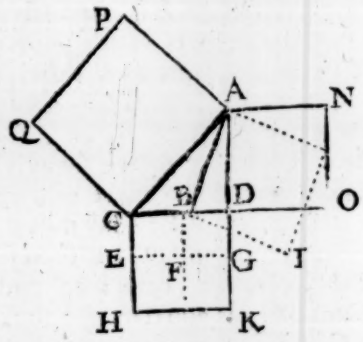
SCHOLIUM.

This Problem cannot in any kind be demonſtrated by numbers: For no number can be divided in ſuch ſort into two parts, as the number produced of the whole multiplyed by one of the parts, may be equal to the ſquare number of the number remaining, as ſhall be demonſtrated in the fourteenth Propoſition of the Ninth Book.

¶ The Demonſtration of the ten foregoing Theorems in numbers, according to *Barlaam*, ſhall follow at the End of this Second Book.

PROP. 12. THEOR. 14.

In Ambligonium *triangles* A B C, *the ſquare* A Q, *of the ſide* A C, *which ſubtendeth the obtuſe angle*

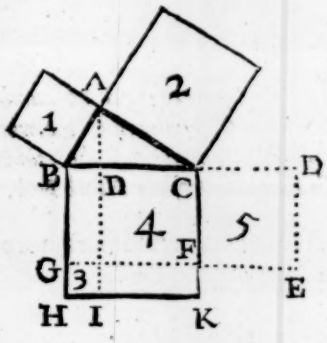


ABC, *is greater than the ſquares* AI *and* HF, *of the ſides* AB *and* BC, *which do contein the obtuſe angle* ABC, *by twice the Rectangle* CF, *conteined under one of the ſides which are about the obtuſe angle, to wit,* CB, *on which being prolonged, there doth fall the perpendicular* AD, *and the line* BD, *taken without, between the perpendicular* AD, *and the obtuſe angle* ABC.

Demonſtration FOr ſeeing that CD is divided at adventure in B, [a] the ſquare of CD, to wit, CK, is equal to the two ſquares of CB and BD, and to twice the Rectangle conteined under CB and BD, adding therefore the common ſquare of AD, to wit, DN, the two ſquares of CD and DA ſhall be equal to the three ſquares of the lines CB, BD, and DA, and to twice the Rectangle conteined under CB and BD: But the ſquare of AC [b] is equal to the two ſquares of CD and DA, the ſame ſquare of AC ſhall be therefore alſo equal to the three ſquares of CB, BD, & DA, and to twice the Rectangle conteined under CB and BD; therefore ſeeing that the ſquare of BA is [c] equal to the ſquares of BD and DA, the ſquare of AC ſhall be equal to the ſquares of CB and BA, and to twice the Rectangle conteined under CB and BD, which is propoſed: Therefore, In, &c. Which was to be demonſtrated.

a) 4. 2.
b) 47. 1.
c) 47. 1.

PROP. 13. THEOR. 12.



In Oxigonium *triangles* ABC, *the ſquare* 1 *of the ſide* AB, *which ſubtendeth the accute angle* ACB, *is leſſe than the ſquares* 2, *and* BK, *of the ſides* AC *and* CB, *which contein the accute angle* ACB, *by twice the Rectangle* BF, *conteined under one of the ſides* BC, *which are about the accute angle* ACB, *to wit, that on which the*

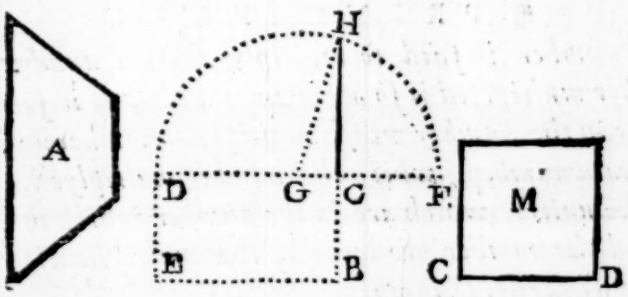
the perpendicular AD *doth fall, and the line* DC *taken from within, between the perpendicular* AD, *and the accute angle* ACB.

Demonſtration FOr ſeeing that the right line BC is divided at adventure in the point D, the ſquares of [a] BC and of CD, are equal to twice the Rectangle conteined under BC and CD, and to the ſquare BD: Therefore if you add the common ſquare of DA, the three ſquares of BC, CD, and DA, ſhall be equal to twice the Rectangle conteined under BC and CD, and to the two ſquares of BD and DA: But [b] the ſquare of CA is equal to the two ſquares of CD and DA; therefore the two ſquares of BC and CA are equal to twice the Rectangle conteined under BC and CD, and to twice the ſquares of BD and DA; Therefore ſeeing that the ſquare of AB is equal to the two ſquares of BD and DA, the two ſquares of BC and CA ſhall be equal to twice the Rectangle conteined under BC and CD, and to the ſquare of AB, which was propoſed. a) 7. 2. b) 47. 1.

In the ſame manner, we ſhall demonſtrate that the ſquares of AB and BC are equal to twice the Rectangle conteined under CB and BD, and to the ſquare of AC, that is to ſay, that the ſquare of the ſide AC is leſſe than the ſquares of the ſides AB and BC, by twice the Rectangle conteined under CB and BD: Therefore, In Oxigonal triangles, &c. Which was to be demonſtrated.

PROP. 14. PROBL. 2.

To conſtitute a ſquare M, *equal to a given right lined figure* A.



Conſtruction MAke [a] the rectangled Parallelogram BCDE, equal to the right lined figure A, and prolong one of its ſides, as DC to F, and make CF equal to CB, and divide DF into two equal parts in the point G, which ſhall be the ſame as the point C, or not; if it fall on the point C, the right line BC, (ſeeing it is made equal to CF,) ſhall be equal to CD; therefore the Rectangle BD ſhall be a ſquare, and ſeeing [b] that the ſides BE and ED are equal to the oppoſite ſides BC and CD, you have what was ſought: But if the point G doth not fall on the point C, from the point G as a center, and at the diſtance GF or GD, deſcribe the ſemicircle FHD, and prolong BC a) 45. 1. b) 34. 1.

B C to the Circumference H, and draw alſo G H: I ſay that the ſquare of the right line C H is equal to the right lined figure A.

Demonſtration FOraſmuch as D F is divided into two equal parts in G, and in two unequal parts in C, the Rectangle conteined under D C and C F, to wit the Rectangle B D, [c] with the ſquare of G C is equal to the ſquare of G F, or of G H equal thereto: But the ſquare of G H is [d] equal to the two ſquares G C and C H: Therefore the Rectangle B D, with the ſquare of G C, is equal to the two ſquares of G C and C H: Therefore if you take away the common ſquare G C, there will remain the Rectangle B D; that is to ſay, the right lined figure A equal to the ſquare of C H: Therefore, We have conſtituted, &c. Which was to be done.

c) 5. 2.

d) 34. 1.

The End of the Second Element of EUCLIDE.

The Demonſtration of the ten firſt Theorems of this Second Book of EUCLIDE in Numbers, according to *Barlaam.*

¶ PRINCIPLES.

1 *A number is ſaid to multiply another number: when the number multiplyed is ſo oftentimes added to it ſelf, as there be unities in the number which multiplyeth: whereby is produced a certain number, which the number multiplyed, meaſureth by the unities which are in the number which multiplieth.*

2 *And the number produced of that multiplication is called a plain or ſuperficial number.*

3 *A ſquare number is that which is produced of the multiplication of any number into its ſelf.*

4 *Every leſſe number compared to a greater, is ſaid to be a part of the greater, whether the leſſe meaſure the greater, or meaſure it not.*

5 *Numbers, whom one and the ſelf ſame number meaſureth equally, that is, by one and the ſelf ſame number are eqnal the one to the other.*

6 Num-

6 Numbers that are equimultiplices to one and the self same number, that is, which contein one and the same number equally and alike, are equal the one to the other.

PROPOSITION I.

C....4
A..2 D..2 E..2 B
F.....24
G.....8 H.....8 I.....8 K

Two numbers being given, if one of them be divided into any numbers, how many soever: the plain or superficial number which is produced of the multiplication of the two numbers first given, the one into the other, shall be equal to the superficial numbers which are produced of the multiplication of the number not divided into every part of the number divided.

Demonstration Suppose that there be two numbers A B and C. And divide the number A B into certain other numbers, how many soever, as into A D, D E, and E B. Then I say that the superficial number which is produced of the multiplication of the number C into the number A B, is equal to the superficial numbers which are produced of the multiplication of the number C into the number A D, and of C into D E, and of C into E B. For let F be the superficial number produced of the multiplication of the number C into the number A B, and let G H be the superficial number produced of the multiplication of C into A D: And let H I be produced of the multiplication of C into D E: and finally, of the multiplication of C into F B, let there be produced the number I K: Now forasmuch as A B multiplying the number C produced the number F: Therefore the number C measureth the number F by the unities which are in the number A B. And by the same reason may be proved that the number C doth also measure the number G H, by the unities which are in the number A D, and then it doth measure the number H I, by the unities which are in the number D F; and finally, that it measureth the number I K by the unities which are in the number E B. Wherefore the number C measureth the whole number G K by the unities which are in the number A B. But it before measured the number F by the unities which are in the number A B, wherefore either of these numbers F and G K is equimultiplex to the number C: But numbers which are equimultiplices to one and the self same numbers, are equal the one to the other [a]: Wherefore the number F is equal to the number G K: But the number F is the superficial number produced of the multiplication of the number C into the number A B: and the number G K is composed of the superficial numbers prodnced of the multiplication of the number C not divided into every one of the numbers A D, D E, and E B: If therefore there be two numbers given, and the one of them be divided, &c. Which was to be demonstrated.

a) Def. 6.

PROPOSITION II.

If a number given, be divided into two other numbers: the su-

ſuperficial numbers, which are produced of the multiplication of the whole into either part, added together, are equal to the ſquare number of the whole number given.

A....4 C..2 B
D.....36
B.....24 F.....12

Demonſtration Suppoſe that the number given be A B: and let it be divided into two other numbers A C and C B. Then I ſay that the two ſuperficial numbers, which are produced of A B into A C, and of A B into B C, thoſe two ſuperficial numbers (I ſay) being added together, ſhall be equal to the ſquare number produced of the multiplication of the number A B into it ſelf. For let the number A B multiplying it ſelf produce the number D. Let the number A C alſo multiplying the number A B, produce the number E F: Again, let the number C B multiplying the ſelf ſame number A B, produce the number F G: Now foraſmuch as the number A C multiplying the number A B, produced the number E F: therefore the number A B meaſureth the number E F by the unites which are in A C. Again, foraſmuch as the number C B multiplyed the number A B, and produced the number F G; therefore the number A B meaſureth the number F G, by the unites which are in the number C B: But the ſame number A B before meaſured the number E F by the unites which are in the number A C: Wherefore the number A B meaſureth the whole number E G, by the unites which are in A B. Farther, foraſmuch as the number A B multiplying it ſelf, produced the number D: therefore the number A B meaſureth the number D by the unites which are in himſelf. Wherefore it meaſureth either of theſe numbers; namely, the number D, and the number E G, by the unities which are in himſelf. Wherefore how Multiplex the number D, is to the number A B, ſo Multiplex is the number E G to the ſame number A B. But numbers which are equimuliplices to one and the ſelf ſame number, are equal the one to the other. Wherefore the number D is equal to the number E G. And the number D is the ſquare number made of the number A B, and the number E G is compoſed of the two ſuperficial numbers produced of A B into B C, and of B A into A C. Wherefore the ſquare number produced of the number A B, is equal to the ſuperficial numbers produced of the number A B into the number B C, and of A B into A C, added together: If therefore a number be divided into two other numbers, &c. Which was to be demonſtrated.

PROPOSITION III.

A....4 C..2 B
D.....12
E.....8 F....4 G

If a number given, be divided into two numbers: the ſuperficial number which is produced of the multiplication of the whole into one of the parts, is equal to the ſuperficial number which is produced of the parts the one into the other, and to the ſquare number produced of the aforeſaid part.

Demon-

Demonſtration Suppoſe that the number given be A B, which let be divided into two numbers A C and C B. Then I ſay that the ſuperficial number which is produced of the multiplication of the number A B into the number B C, is equal to the ſuperficial number which is produced of the multiplication of the number A C into the number C B, and to the ſquare number produced of the number C B. For let the number A B multiplying the number C B, produce the number D. And let the number A C multiplying the number C B, produce the number E F: and finally, let the number C B multiplying himſelf produce the number F G. Now foraſmuch as the number A B multiplying the number C B, produced the number D. Therefore the number C B meaſureth the number D, by the unites which are in the number A B. Again, foraſmuch as the number A C multiplying the number C B, and produced the number E F; therefore the number C B meaſureth the number E F, by the unites which are in A C. Again, foraſmuch as the number C B multiplied it ſelf and produced the number F G: therefore the number C B meaſureth the number F G by the unites which are in it ſelf.

A....4 C..2 B
D.....12
E.....8 F....4 G

But as we have before proved, the ſelf ſame number C B meaſureth alſo the number E F by the unites which are in the number A C; wherefore the number C B meaſureth the whole number E G, by the unites which are in the number A B. And it alſo meaſureth the number D by the unites which are in the number A B. Wherefore the number C B equally meaſureth either number, namely, the number D, and the number E G. But thoſe numbers whom one and the ſelf ſame number meaſureth equally, are equal the one to the other. Wherefore the number D is equal to the number E G. But the number D is a ſuperficial number produced of the multiplication of the number A B into the number B C, and the number E G is the ſuperficial number produced of the multiplication of the number A C into the number C B, and of the ſquare of the number C B. Wherefore the ſuperficial number produced of the multitiplication of the number A B into the number C B, is equal to the ſuperficial number produced of the number A C into the number C B, and to the ſquare of the number C B. If therefore a number be divided into two numbers, the ſuperficial number, &c. Which was required to be proved.

PROPOSITION IV.

If a number given be divided into two numbers, the ſquare number of the whole, is equal to the ſquare numbers of the parts, and to the ſuperficial number which is produced of the multiplication of the parts the one into the other twice.

Demonſtration Suppoſe that the number given be A B; which let be divided into two numbers A C and C B. Then I ſay, that the ſquare number of the whole number A B, is equal to the ſquares of the parts; that is, to the ſquares of the numbers A C and C B, and to the ſuperficial number produced of the multiplication of the numbers A C

and C B, one into the other twice. Let the ſquare number produced of the multiplication of the whole number A B, into himſelf be D. And let C A multiplyed into himſelf produce the number E F: And C B multiplyed into it ſelf let it produce G H: and finally, of the multiplication of the numbers A C and C B, the one into the other twice, let there be produced either of theſe ſuperficial numbers F G and H K. Now foraſmuch as the number A C multiplying it ſelf produced the number E F: therefore the number A C meaſureth the number E F by the unites which are in it ſelf. And foraſmuch as the number C B multiplyed the number C A,

A.....6 C..2 B
D.....64
E.....36 F.....12 G....4 H.....12 K

and produced the number F G: therefore the number A C meaſureth the number F G by the unites which are in the number C B. But it before alſo meaſured the number E F by the unites which are in it ſelf. Wherefore the number A B multiplying the number A C, produceth the number E G. And therefore the number E G is the ſuperficial number produced of the multiplication of the number B A into the number A C. And by the ſame reaſon may we prove that the number G K is the ſuperficial number produced of the multiplication of the number A B into the number B C. Farther the number D is the ſquare of the number A B. But if a number be divided into two numbers, the ſquare of the whole number is equal to the two ſuperficial numbers which are produced of the multiplication of the whole into either the parts, (by the ſecond Theorem.) Wherefore the ſquare number D is equal to the ſuperficial number E K. But the number E K is compoſed of the ſquares of the numbers A C and C B, and of the ſuperficial number which is produced of the multiplication of the number A C & C B the one into the other twice: and the number D is the ſquare of the whole number A B. Wherefore the ſquare number produced of the multiplicatiō of the number A B into its ſelf is equal to the ſquare numbers of the parts, that is, to the ſquare numbers of the numbers A C and C B, and to the ſuperficial number produced of the multiplication of the numbers A C and C B, the one into the other twice: If therefore a number given be divided into two numbers, &c. Which was required to be proved.

PROPOSITION V.

A....4 C..2 D..2 B
E.....16
F.....12 G....4 H
K....4 L....4 M....4 N....4 X

If an even number be divided into two equal parts, and again, alſo into two unequal parts, the ſuperficial number which is produced of the multiplication of the unequal parts the one into the other, together with the ſquare of the number ſet between the parts, is equal to the ſquare of half the number.

Demonſtration Suppoſe that A B be an even number, which let be divided into two equal numbers A C and C B, and into two unequal numbers A D and D B. Then I ſay, that the ſquare number which is produced of the multiplication of the half number C B into it ſelf, is equal to the ſuperficial number produced of the multiplication of the unequal

unequal numbers A D and D B, the one into the other, and to the ſquare number produced of the number C D, which is ſet between the ſaid unequal parts. Let the ſquare number produced of the multiplication of the half number C B into it ſelf be E. And let the ſuperficial number produced of the multiplication of the unequal numbers A D and D B, the one into the other, be the number F G; and let the ſquare of the number D C which is ſet between the parts be G H. Now foraſmuch as the number B C is divided into the numbers B D and D C; therefore the ſquare of the number B C, that is, the number E, is equal to the ſquares of the numbers B D and D C, and to the ſuperficial number which is compoſed of the multiplication of the numbers B D and D C; the one into the other twice, (by the fourth Propoſition of this Book:) Let the ſquare of the number B D be the number K L; and let N X be the ſquare of the number D C: and finally, of the multiplication of the numbers B D and D C, the one into the other twice, let be produced either of theſe numbers L M and M N. Wherefore the whole number K X is equal to the number E. And foraſmuch as the number B D multiplying it ſelf produced the number K L; therefore it meaſureth it by the unites which are in it ſelf. Moreover, foraſmuch as the number C D multiplying the number B D, produced the number L M; therefore alſo D B meaſureth L M by the unites which are in the number C D: but it before meaſured the number K L by the unites which are in it ſelf. Wherefore the number D B meaſureth the whole number K M by the unites which are in C B. But the number C B is equal to the number C A. Wherefore the number D B meaſureth the number K M by the unites which are in C A. Again, foraſmuch as the number C D multiplying the number D B produced the number M N: therefore the number D B meaſureth the number M N, by the unites which are in the number C D: but it before meaſured the number K M by the unites which are in the number A C. Wherefore the number B D meaſureth the whole number K N by the unites which are in the number A D. Wherefore the number F G is equal to the number K N. For numbers which are equimultiplices to one and the ſelf-ſame number, are equal the one to the other. But the number G H is equal to the number N X: for either of them is ſuppoſed to be the ſquare of the number C D. Wherefore the whole number K X is equal to the whole number F H: But the number K X is equal to the number E. Wherefore alſo the number F H is equal to the number E. And the number F H is the ſuperficial number produced of the multiplication of the numbers A D and D B the one into the other, together with the ſquare of the number D C. And the number E is the ſquare of the number C B. Wherefore the ſuperficial number produced of the multiplication of the unequal parts A D and D B, the one into the other, together with the ſquare of the number D C, which is ſet between thoſe unequal parts, is equal to the ſquare of the number C B, which is the half of the whole number A B: If therefore an even number be divided into two equal parts, &c. Which was required to be proved.

PROPOSITION VI.

If an even number be divided into two equal numbers, and unto it be added ſome other numbers, the ſuperficial number which is made of the multiplication of the number compo-

ſed of the whole number and the number added, into the number added, together with the ſquare of the half number, is equal to the ſquare of the number compoſed of the half and the number added.

A...3 C...3 B....4 D
E.....49
F.....40 G.....9 H
K.....16 L.....12 M.....12 N.....9 X

Demonſtration SUppoſe that A B be an even number, and let it be divided into two equal numbers A C and C B: and unto it let there be added an other number B D. Then I ſay, that the ſuperficial number produced of the multiplication of the number A D into the number D B, is equal to the ſquare of the number C D. For let the ſquare number of the number C D be the number E, and let the ſuperficial number produced of the multiplication of the number A D into the number D B, be the number F G; and finally, let the ſquare number of C B be the number G H: And foraſmuch as the ſquare of the number C D is (by the 4th Propoſition) equal to the ſquares of the numbers D B and B C together, with the ſuperficial number which is produced of the multiplication of the numbers D B and B C, the one into the other twice. Let the ſquare of the number B D be the number K L: and let the ſuperficial numbers produced of the multiplication of the numbers D B and B C, the one into the other twice be either of theſe numbers L M and M N: and finally, let the ſquare of the number B C be the number N X. Wherefore the whole number K X ſhall be equal to the ſquare of the number C D: But the ſquare of the number C D is the number E. Wherefore the number K X is equal to the number E. And foraſmuch as the number B D multiplying it ſelf, produced the number K L: therefore the number B D meaſureth the number K L, by the unites which are in it ſelf, but it alſo meaſureth the number L M by the unites which are in the number C B. Wherefore the number D B meaſureth the whole number K M by the unites which are in the number C D. The number D B alſo meaſureth the number M N by the unites which are in the number C B: and the number C B is equal to the number C A by ſuppoſition. Wherefore the number D B meaſureth the whole number K N by the unites which are in the number A D. But the number D B doth alſo meaſure the number F G, by the unites which are in the number A D: for by ſuppoſition, the number F G is the ſuperficial number produced of the multiplication of the numbers A D and D B, the one into the other. Wherefore the number F G is equal to the number K N. But the number H G is equal to the number N X: for either of them is the ſquare number of the number C B. Wherefore the whole number F H is equal to the number K X, and the number K X is proved to be equal to the number E. Wherefore the number F H ſhall alſo be equal to the number E. And the number F H is the ſuperficial number produced of the multiplication of the numbers A D and D B, the one into the other, together with the ſquare of the number C B, and the number E is the ſquare of the number C D: Wherefore the ſuperficial number produced of the multiplication of the numbers A D and D B, the one into the other, together with the ſquare of the number C B, is equal to the ſquare of the number C D: If therefore an even number, &c. Which was to be demonſtrated.

PRO-

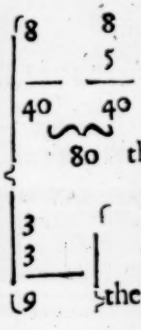
PROPOSITION VII.

If a number be divided into two numbers: the ſquare of the whole number together with the ſquare of one of the parts, is equal to the ſuperficial number produced of the multiplication of the whole number into the foreſaid part twice, together with the ſquare of the other part.

A.....5 C...3 B

Demonſtration SUppoſe that the number A B be divided into the numbers A C and C B. Then I ſay, that the ſquare numbers of the numbers B A and A C are equal to the ſuperficial number produced of the multiplication of the number B A into the number A C twice, together with the ſquare of the number B C. For foraſmuch as (by the 4th. Prop. of this Book,) the ſquare of the number A B is equal to the ſquares of the numbers B C and C A, and to the ſuperficial number produced of the multiplication of the numbers B C and C A, the one into the other twice: add the ſquare of the number A C common to them both. Wherefore the ſquare of the number A B, together with the ſquare of the number A C is equal to two ſquares of the number A C, and to one ſquare of the number C B, and alſo to the ſuperficial number produced of the multiplication of the numbers B C and C A, the one into the other twice. And foraſmuch as the ſuperficial number produced of the multiplication of the numbers B A and C A, the one into the other once, is equal to the ſuperficial number produced of the multiplication of B C into C A once, and to the ſquare of the number C A (by the third Propoſition of this Book): therefore the number produced of the multiplication of B A into A C twice, is equal to the number produced of the multiplication of B C into C A twice, and alſo to two ſquares of the number C A. Add the ſquare number of B C common to them both. Wherefore two ſquares of the number A C, and one ſquare of the number C B, together with the ſuperficial number produced of the multiplication of B C into C A twice, are equal to the ſuperficial number produced of the multiplication of the number B A into the number A C twice together, with the ſquare of the number C B. Wherefore the ſquare of the number A B, together with the ſquare of the number A C is equal to the ſuperficial number produced of the multiplication of the number B A into the number A C twice, together with the ſquare of the number C B. If therefore a number be divided into two numbers, &c. Which was required to be demonſtrated.

8		64	the ſquare of the whole A B.
8		25	the ſquare of the part A C.
64	the ſquare of the whole	89	
5		80	the ſuperficial number.
5		9	the ſquare of the other part B C
25	the ſquare of the part	89	



80 the ſuperficial number produced of the multiplication of the whole into the part twice.

9 the ſquare of the other part.

PROPOSITION VIII.

If a number be divided into two numbers, the ſuperficial number produced of the multiplication of the whole into one of the parts four times, together with the ſquare of the other parts, is equal to the ſquare of the number compoſed of the whole number and the foreſaid part.

A.....6 C..2 B..2 D

Demonſtration Suppoſe that the number A B be divided into two numbers A C and C B. Then I ſay, that the ſuperficial number produced of the multiplication of the number A B into the number C B four times, together with the ſquare of the number A C, is equal to the ſquare of the number compoſed of the numbers A B and C B. For unto the number B C let the number B D be equal. Now foraſmuch as the ſquare of the number A D is equal to the ſquares of the numbers A B and B D, and to the ſuperficial number produced of the multiplication of the numbers A B and B D, the one into the other twice, (by the 4th. Prop. of this Book.) And the number B D is equal to the number B C: therefore the ſquare of the number A D is equal to the ſquares of the numbers A B and B C, and to the ſuperficial number produced of the multiplication of the numbers A B and B C, the one into the other twice. But the ſquares of the numbers A B and B C are equal unto the ſuperficial number produced of the multiplication of the numbers A B and B C, the one into the other twice, and to the ſquare of A C (by the eighth Propoſition.) Wherefore the ſquare of the number A D is equal to the ſuperficial number produced of the multiplication of the numbers A B and B C, the one into the other four times, and to the ſquare of the number A C. But the ſquare of the number A D is the ſquare of the number compoſed of the numbers A B and B C: for the number B D is equal to the number B C. Wherefore the ſquare of the number compoſed of the numbers A B and B C, is equal to the ſuperficial number produced of the multiplication of the numbers A B and B C, the one into the other four times, and to the ſquare of the number A C. If therefore a number be divided into two numbers, &c. Which was to be demonſtrated.

8 8 8 8
2 2 2 2
16 16 16 16

64 the ſuperficial number produced of the multiplication of the numbers AB and BC, the one into the other four times.

6
6
36 the ſquare of AC.

10
10
100 the ſquare of the number compoſed of AB and BC.

64 the ſuperficial nũber produced of the multipl. made 4 times.
36 the ſquare number of AC.
100

PROPOSITION IX.

If a number be divided into two equal numbers, and again be divided into two unequal parts: the ſquare numbers of the unequal numbers, are double to the ſquare which is made of the multiplication of the half number into it ſelf, together with the ſquare which is made of the number ſet between them.

A.....5 C...3 D..2B

Demonſtration FOr let the number AB being an even number be divided into two equal numbers AC and CB: and into two unequal numbers AD and DB. Then I ſay that the ſquare numbers of AD and DB are double to the ſquares which are made of the multiplication of the numbers AC and CD into themſelves. For, foraſmuch as the number AB is an even number, and is divided alſo into two equal numbers AC and CB, and afterward into two unequal numbers AD and DB: therefore the ſuperficial number produced of the multiplication of the numbers AD and DB, the one into the other, together with the ſquare of the number DC, is equal to the ſquare of the number AC, (by the fifth propoſition.) Wherefore the ſuperficial number produced of the multiplication of the numbers AD and DB, the one into the other twice, together with two ſquares of the number CD, is double to the ſquare of the number AC. Foraſmuch as alſo the number AB is divided into two equal numbers AC and CB, therefore the ſquare number of AB is quadruple to the ſquare number produced of the multiplication of the number AC into it ſelf (by the 4th. Propoſition) Moreover foraſmuch as the ſuperficial number produced of the multiplication of the numbers AD and DB, the one into the other twice together, with two ſquares of the number DC, is double to the ſquare number of CA: and foraſmuch

as

as there are two numbers, of which the one is quadruple to one and the ſelf-ſame number, and the other is double to the ſame number: therefore that number which is quadruple, ſhall be double to that number which is double. Wherefore the ſquare of the number A B is double to the number produced of the multiplication of the numbers A D and D B, the one into the other twice together, with the two ſquares of the number D C. Wherefore the number which is produced of the multiplication of the numbers A D and D B, the one into the other twice, is leſſe than half of the ſquare of the number A B, by the two ſquares of the numbers D C. And foraſmuch as the number produced of the multiplication of the numbers A D and D B, the one into the other twice, together with the number compoſed of the ſquares of the numbers A D and D B is (by the fourth Propoſition) equal to the ſquare of the number A B: therefore the number compoſed of the ſquares of the numbers A D and D B is greater than the half of the ſquare number of A B, by the two ſquares of the number D C. And the ſquare of the number A B is quadruple to the ſquare of the number A C. Wherefore the number compoſed of the ſquares of the numbers A D and D B is greater than the double of the ſquare of the number A C by two ſquares of the number D C. Wherefore the ſaid number is double to the ſquares of the numbers A C and C D: If therefore a number be divided, &c. Which was to be demonſtrated.

8 8 — 64 the ſquare of the unequal part A D.

2 2 — 4 the ſquare of the unequal part B D.

5 5 — 25 the ſquare of the half A C.

3 3 — 9 the ſquare of C D, namely, of the number ſet between.

25 9 — 34 the ſquares of the half, and of the number ſet between.

64 4 — 68 the ſquares of the unequal parts.

68 | 2
34

PROPOSITION X.

If an even number be divided into two equal numbers, and unto it be added any other number: the ſquare number of the whole number compoſed of the number and of that which is added, and the ſquare number of the number added: theſe two

ſquare

ſquare numbers (I ſay,) added together, are double to theſe ſquare numbers; namely, to the ſquare of the half number, and to the ſquare of the number compoſed of the half number, and of the number added.

A...3 C...3 B..2 D

Demonſtration Suppoſe that the number A B being an even number, be divided into two equal numbers A C and C B: and unto it let be added an other number B D. Then I ſay, that the ſquare numbers of the numbers A D and D B, are double to the ſquare numbers of A C and C D. For, foraſmuch as the number A D is divided into the numbers A B and B D: therefore the ſquare numbers of the numbers A D and D B are equal to the ſuperficial number produced of the multiplication of the numbers A D and D B, the one into the other twice, together with the ſquare of the number A B, (by the ſeventh Propoſition.) But the ſquare of the number A B is equal to four ſquares of either of the numbers A C or C B (for AC is equal to the number C B): wherefore alſo the ſquares of the numbers A D and D B are equal to the ſuperficial number produced of the multiplication of the numbers A D and D B, the one into the other twice, and to four ſquares of the number B C or C A. And foraſmuch as the ſuperficial number produced of the multiplication of the numbers A D and D B the one into the other, together with the ſquare of the number C B, is equal to ſquare of the number C D, (by the ſixth Propoſition): therefore the number produced of the multiplication of the numbers A D and D B, the one into the other twice together, with two ſquares of the number C B, is equal to two ſquares of the number C D. Wherefore the ſquares of the numbers A D and D B are equal to two ſquares of the number C D, and to two ſquares of the numbers A C. Wherefore they are double to the ſquares of the numbers A C and C D. And the ſquare of the number A D is the ſquare of the whole and of the number added: And the ſquare of D B is the ſquare of the number added: the ſquare alſo of the number C D is the ſquare of the number compoſed of the half and of the number added: If therefore an even number be divided, &c. Which was required to be proved.

8 8 — 64 the ſquare of A D. 2 2 — 4 the ſquare of D B

5 5 — 25 the ſquare of C D, namely, of the number compoſed of the half and of the number added.

3 3 — 9 the ſquare of the half A C.

25 9 — 34 64 4 — 68 68 34 | 2.

The End of the ten firſt Propoſitions according to Barlaam.

THE THIRD ELEMENT OF EUCLIDE.

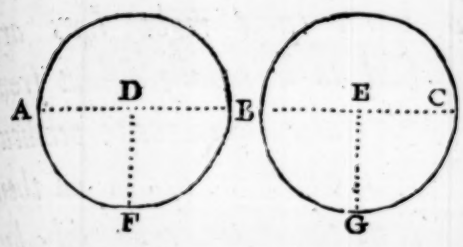
THE ARGUMENT.

THis Third Book of *EUCLIDE* treateth of the circle, which of all figures is the moſt perfect, the properties and paſſions whereof are herein ſet forth and demonſtrated. Here alſo is declared the nature and property of lines applyed to circles, either touching or cutting one another, either within or without the circle, as alſo of right lined angles drawn in the ſegments of circles, &c. With diverſe uſeful Propoſitions applicable to diverſe neceſſary and profitable uſes.

DEFINITIONS.

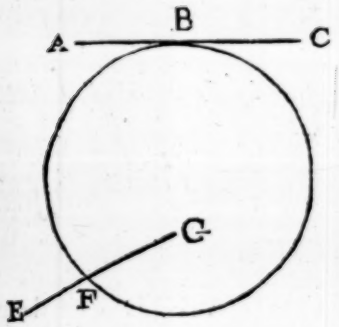
1 *Equal Circles are ſuch whoſe diameters are equal, or whoſe right lines drawn from the centers to the circumferences, are equal.*

FOraſmuch as *EUCLIDE* demonſtrateth in this Third Book diverſe properties of Circles, he unfoldeth in the firſt place certain terms, whoſe uſe is frequent therein: he ſhewes therefore firſt of all, that thoſe circles are equal whoſe diameters or ſemidiameters are equal; for ſeeing the circle is deſcribed by the revolution of the ſemidiameter about one



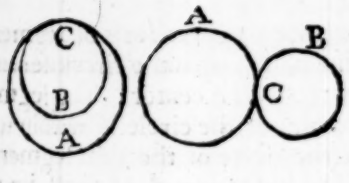
one of its extremities fixed and unmoveable, as is ſhewn in the Firſt Book: It is manifeſt that thoſe circles are equal, whoſe ſemidiameters or right lines, drawn from the centers to the circumferences are equal: as if the diameters A B and B C are equal, or the right lines D F and E G drawn from the centers D and E, the circles A F B and B G C ſhall be equal; and contrarily, if the circles be equal, the diameters or right lines drawn from the centers to the circumferences, ſhall be equal.

Whence it appeares, that the circles (whoſe diameters or right lines drawn from the centers to the circumferences are unequal,) are unequal; and therefore that circle that hath the greater Diameter, is alſo the greateſt circle; and contrarily of unequal circles, the diameters are unequal.

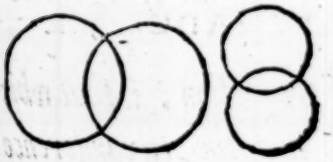


2 *A right line is ſaid to touch a circle, which touching it, if it be prolonged, doth not cut it.*

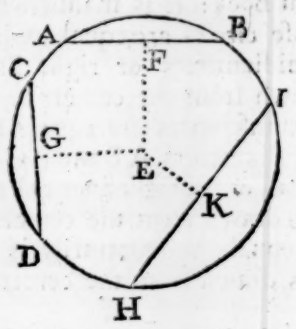
AS A B ſhall be ſaid to touch the circle, if it ſo touch it at B, as that being prolonged to C, doth not cut it; But foraſmuch as E F doth ſo touch touch the circle in the point F, that being prolonged to the point G, it cutteth it, and falls within, it ſhall not be ſaid to touch the circle, but to cut it.



3 *Thoſe circles are ſaid to touch one another, which in touching one another, do not interſect or cut one another.*

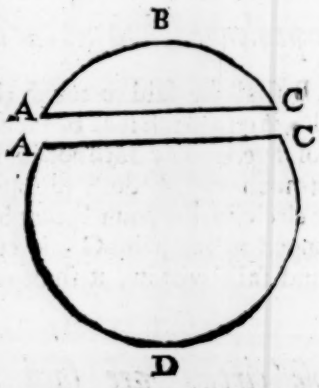


AS the two circles A C and B C are ſaid to touch at C, if they touch in ſuch ſort as that they cut not one another, be the one within the other or without it: But if two circles touch one another otherwiſe, they ſhall be ſaid to cut one another.



4 *In a circle right lines are ſaid to be equally diſtant from the center, when the perpendiculars which are drawn on them from the center, are equal; but that is ſaid to be farther diſtant from the center, on which the greateſt perpendicular doth fall.*

AS the two right lines A B and C D in the circle A B C D ſhall be ſaid to be equally diſtant from the center, if the perpendiculars E F and E G be equal: But the line C D ſhall be ſaid to be more diſtant from the center than the line H I, if the perpendicular E G be greater than the perpendicular E K.



5 *A Segment, or Section of a circle is a figure compriſed under a right line, and the circumference of the circle.*

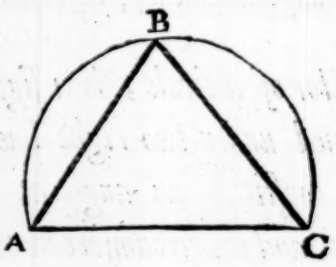
IF in the circle A B C D, let there be drawn a right line by chance as A C, as well the figure A B C conteined under the right line A C, and the circumference A D C ſhall be ſaid to be a Segment, or Section of a circle.

From whence may be gathered, that there are three ſorts of ſegments, to wit, the ſemicircle, when the right line which with the circumference, conſtituteth the ſegment, doth paſſe through the center: The ſegment greater than the ſemicircle, when the center of the circle is within it, as A D C: and the leaſt ſegment is when the figure of the ſaid ſegment is leſſe than the ſemicircle, as A B C. The right line A C ſhall be alſo called a Chord, and the circumference A B C or A D C an Arch.

6 *The angle of the ſegment, or of the ſection, is that which is compriſed under a right line, and the circumference of a circle.*

EUCLIDE now defineth three kinds of angles which are conſidered in circles. In the firſt place the angle of the ſegment; as is the mixt angle B A C or B C A, conteined under the right line A C, and the circumference A B C, and if the ſegment be a ſemicircle, it ſhall be called the angle of the Semicircle, if the ſegment be greater, it ſhall be termed

termed the angle of the greater segment: if lesse than the semicircle, it shall be termed the angle of the lesser segment.

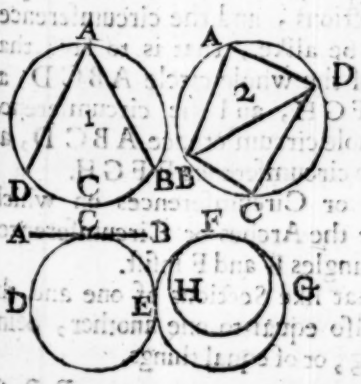


7 *But an angle is in a segment or in a section, when there is taken some point in the circumference of the segment, and from it are drawn two right lines, on the extremities of the right line, which is the base of the segment, and is that (I say,) which is conteined under those drawn right lines.*

LEt A B C be a segment of a circle, and let the right line A C be the base, and from any point as B taken in the circumference, draw the two right lines A B and B C, to the points A and C, which are the extremities of the base A C, the right lined angle A B C conteined of them, is said to be an angle in a segment.

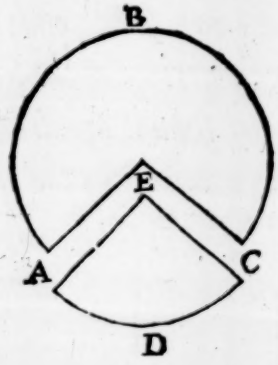
8 *But when the right lines which do contein the angle, do take or receive some circumference, the angle is said to insist upon it.*

AS the angle D A B conteined of the two right lines A B and A D, which do receive the circumference B C D, shall be said to be, or to insist thereon, and differeth not from the afore-going Definition, but only in the term; For if you draw a right line from B to D for the base, it shall be said to be in the segment, according to the precedent Definition, and by this, to rest or insist on the circumference B C D: Nevertheless, it refers to divers things, for the angle in the segment refers to the segment in which it is; and the angle insisting on the circumference refers it self to the circumference which is its base: Therefore if you take some segment of a circle, as B C D, of the circle A B C D, the angle which is in this segment, shall not be the same with that which insisteth or resteth upon it, for the angle which is therein shall be B C D: But the angle which insisteth or resteth on the same circumference, shall be the angle B A D, which is different from it.



Besides, these three sorts of angles aforesaid, the Geometricians do consider also an angle of contingence, which is conteined of a right line, touching the circle, and of the circumference of a circle

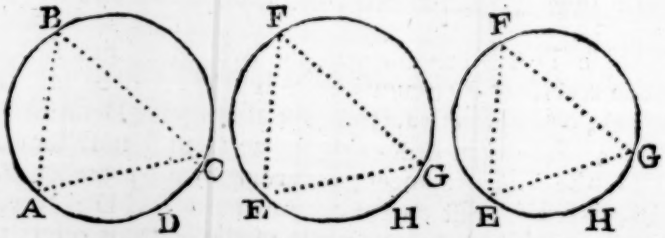
circle, or of two circumferences, touching one another either without or within, as for example, if the right line AB touch the circle CED in the point C, the mixt angle ACD or BCE ſhall be termed the angle of contingence: In like manner, the curvilined angle GFH or EFH, made of the circumferences of two circles touching one another, ſhall be alſo termed the angle of contingence.



9 *A Sector of a circle, is a figure conteined under two right lines, which conſtitute an angle at the center, and the circumference compriſed between the ſame lines.*

AS the figure AECD, conteined under the right lines AE and EC, which make the angle at the center E, and under the circumference ADC, compriſed between A and C, the extremities of the ſame lines, is termed a Sector of a circle.

10 *Like Segments or Sections of Circles, are thoſe which receive equal angles, or in which the angles are equal to one another.*



AS if in the circles ABCD and EFGH, the right lined angles B and F in the Sections ABC and EFG, are equal, thoſe Sections which do receive thoſe ſaid equal angles, or in which thoſe equal angles are, ſhall be ſaid to be like Sections, and the circumferences ABC and EFG in like manner ſhall be alike, that is to ſay, that the Section ABC ſhall be ſuch part of the whole circle ABCD, as the Section EFG is of the circle EFGH, and the circumference ABC ſhall be the ſame part of the whole circumference ABCD, as the circumference EFG is of the whole circumference EFGH.

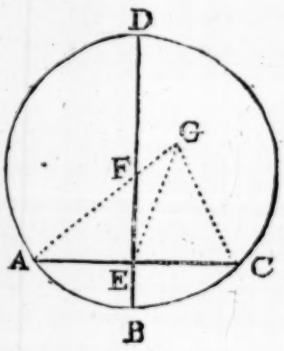
By the ſame reaſon, the Arches or Circumferences on which equal angles inſiſt, are alſo alike, as are the Arches or Circumferences ADC and EHG, on which the equal angles B and F inſiſt.

From theſe things may be gathered that like Sections of one and the ſame circle, or of equal circles, are alſo equal to one another, being the ſame parts of one and the ſame thing, or of equal things.

PRO-

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. PROBLEM 1.



A circle ABCD *being given, to find the center* F.

Conſtruction DRaw any line therein at adventure, as AC, [a] which may be divided in two equal parts in the point E, and by E draw BD at right angles, terminating on both ſides of the circumference in the points B and D, and let it be divided into two equal parts in the point F: I ſay, that F is the center of the propoſed Circle. a) 10. 1.

Demonſtration FOr no other point therein can be the center, conſidering that every other point doth divide it unequally: Therefore if F be not the center of the circle, let G be the center thereof without the line BD, from which draw the right lines GA, GE, and GC.

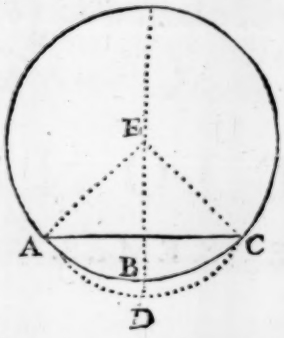
Foraſmuch as, in the triangle AEG, the two ſides AE and EG are equal to the two ſides CE and EG of the triangle CEG, and the baſe AG equal to the baſe CG, (for that they are ſaid to be drawn from the center,) [b] the angles AEG and CEG ſhall be equal; and therefore right angles. But the angle AEF is a right angle by conſtruction; therefore the right angles AEF and AEG ſhall be equal, the part to the whole, which is abſurd: Therefore G is not the center of the circle; nor by the ſame reaſon, is any other point out of the line DB: Therefore F ſhall be the center: Therefore we have, &c. Which was to be done. b) 8. 1.

COROLLARIE.

From tis Propoſition it is evident that if in a circle a right line be divided into two equal parts, and at right angles, by another right line, the center of the circle ſhall be in that dividing line, provided it end and terminate on both ſides of the circumference, or at leaſt, that it be not leſſe than the ſemidiameter of the circle, or that the divided line doth paſſe by the center, conſidering that it is demonſtrated that

that the center of the circle A B C D, *cannot be other-where then in the point* F *of the line* B D, *which divideth* A C *into two equal parts, and at right angles.*

PROP. 2. THEOR. 1.



If in the circumference of a circle A B C, *there be taken two points at pleaſure* A *and* C, *the right line* A C *joyning thoſe points* A *and* C, *will fall within the circle* A B C.

Demonſtration FOr if it fall not within the circle, let it (if poſſible,) fall without ſuch as is the right line
a) 1. 3. A D C, (as the contradictor will have it,) and having [a] found the center thereof E, draw the right lines E A, E C, and E D, to ſome point as at D, of the line A D C, in ſuch ſort as that it cut the circumference of the circle A B C in the point B.

Foraſmuch as the two ſides E A and E C of the triangle E A C, to which the line A D C is put for baſe, are equal, (being drawn from the
b) 5. 1. center,) [b] the angles E A D and E C D ſhall be equal. But the exteriot
c) 16. 1. [c] angle E D A is greater than its interior and oppoſite angle E C D: ſeeing that the ſide C D is prolonged to the point A. Therefore the ſame an-
d) 19. 1. gle E D A [d] ſhall be alſo greater than the angle E A D, which is equal to E C D: Therefore E A oppoſite to the greateſt angle E D A, ſhall be greater than E D, oppoſite to the leaſt E A D: Therefore E B equal to E A ſhould be alſo greater than E D, the part than the whole, which is abſurd: Therefore the right line drawn from A to B, will not fall without the circle, but within it: In like manner, we might ſhew that it will not fall on the circumference: Therefore, If in a circle, &c. Which was to be demonſtrated.

COROLLARIE.

From the Demonſtration of this ſecond Propoſition it is manifeſt, that the right line which toucheth the circle, in ſuch ſort as that it divide not the ſaid circle, touch-
e) 2. 3. *eth it only in one point, for if it touch it in two points,* [e] *the part of the line between thoſe points would fall within the circle; and therefore would divide it, which is contrary to Suppoſition.*

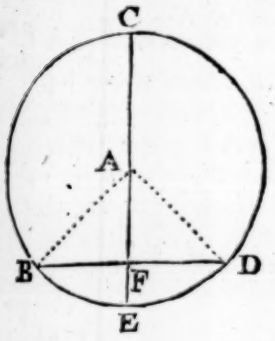
PROP. 3. THEOR. 2.

If in a circle B C D E, *ſome right line* E C *paſſing by the center* A, *doth cut ſome other right line* B D, *which doth not paſſe by the center* A, *into two equal parts in* F, *it will alſo cut it at right angles, and if it divide it at right angles, it will alſo divide it into two equal parts.*

Demonſtration FOr having drawn the two lines A B and A D, the two ſides A F and F B of the triangle A F B, are equal to the two ſides A F and F D of the triangle A F D, and the baſes A B and A D are

are equal [a]: Therefore the angles A F B and A F D ſhall be equal; that is to ſay right angles, which was firſt propoſed. [b]

Now let C E be at right angles to B D: I ſay that B D is divided in two equal parts in the point F, by the right line C E; for the two lines A B and A D, being equal, [c] the two angles A B D and A D B ſhall be equal on the baſe B D, of the Iſoſceles triangle A B D: Foraſmuch therefore as the two angles A F B and A B F of the triangle A B F, are equal to the two angles A F D and A D F of the triangle A D F, and that the ſides A B and A D oppoſite to the right angles in the point F, are in like manner equal: [d] the ſides F D and F B ſhall be equal, which was in the ſecond place propoſed: Therefore, If in a Circle, &c. Which was to be demonſtrated.



This ſecond part may be thus demonſtrated: Let A F be at right angles to B D, the [e] ſquare of A B ſhall be equal to the two ſquares of A F and F B; in like manner, the ſquare of A D ſhall be equal to the two ſquares of A F and F D: but the ſquares of A B and A D are equal, ſeeing the ſame lines are equal: Therefore the ſquares of A F and F B ſhall be equal to the ſquares of A F and F D: Wherefore taking away the common ſquare A F, the ſquares of F B and F D will remain equal, and therefore their ſides F D and F B ſhall be equal.

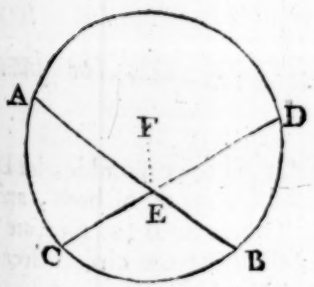
a) 8. 1.
b) 10. def.
c) 5. 2.
d) 26. 1.
e) 47. 1.

COROLLARIE

Hence it follows that in every Iſoſceles or Equilateral triangle, the line which ſhall divide the baſe into two equal parts, ſhall be perpendicular to the baſe: provided it be drawn from the angle oppoſite to the baſe on the baſe, otherwiſe the concluſion would be nothing: For a line may divide another line into two equal parts, and yet nevertheleſſe, not be at right angles to the divided line; and contrarily, the line perpendicular to the baſe drawn from the oppoſite angle, will divide it into two equal parts: For ſeeing that, in the triangle B A D, A F *doth divide* B D *into two equal parts, it hath been demonſtrated that the angles in the point* F *are right angles, and for that the angles in the point* F *are right angles, it hath been ſhewn that* A F *doth divide* B D *in two equal parts.*

PROP. 4. THEOR. 3.

If in a circle A B C D, *be drawn two right lines* A B *and* C D, *dividing one another in* E, *being not drawn by the center* F, *they will not divide one another into two equal parts.*



Demonſtration FOr if one of them paſſe by the center, it is manifeſt, that it ſhall not be divided in two equal

parts, for it can only be divided in two equal parts in the center, by which center the other line is propoſed not to paſſe, but if neither the one nor the other paſſe by the center, although that the one of them may ſometimes be divided into two equal parts by the other, the other nevertheleſſe ſhall not be divided into two equal parts.

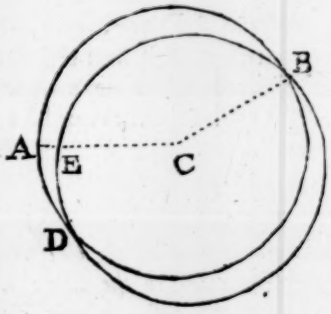
For let them (if poſſible) divide one another in two equal parts, in ſuch ſort as A E may be equal to E B, and C E to E D: and [a] having found the center F of the ſaid Circle, from the ſaid center draw F E to the point E, where they interſect.

a) 1. 3.

b) 3. 3.

Foraſmuch as F E paſſing by the center divides A B, [b] not paſſing by the center into two equal parts in E, it will divide it at right angles: Therefore the angle F E B ſhall be a right angle, by the ſame reaſon it ſhall alſo divide C D at right angles: Therefore the angle F E D ſhall be alſo a right angle, and therefore equal to the angle F E B, the part to the whole, which is impoſſible: Therefore they ſhall not divide one another into equal parts: Therefore, If in the circle, &c. Which was to be demonſtrated.

PROP. 5. THEOR. 4.



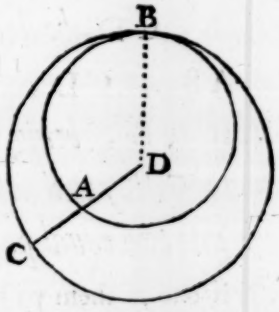
If two circles A B D *and* E B D *do divide one another in* B *and* D, *they ſhall not have the ſame center.*

Demonſtration OTherwiſe, if they have one and the ſame center: Let C be the center of both (if it may be) and joyn C B to the point of Section, and draw C E A, dividing the circumferences in the points E and A.

Foraſmuch as C is the center of A B D, the line C A ſhall be equal to C B. Again, foraſmuch as C is the center of the circle B E D, the line C E ſhall be equal to C B: But C A hath been ſhewn equal to C B: therefore [a] C A and C E ſhall be equal, the part and the whole, which is impoſſible: Therefore C is not the center of the two circles: Therefore, If two circles, &c. Which was to be demonſtrated.

a) 1. c. 5.

PROP. 6. THEOR. 5.



If two circles A B *and* C B, *do touch one another within* B, *they will not have one and the ſame center* D.

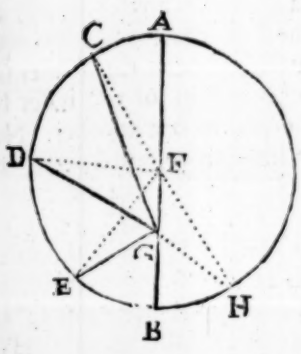
Demonſtration FOr (if it be poſſible, let D be the center of both, and joyn to it the right line D B (at pleaſure) the line D A C dividing the circumferences in the points A and C.

Foraſmuch as D is the center of the circle B C, the line D C ſhall be equal

equal to D B. Again, foraſmuch as D is alſo the center of the circle A B, the line D A ſhall be equal to D B, But D C is demonſtrated alſo to be equal to D B: therefore [a] D A and D C ſhall be equal, the part to the whole, which is impoſſible: Therefore the point D is not the center of both the circles A B and C B: Therefore, If two circles, &c. Which was to be demonſtrated.

a) 1. c. ſ.

PROP. 7. THEOR. 6.



If in the diameter A B *of the circle* A C D E B, *ſome point* G *be taken, which is not the center of the circle, and from this point* G *to the circumference* A C D E B, *there fall right lines* G C, G D, G E, *the greateſt line ſhall be* G A, *that line wherein the center* F *is; but the leaſt ſhall be* G B, *that which remaines (to wit, of the diameter) of the others* G C *and* G A, *alwayes the neereſt* G C, *to that* G A *which paſſeth by the center* F, *is greater than* G D, *that which is more remote; and from this ſame point* G *there will only fall two equal right lines* G E *and* G H, *to the circumference on both ſides of the leaſt line* G B.

Demonſtration DRaw from the center F to the points C, D, and E, the right lines F C, F D, and F E, foraſmuch as the two ſides G F and F C of the triangle G F C, are greater than the other G C: But G F and F C [a] are greater than the other ſide G C, but G F and F C are equal to G F and F A; that is to ſay, to the whole G A, the ſame G A ſhall be alſo greater than G C, and by the ſame reaſon greater than G D, and ſo of the others: Therefore G A ſhall be the greateſt of all thoſe which fall from the point G to the circumference.

a) 20. 1.

Secondly, Foraſmuch as in the triangle E F G, the ſide E F is [b] leſſe than the two ſides E G and G F: but E F is equal to F B; therefore F B ſhall be leſſe than the two ſides F G and G E: Therefore the common part F G being taken away, there will yet remain G B, leſſe than G E, and by the ſame reaſon G B ſhall be ſhewn to be leſſe than any of the others G D and G C: Therefore G B is the leaſt of all thoſe which are drawn from G to the circumference.

b) 20. 1.

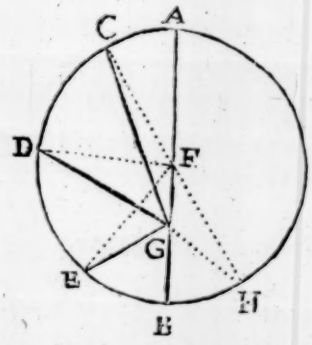
Thirdly, Foraſmuch as the two ſides G F and F C of the triangle G F C, are equal to the two ſides G F and F D of the triangle G F D, and the angle G F C is greater than the angle G F D, the whole than the part, [c] the baſe G C ſhall be greater than the baſe G D; by the ſame

c) 24. 1.

reaſon G C ſhall be greater than G E; therefore the line which is neereſt to that which paſſeth by the center, is greater than that which is more remote.

Laſtly, make the angle B F H on the other ſide of the line F B, equal to the angle B F E, and draw the right line G H: foraſmuch as the two ſides E F and F G of the triangle E F G are equal to the two ſides H F and F G of the triangle H F G, each to his correſpondent ſide, and that the angles E F G and H F G conteined of thoſe ſides are equal by the conſtruction, d the right lines G E and G H, on both ſides of the leſſer line G B, ſhall be equal to one another. Now that no other line can be equal to either of them is manifeſt; for if from the point G you draw another line on that ſide whereon H is, it will fall below H, and therefore is is leſſe than G H, as being more remote from that which paſſeth by the center, as hath been demonſtrated: and therefore alſo is leſſe than G E, equal to G H: If it paſſe above G H, it ſhall be greater by the ſame reaſon: Therefore there cannot be drawn two right lines equal to one another, but only on each ſide of the leaſt line G B: Therefore, If in the diameter, &c. Which was to be demonſtrated.

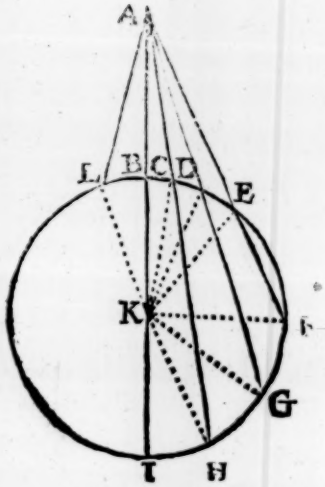
d) 4. 1.



COROLLARIE.

It follows that from every point taken in the Circle, there can be drawn only two right lines to the circumference equal to one another: foraſmuch as that from every point taken in a circle, there can be drawn but one diameter.

PROP. 8. THEOR. 7.



If without the circle B C D E *there be taken ſome point* A, *and from that point there be drawn right lines to the circle, one of which* A I *paſſeth by the center* K, *and the others* A H, A G, A F *at pleaſure; of all the right lines vvhich do fall on the concave circumference, the greateſt* A I, *is that vvhich paſſeth by the center* K, *but of the others alvvayes the neereſt to that vvhich paſſeth by the center, ſhall be greater than that vvhich is more remote: But of thoſe vvhich*

which fall on the convex circumference, the leaſt is that which is put between the point A *and the diameter* BI, *and of the others, that which is the neareſt to the leaſt* AB, *is alwayes leſſe than that which is more remote, and from this point* A *can be drawn only two right lines equal to one another, to the circle on both ſides of the leaſt line.*

Demonſtration 1. FOr from the center K draw the right lines KC, KD, KE, KF, KG, KH: foraſmuch as the two ſides AK and KH of the triangle AKH [a] are greater than AH. But AK and KH are equal to AK and KI, that is to ſay, to the whole AKI, the right line AI ſhall be alſo greater than AH, by the ſame reaſon, AI ſhall be alſo greater than each of the others AG and AF: Therefore AI is the greateſt of all thoſe which are drawn from the point A into the circle. a) 20. 1.

Secondly, foraſmuch as the ſides AK and KH of the triangle AKH, are equal to the two ſides AK and KG of the triangle AKG, and that the whole angle AKH is greater than the angle AKG its part, [b] the baſe AH ſhall be greater than the baſe AG, by the ſame reaſon, AH ſhall be greater than AF: In like manner, AG ſhall be greater than AF: Therefore the line next to that which paſſeth by the center is greater than that which is more remote from the center. b) 24. 1.

Thirdly, foraſmuch as in the triangle ACK the right line AK [c] is leſſe than the two others AC and CK: if the equal lines BK and CK be cut off, there will yet remain AB leſſe than AC: by like reaſon, AB ſhall be leſſe than AD, and leſſe alſo than AE: Wherefore AB is the leaſt of all the lines drawn from the point A without the circle, to the convex circumference. c) 20. 1.

Fourthly, foraſmuch as in the triangle ADK there do fall the two right lines AC and CK, comming from the extremities of the ſide AK, [d] the ſame lines AC and CK ſhall be leſſe than AD and DK: Therefore having taken away the equal lines CK and DK, there will yet remaine AC, leſſe than AD, and in like manner, AC ſhall be leſſe than AE; and AD alſo leſſe than AE: Therefore the line neereſt to the leaſt line AB, is leſſe than that which is more remote. d) 21. 1.

In the laſt place; Make the angle AKL equal to the angle AKD, and draw AL: Foraſmuch as the two ſides AK and KD of the triangle AKD, are equal to the two ſides AK and KL of the triangle AKL: But the angles AKD and AKL conteined of thoſe ſides are equal; [e] the right lines AD and AL on both ſides of the leaſt line AB ſhall be equal to one another, and no other line drawn from the point A to the ſaid circumference, ſhall be equal to any of them, for it would fall above AL towards B, and therefore leſſe than AL; that is to ſay, than AD, equal to AL; or on the other ſide of AL, and therefore greater, as hath been demonſtrated: Therefore there can be only drawn two equal right lines to the circumference on both ſides of the leaſt line AB: Therefore, If you take &c. Which was to be demonſtrated. e) 4. 1.

PROP.

PROP. 9. THEOR. 8.

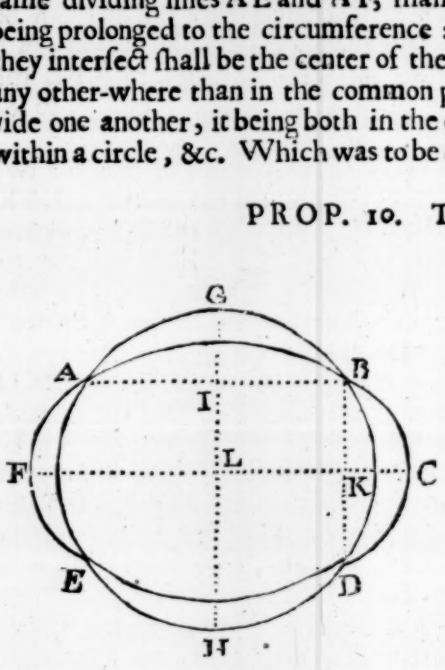


If within a circle BCD, *there be taken a certain point* A, *and from that point* A *there do fall more than two equal right lines* AB, AC, *and* AD, *to the circumference: the point taken* A, *is the center of the circle.*

Demonſtration JOyn the right light lines BC and CD, which lines ſhall be divided in two equal parts in the points E and F, and by the point A draw the right lines AE and AF, and prolong them on both ſides to the circumference: Foraſmuch then as EB is equal to EC, and EA common, the two ſides AE and EB of the triangle AEB ſhall be equal to the two ſides AE and EC of the triangle AEC, and the baſe AB is equal to the baſe AC, by ſuppoſition; [a] the angle AEB ſhall be equal to the angle AEC; [b] and therefore are both right angles, by the ſame reaſon it ſhall be ſhewn that the angles at the point F are right angles: Therefore ſeeing that the right lines AE and AF do divide the right lines BC and CD in two equal parts, and at right angles; in each of the ſame dividing lines AE and AF, ſhall be found the center of the circle, being prolonged to the circumference: Therefore the point A in which they interſect ſhall be the center of the [c] circle, it being not to be found any other-where than in the common point where they interſect and divide one another, it being both in the one and the other: Therefore, If within a circle, &c. Which was to be demonſtrated.

a) 8. 1.
b) 10. def.
c) Cor. 1. 3.

PROP. 10. THEOR. 9.



A circle ABCDEF, *doth not divide another circle* AGBDHE *in more than two points.*

Demonſtration LEt the circle ABCDEF divide the circle AGBDHE in more than two points, A, B, and D, (if poſſible,) and having joyned the right lines AB and BD, divide them in two equal parts in the points I and K, and by thoſe points [a] draw IL and KL at right angles to the lines AB and BD, and let them be prolonged on both ſides to the circumference.

a) 11. 1.

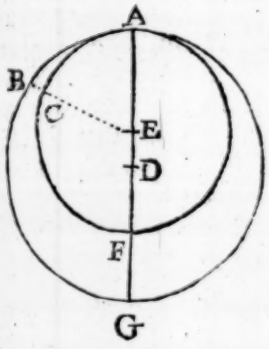
For-

Foraſmuch as the right lines I L and K L do divide the right lines A B and B D in two equal parts, and at right angles in the circle A G B D H E, both the one and the other ſhall paſſe [b] by the center thereof: Therefore the point L by which point they interſect, ſhall be the center of the ſaid circle: In the ſame manner, we might demonſtrate that the point L is the center of the circle A B C D E F. Therefore two circles cutting one another ſhall have one center, [c] which is impoſſible, &c. Therefore &c. Which was to be demonſtrated.

b) Cor. 1. 3.

c) 5. 3.

PROP. 11. THEOR. 10.



If two circles A B G *and* A C F *do touch one another within, (at the point* A,*) and that their centers* E *and* D *be taken, the right line* D E *joyning thoſe centers* E *and* D *being prolonged, will fall on the point of touching* A, *of thoſe circles.*

Demonſtration OTherwiſe it ſhall fall on one ſide or the other of the point of touching: Let it then fall (if poſſible) elſewhere, as at B, cutting the circumferences in the points of C and B, and let B E D be directly prolonged, (which is ſaid to be a right line,) from the other ſide of the circumference in the point G, and ſo B E D G ſhall be the diameter of the circle A B G, and from the point E to the point of touching A, draw E A; foraſmuch then as in the diameter G E B of the circle A B G, there is taken the point E, without the center D, the line E B ſhall be the leaſt [a] of all thoſe that are drawn from the point E, to the circumference of the circle A B G; and therefore leſſe than E A, but E C is equal to E A: Seeing that E C and E A are drawn from the center E of the circle A C F to the circumference A C: Therefore E B ſhall be alſo leſſe than E C, the whole than its part: Therefore D E being prolonged, can fall no other-where then in the point of touching A, as is the line D E A: Therefore, If two circles, &c. Which was to be demonſtrated.

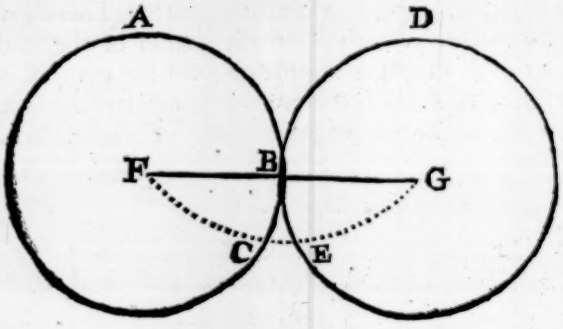
a) 7. 3.

PROP. 12. THEOR. 11.

If two circles A B C *and* D B E *touch one another without in* B, *the right line* F G *drawn from one center* F, *to the other* G, *ſhall paſſe by the point of touching* B.

Demonſtration FOr draw the right line F B, and the right line G B, both to the line of touching B, thoſe two lines do make one only right line: Therefore the right line drawn from F to G, doth paſſe by B: For if it be not ſo, they will make an angle on one ſide or the other of B: Let them then make an angle (if poſſible,) towards G E, and draw from F to G a right line F C E G dividing the circumferences in the points C and

and E: Foraſmuch then as the two ſides BF and BG of the triangle BFG: (for FB and GB are propoſed to make an angle in the point B, and FCEG is a right line, (as the contradictor willhave it,) are [a] greater than the other ſide FCEB: But FC is equal to FB, and GE [b] equal to GB, being drawn from the centers; therefore FC & GE, which are no other then a part of the whole FCEG, ſhall be alſo greater than the ſame FCEG, the part than the whole, (ſeeing that FCEG conteineth yet CE; beſides EC and GE,) which is impoſſible; Therefore if two circles, &c. Which was to be demonſtrated.

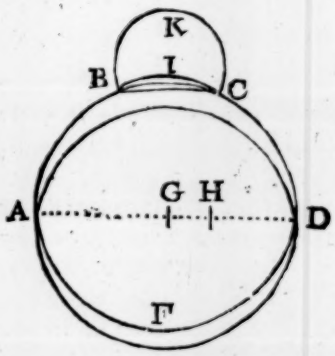


a) 20. 1.

b) 15. def. 1.

PROP. 13. THEOR. 12.

A circle ABCD, *doth not touch a circle* ADF, *in more than one point, whether it touch it without or within.*



Demonſtration LEt the two circles ABCD and ADF, touch one another within, (if it be poſſible,) in more than one point, to wit, in the points A and D: and let there be taken the centers of thoſe circles G and H, [a] which ſhall be different. Therefore the line drawn by the [b] centers G and H prolonged both wayes, ſhall fall on the points of touching A and D, as AGHD: and foraſmuch as G is the center of the circle ABCD, the line AG ſhall be equal to GD; therefore AG ſhall be greater than HD, being equal to GD the greateſt; therefore AH ſhall be yet much grea er than HD: Again, foraſmuch as H is the center of the circle AFD, the line AH ſhall be equal to HD; but it hath been demonſtrated to be much greater (which is impoſſible:) Therefore a circle toucheth not another circle in more than one point within.

a) 6. 3.

b) 11. 3.

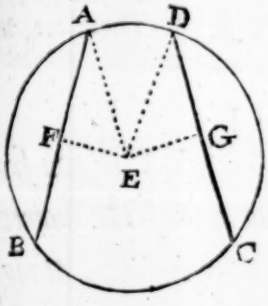
I ſay alſo that it toucheth it not without: For (if it be poſſible,) Let the circle BCK touch the circle ABCD without, in more than one point, to wit, in B and C, and that the right line BC be drawn, it ſhall fall within one of the circles, and therefore without the other circle, which is contrary to the ſame ſecond Propoſition of this Book: Therefore, a circle toucheth not, &c. Which was to be demonſtrated.

PROP. 14. THEOR 13.

In a circle ABCD, *the equal right lines* AB *and* CD, *are*

are equally distant from the center E, *and those that are equally distant from the center are equal to one another.*

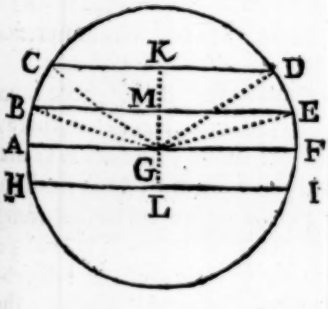
Demonstration FOr from the center E draw EF and EG perpendicular to AB and CD, and joyn EA and ED: the lines EF and EG shall divide [a] AB and CD in two equal parts, which being put equal, their halves AF and DG shall be equal. a) 3. 3.



Demonstration FOrasmuch as the squares of the equal lines EA and ED are equal: and [b] that the square of EA is equal to the squares of AF and FE: In like manner, the square of ED is equal to the squares of DG and GE, the squares of AF and FE shall be likewise equal to the squares of DG and GE; therefore having taken away the equal squares of the equal right lines AF and DG, the square of FE and GE will remain equal: Therefore EF and EG shall be equal: Therefore [c] AB and CD shall be equally distant from the center E. b) 47. 1. c) 4. def. 3.

Again, Let AB and CB be equally distant from the center E: I say that they are equal to one another; for (the construction made as before,) we shall shew that EF and EG [d] are equal, and that they divide [e] AB and CD in two equal parts, and the squares of the equal right lines EA and ED are equal to one another: But the square of AE is [f] equal to the two squares of AF and FE, and that of ED also equal to the two squares of DG and GE: therefore the squares of AF and FE are equal to the squares of DG and GE; having then taken away the equal squares of the equal lines EF and EG: there will remain the squares of AF and DG equal, and therefore their doubles AB and CD are also equal: Therefore, in a circle, &c. Which was to be demonstrated. d) 4. def. 3. e) 3. 3. f) 47. 1.

PROP. 15. THEOR. 14.



In a circle ABCD, *the greatest line is the diameter* AF, *but of the others* CD, *and* HI, *always that* HI *which is nearest to the center* G, *is greater than* CD, *that which is more remote.*

Demonstration 1. FOr draw from the center G the perpendiculars GK and GL to CD and HI: and forasmuch as CD is more remote from the center than HI, the perpendicular GK shall be greater [a] than GL: Therefore let GM be cut off from the line GK equal to GL; and by M draw BME, perpendicular to GK, and joyn the right lines GB, GC, GD, and GE. a) 4. def. 3.

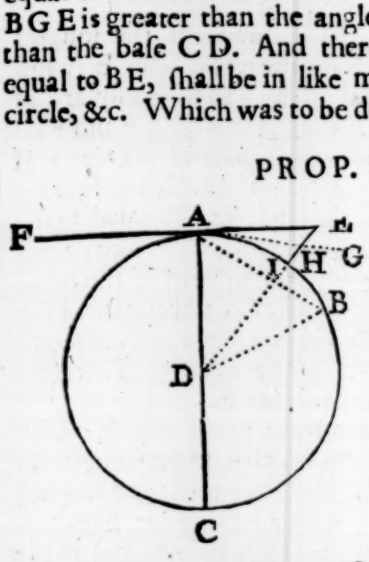
Forasmuch as the perpendiculars G M and G L are equal, B E and H I shall be equally distant from the center, and therefore equal to one another; Again, forasmuch as G B and G E are b greater than B E, but are equal to the diameter A F, the diameter A F shall be also greater than B E; by the same reason it shall be shewn that A F is the greatest of all the other lines.

b) 10. 1.

Lastly, forasmuch as the sides G B and G E of the triangles B G E are equal to the sides G C and G D of the triangle C G D; and the angle B G E is greater than the angle C G D, c the base D E shall be greater than the base C D. And therefore H I which hath been shewn to be equal to B E, shall be in like manner greater than C D: Therefore, In a circle, &c. Which was to be demonstrated.

c) 24. 1.

PROP. 16. THEOR. 15.



The right line E F *drawn from the extremitie* A, *of the diameter* A C *of the circle* A B C, *at right angles to the same diameter, shall fall without the circle* A B C, *in the space comprised between the same right line* E F, *and the circumference* A B C, *and there shall not fall another right line* G A, *and the angle of the semicircle* D A B *is greater than any acute right lined angle: But the rest* E A H *is lesse.*

Demonstration IF it be said that E F doth not fall without the circle: Let it (if possible,) fall within, as the line A B, and draw D B;

a) 5. 1. b) 17. 1.

a the angles D A B and D B A shall be equal; but D A B is a right angle: therefore D B A shall be also a right angle, which is absurd; b seeing that the two angles of a triangle are lesse than two right angles: In like manner we shall shew that it will not also fall on the circumference; therefore it shall fall without at A E.

I say now, that between A E and the circumference B A, there cannot fall another right line drawn from the point A: For let A G fall there (if possible,) and so then the angle D A G shall be acute, and on A G draw D H perpendicular to A G, cutting the circumference in I, D H shall only fall on the side of the acute angle D A G. Forasmuch c as in the triangle D A H d the two angles D H A and D A H are lesse than two right angles, and D H A is a right angle, by the construction, D A H shall be lesse than a right angle; and therefore D A, that is to say, D I equal thereto; shall be e greater than D H, the part than the whole, which is impossible: Therefore, not any right line shall fall between the circumference A B, and the right line perpendicular A E, but will cut the circle.

c) Co. 17. 1. d) 17. 1. e) 19. 1.

I say, Lastly, that the angle of the semicircle conteined of the diameter A C, and the circumfereece B A is greater than any right lined acute angle,

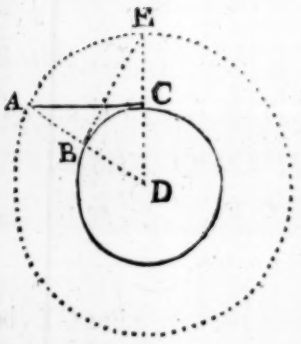
angle: for seeing it hath been shewn that every right line drawn from the point A, below the perdendicular A E, doth fall within the circle, it will of necessity make with the line A C a right lined acute angle, lesse than the angle of the semicircle: But with A E it will constitute a right lined acute angle, greater than the angle of contingence, considering that the angle of contingence is a part of the angle of the semicircle; but this acute angle is some whole, in respect of the angle of contingence, which is manifest, the line A B being drawn at adventure, below A E: for seeing that the same A B doth fall within the circle, as hath been demonstrated; the right lined acute angle C A B is lesse than the angle of the semicircle, conteined under the diameter A C, and the circumference A B C: Seeing that the acute angle is part of the angle of the semicircle: But the angle of contingence conteined under the tangent A E, and the circumference A B C is lesse than the right lined angle B A E, seeing that it is a part thereof: Therefore the angle of the semicircle is greater than any right lined acute angle, and the remainder the angle of contingence lesse: Therefore, the line, &c. Which was to be demonstrated.

COROLLARIE.

It is manifest from hence, that the right line drawn from the extremity of the diameter at right angles, doth touch the circle; for it hath been demonstrated that it falls without the circle: Wherefore it reacheth the circle in this point (the extream of the diameter) only.

As if from the point A *in the circumference of the circle* A B C, *there were to be drawn a line which should touch the circle in the point* A, *we shall draw a line from* A *to the center* D, *and to that draw another*, A E *at right angles, and that line shall touch the circle in the point* A, *as hath been demonstrated.*

PROP. 17. PROBL 2.



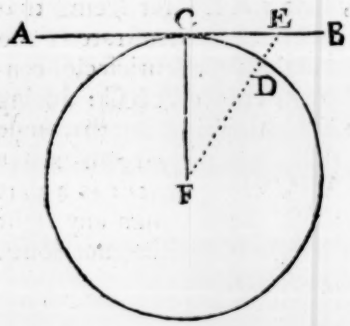
From a given point A, *to draw a right line* A C, *which toucheth a given circle* B C.

Construction FInd the center D, and joyne the line A D, which divideth the circle in the point B, and on the center D, with the distance D A, describe the circle A E, and from the point B draw B E at right angles to A D, dividing the circle A E in the point E; and lastly, draw D E, dividing the circle B C in the point C, and joyn C A: I say, that A C doth touch the given circle in the point C.

Demonstration FOr seeing that the two sides B D and D E of the triangle B D E are equal to the two sides C D and D A of the triangle C D A, each to his correspondent, as appears by the Definition of a circle, and the angle D conteined of those sides is common, the [a] bases B E and C A, and the angles D B E and D C A shall be equal. But D B E is a right angle by Construction: Therefore D C A shall be also a right angle: Therefore seeing that C A is drawn from the extremity C, of a) 4. 1.

b) Co.16.3. C, of the ſemidiameter D C at right angles, it will [b] touch the circle: Therefore from the given point A, &c. Which was to be done.

PROP. 18. THEOR. 16.



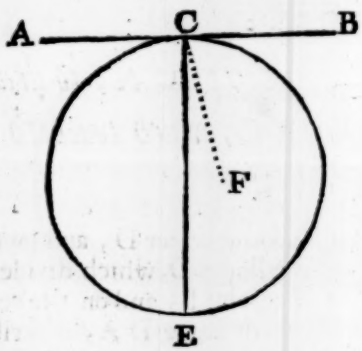
If ſome right line A B, *doth touch a circle* C D, *and from the center* F *to the point of touching* C, *there be joyned ſome right line* F C, *that joyned line* F C *ſhall be perpendicular to the touch line* A B.

Demonſtration FOr if it be denyed, Draw F E perpendicular to A B, cutting the circumference in the point D:

a) 17. 1. Foraſmuch as in the triangle C F E, [a] the two angles F C E and E F C are leſſe than two right angles, and E F C is a right angle, by conſtruction,

b) 18. 1. F C E ſhall be leſſe than a right angle: Therefore [b] C F, that is to ſay, F D its equal, ſhall be greater than F E the part than the whole, which is impoſſible: Therefore F C is perpendicular to A B: Therefore if a right line, &c. Which was to be demonſtrated.

PROP. 19. THEOR. 17.



If ſome right line A B, *doth touch a circle* C E, *and from the point of touching* C, *there be drawn a right line* C E *at right angles to the line touching in that ſame line* C E, *ſhall be the center of the circle.*

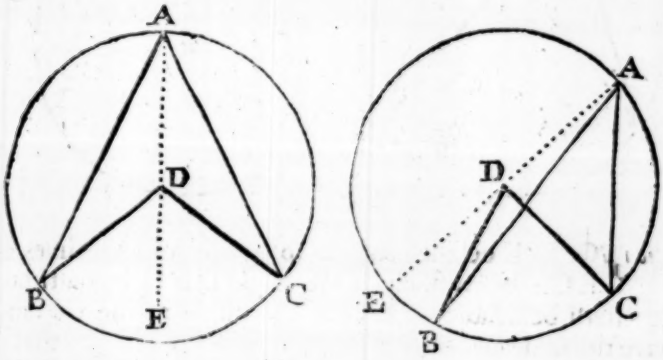
Demonſtration FOr if it be ſaid that the center is not in the line C E, but without; Let it (if poſſible,) be in the point F, from

a) 18. 1. which point F draw the right line F C, [a] which ſhall be perpendicular to A B. Therefore the right angle F C B ſhall be equal to the right angle E C B, the part to the whole, which is impoſſible: Therefore the center of the circle ſhall not be out of the line C E: Therefore, If a right, &c. Which was to be demonſtrated.

PROP. 20. THEOR. 18.

In a circle A B C, *the angle* B D C, *which is at the center* D,

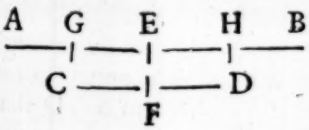
D, *is double to the angle* BAC, *which is at the circumference, when they have for their base one and the same circumference* BC.



Demonstration JOyn AD together, and prolong it to E; forasmuch as the two lines DA and DB are equal, DAD and DBA shall [a] be equal: But the exterior angle BDE is equal [b] to the two interior angles DAB and DBA; and therefore double to each of them, as of DAB: In like manner, we shall shew that CDE is double to the angle DAC: Wherefore the whole angle BDC shall be double to the whole angle BAC which was proposed: Again, let the angle CAB not enclose the angle CDB, and joyn AD, prolonged to E; forasmuch as the two sides DA and DC are equal, [c] the angles DAC and DCA shall be equal, but the exterior angle CDE is equal [d] to them: Therefore it shall be double to DAC, but the angle EDB at the center, is double to EAB, as is demonstrated; Therefore the remainder BDC shall be double to the remainder BAC; [e] for when the whole is double to the whole, and the part cut off to the part cut off, the remainder shall be double to the remainder, as followeth.

a) 5. 1.
b) 32. 1.
c) 5. 1.
d) 32. 1.
e) 10. 1.

LEMMA.

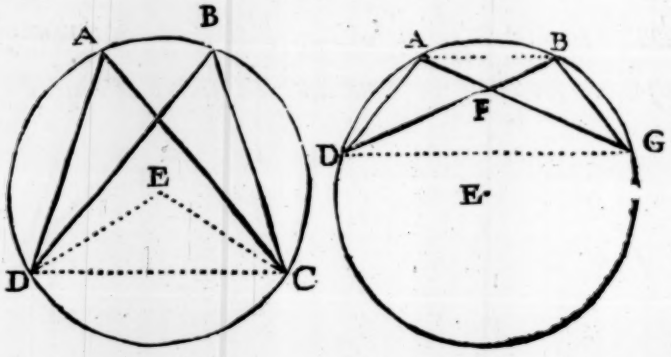


Let the whole AB *be double to the whole* CD, *and* AE *the part cut off, double to* CF *the part cut off, the rest* EB *shall be double to the rest* FD; *for seeing that* AB *is double to* CD, *being divided in two equal parts at* E, *as well* AE *as* EB, *shall be equal to* CD: *Therefore* AE *being also double to* CF, CD *equal to* AE, *shall be also double to* CF, *and therefore double to its other half* FD: *Therefore the rest* EB *equal to* CD, *shall be also double to the rest* FD: *Which was to be demonstrated.*

PROP. 21. THEOR. 19.

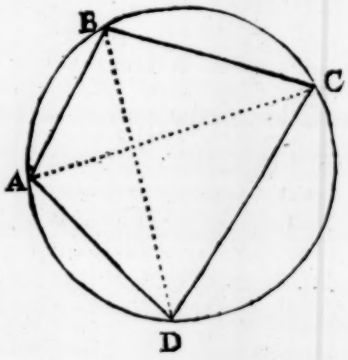
In a circle ABCD, *the angles* DAC *and* CBD, *which are in one and the same segment* CBAD, *are equal to one another.*

De-



Demonſtration FOr let E be the center, to which joyn the lines E D and E C: Foraſmuch as the angle D E C is conſtituted in the
a) 10. 3. center, it [a] ſhall be double to D A C, the angle at the circumference: for they have the ſame circumference D C for their baſe, by the ſame reaſon, it ſhall be double to the angle C B D: Therefore each of the two angles A and B, ſhall be the half of D E C, and therefore equal to one another: Therefore, In a circle, &c. Which was to be demonſtrated.

PROP. 22. THEOR. 20.



The oppoſite angles B A D *and* B C D *of quadrilateral figures* A C, *inſcribed in a circle* A B C D, *are equal to two right angles.*

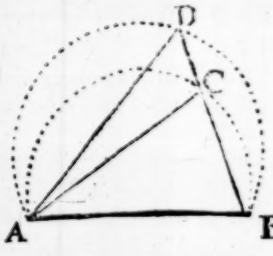
Demonſtration FOr, joyn the two diagonal lines A C and B D; foraſmuch as the two angles A C D and A B D, are in one and the ſame ſegment A B C D, they ſhall be equal to one another, by the ſame reaſon, the two angles B C A and B D A, in the ſegment B C D A, ſhall be alſo equal to one another; therefore the whole angle B C D ſhall be equal to the two angles A B D and A D B; therefore having added the common angle B A D, the two angles B C D and B A D ſhall be equal to the three angles B D A, D B A, and B A D, of the triangle
a) 32. 1. A B D, but the three angles of the triangle A B D [a] are equal to two right angles; therefore the two oppoſite angles B A D and B C D ſhall be alſo equal to two right angles: In like manner, it might be ſhewn that the two oppoſite angles at the points D and B, are equal to two right angles: Therefore the angles, &c. Which was to be demonſtrated.

PROP. 23. THEOR. 21.

On one and the ſame right line A B, *there cannot be conſtituted two ſegments of circles* A D B *and* A C B, *alike and unequal, and on the ſame ſide.*

De-

Demonſtration FOr if it be poſſible, Let two ſegments of circles be conſtituted alike and unequal: On the line A B, and on the ſame part or ſide A C B and A D B, thoſe ſegments will only cut one another in the points A and B; Seeing [a] that a circle cuts not another circle in more than two points: Therefore the circumference of one of the ſegments ſhall be without the circumference of the other. Draw then the right line B C D, cutting the circumference in the points C and D, and joyn A D and A C: Foraſmuch as the ſegments A C B and A D B are put alike ſegments by the tenth Definition of this Book, the angle A C B ſhall be equal to the angle A D B, which is abſurd, [b] for the exterior angle A C B is greater than the interior A D B, they are not then like ſegments: Therefore on one and the ſame, &c. Which was to be demonſtrated.

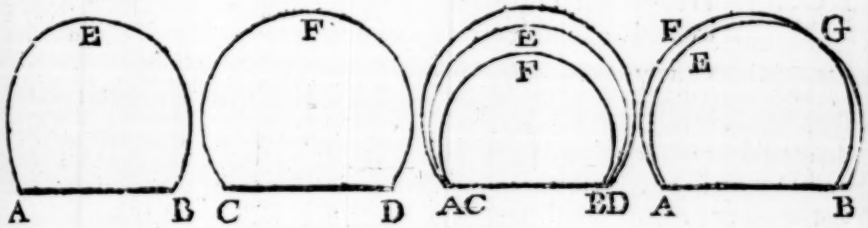


a) 10. 3.

b) 16. 1.

PROP. 24. THEOR. 22.

Like ſegments of circles A E B *and* C F D, *conſtituted on equal right lines* A B *and* C D *are equal to one another.*



Demonſtration FOr the lines A B and C D being equal, they will agree to one another, if it be underſtood that they be placed the one on the other, to wit, A B on C D, and the ſegment A E B will agree with the ſegment C F D: otherwiſe the ſegment A E B ſhall fall without the ſegment C F D, or elſe within it, or a part without, and a part within; and therefore on one and the ſame right line are conſtituted like ſegments of circles, [a] which is abſurd: For if part fall without, and part within, they ſhould cut one another in more than two points, as at B and G, which is impoſſible: For a circle doth not cut another circle in more than two points: Therefore they ſhall agree, and therefore ſhall be equal to one another: Therefore like ſegments, &c. Which was to be demonſtrated.

a) 23. 3.

PROP. 25. PROBL. 3.

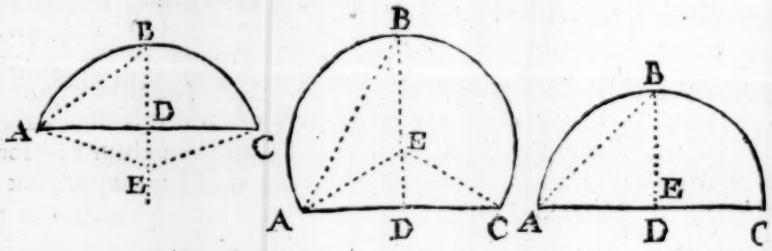
The ſegment A B C, *of a circle, being given, to deſcribe the circle of which it is a ſegment.*

Conſtruction JOyn A C, and divide it into two equal parts in the point D, by which draw D B at right angles to A C, and joyn A B: The angle D B A ſhall be greater than the angle D A B, or equal or leſſe.

Demon-

Demonſtration FIrſt of all, Let it be greater, (which will happen when the ſegment ſhall be leſſe than the ſemicircle,) for then B D prolonged, will paſſe [a] by the center, which is without the ſegment; ſeeing that it is leſſe than the ſemicircle, and [b] D A ſhall be greater than D B, for D B is the reſt of the Diameter: Therefore [c] D B A ſhall be greater than D A B: Let the angle B A E be made equal to D B A, and let the right line A E cut B D prolonged, at E: I ſay, that E is the center of the circle, of which A B C is the ſegment.

a) Co. 1. 3.
b) 7. 3.
c) 18. 1.



For having drawn E C, the two ſides A D and D E of the triangle A D E ſhall be equal to the two ſides C D and D E of the triangle C D E, and the angles conteined of them are right angles; [d] therefore A E and E C are equal, but E A is alſo equal to E B: Seeing [e] that the angles E A B and E B A are equal: Therefore the three ſides E A, E B, and E C are equal: [f] Therefore F ſhall be the center of the circle A B C; ſeeing that from that point there falls more than two equal right lines, to the circumference.

d) 4. 1.
e) 16. 1.
f) 9. 3.

Secondly, Let the two angles D B A and D A B be equal: [g] D A ſhall be equal to D B: But D C is alſo equal to D B. Seeing that A C is divided into two equal parts in the point D, all the three lines D C, D B, and D A ſhall be equal; therefore D ſhall be the center of the circle. and the ſegment propoſed ſhall be a ſemicircle.

g) 6. 1.

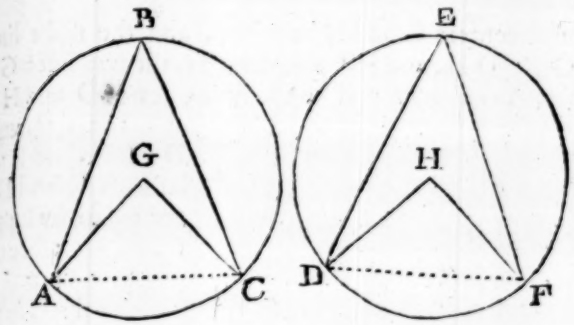
Laſtly, if the angle D B A be leſſe than the angle D A B: Let the angle B A E be made equal to the angle A B E; and let E C be joyned: I ſay that E is the center of the circle: For it will be ſhewn by the ſame reaſons as above, that the three lines E A, E B, and E C, are equal; and therefore alſo the ſegment propoſed will exceed the ſemicircle: Therefore a ſegment, &c. Which was to be done.

PROP. 26. THEOR. 23.

In equal circles A B C *and* D E F, *the equal angles* D H F *and* A G C, *conſiſt on equal circumferences* A C *and* D F, *whether they be conſtituted at the centers* G *and* H, *or on the circumferences* A C *and* D F.

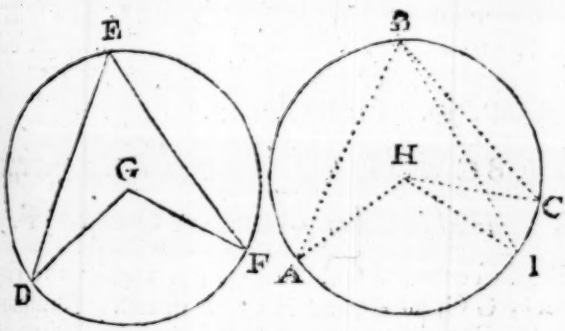
Demonſtration FOr joyn the right lines A C and D F; foraſmuch as the two ſides G A and G C of the triangle A G C, are equal to the ſides H D and H F of the triangle D H F; becauſe of the equality of the circles, and the angles G and H equal by ſuppoſition, [a] the baſes A C and D F ſhall be equal: but the angles B and E conſtituted at the cir-

a) 4. 1.



circumferences, are equal by ſuppoſition, and inſiſt on the ſame baſes: Therefore the ſegments A B C and D E F ſhall be alike [b], and therefore equal; which being ſubtracted from the equal circles, there will remaine the ſegments A C and D F equal; and therefore the circumferences A C and D F equal. Which was to be demonſtrated. b) 24. 3.

PROP. 27. THEOR 24.



In equal circles A B C *and* D E F, *the angles* A H C *and* D G F *that inſiſt on the equal circumferences* A C *and* D F, *are equal to one another, whether they be conſtituted at the centers* H *and* G, *or elſe on the circumferences* A C *and* D F.

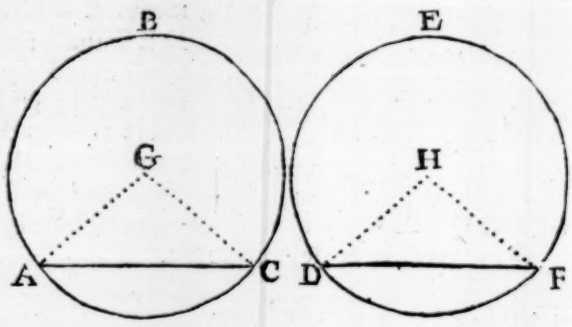
Demonſtration FOr if they be not equal: Let the angle H be the greater, and make the angle A H I equal to the angle D H F [a]: therefore the circumference A I ſhall be equal to the circumference D F: But the circumference A C is equal to the circumference D F, by ſuppoſition; therefore A I and A C ſhall be equal to one another, the part to the whole, which is impoſſible; therefore the angles A H C and D G F are equal. a) 26. 3.

Secondly, Conſtitute the angles B and E at the circumferences, having equal circumferences, A C and D F for baſes; by the ſame reaſon it might be ſhewn that they are equal to one another: Which was to be demonſtrated.

PROP. 28. THEOR. 25.

In equal circles A B C *and* D E F, *equal right lines* A C *and* D F *do take away equal circumferences* A B C *and* D E F, *to wit, the greateſt from the greateſt, and the leaſt from the leaſt.*

Demonſtration FOr let the centers G and H be taken, and the right lines A G, G C, D H, and H F, be drawn; the two ſides G A and G C of the triangle A G C are equal to the two ſides H D and H F, of the triangle D H F, and the baſes A C and D F are equal by ſuppoſition, therefore the angles [a] G and H are equal: Therefore [b] the circumferences A C and D F, on which they inſiſt are equal, which being taken from the equal circles, there will remain the circumferences A B C and D E F, alſo equal: Therefore, In circles, &c. Which was to be demonſtrated.



a) 8. 1.
b) 26. 3.

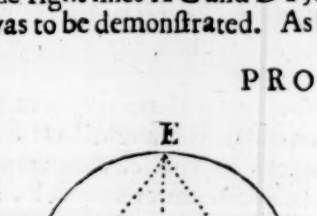
PROP. 29. THEOR. 26.

In equal circles A B C *and* D E F, *equal circumferences* A B C *and* D E F, *ſubtend equal right lines* A C *and* D F.

Demonſtration FOr, Let the centers G and H be taken, and let the right lines A G, G C, D H, and H F, be drawn: Foraſmuch as the two ſides A G and G C of the triangle A G C, are equal to the two ſides D H and H F of the triangle D H F; [a] and the angles G and H equal; ſeeing they inſtſiſt on equal circumferences, [b] the baſes which are the right lines A C and D F, are equal: Therefore in circles, &c. Which was to be demonſtrated. As by the fore-going figure is evident.

a) 27. 1.
b) 4. 1.

PROP. 30. PROBL 4.



To divide a given circumference A E C, *into two equal parts.*

Conſtruction DRaw the right line A C, which divide in two equal parts, [a] at the point D, by which point draw the right line D E, at [b] right angles to A C, and joyn the right lines A E and C E, the circumference ſhall be divided in two equal parts, in the point E.

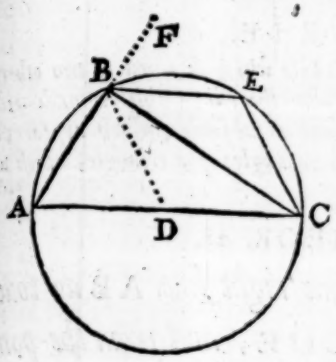
a) 10. 1.
b) 11. 1.

Demonſtration FOr ſeeing that A D and C D are equal, and D E common, the two ſides A D and D E of the triangle A D E ſhall be equal to the two ſides C D and D E of the triangle C D E; and the angles at the point D are equal, to wit, right angles; [c] the baſes A E and C E ſhall be equal; [d] and therefore the circumference A E and C E alſo equal: Therefore we have divided, &c. Which was to be done.

c) 4. 1.
d) 28. 3.

PROP.

PROP. 31. THEOR. 27.



In a circle ABC, *the angle* ABC, *which is in the ſemicircle is a right angle: But that which is in the greateſt ſegment* BCA, *is leſſe than a right angle, and that which is in the leaſt ſegment* BEC, *is greater than a right angle; And moreover, the angle* ABC *of the greateſt ſegment is greater than a right angle: But the angle* EBC *of the leaſt ſegment is leſſe than a right angle.*

Demonſtration. FOr draw BD to the center D, and prolong AB directly towards F: Foraſmuch as DA and DB are equal, the angles [a] DAB and DBA ſhall be equal to one another; by the ſame reaſon, the two angles DBC and DCB ſhall be alſo equal; for DB and DC are alſo equal to one another: Therefore the two angles DBA and DBC, that is to ſay, the whole ABC, is equal to the two angles BAC and BCA: But the exterior angle CBF [b] is equal to the ſame two interior and oppoſite angles BAC and BCA: Therefore the two angles ABC and CBF ſhall be equal to one another; and therefore right angles: Therefore the angle ABC in the ſemicircle is a right angle; which was in the firſt place propoſed. a) 5. 1. b) 32. 1.

I ſay, ſecondly, that the angle BAC in the greateſt ſegment BAC, is leſſe than a right angle: Foraſmuch as in the triangle ABC, the two angles ABC and BAC, are leſſe than two right angles, and ABC is demonſtrated to be a right angle, the angle BAC ſhall be leſſe than a right angle; which is in the ſecond place propoſed.

Thirdly, I ſay, that the angle BEC in the leaſt ſegment BEC, is greater than a right angle: Foraſmuch as in the quadrilateral figure ABEC, deſcribed in the circle, the two oppoſite angles [c] E and A are equal to two right angles, and the angles BAC is ſhewn to be leſſe than a right angle; therefore the other angle BEC ſhall be greater than a right angle; which was in the third place propoſed. c) 22. 3.

Fourthly, I ſay, that the angle of the greateſt ſegment BAC, which is the angle ABC, conteined of the circumference AB, and of the right line BC, is greater than a right angle: for ſeeing that the right angle ABC is conteined of the right lines AB and BC, and is a part thereof, the whole angle ABC, conteined of the circumference AB, and the right line BC, ſhall be greater than a right angle; which was in the fourth place propoſed.

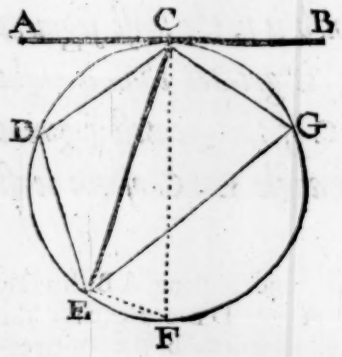
Laſtly, I ſay, that the angle of the leaſt ſegment, which is the angle CBE, conteined of the right line CB, and of the circumference BEC, is leſſe than a right angle; for that it is leſſe than the right angle FBC,

the part than the whole, which was in the laſt place propoſed, &c. Which was to be demonſtrated.

COROLLARIE.

Hence it is manifeſt that the angle of a triangle which is equal to two others, is a right angle; foraſmuch as the angle contiguous thereto, (which is made without the triangle, the ſide being prolonged,) is equal to the ſame; which appeares by the firſt Demonſtration; it is the half of the three angles of a triangle, which are equivalent to two right angles.

PROP. 32. THEOR. 28.



If ſome right line AB *do touch a circle* CDE, *and from the point of touching* C, *there be drawn ſome right line* CE, *to the circle dividing it, the angles* ACE *and* BCE, *that it makes with the touch line* ACB, *ſhall be equal to the angles which are in the alternate ſegments* CDE, *and* CGFE, (VIZ. ACE *equal to* G, *and* BCE *equal to* D.

Demonſtration. FOr, Draw the right line CF by the center, and joyn EF, a) 18. 1. the line [a] CF ſhall be perpendicular to AB, and the angle CEF ſhall be a right angle; and therefore the two remaining angles EFC and ECF ſhall be eqnal to a right angle, as to the right angle ACF: Therefore if the common angle ECF be taken away, there will remain the angle ACE, equal to the angle EFC, which is equal to the angle G: Foraſmnch as they are in one and the ſame ſegment: Therefore the angle ACE ſhall be alſo equal to the angle G.

b) 21. 3. And foraſmuch as in the quadrilateral figure DEGC, inſcribed in the circle, the two oppoſite angles D and G are equal [b] to two right angles, and the two angles ACE and BCE are alſo equal to two right angles; if the two equal angles ACE and CGE be taken away, there will remain the angle BCE, equal to the angle D, in the alternate ſegment CDE: Therefore, if a right line, &c. Which was to be demonſtrated.

PROP. 33. PROBL 5.

On a right line given AB, *to deſcribe a ſegment of a circle, which may receive an angle equal to a given right lined angle* C.

Firſt, Let the angle given C, be a right angle.

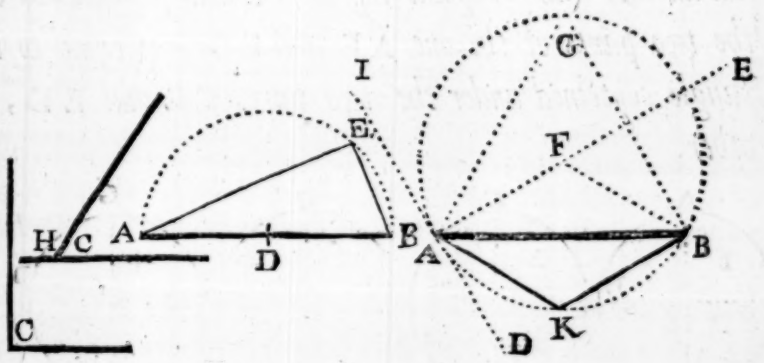
Conſtruction. DIvide the right line AB into two equal parts in the point D, and from the point D as a center, and with the diſtance DA,

D A, deſcribe the ſemicircle, in which make the angle A E B which is a right angle; and therefore [a] equal to the given right angle C. a) 31. 3.

Secondly, Let the given angle C, be acute.

Conſtruction THen from the point A, of the given right line A B, make the angle D A B equal to the acute angle C, and from the point A draw A E at right angles to A D, which ſhall fall above A B; then make the angle A B F equal to the angle F A B, and let B F cut the right line A E in the point F, [b] the lines F A and F B ſhall be equal. Therefore if from the center F, with the diſtance F A, the circle A G be deſcribed, it will paſſe by the point B: I ſay then that the angle which ſhall be made in the ſegment A G B, deſcribed on the line A B, ſhall be equal to the angle C. Therefore let the angle A G B, be made in the ſaid ſegment. b) 6. 1.

Demonſtration FOraſmuch as A E doth paſſe by the center, and that from the extremity A, there is drawn A D at right angles thereto, D A will [c] touch the circle in the point A: Wherefore the angle D A B is to ſay, the given angle C ſhall be equal to the angle G, in the alternate ſegment A G B. c) C. 16. 3.



Thirdly, Let the given angle H be obtuſe.

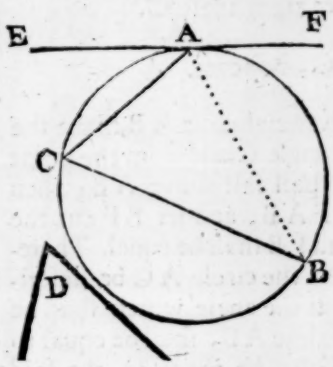
Conſtruction MAke again the angle I A B equal to the angle H, and from the point A, draw A E, at right angles to A I, which will fall above A B: finiſh the reſt as is before taught, and ſo there ſhall be deſcribed on the line A B the ſegment A K B, in which the angle K is equal to the obtuſe angle given H.

Demonſtration FOr the angle I A B, that is to ſay, the given angle H, is equal to the angle K, in the alternate ſegment A K B, for that it is the ſame Demonſtration: Therefore, On a given, &c. Which was to be done.

PROP. 34. PROBL. 6.

From a given circle A B C, *to cut off a ſegment, which may receive an angle, equal to a given right lined angle* D.

Con-



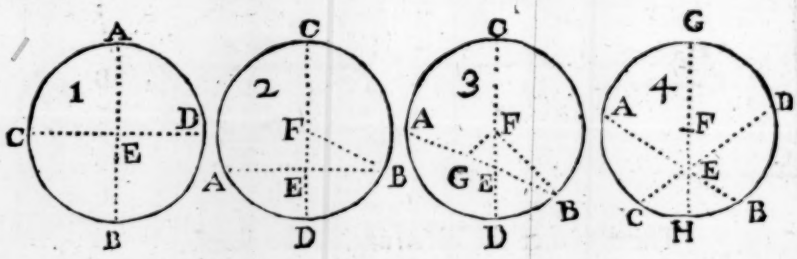
Conſtruction DRaw the right line E F, touching the circle in the point A, then make the angle F A B equal to the given angle D, and draw the right lines A C and B C: I ſay, that the angle A C B in the ſegment A C B is equal to the given angle D.

Demonſtration THe angle F A B, which is made equal to the given angle D, is [a] equal to the angle C, in the alternate ſegment A C B: Therefore the ſame angle C, ſhall be in like manner equal to D: We have therefore cut off from the given circle, the ſegment A C B, &c. Which was to be done.

a) 32. 3.

PROP. 35. THEOR. 29.

If in a circle A B C, *two right lines* A B *and* C D, *do divide one another (in the point* E,*) the Rectangle conteined under the two parts of the one* A E *and* E B, *is equal to the Rectangle conteined under the two parts* C E *and* E D, *of the other.*



Demonſtration LEt the point of interſection E, in the firſt place be the center of the circle, as in the firſt figure, the four ſegments ſhall be equal, each of them being the ſemidiameter of the circle: Therefore it is manifeſt that the Rectangle conteined under the two parts of the one, is equal to the Rectangle conteined under the two parts of the other.

Secondly, Let C D only paſſe by the center, dividing A B into two equal parts in the point E, [a] and therefore at right angles, and joyn the right line B F: Foraſmuch as C D is divided into two equal parts in the center F, and unequally in the point E. The Rectangle conteined under the unequal ſegments C E aud E D, [b] with the ſquare of the mean Section F E, is equal to the ſquare of the halfe F D; that is to ſay, of F B: but the ſquare of F B [c] is equal to the two ſquares of F E and E B: therefore the Rectangle conteined under C E and E D, with the ſquare of F E, ſhall be alſo equal to the two ſquares of F E and E B; therefore if the common ſquare of F E be taken away, there will re-

a) 3. 3.

b) 5. 1.

c) 47. 1.

remain the Rectangle conteined under C E and E D, equal to the ſquare of E B; that is to ſay, to the Rectangle under A E and E B: For ſeeing that they are equal, the Rectangle conteined under the two is a ſquare, according to the Firſt Definition of the ſecond Book. (See the Second Figure.)

Thirdly, Let C D paſſing by the center F, divide A B in two parts unequally, in the point E, and let the ſame A B be divided in two equal parts in the point G, and let G F and F B be drawn; [d] F G ſhall be perpendicular to A B. Foraſmuch then as C D is divided into two equal parts at F, and unequally at E, the Rectangle conteined under C E and E D, with the ſquare of the mean Section F E, ſhall be equal to the ſquare of the half F D; that is to ſay, to the ſquare of F B its equal: But the ſquare of F B [e] is equal to the two ſquares F G and G B: Therefore the Rectangle conteined under C E and E D, with the ſquare of F E, is [f] alſo equal to the two ſquares of F G and G B, and the ſquare of F E is equal alſo to the two ſquares of F G and G E. Therefore the Rectangle under C E and E D, with the two ſquares of F G and G E, is equal to the two ſquares of F G and G B, taking away therefore the common ſquare of F G, there will remain the Rectangle conteined under C E and E D, with the ſquare of G E, equal to the ſquare of G B; but the Rectangle conteined under A E and E B, with the ſquare of G E, is alſo equal to the ſquare of G B: Seeing that A B is divided into two equal parts in the point G; and unequally in the point E. Therefore the Rectangle conteined under C E and E D, with the ſquare of G E, is equal to the Rectangle under A E and E B, with the ſame ſquare of G E, taking away therefore the common ſquare of G E, there will remain the Rectangle conteined under C E and E D, equal to the Rectangle conteined under A E and E B; which is propoſed. (See the third figure.)

d) 3. 3.

e) 47. 1.

f) 47. 1.

Fourthly, Let neither A B nor C D, (cutting one another in the point E,) paſſe by the center, and draw the diameter G H by the point E, (as in the fourth figure,) it ſhall be ſhewn as before, that the Rectangle conteined under A E and E B, is equal to the Rectangle under G E and E H, and the Rectangle under C E and E D, alſo equal to the Rectangle under G E and E H; and therefore equal to one another; whether the one of them A B and C D be divided in two equal parts or not: Therefore, If in a circle, &c. Which was to be demonſtrated.

PROP. 36. THEOR. 30.

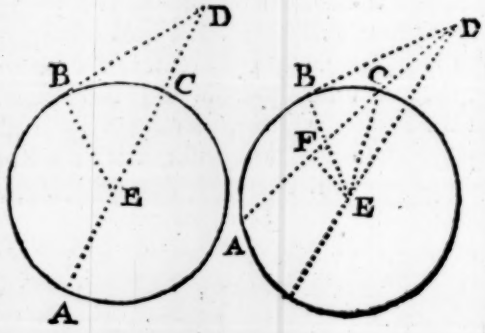
If there be taken ſome point D, *without a circle* A B C, *and from that point there fall two right lines to the circle, one of which* D A, *divides the circle, and the other* D B, *toucheth it; the Rectangle conteined under the whole line dividing (to wit,* A D) *and its part without* D C, *taken between the point* D, *and the convex circumference* C *is equal to the ſquare of the touching line* D B.

Demon-

Demonſtration IN the firſt place, Let D A paſſe by the center E, and then joyn the right line E B, [a] which ſhall be perpendicular to D B. Foraſmuch as C A is divided into two equal parts in E, and to it is added D C, the Rectangle conteined under A D and D C, [b] with the ſquare of E C, that is to ſay, of E B, its equal, is equal to the ſquare of E D: But the ſquare of E D [c] is equal to the two ſquares of D B and B E: Therefore the Rectangle conteined under A D and D C, with the ſquare of B E ſhall be alſo equal to the two ſquares of D B and B E: Therefore taking away the common ſquare of E B, there will remain the Rectangle under A D and D C, equal to the ſquare of the touching line D B; which is propoſed.

a) 18. 3.
b) 6. 2.
c) 47. 1.

Secondly, Let the dividing line D A not paſſe by the center E, and divide A C into two equal parts in the point F, and joyn the right lines E B, E F, E C, and E D, the line E B ſhall be perpendicular to D B, and E F to A C: Foraſmuch then as A C [d] is divided into two equal parts in the point F, and to it is added D C, the Rectangle conteined under A D and D C, with the ſquare of F C, is equal to the ſquare of F D: Therefore adding the common ſquare of E F, the Rectangle conteined under A D and D C, with the two ſquares of C F and F E, is equal to the two ſquares of D F and F E: But the ſquare of E C, that is to ſay, of E B its equal; [e] is equal to the two ſquares of C F and F E; therefore the Rectangle under A D and D C, with the ſquare of E B, is equal to the two ſquares of D F and F E, and the ſquare of D E is alſo equal to the two ſquares of D F and F E; therefore the Rectangle under A D and D C, with the ſquare of B E, ſhall be equal to the ſquare of D E, which is equal [f] to the two ſquares of D B and B E: Therefore the Rectangle under A D and D C, with the ſquare of B E, is equal to the two ſquares of D B and B E, taking away therefore the common ſquare of B E, there will remain the Rectangle under A D and D C, equal to the ſquare D B. Which was to demonſtrated.



d) 6. 2.
e) 47. 1.
f) 47. 1.

COROLLARIE I.

From this Propoſition it is manifeſt that if from any point taken without the circle, there be drawn divers right lines, dividing the circle, the Rectangles compriſed under each of the whole lines, and its exterior part, are equal to one another, for if from the point A (in the firſt figure,) there be drawn A C, A D, and A E, dividing the circle and A B, touching it, the Rectangle under A C and A F, [g] ſhall be equal to the ſquare of A B: in like manner, the Rectangle under A D and A G, and the Rectangle under A E and A H; and therefore equal to one another.

g) 36. 3.

COROLLARIE II.

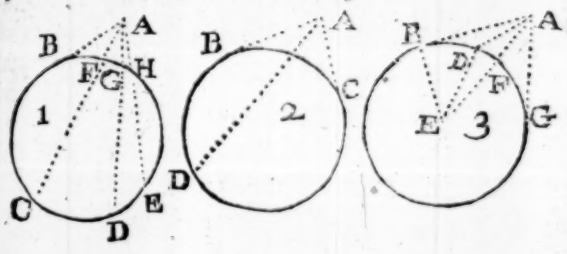
It is alſo manifeſt, that two right lines drawn from one and the ſame point, touching

touching the circle, are equal to one another, as are the two lines AB *and* AC, *which touch the circle in the points* B *and* C *(in the second figure;) I say they are equal to one another, for draw* AD, *dividing the circle in the point* E, *as well the square of* AB *as the square of* AC, [h] *shall be equal to the Rectangle conteined under* AD *and* AE; *and therefore equal to one another; and therefore the lines* AB *and* AC *likewise equal to one another.*

h) 36. 3.

COROLLARIE. III.

It is likewise manifest, that from a point taken without the circle, there can only be drawn two right lines, touching the circle; for if besides the two AB *and* CD, *there may be a third* AD *drawn, touching the same circle; having drawn the right lines* EB, *and* ED, *from the center* E; [i] *the angles* ABE *and* ADE *shall be right angles; and therefore equal to one another, which is absurd: for if you draw the right line* AE, [k] *the angle* ADE *shall be greater than the angle* ABE.



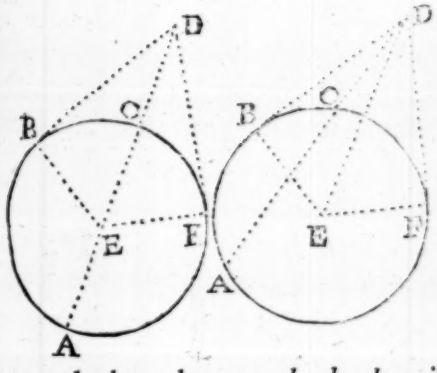
i) 18. 3.

k) 21. 1.

COROLLARIE IV.

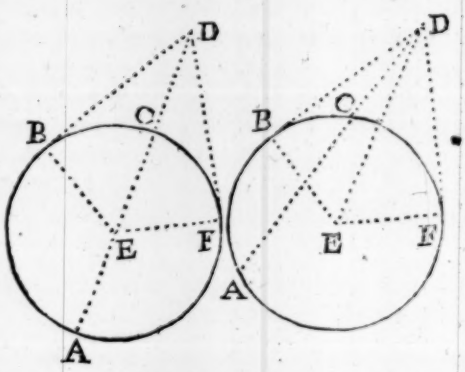
It is in the last place evident, that if two equal right lines be drawn from some point without the circle to the convex circumference, and that the one of them touch the circle, the other shall also touch it, &c.

PROP. 37. THEOR. 31.



If without the circle ABC, *there be taken some point* D, *and from that point there fall to the circle two right lines* DB *and* DA, *one of which* DA, *cuts the circle, and the other reacheth the circle; and that the* Rectangle *conteined under the whole dividing line* DA, *and its part without* DC, *taken between the point* D, *and the convex circumference* C, *be equal to the square of the line* DB, *which reacheth the circle, the same line* DB *shall touch the circle.*

Demonſtration FOr, [a] Let D F be drawn, touching the circle, and let the Center E be found, and joyn E B and E F: and if D A paſſe not by the center E, let D E be alſo joyned, the angle [b] D F E ſhall be a right angle; and foraſmuch as D F toucheth the circle, [c] the ſquare of the ſame D F ſhall be equal to the Rectangle conteined under D A and D C: But the ſquare of D B is equal to the ſame Rectangle under D A and D C, by Suppoſition; therefore the ſquares of D B and D F ſhall be equal to one another; and D B and D F alſo equal; But [d] E B and E F are equal; therefore the two ſides D B and B E of the triangle D B E, are equal to the two ſides D F and F E, of the triangle D F E, each to his correſpondent, and the baſe D E common, and the angles B and F ſhall be equal; but D F E is a right angle; therefore D B E ſhall be alſo a right angle: Therefore by the Corollary of the ſixteenth Propoſition of this Book, D B ſhall touch the circle, which was propoſed: Therefore if there be taken, &c. Which was to be demonſtrated.



a) 17. 3.
b) 18. 1.
c) 36. 3.
d) 15. def.

The End of the Third Element of EUCLIDE.

THE

THE FOURTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

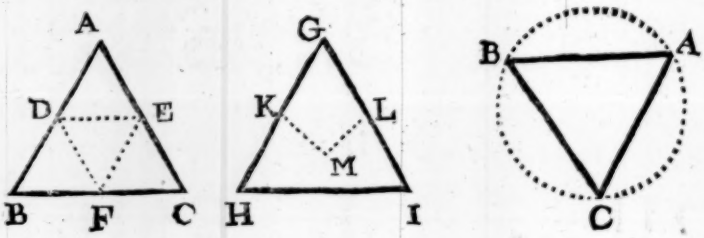
EUCLIDE treating in this Fourth Book of divers inſcriptions of figures in Circles, and the deſcriptions of the ſame figures about the Circle. Alſo of the Inſcriptions of the Circle in the ſame figures, and of the deſcriptions of the Circle about thoſe figures, he expoſeth in a few Definitions, what the meaning is, of a figure inſcribed in a figure, or to be deſcribed about a figure, beginning with right lined figures.

DEFINITIONS.

1 *A right lined figure is ſaid to be inſcribed in a right lined figure, when every one of the angles of the figure inſcribed, doth touch each ſide of the figure in which it is inſcribed.*

AS for Example, if each of the angles D, E, and F, of the interior triangle D E F doth touch each of the ſides of the exterior triangle ABC, the triangle D E F ſhall be ſaid to be inſcribed in the triangle

ABC. But foraſmuch as the angle M of the triangle K L M, (a right line being drawn from K to L,) toucheth not the ſide H I of the triangle G H I, the triangle K L M ſhall not be ſaid to be inſcribed in the other, although it be all within, and that two of the angles of the one, do touch two ſides of the other.



2 *In like manner, a figure is ſaid to be deſcribed about a figure, when each ſide of the figure circumſcribed, doth touch each angle of the figure about which it is deſcribed.*

COntrarily, the triangle A B C ſhall be ſaid to be deſcribed about the triangle D E F; Foraſmuch as each of the ſides of the one, doth touch each of the angles of the other. But the triangle G H I ſhall not be ſaid to be deſcribed about the triangle K L M: Seeing that the ſide H I doth not touch the angle M, and the ſame ought to be underſtood of the inſcriptions and circumſcriptions of other right lined figures, &c. and right lined figures are properly ſaid to be inſcribed or circumſcribed, when the number of the ſides are equal, and although that be not abſolutely neceſſary. (See the precedent figures.)

3 *A right lined figure is ſaid to be inſcribed in a circle, when each angle of the inſcribed figure doth touch the circumference of the circle.*

AS if the angles A, B, and C, of the triangle A B C (See the precedent figure,) do touch the circumference of the circle A B C, the ſaid triangle ſhall be ſaid to be inſcribed in the circle; and if ſome one of the angles do not touch the circumference, the triangle ſhall not be ſaid to be inſcribed in the circle.



4 *But a right lined figure is ſaid to be deſcribed about a circle, when as each ſide of the figure circumſcribed doth touch the circumference of the circle.*

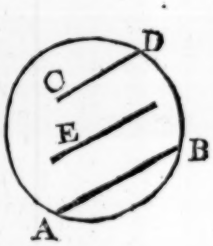
For

FOr Example, If the ſides of the triangle A B C, do touch the circumference of the circle D E F: The triangle ſhall be ſaid to be deſcribed about the circle.

5 *In like manner a circle is ſaid to be inſcribed in a right lined figure, when the circumference of the circle doth touch each ſide of the figure in which it is inſcribed.*

6 *But a circle is ſaid to be deſcribed about a figure, when the Circumference of the circle doth touch every angle of the figure about which it is deſcribed.*

IN like manner, the Circle D E F (in the figure of the fourth Definition) ſhall be ſaid to be inſcribed in the triangle A B C. But the Circle A B C, in the figure belonging to the ſecond Definition, ſhall be ſaid to be deſcribed about the triangle A B C; and the ſame is to be underſtood of other right lined figures, which are ſaid to be inſcribed in the Circle, or deſcribed about the Circle, or in which the Circle is ſaid to be inſcribed or deſcribed about.

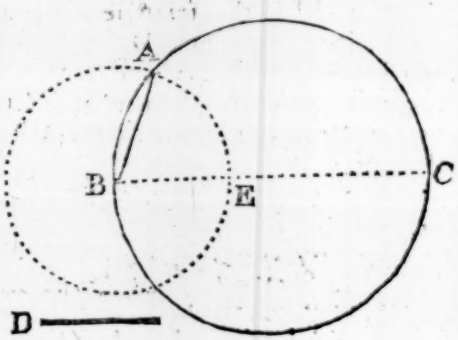


7 *A right line is ſaid to be apted or fitted in a Circle, when the extremities thereof are in the Circumference of the Circle. As appears by this figure.*

PRO-

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. PROBLEM 1.



In a given circle ABC, *to fit a right line* AB, *equal to a given right line* D, *which may not be greater than the diameter* BC, *of the circle* ABC.

Construction. DRaw the diameter BC; if D be equal to BC, then is BC fitted in the circle, equal to D; but if D be leſſe than the diameter BC: Let BE be cut off [a] equal to D, and on B as a center, with the diſtance BE deſcribe the circle AE, cutting AB in the point A, and draw BA, which ſhall be the line fitted in the circle ABC, equal to D.

a) 3. 1.

Demonſtration. FOr BA [b] is equal to BE, and D alſo equal to BE, by Conſtruction: therefore BA and D ſhall be equal to one another, and the extremities of BA are in the circumference, being drawn from the extremity of the diameter, in the point of Section of the circles: Therefore we have, &c. Which was to be done.

b) 25. def.

PROP. 2. PROBL. 2.

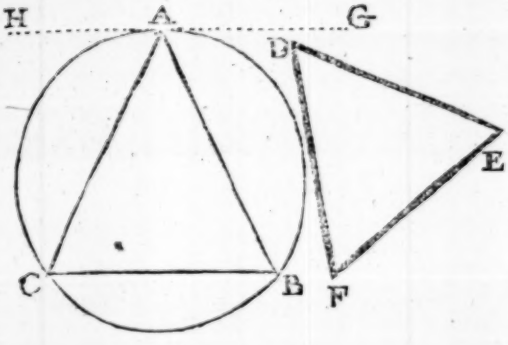
In a given circle ABC, *to inſcribe a triangle equiangled to a given triangle* DFE.

Construction. DRaw HG, touching the circle in the point A, and make the angle GAB equal to the angle F, [a] and the angle HAC equal to the angle E, drawing the right lines AB and AC to the cir-

a) 17. 3.

circumference, at the points B and C, and joyn the right line C B. I say, that the triangle A B C inscribed in the given circle, is equiangled to the triangle D E F.

Demonstration FOr [b] seeing that H G doth touch the circle, and A B cuts it, the angle G A B, which is made equal to the angle F, shall be equal to the angle C, in the alternate segment: therefore the angles C and F shall be equal to one another; In like manner, forasmuch as the angle H A C (which is equal to the angle E, by Construction,) is [c] equal to the angle B, in the alternate segment, the angles B and E shall be equal: therefore seeing that the two angles B and C of the triangle A B C are equal to the two angles E and F, of the triangle D E F, the two others [d] A and D shall be likewise equal to one another: Therefore the triangle A B C is equiangled to the triangle D E F: Therefore, In a Circle, &c. Which was to be done.



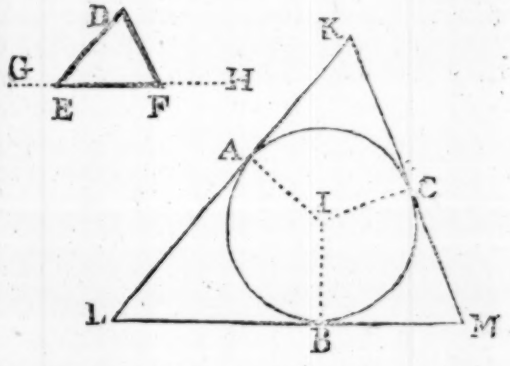
b) 32. 3.

c) 32. 3.

d) 32. 1.

PROP. 3. PROBL 3.

About a given circle A B C, *to describe a triangle equiangled to a given triangle* D E F.



Construction HAving prolonged the side E F on both sides, to the points G and H: Let the center I be taken, and draw I A at the contingent point A, and in the point I of the same line A I, make the angle A I B equal to the angle D E G, and B I C equal to D F H, and from the extremities A, B, and C, draw at right angles the right lines K L, L M, and M K, which lines shall meet in the points K, L, and M: Forasmuch as if you joyn a right line from A to B, the two angles L A B and L B A, shall be lesse than two right angles, being no other than part of the two right angles I A L and I B L; therefore [a] A L and L B shall meet in the point L, and so of the rest: Therefore the triangle L K M is described about the circle A B C, and is equiangled to the given triangle D E F.

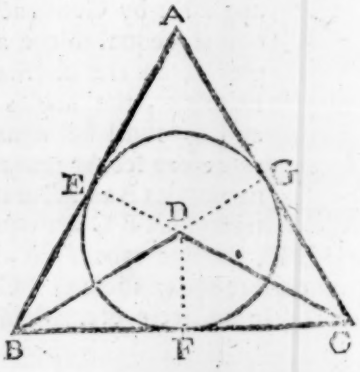
a) 11. def. 1.

Demonstration FOr [b] all the sides do touch the circle in the points A, B, and C, and forasmuch as all the angles of the quadrilateral figure A I B L are equal to four right angles, as is shewn in the

b) C. 16. 3.

3 Propoſition of the firſt Book, and the two angles at the points A and B are right angles; the two angles A I B and A L B will remain equal to two right angles; and therefore equal to the two angles D E G and
c) 13. 1. D E F, which c are alſo equal to two right angles; therefore if the equal angles A I B and D E G be taken away, there will remaine A L B equal to D E F; even ſo the angle M will be ſhewn equal to the angle D F E; Therefore the third angle K ſhall be equal to the third angle D: Therefore the triangle K L M ſhall be equiangled to the triangle D E F: Therefore, &c. Which was to be done.

PROP. 4. PROBL. 4.



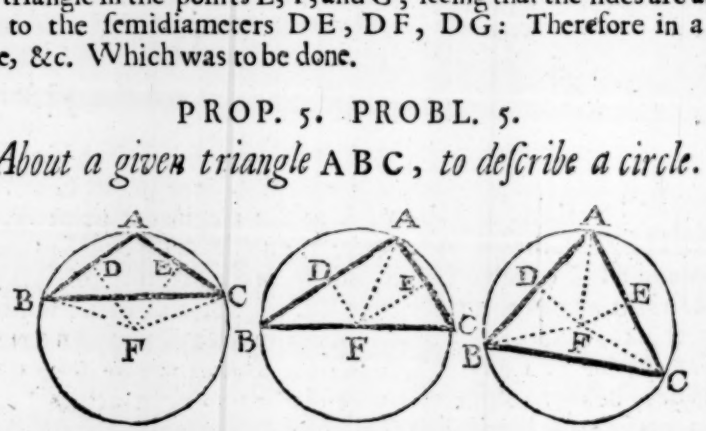
In a given triangle A B C, *to inſcribe a circle.*

a) 9. 1. *Conſtruction* DIvide a the two angles B and C, each in two equal parts, by the right lines B D and C D, meeting one another in the point D, and from that point draw the perpendiculars D G, D E, and D F, on the three ſides A B, B C, and C A.

Demonſtration FOraſmuch as the two angles D B F and D F B of the triangle D F B, are equal to the two angles D B E and D E B of the triangle D E B, each to its correſpondent, and the ſide D B com-
b) 26. 1. mon; b the two ſides D E and D F ſhall be equal, by the ſame reaſon, the ſides D F and D G ſhall be equal in the triangles D C F and D C G: Therefore the three lines D G, D F, and D E, ſhall be equal to one ano-
c) Co. 16. 3. ther, and the circle deſcribed on the point D as a center, at the diſtance D E, ſhall paſſe by the other points G and F, and ſhall c touch the ſides of the triangle in the points E, F, and G; ſeeing that the ſides are at right angles to the ſemidiameters D E, D F, D G: Therefore in a given triangle, &c. Which was to be done.

PROP. 5. PROBL. 5.

About a given triangle A B C, *to deſcribe a circle.*



a) 10. 1. *Conſtruction* DIvide a the two ſides A B and A C in two equal parts in the points D and E, from which points draw D F and E F,
b) 11. 1. b at right angles to thoſe ſides, and meeting in the point F.

Demonſtration FOr if a right line were drawn from D to E, the two angles F D E and F E D ſhould be leſſe than two right angles,)

angles,) and the point F ſhall be in the triangle, or in the ſide, or without. Joyn then the right lines F A, F B, and F C: Foraſmuch as the two ſides A D and D F of the triangle A D F, are equal to the two ſides B D and D F of the triangle B D F, and the angles at the point D right angles: c the baſes F A and F B ſhall be equal, by the ſame reaſon, F A and F C ſhall be alſo equal; therefore the three ſides F A, F B, and F C, ſhall be equal to one another: Therefore the circle deſcribed on the point F as a center, at the diſtance F A, ſhall alſo paſſe by the two other points B and C: Therefore about the triangle, &c. Which was to be done. c) 4. 1.

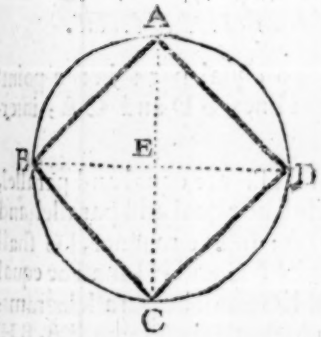
COROLLARIE.

It is manifeſt from this Demonſtration, that if the center fall within the triangle, d *that all the angles are acute, being in the greateſt ſegment of the circle; but if it fall on the ſide, the angle* e *oppoſite to that ſide, ſhall be a right angle in the ſemicircle: Laſtly, if it fall without the triangle, the oppoſite angle* f *ſhall be obtuſe, being in the leaſt ſegment of the circle.* d) 31. 3. e) 31. 3. f) 31. 3.

Contrarily, If the triangle be an Oxigon, the center will fall within it, if a Rectangle, it will fall on the ſide which ſubtends the right angle, and if Ambligon, it will fall without.

PROP. 6. PROBL. 6.

In a given circle A B C, *to deſcribe a ſquare.*



Conſtruction DRaw the two diameters A C and B D, which ſhall divide one another at right angles in the center E, and joyn the right lines A B, B C, C D, and D A: I ſay that A B C D is the ſquare inſcribed in the given circle.

Demonſtration FOraſmuch as the two ſides E A and E B of the triangle A E B are equal to the two ſides E A and E D, of the triangle E A D, being all drawn from the center, and the angles conteined of thoſe ſides being right angles, a the baſes A B and A D ſhall be equal; and in like manner C D and D A, and D A and A B; therefore A B C D ſhall be a ſquare, having all the ſides equal, and all the angles A, B, C, and D, right angles, b each of them being in the ſemicircle: Therefore, In a circle, &c. Which was to be done. a) 4. 1. b) 31. 3.

PROP. 7. PROBL. 7.

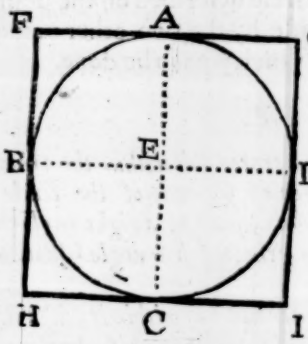
About a given circle A B C, *to deſcribe a ſquare.*

Conſtruction DRaw two diameters, to divide one another at right angles in the center E, and by the points A, B, C, and D, draw the right lines F G, F H, H I, and I G, at right angles to the diameters, which ſhall meet with one another in the points F, H, I, and G: Foraſmuch as if a right line be drawn from B to A, the two angles F B A and F A B, ſhall be leſſe than two right angles, and ſo of the others.

I ſay that F G I H is a ſquare deſcribed about the given circle.

Demonſtration FOr ſeeing that the two angles A E B and F B E are right angles, F H and A C [a] ſhall be parallels, likewiſe G I and A C; therefore F H and [b] G I ſhall be parallels, and by the ſame reaſons F G and H I ſhall be alſo parallel.

a) 28. 1.
b) 30. 1.



c) 34. 1.

Now foraſmuch as A C H F is a parallelogram, the [c] oppoſite ſides A C and F H ſhall be equal, and the oppoſite angles C and F equal: But E C H is a right angle; therefore A F H ſhall be a right angle; by the ſame reaſon, the angles H, I, and G, ſhall be ſhewn to be right angles; and the ſides H I, I G, and G F, equal to the diameters A C and B D; and therefore equal to one another. Therefore F H I G ſhall have the four ſides equal, and the four angles right angles; and therefore ſhall be a ſquare, all whoſe ſides [d] do touch the circle: Therefore about a given circle, &c. Which was o be done.

d) C. 16. 3.

PROP. 8. PROBL. 8.

In a given ſquare F G H I, *to inſcribe a circle.*

Conſtruction HAving divided the ſides in two equal parts in the points B, C, D, and A, draw the right lines B D and C A, interſecting in the point E.

Demonſtration NOw [a] foraſmuch as F G and H I are equal and parallel, their halves F A and H C ſhall be equal and parallel; and therefore F H ſhall be equal and parallel to C A: In like manner, I G ſhall be equal and parallel to the ſame C A: Likewiſe F G and H I ſhall be equal and parallel to B D; therefore F E, E H, I E, and E G, ſhall be parallelograms; and therefore the lines E B, E C, E D, and E A, ſhall be equal to F A, B H, the equal lines G A, and F B; but thoſe lines are equal to one another; being the halves of F G, F H, &c. Therefore E B, E C, E D, and E A, are equal, therefore the circle deſcribed from the point E as a center, and at the diſtance E B: ſhall alſo paſſe by the points C, D, and A, and ſhall touch [b] all the ſides, ſeeing that the angles in the points B, C, D, and A, are right angles, and ſhall be inſcribed in the ſquare F I: Therefore in a given ſquare we have inſcribed a circle: Which was to be done.

a) 34. 1.
b) C. 16. 3.

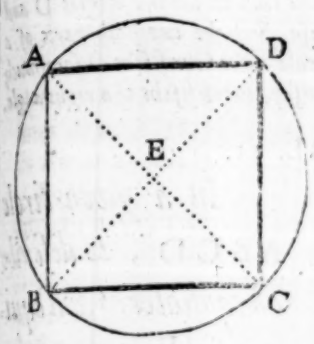
PROP. 9. PROBL. 9.

About a given ſquare A B C D, *to deſcribe a circle.*

Conſtruction DRaw the two diameters C A and B D, interſecting in the point E.

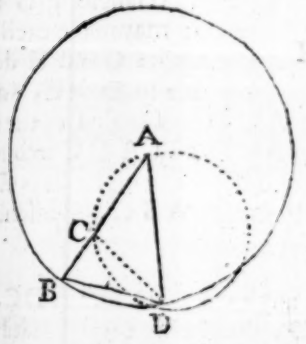
Demonſtration FOraſmuch as the ſides A B and A D, of the triangle A B D are equal, [a] the angles A B D and A D B ſhall be equal: But the angle B A D is a right angle in the ſquare; therefore [b] A B D and A D B ſhall be the halves of right angles: In like manner, we

a) 5. 1.



we ſhall ſhew that all the other angles, which are at the points A, B, C, and D, are the halves of right angles; and therefore equal to one another: Therefore ſeeing that the angles E A D and E D A are equal to one another, the lines E A and E D ſhall be equal, and by the ſame reaſon E A and E B ſhall be equal, and alſo E B and E C; likewiſe E C and E D: Therefore the circle deſcribed on the center E, at the diſtance E A, ſhall paſſe by the other points B, C, and D: Therefore about a given ſquare, &c. Which was to be done.

PROP. 10. PROBL. 10.



To conſtitute an Iſoſceles triangle A B D, *which may have each of the angles* B *and* D, *which are at the baſe* B D, *double to the other* A.

Conſtruction ASſume any line, as A B, which divide in the point C, in ſuch ſort as the Rectangle conteined under [a] A B and B C, may be equal to the ſquare of A C; then on the center A, with the diſtance A B, deſcribe a circle, [b] in which circle fit the right line B D, equal to A C, and joyn A D: I ſay, that the triangle A B D is an Iſoſceles triangle, and hath each of the angles A B D and A D B double to the other angle A.

a) 11. 2.

b) 1. 4.

Demonſtration FOr having drawn C D, deſcribe [c] about the triangle A C D the circle D C A, foraſmuch as the Rectangle conteined under A B and B C is equal to the ſquare of C A, that is to ſay, of B D, its equal; and A B cuts the circle [d] A C D, the line B D ſhall touch it in the point D: Therefore the angle B D C ſhall be [e] equal to the angle A in the alternate ſegment C A D, adding the common angle C D A, the whole angle A D B ſhall be equal to the two angles C A D and C D A; but the exterior angle D C B is [f] equal to the ſame angles A and A D C; therefore the angle B C D is equal to A D B; that is to ſay to [g] A B D, its equal; therefore [h] C D and D B ſhall be equal; But B D is equal to A C by Conſtruction: Therefore C D ſhall be equal to C A, and therefore [i] the two angles C A D and C D A ſhall be equal: Therefore the angle A D B, which is ſhewn equal to the two angles C A D and C D A, ſhall be double to the angle A; and therefore the angle A B D ſhall be alſo double to the ſame angle A: We have therefore conſtituted, &c. Which was to be done.

c) 5. 4.

d) 37. 3.

e) 32. 3.

f) 32. 1.

g) 5. 1.

h) 6. 1.

i) 5. 1.

COROLLARIE.

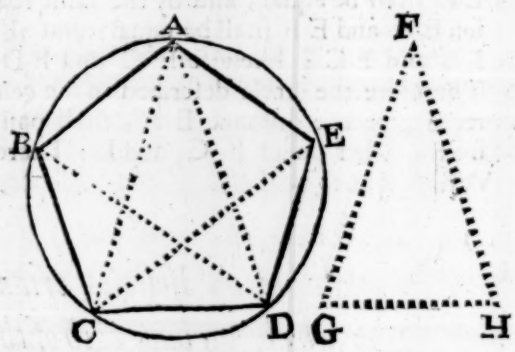
Now foraſmuch as the three angles [k] *of the triangle* A B D, *are equal to two right angles; that is to ſay, to five fifths of two right angles: It is manifeſt that*

k) 32. 1.

the angle A *is the fifth part of two right angles, and each of the two angles* D *and* B *is the two fifth parts of two right angles: Likewise* A *is the two fifth parts of a right angle, and either* B *or* D, *which of them you please is the four fifths;* [l] *forasmuch as all the three are equal to two right angles, that is to say, to ten fifths of a right angle.*

l) 32. 1.

PROP. 11. PROBL. 11.



In a given circle ABCD, *to describe an equilateral and equiangled Pentagon.*

Construction COnstitute the Isosceles triangle FGH; which may have each of the angles G and H double to the angle F, [a] and in the circle ABCD, inscribe the triangle ACD equiangled to the triangle FGH, [b] and divide both the angle ACD and ADC into two equal parts by the lines DB and CE, and joyn the right lines AB, BC, CD, DE, and EA: I say, that the Pentagon ABCDE inscribed in the circle is equilateral and equiangled.

a) 2. 4.

b) 9. 1.

Demonstration FOrasmuch as each of the angles ACD and ADC, is double to the angle CAD, they being each divided into two equal parts, the five angles ADB, BDC, CAD, DCE, and ECA shall be equal; Therefore [c] the Arches AB, BC, CD, DE, and EA, on which those angles are subtended, are equal: Therefore [d] the right lines AB, BC, CD, DE, and EA, which subtend them are equal: Therefore the Pentagon ABCD is equilateral: Again forasmuch as the arches AB and ED are equal, if you adde the common angle BCD, the whole ABCD shall be equal to the whole EDCB: Therefore the [e] angles AED and BAE, insisting on them shall be equal, in the same manner the other angles shall be equal, for they insist on equal arches, each of which is compounded of three equal arches: Therefore the Pentagon is equiangled, the which being also shewn to be equilateral: We have inscribed in a circle, &c. Which was to be done.

c) 26. 1.

d) 29. 1.

c) 27. 1.

COROLLARIE.

Hence it follows that the angle of the equilateral and equiangled Pentagon, is the three fifths of two right angles, or the $\frac{6}{5}$ *of one right angle:* [f] *For seeing that the three angles* BAC, CAD, *and* DAE *are equal, being they insist on the equal arches* BC, CD, *and* DE: *But* CAD *is* $\frac{1}{5}$ *part of two right angles, or* $\frac{2}{5}$ *parts of one right angle, the whole* BAE *shall be the* [3] *of two right angles, or six fifths of one right angle.*

f) 27. 3.

PROP. 12. PROBL. 12.

About a given circle ABC, *to describe an equilateral and equiangled Pentagon.*

Con-

Conſtruction INſcribe [a] in the circle eqnilateral and equiangled Pentagon ABCDE; and let the center E be taken, and draw the right lines FA, FB, FC, FD, and FE, by the extremities of which lines draw GH, HI, IK, KL, and LG, at right angles to them, meeting in the points G, H, I, K, and L.

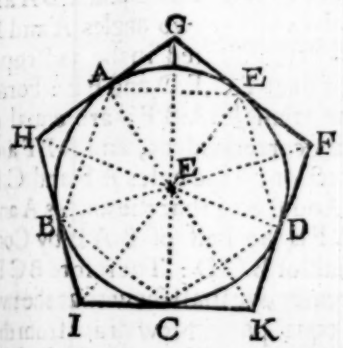
For ſeeing that the angles GAE and GEA are leſſe than two right angles, being part of the two right angles FAG and FEG, [b] the lines AG and EG ſhall meet with one another in the point G, and ſo of the others: And foraſmuch as they [c] touch the circle, the Pentagon GHIKL ſhall be deſcribed about the circle, which I ſay is equilateral and equiangled.

a) 11. 4.

b) 11. c. ſ.

c) C. 16. 3.

Demonſtration FOr having drawn the right lines FG, FH, FI, FK, and FL, the two ſquares of FA and AH [d] ſhall be equal to the ſquare of FH; and in like manner, the ſquares of FB and BH ſhall be equal to the ſame ſquare of FH: therefore the two ſquares of FA and AH ſhall be equal to the two ſquares of FB and BH, taking away then the equal ſquares of the equal lines FA and FB, the ſquares of AH and BH will remaine equal; therefore the lines AH and BH are equal: And foraſmuch as the two ſides AH and FH of the triangle AFH are equal to BF and FH of the triangle BFH, each to his correſpondent, and the baſe AH equal to the baſe BH, as is ſhewn, [e] the angle AFH ſhall be equal to the angle BFH, therefore the angles [f] AHF and BHF ſhall be alſo equal: therefor the angle AFB is double to the angle BFH, and the angle AHB double to the angle BHF.



d) 47. 1.

e) 8. 1.

f) 4. 1.

By the ſame diſcourſe we ſhall ſhew that the angle BFC is double to the angle BFI, and the angle BIC double to the angle BIF: Seeing therefore [g] that the angles AFB and BFC are equal, as inſiſting on the equal circumferences AB and BC, [h] being ſubtended by the equal right lines AB and BC, their halves BFH and BFI, ſhall be equal.

g) 27. 3.

h) 28. 3.

Therefore ſeeing that the two angles BFH and HBF of the triangle BFH, are equal to the two angles BFI and IBF of the triangle IFB, and the ſide BF adjacent common, [i] the ſides BH and BI ſhall be equal, and the angles BHF and BIF equal: Therefore the line HI is double to the line HB.

i) 26. 1.

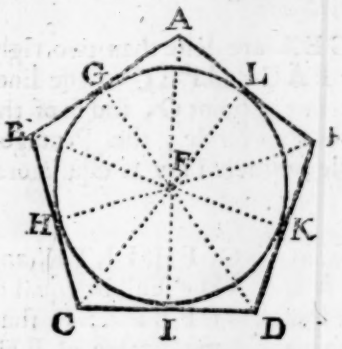
By the ſame reaſon, we ſhall ſhew that GH is double to HA; but HB and HA are ſhewn to be equal: Therefore their double, to wit, HI and HG ſhall be equal. In like manner, we ſhall ſhew, that the right lines IK, KL, and LG are equal to each of the right lines HI and HG: therefore the Pentagon GHIKL, is equilateral.

Again, foraſmuch as it is ſhewn that the angles BHF and BIF are equal, and that they are the halves of the angles BHA and BIC; their doubles BHA and BIC ſhall be equal. By the ſame reaſon, the angles IKL, KLG, and LGH ſhall be equal to each of the angles BHA and BIC: Therefore the Pentagon GHIKL is equiangled:

There-

Therefore being ſhewn to be equilateral, we have deſcribed about a circle a Pentagon, equilateral and equiangled: Which was to be done.

PROP. 13. PROBL 13.



In a given Pentagon ABCDE, *which is equilateral and equiangled, to inſcribe a circle.*

Conſtruction DIvide two of its angles BAE, and ABC, into two equal parts, by the right lines AF and BF, which ſhall meet with one another in the point F.

a) 11. c. ſ. *Demonſtration* SEeing [a] that the two angles FAB and FBA are leſſe than two right angles, being the halves of the two angles A and B, that are leſſe than four right angles, as is demonſtrated in the 32 Propoſition of the firſt Book. Then draw the right lines FC, FD, and FE: Foraſmuch as the two ſides AB and BF of the triangle ABF, are equal to CB and BF of the triangle EBF, each to its correpondent, and ABF and

b) 4. 1. CBF conteined of them, equal by Conſtruction, [b] the baſes AF and CF, and the angles BAF and BCF are equal. And ſeeing that the angles A and C of the Pentagon are put equal, and BAF is the half of BAE by Conſtruction, the angle BCF ſhall be the half of BCD: Therefore BCD divided into two equal parts, by the ſame reaſon, we ſhall ſhew that the two other angles D and E are divided into two equal parts: Now draw from the

c) 12. 1. point F the [c] perpendicular lines FG, FH, FI, FK, and FL, on the ſides of the Pentagon: Foraſmuch as the two angles FGA and FAG of the triangle FAG, are equal to the two angles FLA and FAL of the triangle FAL, and the ſide AF oppoſite to one of the equal common angles, FG and FL, ſhall be equal.

In like manner, it ſhall be ſhewn that the other perpendiculars FH, FI, and FK, are equal to each of the other FG and FL: Therefore the circle deſcribed from the center F, at the diſtance FG, ſhall likewiſe paſſe by all the points H, I, K, and L; and foraſmuch as the ſides of

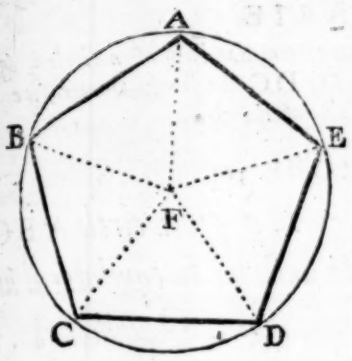
d) C. 16. 1. 0 the Pentagon do [d] touch the circle, ſeeing that they make right angles with the ſemidiameters FG, FH, &c. We ſhall have deſcribed a circle in the given Pentagon. Which was to be done,

PROP. 14. PROBL. 14.

About a given Pentagon ABCDE, *which is equilateral and equiangled, to deſcribe a circle.*

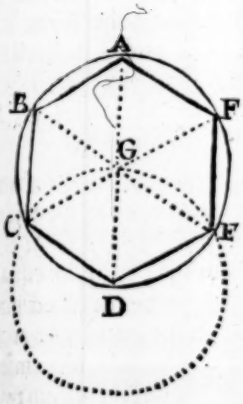
a) 9. 1. *Conſtruction* HAving divided the angles BAE and ABC, into [a] two equal parts, by the right lines AI and BF, which ſhall meet with one another in the point F within the Pentagon, as hath been demonſtrated in the precedent Propoſition, and having joyned the right lines FC, FD, and FE, we ſhall ſhew, as in the precedent Problem, that the other angles C, D, and E, are divided into two equal parts: Therefore all the half angles ſhall be equal to one another; ſeeing that the whole angles are equal.

Demon-



Demonſtration Foraſmuch then as in the triangle A F B, the two angles F A B and F B A, are equal, the lines [b] F A and F B ſhall be equal, by the ſame reaſon, the other lines F C, F D, and F E, ſhall be equal to each of them: Therefore the circle deſcribed from the center F, at the diſtauce of F A, ſhall alſo paſſe by the points B, C, D, and E: Therefore, About a Pentagon, &c. Which was to be done. b) 6. 1.

PROP. 15. PROBL. 15.



In a given circle A B C D E F, *to inſcribe an equilateral and equiangled Hexagon.*

Conſtruction HAving aſſumed the center G, and drawn the diameter A D; deſcribe the [c]ircle C G E from the point D, which cutteth the given circle in the points C and E, from which points draw the right lines C F and E B, by the center G, and joyn the right lines A B, B C, C D, D E, E F, and F A. So ſhall the Hexagon A B C D E F be inſcribed in the giuen circle, which I ſay is equilateral and equiangled.

Demonſtration FOr by the definition of the circle, the three lines G C, C D, and D G ſhall be equal to one another, and the triangle C D G ſhall be equilateral; therefore [a] the three angles thereof ſhall be equal to one another, which [b] being equal to two right angles, which you pleaſe of the two, as C G D ſhall be the third part of two right angles: By the ſame reaſon, the angle D G E ſhall be the third part of two right lines: But the angles C G D, D G E, and E G F, are equal to two right angles: Therefore the other angle E G F ſhall be alſo the third part of two right angles: Therefore the three angles C G D, D G E, and E G F, [c] are equal to one another, to which the angles C G B, B G A, and A G B being [d] equal at the head, the ſix angles at the center G, ſhall be equal to one another: Wherefore [e] the circumferences on which they inſiſt are equal; [f] and therefore the right lines A B, B C, C D, D E, E F, and F A, are equal. Wherefore the Hexagon A B C D E F ſhall be equilateral. a) 5. 1. b) 32. 1. c) 13. 1. d) 15. 1. e) 26. 3. f) 29. 3.

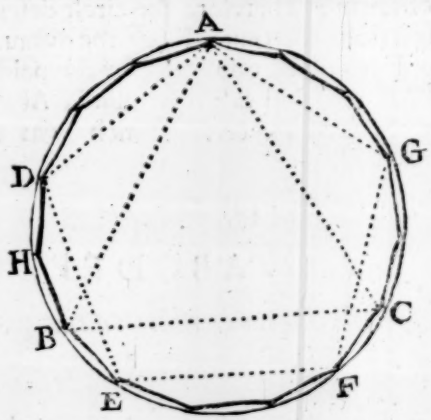
Again, foraſmuch as the circumference B C is equal to A F, if you adde the common circumferences C D E F, the circumferences B C D E F and A F E D C ſhall be equal: Therefore [g] the angles A B C and B A F, which inſiſt on them ſhall be equal: We ſhall ſhew in like manner, that the other angles of the Hexagon C, D, E, and F, are equal to each of the angles A and B, for that each of them inſiſteth as an arch compounded of four equal circumferences: Therefore the Hexagon is alſo equiangled: Therefore, In a given circle: Which was to be done. g) 27. 3.

C O-

COROLLARIE.

From this Propoſition it is manifeſt, that the ſemidiameter of a circle is equal to the ſide of the Hexagon inſcribed therein. For D G *the ſemidiameter, is equal to* D C *the ſide of the Hexagon, by the definition of the circle.*

PROP. 16. PROBL 16.



In a given cirtle A B C, *to inſcribe an equilateral and equiangled Quindecagon.*

Conſtruction INſcribe in the circle, the equilateral triangle A B C, [a] the three arches A B, B C, and C A, ſhall be equal, becauſe of the three ſides, or the three equal angles A, B, and C.

a) 26, 28. 3.

b) 11. 4.

Again, [b] Inſcribe in the ſame circle an equilateral and equiangled Pentagon A D E F G, having one of the angles applyed at the point A, [c] the five arches A D, D E, E F, F G, and G A, ſhall be equal; and ſeeing that the circumference of the circle ought to be divided into 15 equal parts: the arch A B ſhall contein five of them, and the arch A D three, ſeeing the arch A B is the third part of the whole circumference, and A D the fifth part: Therefore the reſt D B, ſhall contein two parts. [d] Therefore having divided the arch D B into two equal parts in the point H, B H ſhall be the fifteenth part of the whole circumference: Therefore drawing the right line B H, it will ſubtend the fifteenth part of the whole circumference: Therefore [e] if you fit in the circle fourteen other right lines, equal to B A, there will be inſcribed in the circle an equilateral Quindecagon, [f] the which is alſo equiangled, ſeeing that each of the angles inſiſt on equal arches, each of which is compounded of thirteen equal arches, as is manifeſt Therefore, In a given circle we have deſcribed a Quindecagon. Which was to be done.

c) 28. 3.

d) 30. 3.

e) 1. 4.

f) 27. 3.

The End of the Fourth Element of EUCLIDE.

THE

THE FIFTH ELEMENT OF EUCLIDE.

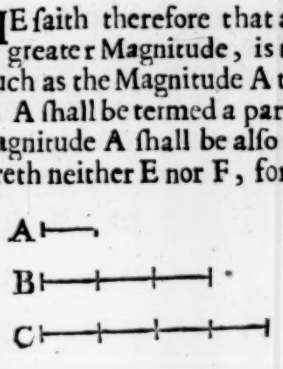
THE ARGUMENT.

IN the four precedent Books *EUCLIDE* hath treated of continued quantity considered, absolutely: But in the two following Books he disputeth of the same quantity, not absolutely; but in as much as the one refers it self to the other, that is to say, in as much as it is compared with another, it hath some Reason. In this Book he teaches the proportions of continued quantities in general, not referring them to any kind of quantity, as to a Line, a Superficie, or to any Body. But in the Sixth Book he shewes especially what reason lines have to one another, the angles, the circumferences of Circles, Triangles, and other plain figures, and to keep his Method, he defineth first the terms needful in the Demonstration.

DEFINITIONS.

1 *A magnitude is part of another magnitude, the lesser of the greater, when as the lesser doth measure the greater.*

HE ſaith therefore that a leſſer Magnitnde which doth meaſure another greater Magnitude, is termed a part thereof; As for Example, foraſmuch as the Magnitude A taken three times, doth meaſure the Magnitude B, A ſhall be termed a part of B, and taken four times meaſureth C, the Magnitude A ſhall be alſo termed a part of C. But foraſmuch as D meaſureth neither E nor F, for taken twice, it exceeds F, and taken thrice it wanteth of E, and taken four times, it exceeds it; D ſhall not be termed a part of the Magnitudes E and F.



Now amongſt Mathematicians, there are two ſorts of parts, the one which meaſureth its whole, in ſuch ſort as being repeated, a certain number of times, it conſtituteth its whole, as 4 compared with 12, 16, 20, &c. is termed an aliquot part, the other doth not meaſure its whole, or being taken a certain number of times, it either exceeds or wants thereof, as 4 compared with 6, 7, 9, 18, 30, &c. is termed an aliquant part.

Therefore here *EUCLIDE* defineth only an aliquot part, as well becauſe it meaſureth only its whole, (for the aliquant part is not ſaid to meaſure its whole,) as for that (as in the Seventh Book,) the aliquant part in numbers is not by *EUCLIDE* called a part, but parts, for 4 is not a part of 6, but is the two third parts thereof, &c.

2 *A magnitude is multiplex of another leſſer magnitude, when the greater is meaſured of the leſſer.*

AS in the precedent Example, as well the Magnitude B, as C, is Multiplex of the Magnitude A; foraſmuch as the Magnitude A meaſureth as well the one as the other; to wit, B and C: But neither the Magnitude E nor F, ought to be termed Multiplex of the Magnitude D; foraſmuch as D meaſureth neither the one or the other: Therefore a part referreth it ſelf to Multiplex, and Multiplex to a part, in ſuch ſort as a leſſer quantity meaſuring a greater, is termed a part of the greater. But the greater, which is meaſured of the leſſer, is termed Multiplex of the leſſer.

It is then manifeſt enough, that the part heretofore defined, is that which exactly meaſures its whole, without any remainder.

Now when two leſſer magnitudes do meaſure equally two other greater Magnitudes, that is to ſay, that if one of the leaſt be conteined ſo many times exactly in one of the greater, as the other leſſer is conteined times in the other greater, thoſe two greater Magnitudes ſhall be termed equimultiplices of the two leſſer, and the ſame is to be ſaid, if divers leſſer Magnitudes do equally meaſure divers other greater Magnitudes.

3 *Reaſon, is an habitude of two magnitudes of the ſame kind, compared the one to the other, according to quantity.*

When

WHen two quantities of the ſame kind, as two numbers, two lines, two ſuperficies, two ſolids, &c. are compared to one another, according to quantity, that is to ſay, according as one is greater or leſſe than the other, or equal thereto, ſuch compariſon or mutual habitude is called reaſon, or (as others will have it,) proportion. Wherefore if you compare a line with a ſuperficie, or a number with a line, that compariſon cannot be called Reaſon; foraſmuch as, neither the line and the ſuperficie, nor the number and the line are quantities of the ſame kind.

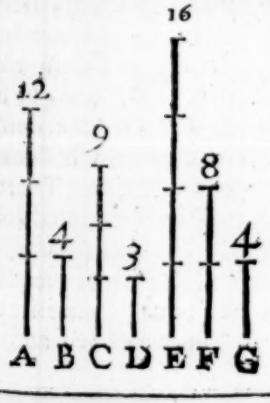
In like manner, if you compare a line with another line, or a ſuperficie with another ſuperfice, according to the quality, that is to ſay, according as the one is white, and the other black, or as the one is hot, and the other cold, &c. Although that both may be of one kind, nevertheleſſe ſuch compariſon ſhall not be termed Reaſon or Proportion, ſeeing that it is not made according to quantity.

Now although that Reaſon or Proportion is properly found betwixt quantities alone, nevertheleſſe the things which do receive in ſome manner the nature of the quantity, as Times, Sounds, Voices, Places, Motions, Weights, and Powers, are likewiſe ſaid to have Reaſon, if you conſider their habitude, according to quantity, as when we ſay that one time is greater than another time, or leſſe, or that two times are equal, &c. this habitude ſhall be called Proportion or Reaſon; for then the times are conſidered according to the manner of ſome quantities.

It remains that in all Proportion, the quantity that refers it ſelf to another, is termed by *EUCLIDE*, and other Geometricians, the Antecedent of the Reaſon. But that to which another refers it ſelf, is termed the conſequent of the Reaſon. As for Example, in the Reaſon of a line of 6 feet to a line of 3 feet, the line of 6 ſhall be termed the Antecedent of the Reaſon, and the line of 3 the conſequent; and contrarily, if you compare the line of 3 to the line of 6, the line of 3 ſhall be termed the Antecedent, and that of 6 the conſequent, otherwiſe the quantity put firſt ſhall be termed the Antecedent, and the ſecond the conſequent.

4. *Proportion is a ſimilitude or likeneſſe of Reaſons.*

THat which I call here Proportion, the Greeks call ἀναλογία, Analogie, and many Latines Proportionality. Therefore even as the habitude of two quantities to one another is termed Reaſon; ſo the compariſon of two or more Reaſons to one another, is called Proportion, as if the Reaſon of the quantity A to the quantity B, be like the Reaſon of C to D, the habitude between the Reaſons ſhall be called Proportion; in the ſame manner, if the Reaſon of E to F, be like to that of F to G, that ſimilitude ſhall be called Proportion.



Now there are divers ſorts of Proportion or habitude of Reaſons, (and we ſhall call the habitude of two quantities Reaſon; but the habitude of Reaſons Proportion,) deſcribed by Authors, principally by *Boëſius* and *Iordanus*, of which thoſe here have been

moſt in uſe amongſt the Ancients; to wit, the proportion Arithmetical, Geometrical, Muſical, or Harmonical. But *EUCLIDE* treateth only of Geometrical proportion in this Book. And of thoſe there are two ſorts, the one continued, in which the intermediate quantities are taken twice, in ſuch ſort as there is not made any interruption of Reaſons. But each intermediate quantity is a conſequent of the precedent quantity, and an antecedent to that which follows, as if you ſhould ſay that there is the ſame reaſon of E to F, as of F to G, that proportion ſhall be called continued Proportion. But the other ſhall be termed diſcontinued proportion, in the which each of the intermediate quantities is taken only once, in ſuch ſort as there is made an interruption of Reaſons, and not any quantity is Antecedent and Conſequent, but Antecedent only, or Conſequent only, as if you ſhall ſay that there is the ſame reaſon of A to B, as of C to D, this proportion ſhall be called Diſcontinual Proportion, or Proportion not continued.



Of the Diviſion of the Reaſon.

Rational. *EUCLIDE* divideth Reaſon in Rational and Irrational, Rational is that which may be expreſſed by numbers, as the reaſon of a line of 20 feet, to a line of 10 feet; for that reaſon is ſhewn by thoſe two numbers 20 and 10.

Irrational. Irrational is that which cannot be expreſſed by numbers, as is the reaſon of the Diameter of a ſquare, to the ſide of the ſame ſquare, for that reaſon cannot be found in Numbers, as *EUCLIDE* doth demonſtrate in his Tenth Book.

Others ſay that Reaſon Rational, is that which is between any two quantities commenſurable; But Irrational is that which is between two quantities incommenſurable; Now thoſe quantities are ſaid to be commenſurable, which have one common aliquot part, or are meaſured by one and the ſame common meaſure, as a line of 20 feet, and a line of 8, for a line of 4 feet is an aliquot part of both of them: In like manner, a line of 2 feet is an aliquot part of them: and therefore as well 4 as 2, ſhall meaſure as well the line of 20, as the line of 8.

Commenſurable quantities.

Incommenſurable. But Quantities incommenſurable, are thoſe which have no aliquot part, or any common meaſure that may meaſure them, as is the diameter of a ſquare, and the ſide of the ſame ſquare. For although that each of thoſe lines have infinite aliquot parts, as the half, the third, and other parts: Nevertheleſſe, not any aliquot part of the one, be it never ſo little, can poſſibly meaſure the other, as *EUCLIDE* doth demonſtrate in the Tenth Book and laſt Propoſition, in which Book are ſhewn divers other incommenſurable lines beſides thoſe two. Therefore only, Reaſon Rational is found in numbers: But as well Rational as Irrational, is found in continued quantity.

Reaſon of equality. Reaſon is alſo divided into Reaſon of equality and Reaſon of inequality. Reaſon of equality is that which is between two equal quantities compared to one another, as between 20 and 20, between 100 and 100, be-

between a line of 12 feet, and a line of 12 feet; but Reason of inequality is that which is between two quantities unequal, compared to one another, as between 20 and 10, between 8 and 40, between a line of 6 feet, and a line of 2, &c.

Now these two kinds of Reason have this in common with the first, that all reason of equality is necessarily rational, and not contrarily. In like manner, all Reason Irrational, is necessarily reason of inequality, and not contrarily, &c.

Reason of equality.

Again, Reason of inequality, (for we have left the reason of equality, considering that it cannot be subdivided; seeing that all equal quantities be they great or small, have alwayes the same reason of equality,) is subdivided into Reason of greater inequality, and of lesser inequality.

Inequality Major. Minor

Reason of greater inequality, is when the greatest quantity is compared to the least, as the reason of 20 to 10, also a line of 8 feet to a line of 6 feet, &c. Reason of lesser Inequality is when the lesser quantity is compared to the greater, as the reason of 10 to 20, also of a line of 6 feet to a line of 8 feet, &c. Now this reason is not vain and superfluous, as divers doubt: for there is not the same reason of 4 to 2, as of 2 to 4, but much different to one another, their use being divers, as is manifest to those that are any wise versed in Geometry and in Algebra.

These are the general Divisions of reason considering it as it conteins all the reasons, none excepted. Now we will subdivide as well the reason of the greater inequality, as of the lesser inequality, for that they only comprehend the Reasons Rational; forasmuch as we ought to speak in the Tenth Book of quantities which have Reason Irrational.

Therefore Reason Rational, of the greater inequality is divided into five kinds, to wit, into Reason Multiplex, Super-particular, Super-partient, Multiplex Super-particular, and Multiplex Super-partient: In like manner, the reason of the lesser inequality is divided according to the same kinds, provided that to each term of the reason there be put before it this Præposition *(Sub)* that is to say, under, as in the Reason Sub-Multiplex, Subsuper-particular, &c.

Now of these five kinds, the three first are simple, but the two last are compounded of those three first, as is manifest.

Of Reason Multiplex.

Reason Multiplex is an habitude of a greater quantity to a lesser, when the greater conteineth the lesser, a certain number of times precisely, as 2 times, 3 times, 10 times, 100 times. In such sort as the lesser measureth the greater, as is the reason of this number 20 to the number 4, for 20 conteins 4 five times, also the reason of a line of 30 feet to a line of 5 feet, &c.

Now this Reason conteineth under it infinite kinds, for if the greater quantity of the Reason Multiplex doth contein the lesser only twice, it shall be called Reason Double: if thrice, Triple, if ten times, Decuple, if a hundred times, Centuple, &c.

From these things, we shall easily define all the kinds of Reason Multiplex; for the Reason Octuple shall be no other thing then an habitude of a greater quantity to a lesser, when the greater doth contein the lesser, 8 times. In the same manner, we may define the other Reasons Multiplices, as

as the Reaſon Quintuple, ſuch as is 40 to 8, of which Reaſon, the greater conteins the leſſer five times, alſo the Reaſon of a line of 10 feet, to a line of 5 feet, is that in which the greater quantity conteineth the leſſer twice, and ſo of the others.

Of Reaſon Super-particular.

Reaſon Super-particular is an habitude of a greater quantity to a leſſer, when the greater conteines the leſſer only once and over and above an aliquot part of the ſaid leſſer, to wit the half, the third or the quarter &c. As is the Reaſon of 3 to 2, for 3 conteins 2 once and over and above Unite which is the half of 2. So in like manner, a line of 12 feet hath Reaſon Super-particular to a line of 9 feet, for the firſt line 12 conteineth the laſt 9 once, and over above a line of 3 feet, which is the third part of 9 feet.

This Reaſon is divided in like manner into infinit kinds: for if the aliquot part conteined in the greateſt quantity, be the halfe of the leſſer quantity, the Reaſon conſtituted ſhall be called Seſquialtera, if it be the third part, it ſhall be called Seſquitertia: if the fourth Seſquiquarta, and if the Thouſandth part, Sequimileſima.

Therefore by this ſame term the definitions of all the Reaſons Super-particular ſhall be eaſie, as the Reaſon ſeſquioctava, is when the greater quantity conteins the leſſer, once and over and above the eighth part of the leſſer, as is the Reaſon of 9 to 8, alſo of 45 to 40, and the ſame is to be underſtood of the reſt.

Of Reaſon Super-partients.

Reaſon Super-partients is the habitude of a greater quantity to a leſſer, when the greater conteins the leſſer once only, and over and above ſome Aliquot parts of the ſame leſſer, which being taken together, do not make an Aliquot part, as is the proportion of 8 to 5, for 8 conteins 5 once, and over and above 3 Unites, each of which is an Aliquot part, to wit, the fifth part of this number 5, but the number 3 compounded of 3 Unites, is not an Aliquot part of 5.

I have ſaid that theſe Aliquot parts ought not to conſtitute an Aliquot part, becauſe of divers reaſons which at firſt ſeeme to be Super-partients, which nevertheleſſe are Super-particulars, as is the reaſon between 10 and 8, for although that the number 10 conteineth 8 once, and over and above 2 Unites, each of which is the eighth part of 8, yet notwithſtanding, forasmuch as the number of 2, compounded of theſe 2 Unites, is the fourth part of 8: this reaſon ought not to be called Super-partients, but Super-particular, to wit Seſquiquarta: therefore, to the end that two quantities may be ſaid to have Reaſon ſuper-partients, it is neceſſary that the greater quantity do contein the leſſer once, and over and above divides Aliquot parts of the ſame leſſer, the which taken together, ought not to conſtitute an Aliquot part; which divers not conſidering do confound many kindes of Reaſons one among another.

Of Reaſon Multiplex Superparticular.

Reaſon Multiplex Super-particular, is the habitude of a greater quantity to a leſſer, when the greater doth contein the leſſer, a certain number of times, as 2, 3, 4, &c. and over and above an Aliquot part of the leſſer, as is the Reaſon of 9 to 4, for 9 conteins 4 twice, and over and above, (in which

which it agrees with the Multiplex as with the double) doth contein Unity, which is the fourth part of the leſſer number (and in that, this Reaſon is like to the Superparticular, to wit, to the Seſquiquarta) in ſuch ſort as this reaſon ſhall be rightly ſaid to be compounded of the Multiplex and of the Superparticular.

Now this Reaſon, even ſo as is the Multiplex, is divided into ſeveral kindes, as into double Superparticular triple Superparticular, &c. conſidering that the greater quantity conteineth the leſſer quantity twice or thrice and over and above an Aliquot part of the leſſer quantity.

Again each of thoſe kindes is ſubdivided into divers, having regard to the Reaſon Superparticular, for Example, the reaſon triple Superparticular, conteineth under it the triple Seſquialtera (to wit, when the greater quantity doth contein thrice the leſſer and over and above the halfe of the ſame leſſer) the triple Seſquitertia, the triple Seſquiquarta and ſo of others.

Of Reaſon Multiplex Superpartients.

Laſtly, Reaſon Multiplex Superpartients is the habitude of a greater quantity to a leſſer, when the greater doth contein the leſſe, a certain number of times, and over and above ſome Aliquot parts of the leſſer, the which being taken together do not make an Aliquot part; as is the reaſon of 11 to 3, I have ſaid [not making an Aliquot part] for the reaſon alleaged in the reaſon Superpartients. For if the ſame Aliquot parts ſhould make an Aliquot part, it ſhould not be Reaſon Multiplex Superpartients but Multiplex Superparticular.

As the reaſon of 20 to 6 ought not to be termed Multiplex Superpartients Sixths, although that 20 contein 6 three times and two Sixths, for as much as $\frac{2}{6}$ parts make $\frac{1}{3}$ part. Wherefore it ſhall be called reaſon triple Seſquitertia.

Now this reaſon is divided, firſt of all, having regard to the reaſon Multiplex; as in Multiplex double, Superpartients triple Superpartients, &c. Again, each of them, (regard being had to the number of the parts,) conteineth under it, infinite kindes: As under the triple Superpartients, is conteined the triple Superbipartients, &c.

Laſtly, each of them, in conſideration of the denomination of the Aliquot parts is again divided into infinite kindes: As the triple Supertripartients is divided into triple Supertripartients fourths and triple Supertripartients fifths, &c. Of all which the definitions and the Examples will be eaſie by what hath been ſaid.

Of Reaſons rational, of leſſer inequality.

All that hath been ſaid hitherto, of the 5 kindes of reaſons Rational, of the greater inequall, ought in like manner to be underſtood of the five Kindes correſponding, of the leſſer inequallity: Nevertheleſſe alwayes adding this Prepoſition (*Sub*) that is to ſay under as hath been ſaid.

For if in the Examples heretofore propoſed, we compare the leſſer quantities with the greater, we ſhall have the Reaſons of the leſſer inequality correſponding, for even ſo as the Reaſon of 100 to 1 is Centuple, ſo the Reaſon of 1 to 100, is Subcentuple, and as the Reaſon of 11 to 3, is Triple Superbipartient thirds, ſo the Reaſon of 3 to 11, is Subtriple, and Superbipartients thirds, and ſo of others.

Of

Of the Denominators of Reaſons Rational.

Foraſmuch as the uſe of the Denominators of Reaſons Rational before expoſed, is not a little profitable, it will not be amiſſe here to ſhew by what numbers they are denominated. The Denominator of any Reaſon, is the number which doth openly and diſtinctly expreſſe the habitude of one quantity to another, as the Denominator of the Reaſon Octuple is 8, for that the number 8 doth ſhew that the greater quantity of the Reaſon Octuple conteineth the leſſer 8 times. In like manner, the Denominator of the Reaſon Seſquiquinta is $\frac{5}{4}$; foraſmuch as this number doth ſignifie that the greater quantity of the Reaſon Seſquiquinta, conteineth the leſſer once, and the fifth part of the ſame leſſer: the ſame ought to be underſtood of the Denominators of the other Reaſons.

Wherefore (in my opinion) *EUCLIDE* in the Sixth Book, and ſundry other Mathematicians, do call the Denominator of any Reaſon whatſoever the quantity thereof: for the Denominator (as we have ſaid) ſhews how great one quantity is in reſpect of another, to wit, of that with which it is compared, as appears by the Examples propoſed.

Now from theſe things which we have ſaid, may eaſily be gathered the Denominator of any Reaſon, for the Denominator of the Reaſon Multiplex whatſoever it be, is a whole number, which doth contein as many unites as the greater quantity doth contein times the leſſer: As for Example, the Denominator of the Reaſon Double 2, of the Reaſon Octuple 8, the Centuple is 100, &c. but the Denominators of the Reaſons Submultiplices, correſpondent to the Multiplices, are aliquot parts, denominated of the Denominators of the Reaſons Multiplices, to which the denominated do correſpond. As the Denominator of the Reaſon Subdupla, is $\frac{1}{2}$, Subquintuple is $\frac{1}{5}$, Subcentuple is $\frac{1}{100}$, &c. and ſo we ſhall find the Denominators of the other Reaſons Submultiplices. Therefore the Denominator of any Reaſon Submultiplex is a broken number, or a fraction of which the Numerator is alwayes an Unite: but the Denominator is a number denominating the Reaſon Multiplex correſponding, as appears by theſe Examples. Therefore it will not be difficult to find the Denominator of any Reaſon Multiplex, or Submultiplex, if what hath been ſaid be well underſtood; foraſmuch as the ſame prolation demonſtrateth the Denominator of the Reaſon, as is manifeſt by the Examples propoſed.

The Denominator of any Reaſon Super-particular is Unity with that aliquot part that the greater quantity ought to contein over and above the leſſer: As the Denominator of the Reaſon Seſquialtera is 1 and $\frac{1}{2}$, Seſquioctava is 1 and $\frac{1}{8}$, Seſquicenteſima is 1 and $\frac{1}{100}$, &c. and it will not be difficult to find the Denominator of any Reaſon Superparticular; ſeeing that the ſame prolation of Reaſon doth expreſſe the Denominator by its aliquot part, as is evident by the Examples propoſed.

But the Denominators of the Reaſons Subſuper-particulars correſpondents, are fractions, whoſe Numerators are leſſe than their Denominators by Unity only, each to his correſpondent, as the Denominator of the Reaſon Subſeſquialtera is $\frac{2}{3}$, Subſeſquioctava is $\frac{8}{9}$, Subſeſquicenteſima $\frac{100}{101}$. Now the Denominator of any Super-particular may be found, if for the Numerator of the fraction you take the Denominator of the aliquot part expreſſed in the reaſon, and for the Denominator of the ſame fraction, you take a number greater than Unity: As the Denominator of the

the Reason Subsesquidecimal is $\frac{10}{11}$; seeing that the Numerator of this fraction is the number denominating the tenth part, to wit 10, but the Denominator of the same fraction exceedeth the Numerator by Unity, the Denominator of the Reason Subsesquiseptimal is $\frac{7}{8}$, that of the reason subsesquiquinta is $\frac{5}{6}$, &c.

In like manner, we shall find the Denominator of any Reason Subsuperparticular, if we reduce the Denominator of the Reason Subsuperparticular, correspondent in a fraction, of the which fraction the Numerator shall exceed alwayes the Denominator by unity, the which is also of the aliquot part, whereof mention is made in the Reason proposed, seeing we transpose the terms of this fraction, in such sort as the Numerator may be made Denominator, and the Denominator Numerator; we shall have the Denominator of the reason Subsuperparticular proposed. As if the Reason Subsesquiseptimal be proposed; forasmuch as the Denominator of the Reason Sesquiseptimal which is correspondent thereto, is 1 and $\frac{1}{7}$, which is reduced into this fraction $\frac{8}{7}$, whose Numerator is greater than the Denominator by the aliquot part of Unity; wherefore if we transpose this fraction in this manner $\frac{7}{8}$, we shall say that the Denominator of the Reason Subsesquiseptimal is $\frac{7}{8}$, &c.

The Denominator of any Reason Superpartients is Unity with the aliquot parts, (not making an aliquot part) which the greatest quantity ought to contein over and above the lesser; As the Denominator of the Reason Supertripartient septimal is 1 and $\frac{1}{7}$, Supertripartient 20 is $1\frac{3}{20}$, &c. and is easie to be found, forasmuch as the prolation of the Reason doth shew the proper Denominator, as appeares by the Examples before mentioned.

But the Denominator of any Reason Super-partient, is a Fraction, whose Numerator is lesse than the Denominator by so many unites, as the greater quantity of the Reason proposed, doth contein aliquot parts, over and above the lesser, as the Denominator of the Reason Subsupertripartient septimal is $\frac{7}{10}$ of the Subsupertripartient 20 is $\frac{20}{13}$, &c. Now the Denominator of any Reason Subsupertripartient shall be found, if for the Denominator of the fraction, you take the Denominator of the aliquot parts conteined in the reason proposed, to which if you adde the number of the same parts, it will give you the Denominator of that fraction.

As the Denominator of the Reason Subsupertripartient 11, to wit 11, to which is added 4, the number of four aliquot parts, to the end that 15 be made the Denominator of the same fraction.

But the Denominator of the Reason Subsupertripartient fifths, is this fraction $\frac{5}{8}$, for the Numerator thereof is the number denominating the fifth parts, to wit 5: But the Denominator of the same fraction 8, is made of the Numerator 5, and of 3, the number of the three aliquot parts, and so of the other Denominators, which may be also found in this manner.

Let the Denominator of the Reason Subsuperpartient, correspondent in a fraction be reduced, whose Numerator may exceed the Denominator, which denominateth also the aliquot parts expressed, by so many unites as there are aliquot parts.

For the numbers of this fraction being transposed in such sort as the Numerator be made Denominator, you shall have the Denominator of the Reason proposed Subsuperpartient, as the Denominator of the Reason Subsupertripartient fifths is $\frac{5}{8}$, for that the Denominator of the

Reason Supertripartient fifths is $1\frac{3}{5}$, which is reduced into this fraction $\frac{8}{5}$, which makes $\frac{5}{8}$ in transposing the numbers, and the same ought to be done for finding the other numbers, &c.

The Denominator of any Reason Multiplex Superparticular, is a whole number denominating the Reason Muliplex proposed, with the aliquot part that the greater quantity ought to contein, over and above the lesser; as the Denominator of the Reason Triplesesquiseptimal is $3\frac{1}{7}$, the Quintuple sesquinoncuple is $5\frac{1}{9}$, &c, and is very easie to expresse the Denominator of any Reason Multiplex Superparticular, forasmuch as the prolation of the Reason sheweth distinctly both the Denominator of the Reason Multiplex, and the aliquot part as may be seen by the examples proposed. Now the Denominator of any Reason Submultiplex superparticular, is a fraction, whereof the Numerator is a number denominating the aliquot parts in the Reason, as the Denominator of the Reason Subtriple sesquiseptimal is $\frac{7}{3}$, the Subquintuple sesquinonal is $\frac{9}{44}$, &c.

Now this Denominator will be found, if for the Numerator of the fraction you take the Denominator of the aliquot part, which being multiplyed by the Denominator of the Reason Multiplex, if you adde unity to the product, you shall have the Denominator of the same fraction.

As for Example, the Denominator of the Reason subquadruple sesquisexta is $\frac{6}{23}$; forasmuch as the Numerator 6 doth denominate the sixth parts, and the same multiplyed by 4 the Denominator of the Reason quadruple, and having added to the product 24 unites, to the end the Denominator of the same fraction may be made 25, &c. otherwise you should find the same denominator, if you reduce into one fraction the Denominator of the Reason Multiplex superparticular, corresponding as is before said.

As if the Reason Subquadruple sesquisexta were proposed; forasmuch as the Denominator of the Reason quadruple sexta correspondent is $4\frac{1}{6}$, which shall be reduced to $\frac{25}{6}$; therefore if you change the order of the terms, of the same fraction you will have $\frac{6}{25}$, for the Denominator of the Reason Subquadruple sesquisexta, and so of the others.

The Denominators of any Reason Multiplex superpartients, is a whole number denominating the Reason Multiplex expressed therein, with the aliquot parts which the greater quantity ought to contein over and above the lesser, which said aliquot parts do not constitute an aliquot part, as the Denominator of the Reason Triple Superquinpartient octa is $3\frac{5}{8}$, of the quadruple superbipartient fifth is $4\frac{2}{5}$.

And to find the Denominators, there is no difficulty, seeing they are expressed distinctly and openly in each Reason proposed Multiplex superpartient, to wit, as well the Denominator of the Reason Multiplex conteined therein, as the aliquot parts, as is manifest by the Examples proposed. But the Denominator of any Reason Submultiple superpartient, is a fraction, of which the Numerator is a number denominating the aliquot parts of the same Reason.

As the Denominator of the Reason Subtriple superquintapartient is $\frac{8}{29}$, subquadruple superbipartient is $\frac{2}{13}$. Now the Denominator of those Reasons submultiplices superpartients will be found, if for the Numerator of the fraction you take the Denominator of the aliquot parts, which being multiplyed by the Denominator of the Reason Multiplex, and having added to the product the number of the aliquot parts, you will obtein the Denominator of the same fraction.

As the Denominator of the Reason subduple superoctupartient is $\frac{17}{18}$, for that

that the Numerator of this fraction 13 doth denominate the thirteenth part which multiplyed by 2, the Denominator of the Reason Duple, and to the number produced 26 there be added the number of 8 parts, it will make 34, the Denominator of the same fraction.

Otherwise there will be found the same Denominator, reducing the Denominator of the Reason correspondent in a fraction, and transposing the terms as is before said.

As for Example, Let the Reason Subquintuple supertripartient tenths be proposed, to find the Denominator, you shall reduce into a fraction the Denominator of the reason correspondent quintuple supertripartient tenth, which is $5\frac{1}{10}$, and you shall have this fraction $\frac{51}{10}$; and in changing the terms, that is to say putting 10 instead of 53, and 53 instead of 10, you will have $\frac{10}{51}$ for the Denominator of the reason subquintuple supertripartient tenth, and so of the rest.

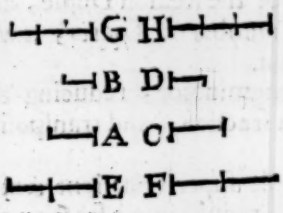
Lastly, the Denominator of the Reason of equality is alwayes unity; forasmuch as in this reason the quantities ought to be equal to one another, and therefore the one ought to contein the other once and no more, which is signified by unity.

5 *Magnitudes are said to have reason to one another, which being multiplyed may exceed one another.*

FOrasmuch as *EUCLIDE* in the third Definition, hath termed the habitude of two magnitudes of the same Kind, Reason, he teacheth now in this fifth Definition, what thing is required, to the end that two quantities of the same kind may be said to have Reason to one another; saying that Magnitudes are said to have Reason to one another, of the which (which you please) being multiplyed, may be augmented in such sort as that it may exceed the other, as the diameter and the side of one and the same square, are said to have Reason, although it be Irrational, which cannot be expressed by any number; forasmuch as the side multiplyed by 2 or taken twice, doth exceed the diameter; for seeing that the two sides of the square, with the diameter do constitute an Isosceles triangle, the two sides of the square shall be greater than the diameter thereof.

So in like manner, the circumference of a circle and the diameter thereof shall have Reason, (although it be not as yet known;) forasmuch as the diameter multiplyed by 4, or taken 4 times, doth exceed the circumference, seeing that every circumference of a circle (as is demonstrated by *Archimedes*) doth contein only three times the diameter of the said circle, and over and above a particle; a little lesse than the seventh part of the diameter.

In the same manner, divers curviline figures will have Reason with right lined figures as *Proclus* hath shewn on the First of *EUCLIDE*. Now as divers Interpreters do say that neither all Lines, nor all Plain Angles have Reason to one another, according to this Definition: for say they, a finite line hath no Reason to an infinite; seeing that in what manner soever it be multiplyed, it cannot exceed the infinite line, neither hath a right lined angle Reason to an angle of contingence, for the angle of contingence cannot exceed a right lined angle.



6 *Magnitudes* A, B, C, *and* D, *are ſaid to be in the ſame Reaſon, the firſt* A *to the ſecond* B, *and the third* C *to the fourth* D, *when the equimultiplices* E *and* F, *of the firſt* A, *and the third* C, *to the equimultiplices* G *and* H *of the ſecond* B, *and the fourth* D, *by whatever multiplication, are either deficient together, or are equal together, or do exceed together each to other, if thoſe be taken which do anſwer one another.*

EUCLIDE ſhews in this place what conditions are required in Magnitudes, that they may be ſaid to have the ſame Reaſon, which to expoſe, he is conſtrained to ſerve himſelf with their equimultiplices, to the end he may comprehend all the Reaſons of Magnitudes, as well Rational as Irrational.

Now if you compare the one to the other, the magnitudes which have been taken equimultiplices, to wit, thoſe which anſwer to one another; as the Multiplex of the firſt and the Multiplex of the ſecond, which are E and G, alſo the Multiplex of the third, and the Multiplex of the fourth, to one another, to wit, F and H, and if it be alwayes found that they have ſuch relation to one another, if E the Multiplex of the firſt Magnitude A, be leſſe than G the Multiplex of the ſecond Magnitude B, that F the Multiplex of the third Magnitude C, be alſo leſſe than H the Multiplex of the fourth C: or if E be equal to G, alſo F is equal to H; or laſtly, if E be greater than G, alſo F is greater than H, which is to be deficient together the one and the other, of the one and the other, or to be equal together, or to exceed together, in ſuch ſort as the contrary cannot be found in any kind of Multiplices; that is to ſay, that E be never leſſe than G, but F alſo be leſſe than H, and that E be never equal to G, if F alſo be not equal to H; and laſtly, that E be never greater than G, if that F alſo be not greater than H.

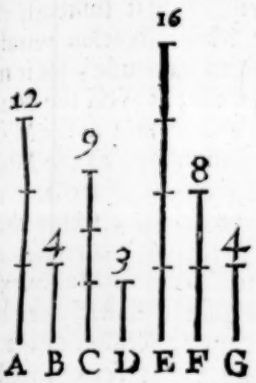
If therefore it be found that the equimultiplex taken in what manner ſoever, have relation perpetually to one another, as aforeſaid, it will be ſaid that there is the ſame Reaſon of the firſt Magnitude A, to the ſecond B, as of the third magnitude C, to the fourth D; as if it were ſometimes found, even in one only kind of Multiplices, that the Multiplex E wanteth of the Multiplex of G; but the Multiplex of F wanteth not of H, or that E is equal to G; but F is not equal to H: Or laſtly, E doth exceed G, but F exceedeth not H, although in infinite other ſorts of Multiplices, the above ſaid condition be found in ſome ſort, the Magnitudes ſhall not be ſaid to have the ſame Reaſon to one another, but divers, as ſhall be made manifeſt by the Eighth Definition.

Contrarily, if there be the ſame Reaſon of the firſt to the ſecond, as of the third to the fourth, it will follow that the equimultiplices of the firſt and

and the third, ſhall be both equal to the Multiplices of the ſecond and the fourth, or both leſſe, or both greater in any Multiplication whatſoever.

And if there were not the ſame Reaſon of the firſt to the ſecond, as of the third to the fourth, alſo the equimultiplices of the firſt and the third exceed not together the equimultiplices of the ſecond and the fourth, or ſhall not be equal, or leſſe, &c.

Now here is to be noted, that this Definition ought alſo to be taken of three Magnitudes, provided that the ſecond be taken twice, to the end you may have 4, &c.



7 *Magnitudes which have the ſame Reaſon, are called proportionals.*

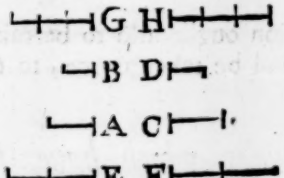
AS if of the Magnitudes A, B, C, and D, there is the ſame reaſon of A to B, as of C to D, thoſe Magnitudes A, B, C, and D, ſhall be termed proportional. In like manner, if there be the ſame reaſon of E to F, as of F to G, the Magnitudes E, F, and G, ſhall be termed proportional. Now there be ſome Magnitudes continually proportional, between which the proportion continued is found, as are the Magnitudes E, F, and G; but ſome are not continually proportional, but ſeperately, as are the Magnitudes A, B, C, and D, for in theſe there is an interruption of Reaſons, but in the others not, as is ſaid in the fourth Definition.

8 *But when of the equimultiplices the multiplex of the firſt magnitude ſhall exceed that of the ſecond. But the multiplex of the third magnitude ſhall not exceed that of the fourth, then the firſt magnitude ſhall be ſaid to have greater Reaſon to the ſecond, than the third to the fourth.*

EUCLIDE declareth in this Definition, what condition four magnitudes ought to have, to the end that the firſt may be ſaid to have greater reaſon to the ſecond, than the third to the fourth; ſaying that if the equimultiplices of the firſt and the third be taken, and likewiſe other equimultiplices of the ſecond and the fourth, and that there be ſometimes found (but not alwayes,) that the Multiplex of the firſt Magnitude be greater than the Multiplex of the ſecond: but the Multiplex of the third is not greater than the Multiplex of the fourth, but leſſe or equal, it will be ſaid that there is greater reaſon of the firſt Magnitude to the ſecond, then of the third to the fourth, as is manifeſt by the following figure propoſed, in which are taken the double of the firſt Magnitude A, and of the third C, to wit, E and F; but of the ſecond B and the fourth D, hath been taken the triples G and H. And foraſmuch as E the Multiplex of the firſt, is greater than G the Multiplex of the ſecond. But F Mul-

the Multiplex of the third, is not greater than H, the Multiplex of the fourth, but leſſe, it may be ſaid that there will be greater Reaſon of A the firſt Magnitude, to B the ſecond Magnitude, then of C the third, to D the fourth.

Now it is not neceſſary (to the end that of four Magnitudes, the firſt be ſaid to have greater Reaſon to the ſecond, than the third to the fourth,) that the equimultiplices according to any Multiplication whatſoever, have ſuch habitude, as the Multiplex of the firſt exceed the Multiplex of the ſecond; But that the Multiplex of the third exceed not the Multiplex of the fourth. But it ſufficeth that according to any Multiplication whaſoever, they have ſuch an habitude, for it may happen ſometimes that as well the Multiplex of the firſt is greater than the Multiplex of the ſecond as the Multiplex of the third, of that of the fourth: In like manner, as the Multiplex of the firſt is leſſe than the Multiplex of the ſecond, that that of the third is alſo leſſe than that of the fourth: Nevertheleſſe, ſeeing that, that happeneth not in every Multiplication; but that ſometimes the Multiplex of the firſt exceeds that of the ſecond, but the Multiplex of the third is leſſe than, or equal to the Multiplex of the fourth, for this cauſe the firſt Magnitude ſhall be ſaid to have greater Reaſon to the ſecond, than the third to the fourth, and not the ſame as is ſeen in the Example propoſed, where the third hath greater Reaſon to the ſecond, than the fourth hath to the third.



15, 9, 12, 3, 2, 10, 18, 14.
20, 12, 16, 4, 3, 15, 12, 21.

Therefore to the end that four Magnitudes may be ſaid to be proportional, it is neceſſary that the equimultiplices of them taken, according to any Multiplication whatſoever, be deficient together, equal together, or do exceed together, as hath been expoſed in the Sixth Definition.

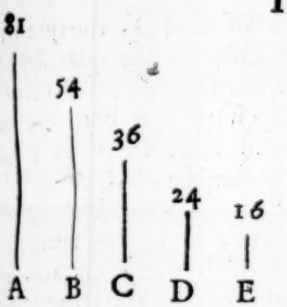
But to the end that the firſt may be ſaid to have greater Reaſon to the ſecond, than the third to the fourth, it ſufficeth that according to ſome Multiplication, the Multiplex of the firſt exceed the Multiplex of the ſecond; but that of the third exceedeth not that of the fourth, although that according to infinite other Multiplications, the equimultiplices of the firſt and of the third, exceed together the equimultiplices of the ſecond, and of the fourth.

But if contrarily, the Multiplex of the firſt Magnitude wanteth of the Multiplex of the ſecond, the Multiplex of the third wanteth not of the Multiplex of the fourth, the firſt Magnitude ſhall be ſaid to have leſſer Reaſon to the ſecond, than the third to the fourth; although that according to divers other Multiplications, the equimultiplices of the firſt and of the third do want together, of the equimultiplices of the ſecond and of the fourth, as may be ſeen in the ſame numbers of the Example propoſed, the Reaſon of 2 to 3 ſhall be leſſe than that of 3 to 4, &c.

9 *Proportion cannot be conſtituted in leſſe than three terms.*

FOraſmuch as all Analogie which is termed Proportion, is a ſimilitude or likeneſſe of Reaſons, and ſeeing that in all Reaſons there is the term ante-

antecedent, and the term consequent, it is necessary, that in all proportion there may be at least two terms antecedents, and two consequents: Wherefore if the proportion be not continued, there will be required at least four terms or Magnitudes: But if the Proportion be continued, there will be at least three terms; forasmuch as the middle term is taken twice; seeing that it is the consequent of one Reason, and the antecedent the other, and this here is the least number of the terms of proportion; for in any two terms is only found Reason, and not Proportion.



10 *When there are three magnitudes* A, B, C, *proportional, the first* A *to the third* C, *is said to have duplciate reason of the first* A *to the second* B: *But when four magnitudes are proportional, the first* A *to the fourth* D, *is said to have triplicate reason of the first* A *to the second* B, *and alwayes after the same order, one more, until that the proportion be accomplished.*

AS if the Magnitudes A, B, C, D, and E, are continually proportional in such sort, as there may be the same reason of A to B, as of B to C, and of C to D, and of D to E; the reason of the first Magnitude A, to the third Magnitude C, shall be termed the duplicate of the reason that the first Magnitude A hath to the second Magnitude B: Forasmuch as between A and C, there are two reasons, which are equal to the reason of A to B; to wit, the reason of A to B, and of B and to C, in such sort as the reason of A to B is doubled, that is to say, placed twice in order. But the reason of the first A to the fourth D, is said to be triplicate of the reason that the first Magnitude A hath to the second Magnitude B; forasmuch as between A and D are three reasons, which are equal to the reason of A to B; to wit, the reason of A to B, of B to C, and of C to D, insomuch as the reason of A to D encloseth or comprehendeth in some sort the reason of A to B triplicated, that is to say, put three times in order. So in like manner the reason of A to E, shall be said to be quadruplicate of the reason of A to B; forasmuch as four reasons are found between H and F, which are equal to the reason of A to B, &c.
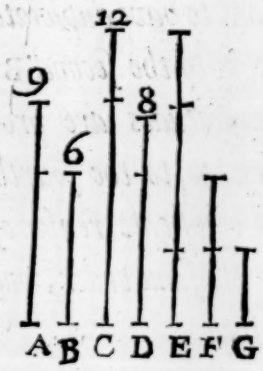
And if contrarily, there be the same reason of E to D, as of D to C, of C to B, as of B to A, the reason of E to C shall be termed the duplicate of the reason of E to D: But the reason of E to B shall be said to be the triplicate of the reason of E to D: So in like manner, the reason of E to A shall be said to be quadrupled of the reason of E to D.

11 *Magnitudes are said to be Homologal, or of like reason, the antecedents to the antecedents, and the consequents to the consequents.*

He

HE hath defined heretofore that proportion is a ſimilitude of reaſons, and now he teacheth that not only in ſome proportion the Reaſons are ſaid to be alike, but alſo the terms themſelves or quantities, are ſaid to be alike or homologal, ſaying that the Magnitudes antecedents are termed homologal, or alike to one another in reaſon, and the ſame of the conſequents to one another, to the end that it may be underſtood in ſeveral Demonſtrations what ſides of figures compared to one another, ought to be the antecedents of the reaſons, and which the conſequents, as ſhall be declared in the Sixth Book.

If there be then the ſame reaſon of A to B, as of C to D, the quantity A ſhall be ſaid to be alike to the quantity C and B alike to D, for becauſe of the ſimilitude of Reaſons, it is neceſſary that both the one and the other Magnitude antecedent be equal to one and the other conſequent, or in the ſame ſort, greater or leſſe, otherwiſe both the one and the other conſequent, or in the ſame ſort greater or leſſe, otherwiſe both the one and the other antecedent ſhould not have the ſame reaſon to the one and the other conſequent: Behold the Example.



In the Magnitudes propoſed, to the which the antecedents are in the ſame ſort greater than the conſequents, to wit, by the halfe: See another Example.

In the Magnitudes continually proportional E, F, and G, where as well E and F are homologal as F and G, as is manifeſt, and for this cauſe *EUCLIDE* in the ſixth and eighth Definition, enjoynes to take the equi-multiplices of the firſt and third Magnitude, that is is to ſay the antecedents.

In like manner, to take the other Multiplices of the ſecond and the fourth Magnitudes, to wit, the conſequents, for they are alike to the Magnitudes proportioual, as is manifeſt by this Definition; but to the Magnitudes that are not proportional, they are unlike.

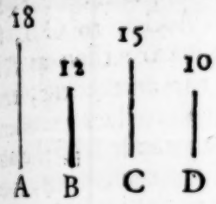
12 *Reaſon alternate, is to take the antecedent compared to the antecedent, and the conſequent to the conſequent.*

HE here unfoldeth ſome certaine kinds of arguing in proportions, whoſe uſe is very frequent amongſt Geometricians, which kinds of arguings are ſix in number; The firſt is termed alternate reaſon or reaſon by permutation, the ſecond is termed inverſe, or contrary reaſon, the third compoſition of reaſon, or conjunct proportion, the fourth diviſion of reaſon, or diſjunct or ſeparated proportion, the fifth converſion of reaſon, or reverſed proportion: Laſtly, the ſixth is termed reaſon of equality, or equal proportion.

Therefore, (ſaith he,) alternate reaſon, or reaſon by permutation, is when four proportional Magnitudes being propoſed, it be inferred that there is the ſame reaſon of the antecedent of the firſt reaſon, to the antecedent of the laſt, as of the conſequent of the firſt, to the conſequent of the laſt; as if we propoſe the ſame reaſon of A to B, as of C to D; and

and therefore conclude that there is the same Reason of A to C as of B to D; we are said to argue according to Permutation of Reason.

The Greek Authours do use in this kinde of arguing; neer this manner of expression; to wit, as A is to B, so C is to D: Therefore alternately, or contrarily, A shall be to C as B to D. Now we shall demonstrate the truth of this Illation at the sixteenth proposition of this Book. And in this kinde of arguing, it is necessary that all the four Magnitudes be of one and the same kind, to the end they may have reason to one another, being taken two and two at pleasure, for it is no arguing to say as the line A is to the line B, so the number C is to the number D; then alternately, as the line A is to the number C, so the line B is to the number D, considering that there is no proportion of a line to a number, and contrarily, as is manifest by the fifth definition.



But as to the other following kindes of arguing, the first Magnitudes may be of one kinde of quantity and the last Magnitudes of another, as shall be made manifest by the demonstrations of this fifth Book.

13 *Inverse or transposed Reason, is to take the consequent as antecedent, to compare it to the antecedent, as if it were the consequent.*

AS if (seeing that A is to B as C is to D) we should inferre that B is to A as D is to C (see the preceedent figure) we shall be said to argue according to reason Inverse, in this kinde of arguing Authours expresse themselves after this manner, to wit, as A is to B, so C is to D then inversely B shall be to A as D to C, which manner of arguing shall be demonstrated to be true by the Corollarie of the eighth Proposition of this Book.

Lastly, the two first Magnitudes, may be of one kinde and the latter of another, for it may be very well inferred, as the line A is to the line B, so the Triangle or the number C, is to the Triangle or number D, then inversely, as the line B is to the line A, so the Triangle or the number D, is to the Triangle or number C, as shall be made evident by the Corollary of the fourth Proposition of this Book.

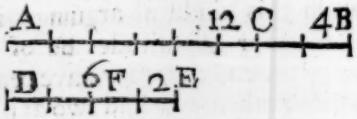
14 *Composition of reason is when the antecedent with the consequent are taken together as one to be compared to the same consequent.*

AS if there were the same reason of A C to C B, as of D F, to F E, and from that may be gathered, that there is the same reason, of the whole A B, to wit, of the antecedent with the consequent; to C B the consequent; as of the whole D E (to wit, the antecedent with the consequent,) to F E the consequent, this sort of arguing shall be called Composition of reason for as much as, of the antecedent and of the consequent there is compounded another new antecedent.

Now the Greek Authours use this manner of expression in this kind of arguing, to wit, as A C is to C B, so D F is to F E: then in compounding;

as AB ſhall be to CB, ſo DE to FE, this manner of arguing ſhall be demonſtrated in the eighteenth Propoſition of this Book.

To this, there may yet be added two other means of arguing, the firſt may be termed compoſition of Reaſon converſe; to wit, when the antecedent and the conſequent are taken as one, to be compared to the antecedent, as if (foraſmuch as AC is to CB, ſo DF to FE) we inferre then as AB compounded of the antecedent, and of the conſequent, is to the antecedent AC; ſo DE compounded of the antecedent, and of the conſequent, is to the antecedent DF, which manner of arguing we ſhall ſhew to be of worth by the eighteenth Propoſition of this Book, in which we ſhall uſe this manner of expreſſion, when we ſhall ſpeak of compoſition of converſe reaſon.



The other manner of arguing, may be termed compoſition of contrary reaſon, which is done when you compare one and the ſame Magnitude the antecedent, to the antecedent, and to the conſequent, as to one only; as if AC be to CB, ſo DF is to FE, and we inferre by compoſition of contrary reaſon: Therefore as AC antecedent, is to the whole AB, compounded of the antecedent, and of the conſequent; ſo DF antecedent ſhall be to DE, compounded of the antecedent, and of the conſequent; and we ſhall alſo demonſtrate this kind of arguing to be of worth in the eighteenth Propoſition of this Book.

15 *Diviſion of Reaſon is when you take the exceſſe by which the antecedent ſurmounts the conſequent, to compare it to the ſame conſequent.*

AS if you ſhall ſay, that there is the ſame reaſon of the whole AB to to CB, as of the whole DE to FE (ſee the precedent figure): Therefore there will be the ſame reaſon of AC, (the exceſſe by which the antecedent ſurmounts the conſequent,) to CB conſequent, as of DF, (the exceſſe, by which the antecedent ſurmounts the conſequent,) to FE conſequent.

Now in this Diviſion of Reaſon Authors ſpeak thus: [Therefore in dividing, &c.] But this Illation ſhall be demonſtrated in the ſeventeenth Propoſition of this Book.

Now to this manner of arguing there may be added two others, the firſt of which may be called Diviſion of Converſe Reaſon, to wit, when the conſeqnent is compared to the exceſſe, by which the antecedent ſurmounts the conſequent, as if AB be to CB as DE is to FE, and we conclude by Diviſion of Reaſon converſe; Therefore as CB conſequent ſhall be to AC (the exceſſe by which the antecedent ſurmounts the conſequent,) ſo FE conſequent ſhall be to DF the exceſſe by which the antecedent ſurmounts the conſequent; but we ſhall demonſtrate the certainty of this manner of arguing, in the ſeventeenth Propoſition of this Book.

Now it is manifeſt that to the end that both the one and the other of thoſe arguings may take place, (to wit, this, and that of *EUCLIDE*) that the antecedent ought to be greater than the conſequent, otherwiſe the Definition cannot be made.

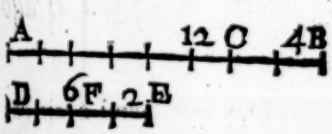
The

The other manner of arguing may be called Diviſion of Contrary Reaſon, which is made when the antecedent is compared with the exceſſe by which the conſequent ſurmounts the antecedent, as when we ſay, as AC is to AB, ſo DF is to DE: Then by Diviſion of Contrary Reaſon AC the antecedent ſhall be in like manner to CB (the exceſſe by which the conſequent ſurmounts the antecedent,) as DF the antecedent is to FE the exceſſe by the which the conſequent ſurmounts the antecedent; which manner of arguing we ſhall alſo demonſtrate in the ſeventeenth Propoſition of this Book.

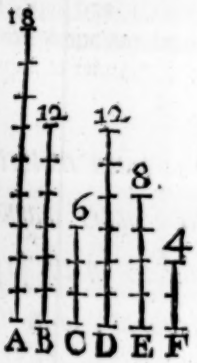
Laſtly, it is manifeſt, that in this diviſion of contrary Reaſon, the conſequent ought to be greater than the antecedent, to the end the exceſſe may be taken, by which the conſequent ſurmounts the antecedent.

16 *Converſion of Reaſon, is to take the antecedent to compare it to the exceſſe, by which the antecedent ſurmounts the ſame conſequent.*

AS if we ſay after this manner, as the whole Magnitude AB is to CB, ſo the whole DE is to FE; therefore alſo the ſame AB ſhall be to AC, the exceſſe by which the antecedent ſurpaſſeth the conſequent, as DE to DF; we ſhall be ſaid to argue according to converſion of Reaſon: Therefore Authors do expreſſe themſelves almoſt after this manner: [Therefore by Converſion of Reaſon, &c.] But this manner of arguing ſhall be confirmed in the Corollary of the nineteenth Propoſition of this Book. See the precedent figure.



¶ It is manifeſt alſo than in this manner of arguing by converſion of Reaſon, the antecedent ought to exceed the conſequent; to the end the exceſſe by which the antecedent ſurmounts the conſequent may be taken.

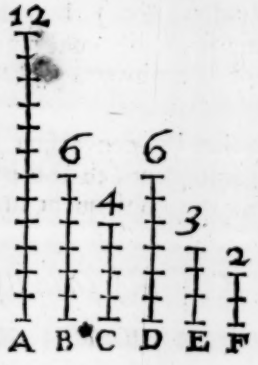


17 *Equal Reaſon, or Reaſon of equality, is when there be divers magnitudes, and other magnitudes equal to them in multitude, which may be taken two and two, and in the ſame reaſon: When as in the firſt magnitudes, the firſt is to the laſt of the ſame magnitudes; ſo in the ſecond magnitudes, the firſt is to the laſt of the ſame. Otherwiſe, it is to take the extreams by the ſubtraction of the meanes.*

LEt A B C be ſeveral Magnitudes of one part, and D, E, and F, as many Magnitudes of the other part, which being taken two and two, in the ſame Reaſon; that is to ſay, as A is to B, ſo D is to E, and as B is to C, ſo E is to F, if therefore for this cauſe it be inferred, that there is the ſame Reaſon of A the firſt of the firſt Magnitudes, to C the laſt of the ſame Magnitudes, as of D the firſt of the ſecond Magnitudes, to F the laſt of the ſame Magnitudes, this manner of arguing ſhall be called equal Reaſon or Reaſon of equality, to wit, in which the extream Magnitudes being ſubſtracted from the meanes, are ſaid to have one and the ſame Reaſon to one another, as appears by the other Definitions.

But foraſmuch as it is permitted to argue in proportions by two ſeveral manners, by the equality; to wit, in the one, when we take the Magnitudes two and two in the ſame Reaſon, in proceeding by order, and in the other, when the order is changed. *EUCLIDE* explaines in the two following Definitions, what is meant by Ordinate Proportion, and Perturbated or troubled Proportion.

18 *Ordinate proportion is, when as the antecedent is to the conſequent, as the antecedent is to the conſequent; and as the conſequent is to ſome other, ſo the conſequent is alſo to ſome other.*



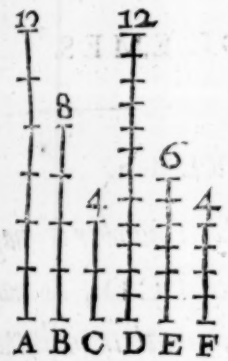
AS if A be to B, as D is to E; and again, as B the conſequent is to ſome other, to wit, to C, ſo E the conſequent is to ſome other, as to F; ſuch manner of arguing ſhall be termed Ordinate: and foraſmuch as the ſame order is obſerved, as well in the firſt Magnitudes, as in the ſecond; ſeeing that on both parts the firſt is firſt of all compared with the ſecond, and after that the ſecond with the third. Therefore when in the manner of arguing by equality, the Ordinate Proportion is obſerved and kept, it is demonſtrated in the two and twentieth Propoſition of this Book, that this manner of arguing is good.

19 *Perturbated or troubled Proportion is, when as three magnitudes are propoſed on one part, and other magnitudes, equal unto them in multitude, and as in the firſt magnitudes, the antecedent is to the conſequent; ſo in the ſecond magnitudes the antecedent is to the conſequent: But as in the firſt magnitudes, the conſequent is to ſome other, ſo in the ſecond magnitudes, ſome other is to the antecedent.*

That

THat is to say, when three Magnitudes are proposed on the one part and as many on the other, the first of the first is to the second of the same; as the second of the seconds, is to the third of the same; and as the second of the first is to the third of the same; so the first of the seconds is to the seconds of the same.

As if there be the same Reason of A to B, as of E to F; then afterward in the first Magnitudes, as B consequent, is to some other, to wit C; so in the second Magnitudes, some other, to wit D, is to E the antecedent. And this sort of arguing shall be termed disturbed Proportion, forasmuch as the same order in the proportions of the Magnitudes is not observed; seeing that in the first magnitudes, the first is compared with the second: but in the second Magnitudes, the second is compared with the third: And again, in the first Magnitudes, the second is compared to the third, but in the second Magnitudes, the first is compared with the second. Therefore, when in the manner of arguing by equality, the proportion disturbed is kept and observed, it is demonstrated that this manner of arguing is good, as in the three and twentieth Proposition of this Book.



Lastly, as well the disturbed Proportion, as the Ordinate, inferre alwayes by equality, that there is the same proportion of the extreams, although more than three Magnitudes be proposed, as shall be made manifest by the two and twentieth and three and twentieth Propositions of this Book.

PRO-

PROPOSITIONS,

PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.



If there be a number of magnitudes A B *and* C D, *as many as you pleaſe, equal in multitude to other magnitudes* E *and* F, *and equimultiplices to them, each to his correſpondent, as the one of the magnitudes* A B, *ſhall be multiplex of one* E, *ſo the whole* A B *and* C D, *ſhall be Multiplex of the whole* E *and* F.

Demonſtration FOr ſeeing that A B and C D are equimultiplices of E and F, if A B be divided according to the Magnitudes A G, G H, and H B, equal each of them to E, and C D in like manner according to C I, I K, and K D, equal each to F. Now each of them may be divided into parts totally equal to E and F; ſeeing that A B and C D are equimultiplices of E and F; and therefore E ſhall be conteined exactly as many times in A B, as F in C D, as appears by the ſecond Definition of this Book; the Magnitudes A G, G H, and H B, ſhall be as many in number as C I, I K, and K D. But foraſmuch as A G and E, are equal to one another, if to them be added the equal Magnitudes C I and F, A G and C I together, they ſhall be equal to E and F together. In like manner, G H and I K together ſhall be equal to E and F together; and alſo H B and K D to the ſame E and F; therefore as many times as E is contained in A B, or F in C D, ſo many times E and F together ſhall be conteined in A B and C D together; therefore as A B is Multiplex of E, ſo A B and C D together, ſhall be Multiplex of E and F together, by the ſecond Definition of this Book: Therefore, If there be, &c. Which was to be demonſtrated.

PROP.

PROP. 2. THEOR. 2.



If the first AB, *be as much multiplex of the second* C, *as the third* DE *is of the fourth* F, *and that the fifth* BG *be also as much Multiplex of the second* C, *as the sixth* EH *is of the fourth* F. *The magnitude* AG, *compounded of the first* AB, *and of the fifth* BG, *shall be as much multiplex of the second* C, *as the magnitude* DH, *compounded of the third* DE, *and of the sixth* EH, *is of the fourth* F.

Demonstration FOr seeing that AB and DE are equimultiplices of C, and of F, there will be in AB as many Magnitudes equal to C, as there are in DE, equal to F: In like manner, there will be in BG as many Magnitudes equal to C, as there are in EH, equal to F. If therefore to the equal Multitudes AB and DE, the equal Multitudes BG and EH, all the Multitudes AG and DH shall be equal in number: Therefore C shall be conteined as many times in AG as F is conteined in DH: Therefore AG (compounded of the first and fifth,) shall be as much Multiplex of C the second, as DH (compounded of the third and the sixth,) is of F the fourth: Therefore, If the first, &c. Which was to be demonstrated.

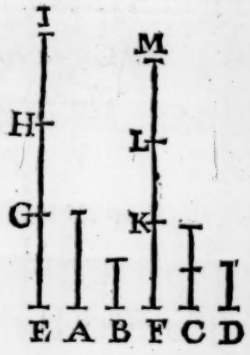
PROP. 3. THEOR. 3.



If the first A, *be as much Multiplex of the second* B, *as the third* C, *is of the fourth* D, *and there be taken the equimultiplices* E *and* F, *of the first* A, *and of the third* C, *in equal reason, the magnitude* E, *taken multiplex of the first* A, *shall be also so much multiplex of the second as the magnitude taken* F, *multiplex of the third* C, *shall be of the fourth* D.

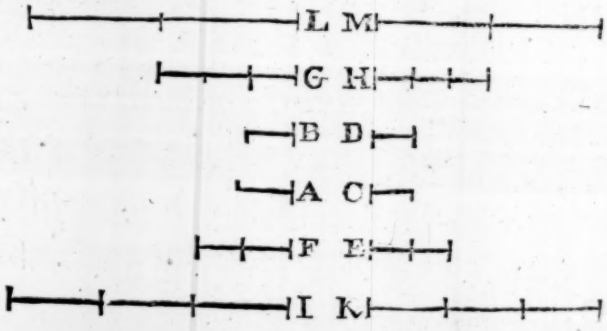
Demonstration FOr seeing that E and F are equimultiplices of A and C, if E be divided into Magnitudes equal to A, as into EG, GH, and HI, also F into Magnitudes, equal to C, as into FK, KL, and

and L M; there will be in E as many parts equal to A, as there are in F, equal to D; foraſmuch therefore as E G is equal to A, and F K equal to C: But A and C are equimultiplices of B and D, by ſuppoſition; E G and F K ſhall be equimultiplices of the ſame B and D: In like manner, G H, and K L, and H I and L M ſhall be equimultiplices of the ſame B and D: foraſmuch therefore as E G the firſt magnitude, is ſo much Multiplex of the ſecond B, as F K the third is Multiplex of the fourth D, alſo G H the fifth, is as much Multiplex of B the ſecond, as K L the ſixth is Multiplex of the fourth D; alſo G H the fifth is as much Multiplex of B the ſecond, as K L the ſixth is Multiplex of the fourth D; E H compounded of the firſt and of the fifth, ſhall be as much Multiplex of the ſecond B, as F L compounded of the third, and the ſixth is Multiplex of the fourth D. Again, foraſmuch as E H the firſt is as much Multiplex of B the ſecond, as F L the third, is of D the fourth, as we even now demonſtrated: But H I the fifth is ſo much Multiplex of the ſecond B, as L M the ſixth is of D, the fourth E I, compounded of the firſt and fifth, ſhall be as much Multiplex of the ſecond B, as F M compounded of the third, and of the ſixth, is of D the fourth, and ſo of the others, if there be more parts in E and F: Therefore, If, &c. Which was to be demonſtrated.



PROP. 4. THEOR. 4.

If the firſt A, *hath the ſame reaſon to the ſecond* B, *as the third* C, *to the fourth* D, *alſo the equimultiplices* E *and* F, *of the firſt* A, *and of the third* C, *ſhall have the ſame reaſon to the equimultiplices* G *and* H, *of the ſecond* B, *and of the fourth* D, *according to any Multiplication, if they be taken ſo as that they anſwer to one another.*



Demonſtration FOr if you conſtitute E the firſt Magnitude, and G the ſecond, F the third, and H the fourth, and that I and K any what ever equimultiplices of E and F, the firſt and third alſo L and M, ſome equimultiplices of G the ſecond, and H the fourth; foraſmuch as

as E the first is as much Multiplex of A the second, as F the third, is of C the fourth, and that I and K be taken equimultiplices of E and F, the first and third, in equal reason, [a] I and K shall be also equimultiplices of A and C, the second and fourth. By the same reason L and M are equimultiplices of B and D; and forasmuch as A is to B, as C to D, and that I and K are taken equimultiplices of A and C, the first and third, [b] if I the Multiplex of the first A, wanteth of L the Multiplex of the second B; also K the Multiplex of the third C, shall want of M the Multiplex of the fourth D: if I be equal to L, also K shall be equal to M: and lastly, if I exceed L, also K shall exceed M, according to any whatsoever Multiplication they be taken. Therefore, seeing that I and K are equimultiplices of E the first, and F the third, also L and M equimultiplices of G the second, and H the fourth, and that it hath been demonstrated that if I the Multiplex of E the first, be lesse than L the Multiplex of G the second, that K the Multiplex of F the third, is lesse than M the Multiplex of H the fourth, and if I be equal to L, K is equal to M, and if I be greater than L, K is greater than M, and that this doth happen in every Multiplication, there [c] shall be the same reason of E the first, to G the second, as of F the third, to A the fourth; Therefore if the first, hath the same reason to the second, as the third to the fourth, &c. Which was to be demonstrated.

a) 3.

b) 6. def.

c) 6. def.

COROLLARIE.

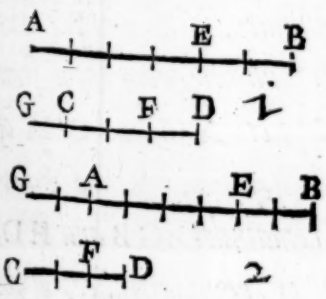
By this Demonstration the proof of inverse reason explained by EUCLIDE, *is manifest;* [d] *which is if four Magnitudes are proportional, they shall be the same taken backwards, or contrarily, to wit, if* A *be to* B, *as* C *to* D, *in inverse reason as* B *shall be to* A, *so* C *to* D; *for seeing that as* A *is to* B, *so* C *is to* D; [e] E *and* F *equimultiplices of* A *and* C *first and third, shall be both greater, equal, or lesse than* G *and* H, *equimultiplices of* B *and* D, *the second and fourth, in any Multiplication whatever: Therefore contrarily,* G *and* H *shall be both lesse, equal, or greater than* E *and* F. *Therefore* [f] *there will be the same reason of* B *to* A, *as of* D *to* C.

d) 13. def.

e) 6. def.

f) 6. def.

PROP. 5. THEOR. 5.



If a magnitude AB *be as much Multiplex of a magnitude* CD, *as the part cut off* AE, *is of the part cut off* CF; *also the remainder* EB *shall be as much Muliplex of the remainder* FD, *as the whole* AB, *is of the whole* CD.

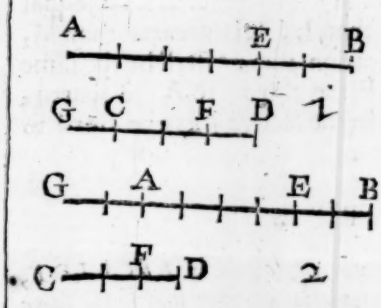
Demonstration ASsume CG, of which EB may be so much Multiplex, as AE is of CF, or the whole AB of the whole CD: forasmuch as AE and EB are equimultiplices of CF and GC, the [a] whole AB, shall be so much Multiplex of the whole GF, as AE is of CF; that is to say, the whole of the whole, as one only Magnitude, of one only Magnitude. But AB is proposed so much Multiplex of CD,

a) 1. 5.

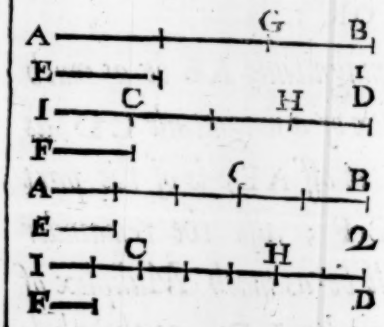
as AE of CF; therefore AB shall be as much Multiplex of GF, as of
b) 6. c. f. CD; therefore [b] GF and CD are equal; therefore taking away the common CF, there remains DF, equal to GC; therefore EB shall be as much Multiplex of FD, as of CG, but EB is as much Multiplex of CG, by supposition, as AE is of CF; that is to say, as the whole AB of the whole CD: Therefore the remainder EB shall be as much Multiplex of the remainder FD, as the whole AB, of the whole CD. Which was proposed.

Otherwise, Let AB be as much Multiplex of the whole CD, as the part cut off AE, of the part cut off CF. I say, that the rest EB, is as much Multiplex of the rest FD, as the whole AB, of the whole CD.

For having taken GA as much Multiplex of FD, as AE is of CF, or the whole AB of the whole CD: Forasmuch as AE and GA are
c) 1. 5. equimultiplices of GF and FD, [c] the whole GE shall be as much Multiplex of the whole CD, as AE of CF; But AB is in like manner, also as much Multiplex of CD, as AE of CF, by supposition: Therefore GE and AB are equimultiplices of CD; [d] and therefore equal to one another, taking away
d) 6. a. f. then the common AE, there will remain GA and EB equal; and therefore equimultiplices of FD; seeing that GA is taken as much Multiplex of FD as AE of CF, or the whole AB of the whole CD; therefore the remainder EB shall be as much Multiplex of the remainder FD, as the whole AB of the whole CD, which was proposed: Therefore, If a Magnitude, &c. Which was to be demonstrated.



PROP. 6. THEOR. 6.



If two magnitudes AB *and* CD, *are equimultiplices of two other magnitudes* E *and* F, *and there be cut off from them some equimultiplices* AG *and* CH, *of the same magnitudes* E *and* F, *either the remainders* GB *and* HD, *shall be equal to the same* E *and* F, *or equimultiplices of the same* E *and* F.

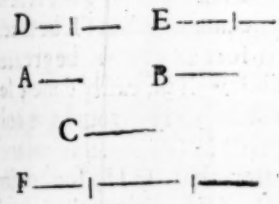
Demonstration FOr in the first place, Let GB be equal to E: I say, that HD shall be equal to F: Let CI be put equal to F; forasmuch as AG the first, is as much Multiplex of E the second, as CH the third, is of F the fourth, & that GB the fifth, is equal to E, and IC the sixth, equal to F the
a) 2. 5. fourth, [a] AB compounded of the first and of the fifth, shall be as much Multiplex of

of E the second, as I H compounded of the third and sixth, is of F the fourth. But C D is as much Multiplex of F by Supposition, as A B is of E: therefore H I and C D are equimultiplices of F. Therefore [b] equal to one another; therefore taking away the common part C H, there will remain C I and H D equal: Now seeing that C I is put equal to F, H D shall be likewise equal to F, which was proposed. In like manner we shall demonstrate, that if G B be Multiplex of E, that H D shall be as much Multiplex of F, in putting I C as much Multiplex of F, as G B is of E, &c. b) 6. c. 5.

Otherwise, forasmuch as A B and C D are equimultiplices of E and F, there will be in A B as many Magnitudes equal to E, as there are in C D equal to F. Again, forasmuch as A G and C H are equimultiplices of the same E and F, [c] there will be in like manner in A G as many magnitudes equal to E, as there are in C H equal to F. c) 1, 2. def.

If therefore from the equimultiplices A B and C D, you take away the equimultiplices A G and C H, there will remaine the equimultiplices G B and H D equimultiplices. Therefore G B shall contein as many times E, as H D shall contein F, if once only, G B and H D shall be equal to E F, if more than once, they shall be equimultiplices. Which was proposed.

PROP. 7. THEOR. 7.



Equal magnitudes A *and* B, *have the same reason to one and the same magnitude* C, *and one and the same magnitude* C *hath the same reason to the equal magnitudes* A *and* B.

Demonstration. FOr assume D and E the equimultiplices of the equal Magnitudes A and B, [a] the same D and E shall be equal to one another. Again, Let F be any Multiplex of C; forasmuch then as D and E are equal, and equimultiplices of A and B, the first and third, both the one and the other shall be lesse than F, or equal, or greater, after what Multiplication soever they be taken. a) 6. c. 5.

Therefore seeing that D and E equimultiplices of the first A, and of the third B, are both greater than F, the Multiplex of the second and of the fourth, (for C is put as for two Magnitudes) or equal, or lesse; [b] there will be the same reason of A the first, to C the second, as of B the third, to D the fourth. b) 6. def. 5.

In the same manner, we shall demonstrate that F is lesse than each of the Magnitudes D and E, or equal, or greater: Therefore seeing that F the Multiplex of the first, and of the third C, is lesse than either D or E, equimultiplices of the second, and the fourth A and B, or equal, or greater, there will be likewise the same reason of the first C, to the second A, as of the third C, to the fourth B. Which was proposed.

SCHOLIUM.

This second part may be more briefly demonstrated by Inverse Reason, it having been already demonstrated that there is the same reason of A *to* C, *as of* B *to* C; *alternately,* [c] *there shall be the same reason of* C *to* A, *as of* C *to* B. *Therefore the equal Magnitudes, &c. Which was to be demonstrated.* c) Cor. 4. 5

PROP. 8. THEOR. 8.



Of unequal magnitudes A, B *and* C, *the greater* A B, *hath greater reaſon to one and the ſame magnitude* D, *than the leſſer* C, *and one and the ſame magnitude* D, *hath greater reaſon to the leſſer* C, *than to the greater* A B.

Demonſtration FOr from the greateſt A B cut off A E equal to C; and let A B be propoſed the firſt Magnitude, C the third, and D the ſecond and fourth, then take H G and G F, equimultiplices of A E and E B, in ſuch ſort as G H the Multiplex of A E be greater than D, and F G the Mutiplex of E B, be not leſſe than the ſame D, but alſo greater.

Now foraſmuch as H G and G F are equimultiplices of A E and E B; a) 2. 5. a the whole H F ſhall be as much Multiplex of the whole A B, as G H is of A E; that is to ſay of C, equal to A E. In like manner, Let I K be taken Multiplex of D the ſecond and fourth, in ſuch ſort as that it be greater than H G, but leſſe than the whole H F, which will be eaſily done; for ſeeing that D is leſſe than F G, and alſo leſſe than G H, you have only to adde D ſo many times as its product I K exceeds H G; by this meanes I K ſhall be leſſe than the whole H F, and greater than G H, for having cut off K L equal to D, it ſhall be leſſe than F G, (F G being taken greater than D,) and the remainder L I leſſe than G H, or equal, (the ſame G H being put greater than D,) and D added only ſo many times as its product I K exceeds G H. Therefore I K ſhall be greater than G H, and leſſe than the whole H F. Therefore ſeeing that F H and H G are equimultiplices of A B the firſt, and C the third, and I K Multiplex of D, put for the ſecond and the fourth; but F H the Multiplex of the firſt, is not b) 8. def. greater than I K the Multiplex of the fourth, but leſſe by ſuppoſition, (b for I K the Multiplex of D, is taken greater than H G,) there ſhall be greater reaſon of A B the firſt, to D the ſecond, then of C the third, to D the fourth.

Secondly, foraſmuch as I K the Multiplex of D the firſt; (Let D be taken for firſt, and third, and C the ſecond, and A B fourth,) is greater than H G Multiplex of C the ſecond: But I K the Multiplex of D the third, is not greater than F H Multiplex of A B the fourth, but leſſe; F H being greater than I K, as hath been ſhewn, there will be greater reaſon of D the firſt, to C the ſecond, than of D the third, to A B the fourth; which was propoſed: Therefore, The Magnitudes, &c. Which was to be demonſtrated.

PROP. 9. THEOR. 9.

Magnitudes A *and* B, *which have the ſame reaſon to one and the ſame magnitude* C, *are equal to one another: and thoſe mag-*

magnitudes A B, *to which one and the ſame magnitude* C, *hath the ſame reaſon, are equal to one another.*

Demonſtration OTherwiſe, they ſhould be unequal, and the one greater than the other. Let A then be the greater, and B the leſſer, (if poſſible:) Therefore [a] there will be greater reaſon of A the greater, to C, then of B the leſſer, to the ſame C, which is contrary to Suppoſition: Therefore A and B are not unequal, but equal. a) 8. 5.

Secondly, Let C have the ſame reaſon to A, as to B. I ſay again, that A and B are equal to one another: for if the one were greater, to wit A, and B the leſſer, C [b] would have greater reaſon to B the leſſer, then to A the greater, which is contrary to Suppoſition: Therefore A ſhall not be greater than B, but equal thereto. Therefore Magnitudes, &c. Which was to be demonſtrated. b) 8. 5.

A——
B——
C——

PROP. 10. THEOR. 10.

Of magnitudes A, *and* B, *which have reaſon to one and the ſame magnitude* C, *that which hath the greateſt reaſon* A, *is the greater; but that* B *to which one and the ſame magnitude* C, *hath the greateſt reaſon, is the leaſt.*

A——
B——
C——

Demonſtration FIrſt, Let A have greater reaſon to C, then B to the ſame C: I ſay that A is greater than B, for if A were equal to B, the ſame [a] A and B would have the ſame reaſon to C, and if A were leſſe than B, [b] the greateſt B would have greater reaſon to C, than the leaſt A, which is contrary to ſuppoſition; therefore A is not equal to B, nor leſſe, but greater. a) 7. 5. b) 8. 5.

Secondly, Let C have greater reaſon to B then to A. I ſay that B is leſſe than A; [c] for then C would have the ſame reaſon to A as to B, which is contrary to ſuppoſition: B alſo ſhall not be greater then A, foraſmuch as C ſhould have greater reaſon to the leaſt A, then to B the greater, which is contrary to ſuppoſition: Therefore B is leſſe than A, which was propoſed: Therefore the Magnitudes, &c. Which was to be demonſtrated. c) 7. 5.

PROP. 11. THEOR. 11.

Reaſons which are the ſame to one and the ſame reaſon, are alſo the ſame to one another.

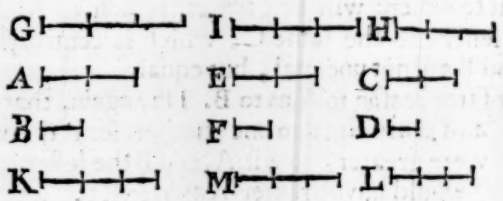
LEt the reaſon of A to B, be as that of E to F, and E be to F, as C to D: I ſay that as A is to B, ſo C is to D.

Demonſtration FOr let there be taken any equimultiplices whatſoever, of all the antecedents A, E, and C, which may be G, I, and H: In like manner, Let there be alſo taken any whatſoever equimultiplices K,

K, M, and L, of the conſequents B, F, and D, foraſmuch as A the firſt is to B the ſecond, as E the third, to F the fourth, [a] if G the Multiplex of A the firſt, wanteth of K the Multiplex of B the ſecond, I the Multiplex of E the third, ſhall alſo want of M the Multiplex of F the fourth, and if G be equal to K, I ſhall be equal to M, and if G be greater than K, I, ſhall be greater than M. [b] But (as ſhall be demonſtrated in the ſame manner,) if I, be leſſe than M, or equal, or greater: Likewiſe H ſhall be leſſe than L, or equal, or greater; foraſmuch as by Suppoſition there is the ſame reaſon of E the firſt, to F the ſecond, as of C the third, to D the fourth. Wherefore if G the Multiplex of A the firſt, be leſſe than K the Multiplex of B the ſecond; H the Multiplex of C the third, ſhall be leſſe than L the Multiplex of D the fourth, and if G be equal, or greater than K, alſo H ſhall be equal or greater than L, and ſhall be demonſtrated that it will happen ſo in any whatever other equimultiplices: Therefore [c] there ſhall be the ſame reaſon of A the firſt, to B the ſecond, as of C the third, to D the fourth. Therefore, Reaſons which are the ſame, &c. Which was to be demonſtrated.

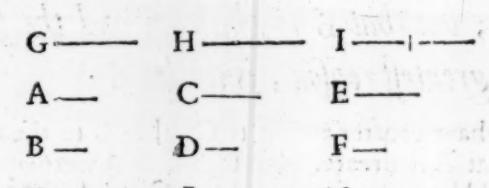
a) 6. def. 5.

b) 6. def. 5.



c) 6. def. 5.

PROP. 12. THEOR. 12.



If as many magnitudes as you pleaſe A, B, C, D E, *and* F, *are proportional, as one of the antecedents* A, *ſhall be to one of the conſequents* B; *ſo all the antecedents* A, C, *and* E, *ſhall be to all the conſequents* B, D, *and* F.

Demonſtration FOr aſſume G, H, and I, equimultiplices of the antecedents A, C, and B, and K, L, and M, equimultiplices of the conſequents B, D, and F, [a] the whole G, H, and I, together, ſhall be as much Multiplex of the whole A, C, and E, together, as one of them alone, is of one alone, to wit, as G is of A, and the whole K, L, and M together, ſhall be as much Multiplex of the whole B, D, and F, together, as one alone, is of one alone, to wit, as K is of B: but foraſmuch as by ſuppoſition there is the ſame reaſon of A the firſt, to B the ſecond, as of C the third, to D the fourth, and as another E the third, to F the fourth, it will happen that if G the Multiplex of A the firſt, doth want of K the Multiplex of B the ſecond. In like manner, H the Multiplex of C the third, will want of L the Multiplex of D the fourth, and I of M, and if G be equal to K, or greater. Likewiſe, H ſhall be equal to L, and I to M, or greater; and therefore if G be leſſe or equal, or greater than K, the whole G, H, and I, together, ſhall be leſſe than the whole K, L, and M, together, or equal, or greater. Wherefore

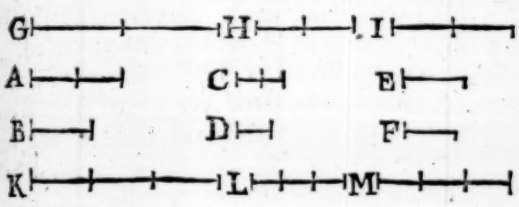
a) 1. 5.

6. def. 5.

as

as A the first, is to B the second, so A, C, and E, the third, shall be to B, D, and F, the fourth: Therefore if there be, &c. Which was to be demonstrated. 6. def. 5.

PROP. 13. THEOR. 13.



If the first A, *hath the same reason to the second* B, *as the third* C, *hath to the fourth* D; *But the third* C, *hath greater reason to the fourth* D, *than the fifth* E, *to the sixth* F, *also the first* A, *shall have greater reason to the second* B, *than the fifth* E, *to the sixth* F.

Demonstration FOr, assume G, H, and I, equimultiplices of the antecedents A, C, and E, and K, L, and M, equimultiplices of the consequents B, D, and F: forasmuch as there is the same reason of A the first, to B the second, as of C the third, to D the fourth, [a] it will happen, that if G, the Multiplex of A the first, exceed K, the Multiplex of B the second, that H the Multiplex of C the third, will exceed L the Multiplex of D the fourth, &c. But when H exceeds L, [b] it is not necessary that I exceed M, but shall be sometimes equal, or lesse; forasmuch as there is greater reason of C the first, to D the second, then of E the third, to F the fourth; therefore if G exceed K, of necessity I shall exceed M: and [c] therefore there is greater reason of A the first, to B the second, then of E the fifth, to F the sixth: Therefore, &c. Which was to be demonstrated.

a) 6. def. 3.
b) 6. def. 5.
c) 8. def. 5.

PROP. 14. THEOR. 14.



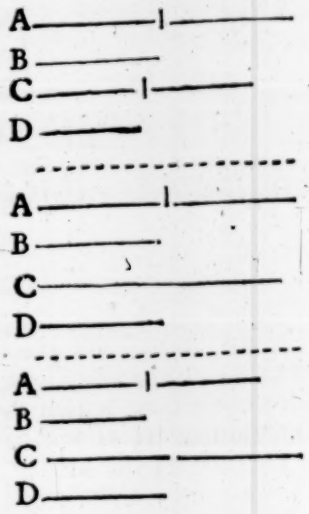
If the first A, *hath the same reason to the second* B, *as the third* C, *to the fourth* D, *and that the first* A, *be greater than the third* C, *the second* B, *shall be also greater than the fourth* D, *and if the first* A, *be equal to the third* C, *also the second* B, *shall be equal to the fourth* D, *and if lesse, lesse.*

Demonstration IN the first place, Let A the first, be greater than C the third,

third, there [a] will be greater reaſon of A the greater, to B, then of C the leſſer, to the ſame B: Therefore foraſmuch as C the firſt, is to D the ſecond, as A the third, to B the fourth: But the reaſon of A the third, to B the fourth, is greater than the reaſon of C the fifth, to B the ſixth, as we have ſhewn; [b] there will be therefore greater reaſon of C the firſt, to B the ſecond, then of C the fifth, to B the ſixth; therefore D ſhall be leſſe than B, therefore B ſhall be greater than D, which was propoſed.

a) 8. 5.

b) 13. 5.



Secondly, Let A be equal to C, there [c] will be the ſame reaſon of A to B, as of C to B; foraſmuch then as the reaſons of C to D, and of C to B, are the ſame as the reaſon of A to B, [d] the reaſons of C to D, and of C to B, ſhall be the ſame to one another: Therefore B and D, ſhall be equal, which was propoſed.

c) 7. 5.

d) 11. 5.

Laſtly, Let A be leſſe than C, [e] there will be greater reaſon of the greateſt C, to B, then of the leaſt A, to the ſame B. Foraſmuch then as C the firſt, is to D the ſecond, as A the third, to B the fourth. But the reaſon of A the third, to B the fourth, is leſſe than that of C the fifth, to B the ſixth; there [f] will be likewiſe leſſe reaſon of C the firſt, to D the ſecond, than of C the fifth, to B the ſixth: Therefore B ſhall be leſſe than D, which was propoſed: [g] Therefore, If, &c. Which was to be demonſtrated.

e) 8. 5.

f) 13. 5.

g) 10. 5.

PROP. 15. THEOR. 15.



The parts C *and* F *are to one another, as are their equimultiplices* AB *and* DE, *to one another, if they be taken ſo as they mutually anſwer to one another.*

Demonſtration. FOr ſeeing that A B is as much Multiplex of C, as D E is of F, the Magnitude C ſhall be as oftentimes conteined in A B, as F is conteined times in D E.

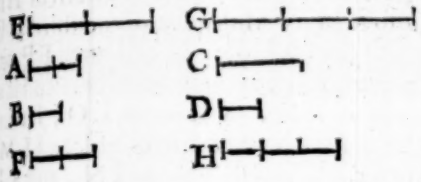
Let therefore A B be divided according to the parts A G, G H, and H B, each equal to C, and D E according to the parts D I, I K, and K E, each equal to F. And foraſmuch as A G and C are equal, and D I and F, alſo equal, A G [a] ſhall be to D I, as C ſhall be to F, by the ſame reaſon, G H ſhall be to I K, and H B to K E, as C to F; and therefore A G, G H, and H B, ſhall have the ſame reaſon to D I, I K, and K E: Wherefore as A G ſhall be to D I, that is to ſay, as C to F, ſo [b] A B ſhall be to D E, to wit, all the antecedents together, A G, G H, and H B, to all the conſequents together D I, I K, and K E, which was propoſed. Therefore, The parts, &c. Which was to be demonſtrated.

a) 7. 5.

b) 12. 5.

PROP.

PROP. 16. THEOR. 16.



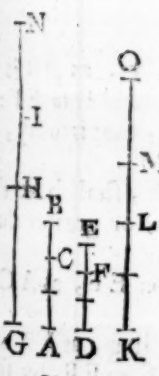
If four magnitudes A, B, C, *and* D, *be proportional, they ſhall be alſo proportional alternately.*

Demonſtration FOr, Let A be to B, as C is to D; I ſay that alternately, or by permutation, A ſhall be to C, as B to D.

Aſſume E and F equimultiplices of A and B, the firſt and ſecond; Alſo G and H equimultiplices of C the third, and D the fourth, E ſhall be to F, as A is to B; [a] Seeing that E and F are equimultiplices of A, and by the ſame reaſon, G ſhall be to H, as C to D. a) 15. 5.

Now ſeeing that the reaſons of E to F, and C to D, are the ſame as the reaſon of A to B, [b] they ſhall be the ſame to one another. Again, foraſmuch as the reaſons of E to F, and G to H, are the ſame as the reaſon of C to D, [c] they ſhall be alſo the ſame to one another: That is to ſay, as E the firſt, ſhall be to F the ſecond, ſo G the third, to H the fourth: Therefore [d] if E the firſt be greater than G, the third, F the ſecond, ſhall be greater than H the fourth, and if E be equal or leſſe than G, alſo F ſhall be equal or leſſe than H, in whatſoever Multiplication they be taken E F and G H. Therefore [e] A the firſt, ſhall be to C the ſecond, as B the third, to D the fourth; ſeeing that E and F, equimultiplices of the firſt and third A and B, are both greater than G and H, equimultiplices of the ſecond and the fourth C and D, or both equal, or both leſſe, &c. Which was propoſed. Therefore, If four Magnitudes, &c. Which was to be demonſtrated. b) 11. 5. c) 11. 5. d) 14. 5. e) 6. def. 5.

PROP. 17. THEOR. 17.



If Compounded Magnitudes AB, CB, DE, *and* FE, *are proportional, they ſhall be alſo proportional, being divided.*

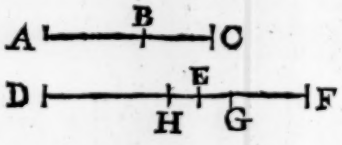
Demonſtration FOr, Aſſume the equimultiplices of AC, CF, DF, and FE, of the ſame order, to wit, GH, HI, KL, and LM, and [a] GI ſhall be as much Multiplex of AB, as GH is of AC; that is to ſay, as KL is Multiplex of DF, [b] KM is likewiſe Multiplex of DE: Therefore GI and KM are equimultiplices of AB and DE. Again, aſſume IN and MO, equimultiplices of CB and FE. Foraſmuch as HI the firſt, is as much Multiplex of CB the ſecond, as LM the third, is of FE the fourth, alſo IN the fifth, as much multiplex of CB the ſecond, as MO the ſixth, is of FE the fourth, HN compounded of the firſt and the fifth, ſhall be as much multiplex of the ſecond CB, as LO compounded of the third and ſixth, is of FE the fourth. Therefore ſeeing that AB the firſt, is to CB the ſecond, as DE the third, to FE the a) 1. 5. b) 1. 5.

fourth, and that the equimultiplices of the first and of the third, A B and D E, have been assumed; to wit, G I and K M. In like manner, of the second, and the fourth, C B and F E, the equimultiplices H N and L O,
c) 6. def. 5. c it will happen that if G I the Multiplex of the first A B, wanteth of H N the Multiplex of the second C B, also K M the Multiplex of the third D E,



shall want of L O the Multiplex of the fourth F E, and if it be equal, equal; and if greater, greater. And if as well G I wanteth of H N, as K M of L O, having taken away the common magnitudes H I and L M; in the like manner, G H will want of I N, and K L of M O; and if G I be equal to H N, and K M to L O, having taken away the common magnitudes H I and L M; G H shall be equal to I N, and K L to M O. And if lastly, G I exceed H N, and K M, L O, taking away the common parts H I and L M; G H shall likewise exceed I N, and K L, M O. Therefore seeing that G H and K L have been assumed equimultiplices of A C the first, and of D F the third. In like manner, I N and M O, equimultiplices of C B the second, and F E the fourth, and it is shewn in any multiplication whatsoever, that the equimultiplices of the first and third, do want together of the equimultiplices of the second and of the fourth, or together are equal, or do
d) 6. def. 5. exceed them. d As A C the first, to C B the second, so D F the third, shall be to F E the fourth. Which was propose[d]. Therefore, If the magnitudes compounded, &c. Which was to be demonstrated.

PROP. 18. THEOR. 18.



If Magnitudes divided A B, B C, D E, *and* E F, *are proportional, being compounded they shall be also proportional.*

Demonstration LEt it be, as A B is to B C, so D E to E F: I say, that being compounded, they shall be proportional, that is to say, as A C is to B C, so D F is to E F.

For if A C be not to B C, as D F is to E F and D F, shall have the same reason to some other magnitude, lesse than E F, or greater than the same E F, as A C to B C.

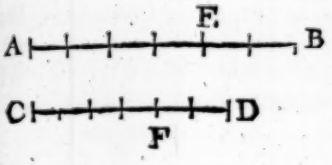
17. 5. First, Let D F have the same reason to G F, lesse than E F, as A C to B C, (if it may be done.)

Forasmuch as A C is to B C, as D F to G F, by dividing, as A B is to B C, so D G shall be to G F: But as A B to B C, so D E to E F, by sup-
11. 5. position. Therefore as D G the first, to G F the second, so D E the third, to E F the fourth; and seeing that D G the first, is greater than D E the third; likewise G F the second, shall be greater than E F the fourth, the part than the whole, which is impossible.

Secondly, Let D F have the same reason to H F, greater than E F, (if it may be) as A C hath to B C. Forasmuch as A C is to B C, as D F to
17. 5. H F; by dividing, as A B shall be to B C, so D H to H F. But as A B to B C,

BC; ſo is DE to EF, by ſuppoſition: Therefore as DH the firſt, to HF the ſecond, ſo DE the third, to EF the fourth. And ſeeing DH the firſt, is leſſe than DE the third. Likewiſe, HF the ſecond, ſhall be leſſe than EF the fourth, the whole than its part, which is abſurd: Therefore DF ſhall not have the ſame reaſon to GE the leſſer, as to EF, or to HF the greater, as AC to BC: Therefore DF ſhall be to EF, as AC to BC; Which was propoſed. Therefore, If the Magnitudes, &c. Which was to be demonſtrated. 11. 5.

PROP. 19. THEOR. 19.



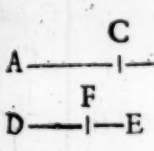
If as the whole AB, *is to the whole* CD, *ſo the part cut off* AE, *to the part cut off* CF, *then the remainder* EB, *ſhall be to the remainder* FD, *as the whole* AB, *to the whole* CD.

Demonſtration FOr ſeeing that AB is to CD, as AE to CF, alternately, AB ſhall be to AE, as CD to CF: Therefore divided-ly, EB ſhall be to AE, as CD to CF; therefore again alternately, EB ſhall be to FD, as AE to CF; that is to ſay, as the whole AB, to the whole CD; ſeeing that AB is put to CD, as AE to CF: Therefore, If as as the whole, &c. Which was to be demonſtrated. 16. 5. 17. 5.

COROLLARIE.

Hence ſhall be demonſtrated the manner of arguing in proportions, which is taken from the converſion of reaſon, according to the ſixteenth Definition.

For, Let it be as AB *to* CB, *ſo* DE *to* FE. *I ſay, by converſion of reaſon, as* AB *is to* AC, *ſo* DE *to* DF: *For ſeeing that as* AB *is to* CB, *ſo* DE *is to* FE, *in dividing,* AC *ſhall be to* CB, *as* DF *to* FE. *Therefore, alternately, as* CB *to* AC, *ſo* FE *to* DF; *and therefore in compounding, as* AB *ſhall be to* AC, *ſo* DE *to* DF. *Which was propoſed.* 17. 5.



PROP. 20. THEOR. 20.



If there be three magnitudes A, B, *and* C, *and others equal to them in number* D, E, *and* F, *which being taken two and two, and in the ſame reaſon, as* A *to* B, *ſo* D *to* E,

and B *to* C, *as* E *to* F, *and that in equal reaſon, the firſt* A, *be greater than the third* C, *alſo the fourth* D *ſhall be greater than the ſixth* F, *if equal, equal, and if leſſe, leſſe.*

13. 5. *Demonſtration* FOr ſeeing that A is greater than C, there will be greater reaſon of A to B, than of C to B. But as A is to B, ſo D is to E, there will be therefore greater reaſon of D to E, then of C to B. But as C is to B, ſo F to E; (for ſeeing that as B to C, ſo E to F; alternately, as C ſhall be to B, ſo F ſhall be to E,) there will be therefore in
10. 5. like manner, greater reaſon of D to E, than of F to E. Therefore D ſhall be greater than F. Which was propoſed.

7. 5. Secondly, Let A be equal to C: I ſay that D ſhall be equal to F. For ſeeing that A is equal to C, as A ſhall be to B, ſo C to B; but as A is to B,
11. 5. ſo D is to E: Therefore D ſhall be to E, as C to B; but as C is to B, ſo F
9. 5. is to E, (by inverſe reaſon, as before) therefore D ſhall be likewiſe to E, as F to E; therefore D and F ſhall be equal; which was propoſed.

Thirdly, Let A be leſſe than C: I ſay that D is leſſe than F: For ſeeing that A is leſſe than C, there will be leſſe reaſon of A to B, than of C to B: but
8. 5. as A is to B, ſo D is to E; there will be therefore leſſe reaſon of D to E,
15. 3. than of C to B; but by inverſe reaſon as before, as C is to B, ſo F is to E: Therefore there is leſſe reaſon of D to E, then of F to E; therefore D
10. 5. ſhall be leſſe than F. Which was propoſed. Therefore, If there be three magnitudes, &c. Which was to be demonſtrated.

PROP. 21. THEOR. 21.



If there be three magnitudes A, B, *and* C, *and others equal to them in number,* D, E, *and* F, *which being taken two and two, and in the ſame reaſon, and that their proportion be perturbate, or without order, (that is, as* A *to* B, *ſo* E *to* F, *and as* B *to* C, *ſo* D *to* E:) *But that in equal reaſon, the firſt* A *be greater than the third* C, *the fourth* D, *ſhall be alſo greater than the ſixth* F, *if equal, equal, if leſſer, leſſer.*

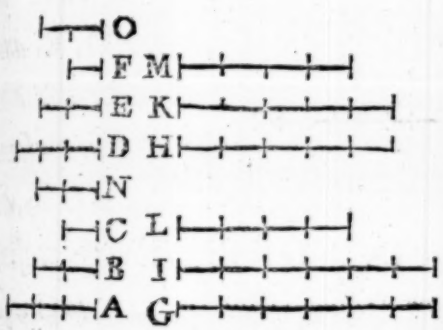
5. C. 13. 5. *Demonſtration* FOr ſeeing that A is greater than C, there will be greater reaſon of A to B, than of C to B, but as A is to B, ſo E is to F; there will be therefore in like manner, greater reaſon of E to F, than of C to B. But ſeeing that as B is to C, ſo D is to E, by inverſe reaſon, as

as C is to B, ſo E is to D. Wherefore there will be alſo greater reaſon of E to F, than of E to D; and therefore D ſhall be greater than F. Which was propoſed. 10. 5.

Secondly, Let A be equal to C; I ſay that D ſhall be equal to F. For ſeeing that A is equal to C, as A ſhall be to B, ſo C ſhall be to B; but as A is to B, ſo E is to F; Therefore as C is to B, ſo E ſhall be to F; but by inverſe reaſon, as C is to B, ſo E is to D, as before; therefore as E ſhall be to F, ſo E ſhall be to D; and therefore D ſhall be equal to F. Which was propoſed. 7. 5. 11. 5. 9. 5.

Laſtly, Let A be leſſe than C: I ſay that D ſhall be leſſe than F: for ſeeing that A is leſſe than C, there will be leſſe reaſon of A to B, than of C to B: But as A is to B, ſo E is to F; therefore there will be leſſer reaſon of E to F, than of C to B: But foraſmuch as before, by inverſe reaſon, as C is to B, ſo E is to D, there will be leſſer reaſon of E to F, then of E to D. Wherefore D ſhall be leſſe than F. Which was propoſed. Therefore, If there be, &c. Which was to be demonſtrated. 8. 5. 10. 1.

PROP. 22. THEOR. 22.



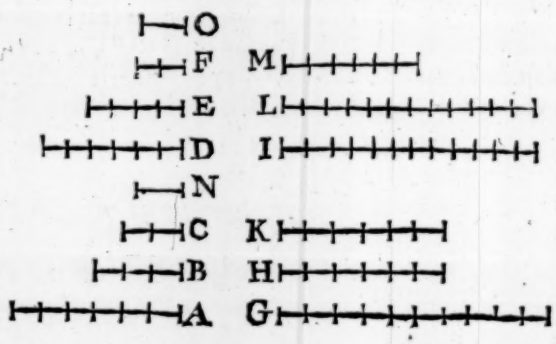
If there be as many magnitudes as you pleaſe, A, B, *and* C, *and others equal to them in number* D, E, *and* F, *which being taken two and two, and in the ſame reaſon*, (*that is as* A *to* B, *ſo* D *to* E, *and* B *to* C, *as* D *to* F,) *thoſe magnitudes in equal reaſon, ſhall be proportional, that is as* A *to* C, *ſo* D *is to* F.

Demonſtration FOr, aſſume G and H equimultiplices of A and D, and I and K, equimultiplices of B and E, and L and M of C and F: And ſeeing that A the firſt, is to B the ſecond, as D the third, is to E the fourth. In like manner, G the Multiplex of A the firſt, ſhall be to I the Multiplex of B the ſecond, as H the Multiplex of D the third, to K the Multiplex of E the fourth; by the ſame reaſon, ſeeing that B the firſt, is to C the ſecond, as E the third, to F the fourth, I the Multiplex of B the firſt, ſhall be to L the Multiplex of C the ſecond, as K the Multiplex of E the third, ſhall be to M the Multiplex of F the fourth: Foraſmuch then as there are three magnitudes G, I, and L, and other three H, K, and M, which are taken two and two, and in the ſame reaſon. It will happen, that if G the firſt, exceed L the third, that H the fourth, will exceed M the ſixth, and if equal, equal, and if leſſe, leſſe. Wherefore ſeeing that G and H, the equimultiplices of the firſt A, and of the third D, either want together, of L and M equimultiplices of C the ſecond and of F the fourth, or together are equal, or together do exceed, according to whatſoever multiplications thoſe equimultiplices be taken; A the firſt, ſhall be to C the ſecond, as D the third, is to F the fourth, which was propoſed. 4. 5. 4. 5. 20. 5. 6. def. 5.

Again,

Again, Let them be more than three magnitudes, in ſuch ſort as C be alſo to N, as F to O, I ſay moreover, that as A ſhall be to N, ſo D ſhall be to O; for ſeeing it hath been already ſhewn in three magnitudes that A is to C, as D to F. But C is put to N, as F to O, there will be three magnitudes to A, C, and N, and other three D, F, and O, which are taken two and two, in the ſame reaſon: Therefore in equal reaſon demonſtrated in three magnitudes: Again, as A ſhall be to N, ſo D to O. In like manner, it may be ſhewn the ſame in five magnitudes by four, as it hath been demonſtrated in four magnitudes by three, and ſo of divers: Therefore, If there be as many Magnitudes, &c. Which was to be demonſtrated.

PROP. 23. THEOR. 23.

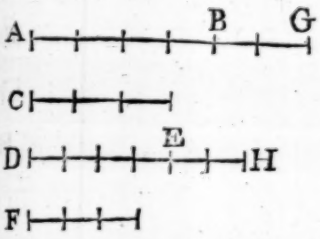


If there be three magnitudes A, B, *and* C, *and others equal to them in numbers* D, E, *and* F, *taken two and two, and in the ſame reaſon, and that their proportion be perturbed (that is to ſay, as* A *to* B, *ſo* E *to* F, *and* B *to* C, *as* D *to* E,) *in equal reaſon, they ſhall be alſo in the ſame reaſon (that is, as* A *to* C, *ſo* D *to* F.)

15. 5. 11. 5. 15. 5. 11. 8. 4. 5. 21. 5. *Demonſtration* Aſſume G, H, and I, equimultiptices of A, B, and D. Likewiſe K, L, and M, equimultiplices of C, E, and F, as A ſhall be to B, ſo G ſhall be to H; ſeeing that G and H are equimultiplices of A and B. But as A is to B, ſo E is to F: Therefore as G is to H, ſo E is to F: But as E is to F, ſo L is to M; ſeeing that L and M are equimultiplices of E and F. Therefore G ſhall be likewiſe to H, as L to M. Again, foraſmuch as B the firſt, is to C the ſecond, as D the third, to E the fourth; likewiſe H the Multiplex of the firſt B, ſhall be to K the Multiplex of the ſecond C, as I the Multiplex of D the third, is to L the Multiplex of E the fourth. Now foraſmuch as there are three magnitudes G, H, and K, and other three I, L, and M, which taken two and two in the ſame reaſon, their proportion being perturbated: Seeing that it is ſhewn, that G is to H, as L is to M, and as H is to K, ſo I is to L; if G the firſt, exceed K the third, I the fourth ſhall likewiſe exceed M the ſixth, and if equal, equal, and if leſſe, leſſe, therefore ſeeing that G and I equimulriplices of the firſt A, and of the third D, are both greater, or equal, or leſſe, then K and M equimultiplices of the ſecond C, and of the fourth F, as A the firſt, ſhall be to C the ſecond, ſo D the third, ſhall be to F the fourth. Which was propoſed. Therefore, If there be three Magnitudes, &c. Which was to be demonſtrated.

PROP.

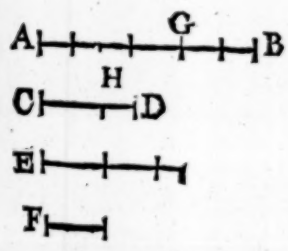
PROP. 24. THEOR. 24.



If the firſt A B, *hath the ſame reaſon to the ſecond* C, *as the third* D E, *hath to the fourth* F, *and that the fifth* B G, *hath alſo the ſame reaſon to the ſecond* C, *as the ſixth* E H, *hath to the fourth* F, *alſo* A G *the compound of the firſt* A B, *and the fifth* B G, *ſhall have the ſame reaſon to the ſecond* C, *as the compound* D H, *of the third* D E, *and of the ſixth* E H, *hath to the fourth* F.

Demonſtration FOr ſeeing that B G is to C, as E H to F; alternately, as C ſhall be to B G, ſo F ſhall be to E H; therefore foraſmuch as A B is to C, as D E is to F, and that C is to B G, as F is to E H, 22. 5.
inequal reaſon, A B ſhall be to B G, as D E is to E H. Therefore by compounding, the whole A G ſhall be to B G, as the whole D H to E H, and B G to C, as E H is to F; inequal reaſon, A G ſhall be to C, as 22. 5.
D H is to F. Which was propoſed. Therefore, If the firſt, &c. Which was to be demonſtrated.

PROP. 25. THEOR. 25.

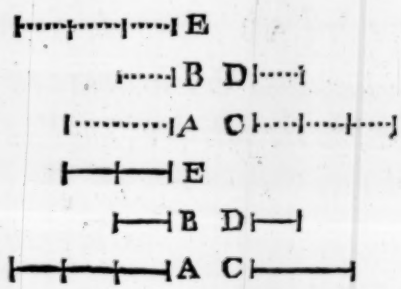


If four magnitudes are proportional, A B *to* C D, *as* E *to* F, *the greateſt* A B, *and the leaſt* F, *are greater than the two others* C D *and* E.

Demonſtration FOr, From the magnitude A B cut off A G equal to E, and from C D cut off alſo C H, equal to F. Then A G ſhall be to C H, as E to F; that is to ſay, as A B to C D. Therefore ſeeing that the whole A B, is to the whole C D, as the part cut off A G, is to the part cut off C H. In like manner, as the whole A B, to the whole 91. 5.
C D, ſo the remainder G B, to the remainder H D. But A B (ſeeing that it is the greateſt of all,) is greater than C D. Therefore G B ſhall be alſo greater than H D. But foraſmuch as A G and E are equal, if you adde to them the equals F and C H; to wit, F to A G, and C H to E; A G and F together, ſhall be equal to E and C H together: Therefore adding the unequals G B and H D, A B and F together, ſhall be greater than E and C D together; ſeeing that G B is greater than H D. Which was propoſed. Therefore, If four Magnitudes, &c. Which was to be demonſtrated.

PROP

PROP. 26. THEOR. 26.



If the first A, *hath greater reaſon to the ſecond* B, *than the third* C, *hath to the fourth* D, *by inverſe reaſon, the ſecond* B, *ſhall have leſſe reaſon to the firſt* A, *than the fourth* D, *hath to the third* C.

Demonſtration FOr, Let it be underſtood that E is to B, as C is to D, the reaſon of A to B ſhall be likewiſe greater than that of E to to B; and a therefore A ſhall be greater than E: Wherefore b there will be leſſe reaſon of B to A the greater, than of B to E the leſſer: But as B is to E, ſo by inverſe reaſon, D is to C. Therefore the reaſon of B to A, is likewiſe leſſe than that of D to C. Which was propoſed. Therefore, If the firſt, &c. Which was to be demonſtrated.

a) 10. 5.
b) 8. 5.

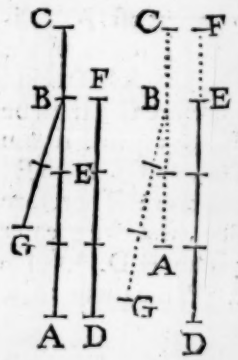
PROP. 27. THEOR. 27.

If the firſt A, *hath greater reaſon to the ſecond* B, *than the third* C, *hath to the fourth* D; *alſo alternately, the firſt* A, *ſhall have greater reaſon to the third* C, *than the ſecond* B, *ſhall have to the fourth* D.

Demonſtration FOr, Let E be put to B, as C to D, A ſhall likewiſe have greater reaſon to B, than E to B. Wherefore a A ſhall be greater than E, and b therefore there will be greater reaſon of A to C, than of E to C; but c foraſmuch as alternately as E is to C, ſo B is to D; (E being put to B, as C to D.) Therefore A ſhall alſo have greater reaſon to C, then B hath to D, which was propoſed. Therefore If the firſt, &c. Which was to be demonſtrated. (As by the precedent figure is manifeſt.)

a) 10. 5.
b) 8. 5.
c) 6. 5.

PROP. 28. THEOR. 28.



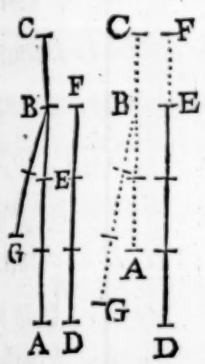
If the firſt AB, *hath greater reaſon to the ſecond* BC, *than the third* DF, *hath to the fourth* EF, *the compounded* AC, *of the firſt* AB, *with the ſecond* BC, *ſhall have alſo greater reaſon to the ſecond* BC, *than the compounded* DF, *of the third* DE, *with the fourth* EF, *hath to the fourth* EF.

Demon-

Demonſtration FOr, Let G B be put to B C, as D E to E F, there will be alſo greater reaſon of A B to B C, then of G B to B C: Therefore [a] A B ſhall be greater than G B, adding therefore the common part B C, A C ſhall be greater than G C; and therefore A C ſhall have greater reaſon to B C, than G C to B C; but in compounding, G C is to B C, as D F to E F; (ſeeing G B is put to B C, as D E to E F;) therefore A C ſhall have greater reaſon to B C, than D F to E F. Which was proſed. Therefore, &c. Which was to be demonſtrated.

a) 10. 5.
17. 5.

PROP. 29. THEOR. 29.

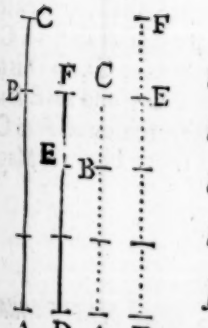


If the compound A C, *of the firſt* A B, *with the ſecond* B C, *hath greater reaſon to the ſecond* B C, *than the compound* D F, *of the third* D E, *with the fourth* E F, *hath to the fourth* E F. *In dividing, the firſt* A B, *ſhall have alſo greater reaſon to the ſecond* B C, *than the third* D E, *to the fourth* E F.

Demonſtration FOr, Let G C be put to B C, as D F to E F, the reaſon of A C to B C, ſhall be alſo greater than of G C to B C: Wherefore [a] A C ſhall be greater than G C, Therefore taking away the common part B C, there will remain A B, greater than G B. [b] Therefore there will be greater reaſon of A B to B C, than of G B to B C. But in dividing, as G B is to B C, ſo D E is to E F, (G C being put to B C, as D F to E F,) there will be therefore alſo greater reaſon of A B to B C, than of D E to E F. Which was propoſed. Therefore, If the compound, &c. Which was to be demonſtrated.

a) 10. 5.
b) 17. 5.

PROP. 30. THEOR. 30.

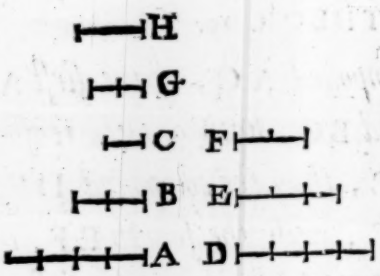


If the compound A C, *of the firſt* A B, *with the ſecond* B C, *hath greater reaſon to the ſecond* B C, *than the compound* D F, *of the third* D E, *with the fourth* E F, *hath to the fourth* E F, *by converſion of reaſon, the firſt* A B, *with the ſecond* B C; (*that is* A C) *ſhall have leſſer reaſon to the firſt* A B, *than the third* D E, *with the fourth* E F, (*that is* D F,) *hath to the third* D E.

Demonſtration FOr A C having greater reaſon to B C, than D F to E F; a in dividing, A B ſhall have greater reaſon to B C, than D E to E F. Therefore by converſion of reaſon, there ſhall be leſſe reaſon of B C to A B, than of E F b to D E, and in compounding, there will be leſſer reaſon of the whole A C to A B, than of the whole D F to D E. Which was propoſed. Therefore, If the compound, &c. Which was to be demonſtrated.

a) 29. 5.
b) 26. 5.

PROP. 31. THEOR. 31.



If there be three magnitudes A, B, *and* C, *and other three* D, E, *and* F, *equal to them in number, and that there be greater reaſon of the firſt* A *of the firſt, to the ſecond* B, *then of the firſt* D, *of the laſt, to the ſecond* E. *Likewiſe that there be greater reaſon of the ſecond* B, *of the firſt, to the third* C, *than of the ſecond* E, *of the laſt, to the third* F. *In equal reaſon there will be alſo greater reaſon of the firſt* A, *of the firſt, to the third* C, *than of the firſt* D, *of the laſt, to the third* F.

Demonſtration FOr, Let G be put to C, as E to F: Therefore there ſhall be alſo greater reaſon of B to C, than of G to C: Wherefore a B ſhall be greater than G: Therefore b there will be greater reaſon of A to G the leſſer, then of A to B the greater: But the reaſon of A to B, is put greater than of D to E: Therefore there will be yet greater reaſon of A to G, then of D to E.

a) 10. 5.
b) 8. 5.

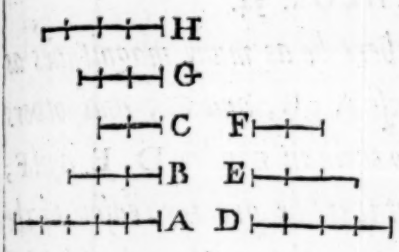
Again, Let H be put to G, as D to E, there will be alſo therefore greater reaſon of A to G, then of H to G: Therefore c A ſhall be greater than H: Therefore d A the greateſt, ſhall have greater reaſon to C, then H the leſſer, ſhall have to the ſame C. But as H is to C, ſo D is to F, in equal reaſon, (ſeeing that as D is to E, ſo H is to G, and as E is to F, ſo G is to C;) Therefore there ſhall be alſo greater reaſon of A to C, then of D to F. Which was propoſed. Therefore, If there be three Magnitudes, &c. Which was to be demonſtrated.

c) 10. 5.
d) 8. 5.
e) 22. 5.

PROP. 32. THEOR. 32.

If there be three magnitudes A, B, *and* C, *and others equal to them in number* D, E, *and* F, *and that there be greater reaſon of the firſt* A, *of the firſt, to the ſecond* B, *then of the ſecond*

E,



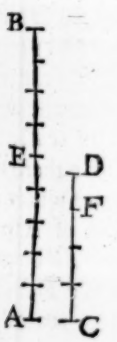
E, *of the laſt, to the third* F; *likewiſe that there be greater reaſon of the ſecond* B, *of the firſt, to the third* C, *then of the firſt* D, *of the laſt, to the ſecond* E: *In equal reaſon, there will be alſo greater reaſon of the firſt of the firſt* A, *to the third* C, *then of the firſt of the laſt* D, *to the third* F.

Demonſtration FOr put G to C, as D to E; therefore alſo B ſhall have greater reaſon to C, than G to C; therefore [a] B ſhall be greater than G. Wherefore [b] there will be greater reaſon of A to G the leſſe, than of the ſame A to B the greater. But there is greater reaſon of A to B, than of E to F: Therefore there will be yet greater reaſon of A to G, than of E to F.

Again, put H to G, as E to F; there will be then greater reaſon of A to G, than of H to G. Therefore [c] A ſhall be greater than H. Wherefore [d] A ſhall have greater reaſon to C, than H the leſſer, ſhall have to the ſame C: But [e] as H is to C, ſo D is to F, in equal reaſon, (D being to E, as G to C, and as E to F, ſo H to G:) there is therefore alſo greater reaſon of A to C, than of D to F. Which was propoſed. Therefore, If there be three Magnitudes, &c. Which was to be demonſtrated.

a) 10. 5.
b) 8. 5.
c) 10. 5.
d) 8. 5.
e) 23. 5.

PROP. 33. THEOR. 33.

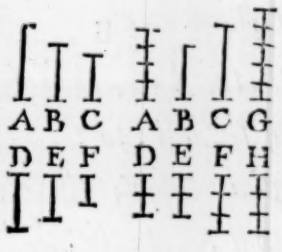


If there be greater reaſon of the whole AB, *to the whole* CD, *then of the part cut off* AE, *to the part cut off* CF; *there will be alſo greater reaſon of the remainder* EB, *to the remainder* FD, *then of the whole* AB, *to the whole* CD.

Demonſtration FOr ſeeing that there is greater reaſon of AB to CD, than of AE to CF; [a] by permutation, there ſhall be alſo greater reaſon of AB to AE, than of CD to CF; therefore [b] by converſion of reaſon, there will be leſſer reaſon of AB to EB; than of CD to FD. Therefore [c] alternately, there ſhall be alſo leſſer reaſon of AB to CD, than of EB to FD; to wit, the remainder EB, ſhall have greater reaſon to the remainder FD, than the whole AB, to the whole CD. Which was propoſed. Therefore, If there be greater reaſon, &c. Which was to be demonſtrated.

a) 27. 5.
b) 30. 5.
c) 27. 5.

PROP. 34. THEOR. 34.



If there be as many magnitudes as you please A, B, *and* C, *and others equal to them in number* D, E, *and* F, *and that there be greater reason of the first of the first* A, *to the first of the last* D, *than of the second* B, *to the second* E; *and of the second* B, *to the second* E, *then of the third* C, *to the third* F, *and so on; all the first* A, B, *and* C *together, shall have greater reason to all the last* D, E, *and* F *together; then all the first* B *and* C, *the first* A *being taken away, to all the last* E F, *the first* D *also taken away; But lesse reason than the first of the first* A, *to the first of the last* D; *and lastly, also greater reason than the last of the first* C, *to the last of the last* F.

Demonstration FOr seeing that there is greater reason of A to D, than of B to E; [a] by permutation, there shall be greater reason of A to B, then of D to E; therefore [b] in compounding, there shall be greater reason of A and B together to B, then of D and E together to E; and [c] again, by permutation there will be greater reason of A & B together, to D and E together, than of B to E. Therefore the whole A & B having greater reason to the whole D & E; than the part cut off B, to the part cut off E; [d] the remainder A shall also have greater reason to the remainder D, than the whole A & B, to the whole D & E. In like manner, there will be greater reason of B to E, than of the whole B and C, to the whole E and F; therefore there will be yet greater reason of A to D, than of the whole B and C, to the whole E and F. Therefore [e] by permutation, there will be greater reason of A to B and C, than of D to E and F. Therefore [f] in compounding, there will be greater reason of the whole A, B, and C, to B and C, than of the whole D, E, and F, to E and F, and again, [g] by permutation, there will be greater reason of the whole A, B, and C, together, to the whole D, E, and F, together, then of B and C to E and F. Which was in the first place proposed.

Therefore there being greater reason of the whole A, B, and C, to the whole D, E, & F than of the part cut off B and C, to the part cut off E and F; There [h] will be also greater reason of the remainder A, to the remainder D, then of the whole A, B, and C, to the whole D, E, and F. Which was in the second place proposed.

But forasmuch as there is greater reason of B to E, than of C to F; alternately, [i] there will be also greater reason of B to C, than of E to F, and in compounding, greater reason of the whole B and C to C, than of the whole

a) 27. 5.
b) 28. 5.
c) 27. 5.
d) 33. 5.
e) 27. 5.
f) 28. 5.
g) 27. 5.
h) 33. 5.
i) 28. 5.

whole E & F to F; and again, by permutation, greater reaſon of B and C, to E & F, than of C to F. But there is greater reaſon of A, B and C, to D, E, & F, as hath been ſhewn, than of B and C to E and F: Therefore there will be yet greater reaſon of the whole A, B, and C, to the whole D, E, and F, than of the laſt C, to the laſt F. Which was in the third place propoſed.

Let there be afterward propoſed four magnitudes on both parts, by the ſame ſuppoſition; that is to ſay, that there be greater reaſon of C the third, to F the third, than of G the fourth, to H the fourth; I ſay that the ſame thing will follow, for (as hath been already ſhewn in three Magnitudes, there is greater reaſon of B to E, than of B, C, and G, to E, F, and H. Therefore there will be yet greater reaſon of A to D. Then of B, C, and G, to E, F, and H, Therefore [k] alternately, there will be greater reaſon of A to B, C, and G, than of D to E, F, and H. And [l] in compounding, greater reaſon of A, B, C, and G, to B, C, and G, than of D, E, F, and H, to E, F, and H; [m] and alternately, greater reaſon of A, B, C, and G, to D, E, F, and H, than of B, C, and G, to E, F, and H. Which was in the firſt place propoſed.

k) 27. 5.
l) 28. 5.
m) 27. 5.

Therefore having greater reaſon of the whole A, B, C, and G, to the whole D, E, F, and H, than of the part cut off B, C, and G, to the part cut off E, F, and H, [n] the remainder A, ſhall have greater reaſon to the remainder D, than the whole A, B, C, and G, to the whole D, E, F, and H. Which was in the ſecond place propoſed.

n) 33. 5.

But (as is demonſtrated in three magnitudes,) foraſmuch as there is greater reaſon of B, C, and G, to E, F, and H, than of G to H; and greater of A, B, C, and G, to D, E, F, and H, than of B, C, and G, to E, F, and H, as hath been ſhewn; there will be much greater reaſon of A, B, C, and G, to D, E, F, and H, than of the laſt G, to the laſt H. Which was in the third place propoſed. The ſame may be ſhewn in five Magnitudes, by four, and in ſix by five, &c. as hath been ſhewn in four by three. Therefore, If, &c. Which was to be demonſtrated.

The End of the Fifth Element of EUCLIDE.

THE

THE SIXTH ELEMENT OF EUCLIDE.

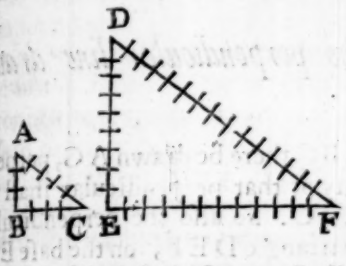
THE ARGUMENT.

THis Sixth Book treateth of the proportion of figures among themſelves, of like and reciprocal figures, of proportional right lines, of the application of Parallelograms to right lines, which may either want or exceed by like Parallelograms, and how a terminated right line may be divided by extream and mean proportion, and of the proportions of circumferences and angles, alſo of Sectors in equal circles.

DEFINITIONS.

1 *Like right lined figures are thoſe which have the angles equal each one to his correſpondent, and the ſides which are about the equal angles proportional.*

AS the triangles A B C, and D E F, ſhall be ſaid to be alike, if they be equiangled; that is to ſay, that the angle A, be equal to the angle D, and the angle B to the angle E, and the angle C to the angle F; and in



in like manner, that the ſides about the equal angles be proportional; to wit, as A B is to A C, ſo D E may be to D F, and as A B to B C, ſo D E to E F; and as A C to C B, ſo D F to E F.

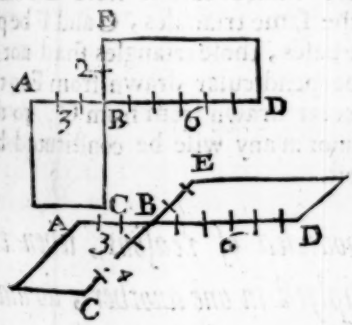
And if each of the angles of one of the figures be equal to each of the angles of the other. But the ſides about the equal angles are not proportional; or contrarily, ſuch figures ſhall not be ſaid to be alike; as are the ſquare and the oblong, or long ſquare; for thoſe figures have the angles equal; to wit, right angles, but the ſides of the one, are not proportional to the ſides of the other; the ſides of the ſquare having reaſon of equality; and thoſe of the oblong about the right angle, reaſon of inequality; from whence it appears that all right lined figures equiangled, and equilateral, having the ſides and the angles equal in number, are alike, although they be unequal.

2 *Reciprocal figures are ſuch, when the reaſons antecedents and conſequents are in both the figures.*

Or thus,

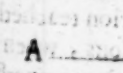
The figures are Reciprocal, when the terms, antecedents and conſequents of the reaſons, are in both the figures.

FOr by theſe reaſons, antecedents and conſequents ought to be underſtood, the terms antecedent and conſequent of the proportion; as if there be two figures rectangled, or not rectangled, A C and E D; and that as A B is to B D, ſo E B is to B C, thoſe figures ſhall be termed reciprocal: Foraſmuch as in the one is the term antecedent of the firſt reaſon, to wit, A B, and the conſequent of the ſecond reaſon, to wit, B C, and in the other is the term conſequent of the firſt reaſon B D, and the antecedent of the ſecond E B, ſuch figures are alſo ſaid to be equal to one another, as ſhall be hereafter demonſtrated.



3 *A right line is ſaid to be divided according to mean and extream reaſon, when as the whole is to the greateſt ſegment, as the greateſt ſegment is to the leſſer.*

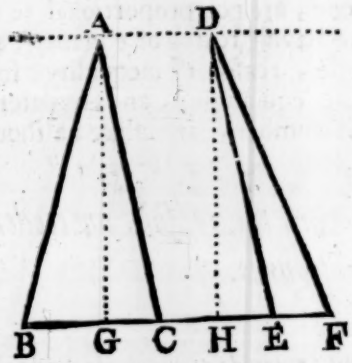
IF any right line, as A B, be divided, in ſuch ſort unequally in the point C, as that the whole A B, be to the greateſt ſegment A C, as the greateſt ſegment A C, is to the leaſt ſegment C B, it ſhall be ſaid to be divided according to mean and extream reaſon, which ſaid diviſion *EUCLIDE* teacheth at the thirtieth Propoſition of this Book.



4 *The*

4 *The height of any figure is the perpendicular line drawn from the top to the base.*

IF from the vertex A, of the triangle A B C, there be drawn A G, perpendicular, to the base B C, the quantity of that perpendicular shall be the height or altitude of the triangle A B C. So also the perpendicular D H, drawn from the vertex D, of the triangle D E F, on the base E F, prolonged towards E, shall be the height of that triangle D E F. Therefore if the perpendiculars of the two figures, drawn from their tops to their bases, (be the bases prolonged, or not,) are equal; such figures shall be said to have the same height. Now such perpendiculars shall be equal, when the bases of the figures and the tops shall be constituted between the same parallels, as are A G and D H.



For seeing that the interior angles A G H, and D H G, on the same part, are equal to two right angles, or to expresse it better, are right angles; [a] A G and D H shall be parallels, but A D and G H are also parallels; forasmuch as the triangles are proposed to be constituted between the same parallels; Therefore A D and H G, shall be a parallelogram, [b] and therefore the sides A G and D H, shall be equal: Therefore those triangles shall be said to have the same height the one as the other. And if in the same triangles, C and F be put for the tops, and A B and D E for the bases, those triangles shall not be said to have the same height; for the perpendicular drawn from F to the base D E, is not equal to the perpendicular drawn from from C, to the base A B, those triangles therefore cannot in any wise be constituted between the same parallels, as is manifest.

a) 28. 1.

b) 34. 1.

5 *A reason is said to be compounded of reasons, when the quantities of the reasons multiplyed in one another, do make some reason.*

FOrasmuch as the denominator of any reason whatsoever, doth expresse the quantity of the Magnitude antecedent, in respect of the consequent, (as the denominator of the reason quadruple; to wit 4, shewes that in all reason quadruple, the Magnitude antecedent, conteineth four times the consequent. But the denominator of the reason subquadruple, to wit $\frac{1}{4}$, demonstrateth that the Magnitude antecedent is the fourth part of the consequent, &c. (Geometricians use to term the denominator the quantity of the reason; so as that the quantity of the reason, and the denominator of the reason, is the same thing;) therefore this definition teacheth us that a reason is said to be compounded of two or more reasons, when the denominators or quantities of those reasons multiplyed in one another, shall make that reason, or (according to *Zambert*,) shall make the quantity of

or denominator of that reaſon: As the reaſon duodecuple is ſaid to be compounded of the double and the ſextuple: Foraſmuch as the denominator of the reaſon duodecuple, to wit 12, is produced of the multiplication of the denominator of the reaſon double, to wit of 2, by the denominator of the ſextuple, that is to ſay, by 6. So the ſame reaſon duodecuple is ſaid to be compounded of the triple and of the quadruple; For of the Multiplication of 3 by 4, is produced the denominator 12, of the reaſon duodecuple, In like manner, the reaſon trigecuple is ſaid to be compounded of the double, triple, and quintuple, for the denominators of that reaſon 2, 3, and 5, multiplyed in one another, produce 30, the denominator of that reaſon trigecuple. So the raaſon double is ſaid to be compounded of the ſeſquialtera and ſeſquitertia, foraſmuch as the denominator of the ſeſquialtera, which is $1\frac{1}{2}$, multiplied by $1\frac{1}{3}$, denominator of the ſeſquitertia is produced of 2, the denominator of the double.

Again, the ſame reaſon double, is compounded of the ſeſquiſeptuple, and the ſupertripartient fourths; for the denominator of thoſe reaſons $1\frac{1}{7}$, and $1\frac{1}{4}$, multiplyed in one another, do produce two denominators of the double: In like manner, the ſame reaſon double ſhall be ſaid to be compounded of the ſubſeſquiquarta, and of the double ſeſquialtera; foraſmuch as their denominators $\frac{4}{5}$, and $2\frac{1}{2}$ multiplyed in one another, do produce in like manner the ſame denominator of the reaſon double, to wit 2, &c.

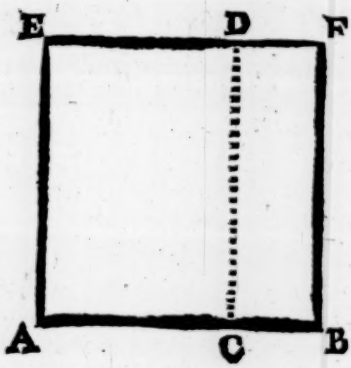
Therefore if you propoſe as many Magnitudes as you pleaſe in order, as A, B, C, and D, to wit, A to B in reaſon double, B to C triple, and C to D quintuple, the reaſon of the extreams A and D, ſhall be trigecuple, which ſhall be ſaid to be compounded of the mean reaſons, for if you multiply the denominator of A by B, which is 2, by that of B to C 3, you ſhall have 6 for the denominator of the reaſon of A to C.

A.	B.	C.	D
60	30	10	2

Therefore A to C, hath reaſon ſextuple. Again, multiplying 6 the denominator of the reaſon of A to C, by that of C to D 5, you ſhall have 30, for the denominator of the reaſon of A to D. Wherefore A ſhall be to D, in reaſon trigecuple, &c.

And thus much may be ſaid of the five definitions of *EUCLIDE*, on this Sixth Book, to which ſhall be added this following definition; which will render as well the 27, 28, 29, and 30th. Propoſitions of this Book, as alſo divers others of the Tenth Book, more eaſie and intelligible, and is as followeth.

6 *A parallelogram being applyed according to ſome right line is ſaid to want thereof by a parallelogram, when it occupieth not all that right line. But is ſaid to exceed it, when it occupieth a greater line than that according to which it is applyed: In ſuch ſort neverthelesſe, that the parallelogram exceeding, or wanting, hath the ſame height as the parallelogram of which it wanteth, or which it exceedeth, and doth conſtitute therewith one only parallelogram.*



LEt the line AB, be the line on which the Parallelogram A D is to be constituted, not imploying the whole line AB, but leaving C B, having finished the parallelogram A F, the parallelogram A D, applyed according to the line A B, shall be said to want thereof by the parallelogram C F, so as that C F shall be called the want.

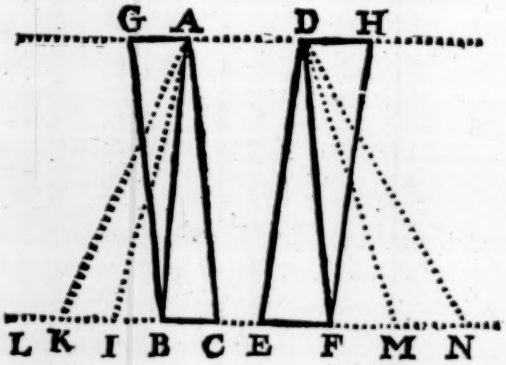
Again, Let A C be the line on which the parallelogram A F is to be constituted, having the side A B, greater than A C, to which let C D be drawn parallel to B F, the parallelogram A F, applyed according to A C, shall be said to exceed A C, by the parallelogram C F, in such sort as C F shall be termed the excesse.

Now this want, or excesse in rectangles may be a square, or an oblong, and in parallelograms not rectangled, a Rhombe or Rhomboides, as is manifest.

PROPOSITIONS, PROBLEMES, and THEOREMES.

PROPOSITION 1. THEOREM 1.

Triangles A B C *aud* D E F, *and parallelograms* C G *and* E H, *which have the same height, are to one another, as their bases* B C *and* E F.



I Say that the triangle A B C, is to the triangle D, E, and F, and the parallelogram C G, to the parallelogram E A, as the base B C is to the base E F; that is to say, that if the base B C be taken for the first magnitude, and

and the baſe E F for the ſecond. But the triangle A B C, or the parallelogram C G for the third, and the triangle D E F, or the parallelogram E H, for the fourth, the equimultiplices of the firſt and the third Magnitudes ſhall be both greater, or both leſſer, or equal to the equimultiplices of the ſecond and fourth, according to the meaning of the ſixth definition of the fifth Book.

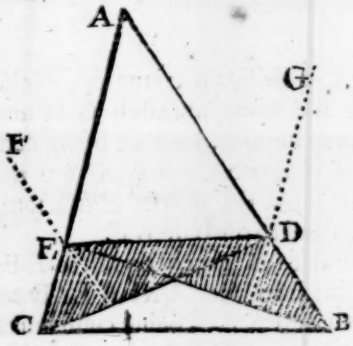
Demonſtration. For let there be conſtituted as well the triangles, as the parallelograms, between the ſame parallels G H and L N, (then [a] they will have one and the ſame height, for that being conſtituted between the ſame parallels, the perpendiculars drawn from the tops, on the baſes, ſhall be equal, as is ſaid in the fourth definition,) and from B L take B I, I K, and K L, each equal to the baſe B C. a) 4. def.

In like manner, from F N, take F M and M N, each equal to E F; and from the points A and D, draw the right lines A I, A K, A L, D M, and D N; [b] the triangles A B C, A I B, A K I and A L K, being conſtituted on equal baſes, and between the ſame parallels, ſhall be equal to one another; by the ſame reaſon, the triangles D E F, D F M, and D M N, ſhall be alſo equal to one another. b) 38. 1.

Now foraſmuch as C L conteineth as many parts equal to B C, as the triangle A C L doth contein triangles equal to the triangle A B C, as hath been ſhewn, C L ſhall be as much Multiplex of C B, as the triangle A C L is of the triangle A B C. Likewiſe, foraſmuch as E N doth contein as many parts equal to E F; as the triangle D E N conteineth triangles equal to the triangle D E F, as hath been alſo ſhewn, E N ſhall be as much Multiplex of E F, as the triangle D E N is of the triangle D E F. But foraſmuch as if the baſe C L be equal to the baſe E N, [c] of neceſſity, the triangle A C L ſhall be equal to the triangle D E N, being between the ſame parallels; and therefore if C L be greater than E N, the triangle A C L ſhall be greater than the triangle D E N, and if leſſe, leſſe, as appears by the ninth Common Sentence. Therefore the right line C L, and the triangle A C L equimultiplices of the firſt Magnitude B C, and of the third A B C, will both want of the right line E N, and of the triangle D E N, equimultiplices of the ſecond Magnitude E F, and of the fourth D E F, or ſhall be both equal, or both greater, the Magnitudes being ſo taken, as that they anſwer one another; [d] wherefore there will be greater reaſon of the firſt B C, to the ſecond E F, than of the third A B C, to the fourth D E F, to wit, of the triangle to the triangle: Therefore as the baſe is to the baſe, ſo is the triangle to the triangle. Which was propoſed. c) 38. 5. d) 6. def. 5.

For the ſecond part; Seeing that as [e] the triangle A B C is to the triangle D E F, ſo the parallelogram C G [f] (which is double to the triangle A B C,) is to the parallelogram E H; (which [g] is double to the triangle D E F,) it is manifeſt that the parallelogram ſhall [h] be alſo to the parallelogram, as the baſe to the baſe, which may be proved by the ſame diſcourſe which we have uſed in the Demonſtration of triangles, if firſt of all, from the points I, K, and L, you draw parallels to B G, and alſo from M and N, to F and H: Therefore the triangles, &c. Which was to be demonſtrated. e) 15. 5. f) 34. 1. g) 34. 1. h) 11. 5.

PROP. 2. THEOR. 2.



If to one of the ſides B C, *of the triangle* A B C, *be drawn ſome parallel right line* D E, *it will divide the ſides* A B *and* A C, *of the triangle* A B C, *proportionally; and if the ſides* A B *and* A C *are divided proportionally, the right line* D E, *joyning the points of the ſections ſhall be parallel to the other ſide* C B, *of the triangle* A B C.

Demonſtration FOr having drawn the right lines C D and B E, [a] the triangles D E B and D E C, conſtituted on the baſe D E, and between the ſame parallels D E and B C, ſhall be equal to one another: Therefore as the triangle A D E, is to the triangle D E B; ſo the ſame A D E ſhall be to the triangle D E C. But [b] as A D E is to D E B, ſo the baſe A D is to the baſe D B; (ſeeing that thoſe triangles are of the ſame height, as appeareth, if by the point E, be drawn E F, parallel to A B,) and by the ſame reaſon, as the triangle A D E, is to the triangle D E C, ſo the baſe A E, to the baſe E C, (A E D and D E C, being between the ſame parallels A C and D G,) therefore [c] as A D is to D B, ſo A E is to E C, (ſeeing that theſe two reaſons are the ſame to the reaſon of the triangle A D E) ſo the triangle D E B, and of the ſame triangle A D E, to the triangle D E C. Which was propoſed.

a) 37. 1.
b) 1. 6.
c) 11. 5.

Secondly, Let the right line D E divide the ſides A B and A C proportionally: I ſay that D E is parallel to the other ſide C B.

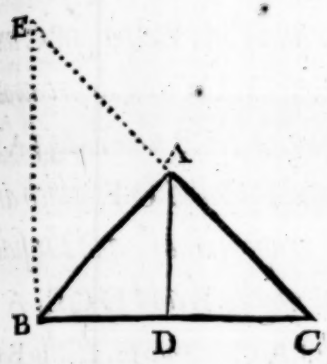
For again, having drawn B E and C D, as the [d] baſe A D ſhall be to the baſe D B, ſo the triangle A D E, to the triangle D E B; ſeeing they are both of one height; but as A D is to D B, ſo is A E to E C, by ſuppoſition. Therefore as the triangle A D E, ſhall be to the triangle D E B, ſo A E ſhall be to E C; but again, as the baſe A E is to the baſe E C, ſo the triangle A D E, to the triangle D E C, being of the ſame height. Therefore [e] as the triangle A D E, is to the triangle D E B, ſo the ſame triangle A D E, is to the triangle D E C; therefore [f] the triangles D E B and D E C ſhall be equal; and therefore ſeeing they are conſtituted on the ſame baſe, [g] they ſhall be alſo between the ſame parallels: Therefore D E is parallel to B C. Which was propoſed. Therefore, If to one of the ſides, &c. Which was to be demonſtrated.

d) 16. 5.
e) 11. 5.
f) 9. 5.
g) 39. 2.

PROP.

PROP. 3. THEOR. 3.

If an angle B Â C, *of a triangle* A B C, *be divided into two equal parts, and that the right line* A D, *which divideth the angle, doth alſo divide the baſe* B C, *the ſegments of the baſe* B D *and* D C, *ſhall have the ſame reaſon to one another, as the other ſides of the triangle* B A *and* A C, *and if the ſegments of the baſe* B D *and* D C, *have the ſame reaſon to one another, as the other ſides of the triangle* B A *and* A C, *the right line* A D, *drawn from the top* A, *to the point of ſection* D, *divideth the angle* B A C, *of the triangle* A B C, *into two equal parts.*



Demonſtration FOr, Let B E be drawn parallel to D A, meeting C A prolonged towards E. Now they will meet, ſeeing that the angles C and C B E, are leſſe than two right angles, for [a] C and C D A, are leſſe than two right angles, and [b] C D A is equal to C B E, the exterior angle to the interior, C and C E B, ſhall be likewiſe leſſe than two right angles, and the angle E B A [c] ſhall be equal to its alternate angle B A D, and the angle E, equal to the exterior angle D A C; therefore ſeeing that the two angles B A D and D A C, are equal by ſuppoſition; the angles E B A and E, ſhall be equal to one another; [d] therefore the lines A B and A E equal to one another [e].

a) 17. 1.
b) 29. 1.
c) 29. 1.
d) 6. 1.
e) 7. 5.

Therefore as E A ſhall be to A C, ſo B A ſhall be to the ſame A C: But [f] as E A is to A C, ſo B D is to D C, ſeeing that in the triangle B E C, the right line D A is parallel to the ſide B E. Therefore as B A ſhall be to A C, ſo B D ſhall be to D C. Which was propoſed.

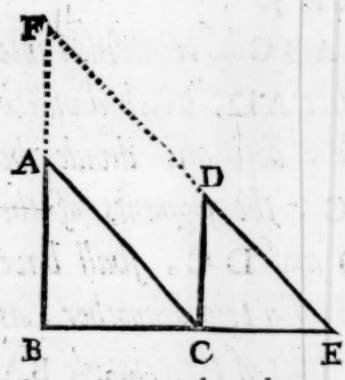
f) 2. 6.

Secondly, Let it be as B A is to A C, ſo B D to D C; I ſay that the right line A D doth divide the angle B A C, into two equal parts: For again, draw the right line B E, by the point B, parallel to D A, meeting C A prolonged in the point E; foraſmuch therefore as B A is to A C, ſo B D is to D C, by ſuppoſition; but [g] as B D is to D C, ſo E A is to A C, (ſeeing that in the triangle B C E, the line A D is parallel to the ſide B E,) as B A [h] ſhall be to A C, ſo E A to the ſame A C. Therefore [i] B A and A E ſhall be equal to one another; [k] therefore the angles A B E and E, are equal: [l] Therefore ſeeing that the angle A B E is equal to its alternate angle B A D, and the angle E equal to its exterior angle D A C, the two angles B A D and D A C, ſhall be equal to one another. Which was propoſed. Therefore, If the angle, &c. Which was to be demonſtrated.

g) 2. 6.
h) 11. 5.
i) 9. 5.
k) 5. 1.
l) 29. 1.

PROP.

PROP. 4. THEOR. 4.



Of equiangled triangles A B C *and* D E C, *the ſides* A B *and* B C, D C *and* C E *which are about the equal angles* A B C *and* D C E, *as alſo* B C *and* C A, C E *and* E D, *which are about the equal angles* A C B *and* E, *and* A B *and* A C, *and* D C *and* D E, *which are about the equal angles* A *and* D, *are proportional, and the ſides* A C *and* D E, *which ſubtend the equal angles* B *and* D C E, A B *and* D C, *which ſubtend the equal angles* A C B *and* E, *as alſo* B C *and* C E, *which ſubtend the equal angles* A *and* D, *are homologal, or of the ſame reaſon.*

Demonſtration CONſtitute B C and C E, in a right line, in ſuch ſort as the exterior angle D C E may be equal to the interior angle A B C. Likewiſe the exterior angle A C B equal to the interior D E C; and foraſmuch as the two angles A B C and A C B, are leſſe than two right angles; but D E C is equal to A C B, the two angles B and E ſhall
a) 11. c. f. be leſſe than two right angles. Wherefore [a] B A and E D, prolonged towards A and D, ſhall meet, let them then be prolonged, and they ſhall meet in the point F.

But foraſmuch as the exterior angle D C E is equal to the interior and
b) 28. 1. oppoſite angle A B C, [b] C D and B F ſhall be parallels; and by the ſame reaſon C A and E F ſhall be alſo parallels, the exterior angle A C B being equal to the interior D E C. Therefore A C D F is a parallelogram: Therefore
c) 34. 1. [c] A F is equal to C D, and C A to D F. Therefore foraſmuch as in the tri-
d) 2. 6. angle B E F, the line A C is parallel to E F, [d] A B ſhall be to A F, that is to ſay to D C, its equal, as B C to C E: Therefore alternately, as A B to B C, ſo D C to C E.

Again, ſeeing that in the ſame triangle B E F, the line C D is parallel to the ſide B F, as B C ſhall be to C E, ſo F D, that is to ſay C A, (which
e) 22. 5. is equal to F D,) to E D. Therefore [e] alternately, as B C to C A, ſo C E to E D: Therefore ſeeing that as A B to B C, ſo D C to C E, and as B C to C A, ſo C E to E D: In equal reaſon, A B ſhall be to C A, as D C to E D. Which was propoſed. Therefore the triangles, &c. Which was to be demonſtrated.

COROLLARIE. I.

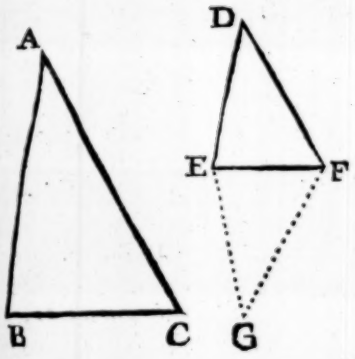
From hence it followes, that if in a triangle there be drawn a line parallel to one of the ſides, it will take away a triangle reſembling the whole, as in the triangle F B E, *where the line* C D *is drawn parallel to* B F. *I ſay that the triangle* D C E *is alike to the*
f) 29. 1. *triangle* F B E, *for they are equiangled;* [f] *ſeeing that the angles* E D C *and* E C D, *are equal to the exterior angles* F *and* B, *each to his correſpondent angle, and the angle*

E *common; therefore* g *(as hath been demonſtrated) they have the ſides about the equal angles proportional; therefore according to the Definition they are proportional.* g) 4. 6.

COROLLARIE II.

It follows moreover, that if two triangles equiangled, that is to ſay, which have the ſides proportional, are diſpoſed according to one angle, in ſuch ſort, as that two ſides of the ſame reaſon may be parallels, the other ſides will directly meet with one another, as in the triangles before mentioned; where AB *and* DC, *and* AC *and* DE, *being parallels, the other ſides* BC *and* CD, *will directly meet with one another, as is manifeſt by the fourteenth Propoſition of the Firſt Book; all the angles at the point* C *being ſhewn to be equal to the three angles of the triangle* ABC; *that is to ſay, to two right angles, by the* 32 *Propoſition of the Firſt Book; which* EUCLIDE *demonſtrateth again in the* 32 *Propoſition of this Book.*

PROP. 5. THEOR. 5.



If two triangles ABC, *and* DEF, *have the ſides proportional; that is,* AB *to* BC, *as* DE *to* EF, *and as* BC *to* CA, *ſo* EF *to* FD, *and as* AB *to* AC, *ſo* DE *to* DF; *thoſe triangles ſhall be equiangled, and ſhall have the angles equal, under which the ſides of the ſame reaſon are ſubtended.*

Demonſtration LEt the angle FEG be made equal to the angle B, and EFG equal to C, the two lines EG and FG, ſhall meet in the point G; and [a] the other angle G ſhall be equal to the other angle A. Therefore the triangles ABC and GEF are equiangled: Therefore [b] as AB to BC, ſo GE to EF; but as AB to BC, ſo DE to EF, by ſuppoſition: Therefore [c] as GE to EF, ſo DE to the ſame EF. Wherefore GE and DE ſhall be equal. a) 32. 1. b) 4. 6. c) 11. 5.

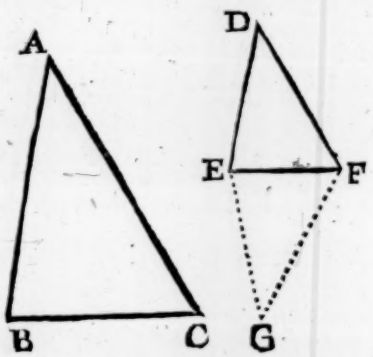
Again, ſeeing that BC is to CA, as EF to FG; but [d] as BC is to CA, ſo EF to FD, by ſuppoſition, as EF ſhall be to FG; ſo the ſame EF ſhall be to FD, therefore [e] FG and FD, ſhall be equal: Therefore ſeeing that the ſides GE and GF are equal to the ſides DE and DF, each to its correſpondent, and the baſe EF [f] common, the angles G and D ſhall be equal. Therefore [g] the other angles GEF and GFE ſhall be equal to the other angles DEF and DFE, each to his correſpondent. Wherefore ſeeing that the angle G is equal to the angle A, alſo D its equal, ſhall be equal to A, by the ſame reaſon, DEF equal to B, and DFE equal to C. Which was propoſed. Therefore, If two triangles, &c. Which was to be demonſtrated. d) 4. 6. e) 9. 5. f) 8. 1. g) 4. 1.

PROP. 6. THEOR. 6.

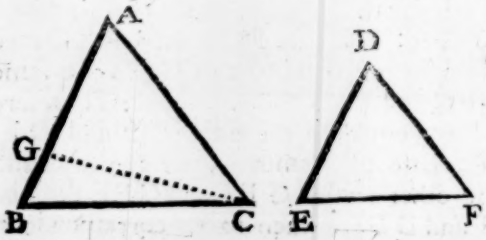
If two triangles ABC *and* DEF, *have an angle* B, *equal to*

to an angle E, *and the ſides about thoſe equal angles proportional; that is, as* AB *to* BC, *ſo* DE *to* EF, *thoſe triangles ſhall be equiangled, and ſhall have the angles equal, under which the homologal ſides, or ſides of the ſame reaſon are ſubtended; that is, the angle* A *equal to the angle* D, *and the angle* C *to the angle* F.

Demonſtration. FOr, Let the angle FEG be made equal to B, and EFG equal to C, the triangle GEF ſhall be equiangled to the
a) 4. 6. triangle ABC, as is ſhewn in the precedent Propoſition. Therefore[a] as AB to BC, ſo GE to EF. But as AB to BC, ſo DE to EF by ſuppo-
b) 11. 5. ſition: Therefore[b] as DE to EF, ſo GE to the ſame EF; therefore DE and GE ſhall be equal: Wherefore ſeeing that the ſides DE and EF are equal to the ſides GE and EF,
9. 5. and the angles conteined of thoſe ſides alſo equal, (for the angle B, to which the angle GEF is made equal, is equal to DEF by ſuppoſition: therefore the two angles at the point E ſhall be e-
c) 4. 1. qual,) [c] the other angles D, and EFD ſhall be equal to the two others G, and EFG: Therefore ſeeing that the angle G is equal to the angle A, and EFG equal to C, the angles D and EFD, ſhall be alſo equal to the angles A and C; and therefore the triangles ABC and DEF, ſhall be equiangled. Which was propoſed. Therefore, If two triangles, &c. Which was to be demonſtrated.



PROP. 7. THEOR. 7.



If two triangles ABC, *and* DEF, *have an angle* A, *equal to an angle* D, *and about another angle* ACB, *the ſides proportional; that is,* AC *to* BC, *as* DF *to* EF. *But the other angles* B *and* E *together, the one and the other be leſſe or not leſſe than a right angle: The triangles* ABC *and* DEF *ſhall be equiangled, and ſhall have the angles equal, about which the ſides are proportional; that is to ſay, the angles* ACB *and* F, *and* B *and* E.

De-

Demonſtration FOr firſt of all, Let as well B as E, be leſſe than a right angle, that being, if the angles A C B and F are equal, the Propoſition is manifeſt: But if A C B and F are not equal; Let A C B be greater than F, and let A C G be made equal to F: Therefore ſeeing that A is put equal to D, [a] the other angle A G C ſhall be equal to E. Therefore the triangles A G C and D E F ſhall be equiangled. Wherefore [b] as A C ſhall be to C G, ſo D F to E F. But as D F to F E, ſo A C to C B, by ſuppoſition: Therefore [c] as A C to C G, ſo the ſame A C to C B; and therefore [d] C G and C B ſhall be equal; [e] and the angles C B G and C G B ſhall be equal.

a) 32. 1.
b) 4. 6.
c) 11. 5.
d) 9. 5.
e) 5. 1.

Therefore ſeeing that the angle B is put leſſe than a right angle, C G B ſhall be alſo leſſe than a right angle; and therefore A G C ſhall be greater than a right angle; ſeeing that [f] A G C and C G B are equal to two right angles. Now the angle A G C is ſhewn to be equal to the angle E. Therefore the angle E ſhall be alſo greater than a right angle. But it is leſſe than a right angle by ſuppoſition, which is impoſſible.

f) 13. 1.

Secondly, Let as well the angle B, as the angle E, not be leſſe than a right angle, as before, the angle B ſhall be equal to the angle C G B; and therefore C G B ſhall not be leſſe than a right angle; and ſo the angles C B G and C G B, in the triangle B C G, ſhall not be leſſe than two right angles, but greater, or equal to two right angles, which is abſurd; [g] for they are leſſe than two right angles. Therefore the angles A C B and F are not unequal, but equal; and therefore [h] the other angles B and E are equal. Which was to be demonſtrated. Therefore, If two triangles, &c.

g) 17. 1.
h) 32. 1.

PROP. 8. THEOR. 8.

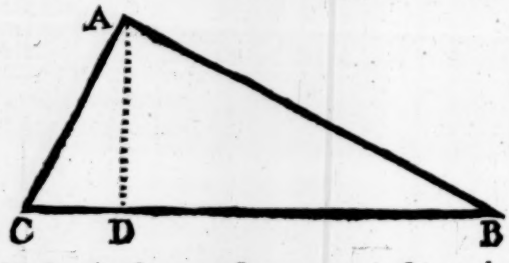


If in a rectangled triangle A B C, *there be drawn a perpendicular line* A D, *from the right angle* B A C, *on the baſe* C B, *the triangles* A D B, *and* A D C, *which are on both ſides of the perpendicular, are alike to the whole, and alike to one another.*

Demonſtration FOr ſeeing that in the triangles A B C and D B A, the angles B A C and A D B are right angles, and A D B are right angles, and the angle B common, [a] the other angles A C B and D A B ſhall be equal: Therefore the triangle D B A is equiangled to the triangle A B C; and [b] therefore they will have the ſides about the equal angles proportional, &c, that is to ſay, as C B ſhall be to B A, ſo B A to B D, and as B A to A C, ſo B D to D A, and as B C to C A, ſo B A to A D, for ſo the ſides having the ſame reaſon, are oppoſite to the equal angles, as by the fourth Propoſition of this Book. Wherefore the triangle A D B is alike to the whole triangle A B C, by the ſame reaſon it may be ſhewn

a) 32. 1.
b) 4. 6.

that the triangle A D C is alike to the ſame triangle A B C; for the angles B A C and A D C, are right angles, and the angle C common: Therefore [c] the other angles A B C and C A D are equal; [d] wherefore as B C to C A, ſo C A to C D, and as C A to A B, ſo C D to D A, and as C B to B A, ſo C A to A D; for ſo the ſides having the ſame reaſon, are oppoſite to the equal angles, as by the fourth Propoſition of this Book.



c) 32. 1.

d) 4. 6.

Even ſo it will be demonſtrated that the triangles A D B and A D C are alike to one another, ſeeing that the angles A D B and A D C are right angles, and A B D and C A D are ſhewn to be equal, and alſo the angles B A D and A C D equal; and [e] therefore as B D to D A, ſo D A to D C; and as D A to A B, ſo D C to C A; and as A B to B D, ſo C A to A D. Therefore, If in a triangle, &c. Which was to be demonſtrated.

e) 4. 6.

COROLLARIE.

From this Propoſition it is evident, that the perpendicular drawn from the right angle on the baſe, in a rectangle triangle, is a mean proportional between the two ſegments of the baſe. Likewiſe each of the ſides which contein the right angle, is a mean proportional between the whole baſe and the ſegment of the baſe, which is adjacent, or toucheth the ſame ſide.

For it is demonſtrated that as B D *is to* D A, *ſo* D A *is to* D C; *and therefore* D A *is a mean proportional between* B D *and* C D. *Likewiſe as* C B *is to* B A, *ſo* B A *to* B D; *and ſo* B A *is a mean proportional between* C B *and* B D: *Laſtly, it is demonſtrated that as* B C *is to* C A, *ſo* C A *is to* C D; *and therefore* C A *is a mean proportional between* B C *and* C D. *Which was propoſed.*

PROP. 9. PROBL. 1.

From a given right line A B, *to take away a required part, as* A G.



Conſtruction FRom A draw A C, making any angle whatſoever with A B, as C A B, and from A C take ſo many equal parts of ſuch a Magnitude as you would that the part to be ſubſtracted ſhall denote; (for A C ought to be ſo great as need requires,) as in the example propoſed, you ought to take three equal parts, A D, D E, and E F, then joyn F B, to which by the point D, draw D G, parallel to F B: I ſay that A G is the third part of A B required.

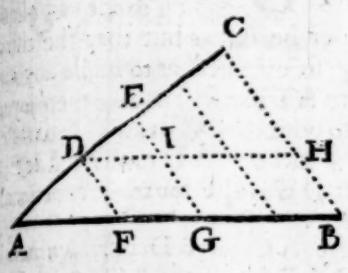
Demonſtration FOr ſeeing in the triangle A B F, D G is parallel to the ſide F B; [a] as F D to D A, ſo B G to G A; [b] therefore in compounding, as F A to D A, ſo B A to G A; but F A is triple to A D, by Conſtruction, therefore B A ſhall be alſo triple to A G; and therefore A G ſhall be the required third part of A B. Therefore, From a given right line, &c. Which was to be done.

a) 2. 6.

b) 18. 5.

PROP.

PROP. 10. PROBL. 2.



To divide a given right line undivided AB, *alike to a given right line* AC, *which is divided, as in* D *and* E.

Construction JOyne the two given lines in the point A, making any angle whatsoever, as BAC, and joyn the right line BC, then from D and E, draw DF and EG, parallel to BC. I say that AB is divided alike at F and G, as AC is divided in D and E, [a] that is to say, as AD to DE, so AF to FG. Therefore the parts AF and FG are proprtional to AD and DE, and if you draw DH parallel to FB, dividing EG in the point I.

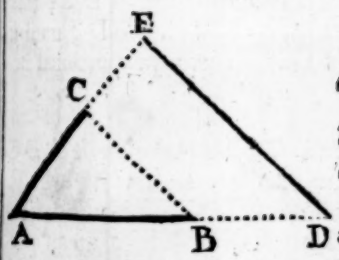
Again, [b] as DE shall be to EC, so DI to IH; that is to say, so FG to GB [c].

a) 2. 6.

b) 2. 6.
c) 34. 1.

Demonstration FOrasmuch as FG is equal to DI, and GB to IH. Wherefore the parts FG and GB shall be also proportional to DE and EC. And so by the same reason if there were more parts. Therefore, We have divided a line, &c. Which was to be done.

PROP. 11. PROBL. 3.



To two given right lines AB *and* AC, *to find a third proportional line* CE.

Construction DIspose the right lines AB and AC, so as to make any angle A, to which a third proportional ought to be found; that is to say, as AB is to AC, so AC is to a third.

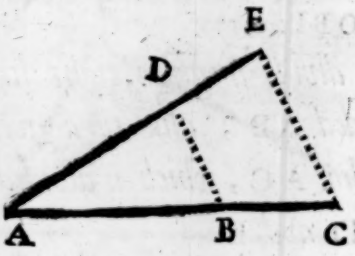
Prolong AB the antecedent line proposed to D, and assume BD, equal to AC the consequent or mean; then having drawn BC; by D draw DE, parallel to BC, meeting AC prolonged in E; I say that CE is the third proportional; that is to say, that as AB is to AC, so AC is to CE.

Demonstration FOr seeing that in the triangle ADE, BC is parallel to DE, as [a] AB shall be to BD, so AC shall be to CE. But [b] as AB to BD, so the same AB to AC, equal to BD: Therefore as AB to AC, so AC to CE. Which was proposed. Therefore, To two given right lines, &c. Which was to be done.

a) 2. 6.
b) 7. 5.

PROP. 12. PROBL. 4.

To three given right right lines AB, BC, *and* AD, *to find a fourth proportional; that is, as* AB *to* BC, *so* AD *to a fourth* DE.

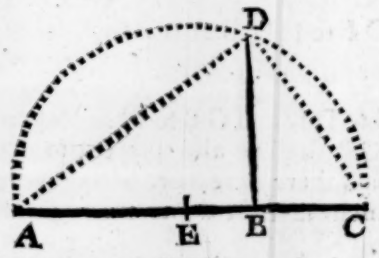


Conſtruction DIſpoſe the two firſt according to one right line, which let be A C; but that the third A D be ſo diſpoſed as to make any angle with A B the firſt as A, then joyne B D, to which by C draw C E parallel thereto, meeting A D prolonged at E; I ſay that D E is the fourth proportional.

Demonſtration SEeing that in the triangle A C E, B D is drawn parallel to the ſide C E; as [a] A B ſhall be to B C, ſo A D to D E: Therefore D E is the fourth proportional. Wherefore to three given right lines, &c. Which was to be done.

a) 2. 6.

PROP. 13. PROBL. 5.



To two given right lines A B *and* B C, *to finde a mean proportional.*

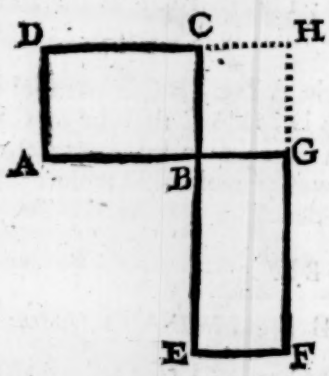
Conſtruction HAving diſpoſed the right lines A B and B C, according to one right line A C, divide A C into two equal parts in the point E, and from the point E as a center, and the diſtance E A or E C, deſcribe the ſemicircle A D C; then from the point B, in the line A C, raiſe the perpendicular B D, until it touch the circumference at D. I ſay that B D is a mean proportional between A B and B C.

Demonſtration FOr having drawn A D and C D, [a] the angle A D C ſhall be a right angle in the ſemicircle; and ſeeing that from the right angle of the rectangle triangle A D C, the perpendicular D B, is drawn on the baſe A C, the ſame B D ſhall be a mean proportion [b] between A B and B C. Therefore, To two right lines, &c. Which was to be done.

a) 31. 1.

b) Cor. 8. 6.

PROP. 14. THEOR. 9.



The ſides A B, B G, E B, *and* B C, *which are about the equal angles of equal parallelograms* A C *and* B F, *which have an angle* A B C, *equal to an angle* E B G, *are reciprocal; and the parallelograms which have an angle equal to an*

an angle, and the ſides which are about the equal angles reciprocal, are equal.

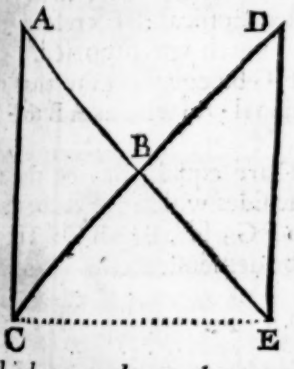
Demonſtration FOr joyn thoſe parallelograms according to the equal angles, in ſuch ſort as that A B and B G may make one right line E B and B C [a] ſhall alſo make a right line, as is evident; and let D C and F G be prolonged, untill they meet with one another in H. a) 2. c. ſ. 13, 14. 1.

Foraſmuch then as the parallelograms D B and B F are equal; as [b] D B ſhall be to B H, ſo B F to the ſame B H; but as D B to B H, ſo the baſe [c] A B to the baſe B G; foraſmuch as the parallelograms are of the ſame height. Likewiſe as B F is to B H, ſo the baſe E B to the baſe B C: Therefore [d] as A B to B G, ſo E B to B C. Which was propoſed. b) 7. 5. c) 1. 6. d) 11. 5.

Contrarily, Let the ſides about the ſaid equal angles be reciprocal; to wit, as, A B to B G, ſo E B to B C: I ſay that the parallelograms D B and B F are equal.

Uſe the ſame Conſtruction, ſeeing that A B is to B G, as E B to B C. But [e] as A B to B G, ſo D B to B H; and as E B to B C, ſo B F to the ſame B H. Likewiſe [f] D B ſhall be to B H, as B F to the ſame B H; and therefore [g] the parallelograms D B and B F ſhall be equal. Therefore, The ſides, &c. Which was to be demonſtrated. e) 1. 6. f) 11. 5. g) 9. 5.

PROP. 15. THEOR. 10.



The ſides which are about the equal angles of the equal triangles A B C *and* D B E, *which have an angle* B *equal to an angle* B, *are reciprocal; that is, as* A B *to* B E, *ſo* D B *to* B C. *And the triangles which have an angle equal to an angle, and the ſides which are about the equal angles reciprocal, are equal.*

Demonſtration JOyne thoſe ſides according to the equal angles, in ſuch ſort as that A B and B E may make one right line, that being D B, B C ſhall alſo make one right line; as is ſaid in the precedent Propoſition; and let C E be drawn; foraſmuch as the triangles A B C and D B E are equal, as A B C ſhall be to B C E, ſo D B E ſhall be to the ſame B C E; but as the triangle A B C, to the triangle B C E; [a] ſo the baſe A B to the baſe B E, they being of one and the ſame height: And in like manner, as D B E to B C E, ſo is the baſe D B, to the baſe B C; therefore [b] as A B to B E, ſo is D B to B C. Which was propoſed. a) 1. 6. b) 11. 5.

Now let the ſides about the equal angles in the point B, be reciprocal; to wit, as A B to B E, ſo D B to B C; I ſay that the triangles A B C and D B E, are equal.

Let the ſame Conſtruction be made, foraſmuch as A B is to B E, as D B to B C; but as A B to B E, [c] ſo the triangle A B C to the triangle B C E; c) 1. 6.

BCE, and as DB to BC, ſo the triangle DBE to the ſame triangle BCE: Therefore [d] as ABC ſhall be to BCE, ſo DBE ſhall be to the ſame BCE; and therefore the triangles ABC and DBE ſhall be equal. Therefore the ſides, &c. Which was to be demonſtrated.

d) 11. 5.

PROP. 16. THEOR. 11.



If four right lines AB, FG, EF, *and* BC, *are proportional; the rectangle* AC, *conteined under the extreams* AB, *and* BC, *is equal to the rectangle* EG, *conteined under the means* EF *and* FG, *and if rectangle conteined under the extreams, be equal to the rectangle conteined under the means, thoſe four lines ſhall be proportional.*

Demonſtration FOr ſeeing that the angles B and F are equal, being right angles, and that AB is to FG, as EF to BC; thoſe ſides which are about the equal angles B and F, are reciprocal: Therefore [a] the Parallelograms AC and EG ſhall be equal. Which was propoſed.

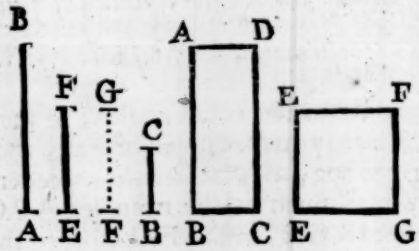
a) 14. 6.

Contrarily, Let the rectangles AC and EG be equal; I ſay that the four lines AB, FG, EF and BC are proportional; to wit, as AB to FG, ſo EF to BC.

For ſeeing that the rectangles AC and EG are equal; having the angles B and F equal; to wit, right angles, [b] the ſides which are about thoſe angles ſhall be reciprocal; to wit, as AB to FG, ſo EF to BC: Therefore, If four right lines, &c. Which was to be demonſtrated.

b) 14. 6.

PROP. 17. THEOR. 12.



If three right lines AB, EF, *and* BC, *be proportional: The rectangle* AC *conteined under the extreams* AB *and* BC, *is equal to the ſquare* EG, *of the mean* EF *and if the rectangle conteined under the extreams, be equal to the ſquare of the mean: thoſe three right lines ſhall be proportional.*

Demonſtration FOr aſſume the line FG, equal to EF, the four right lines AB, EF, FG, and BC ſhall be proportional; to wit, as AB to EF, ſo FG to BC; and the ſquare EG ſhall be conteined under the

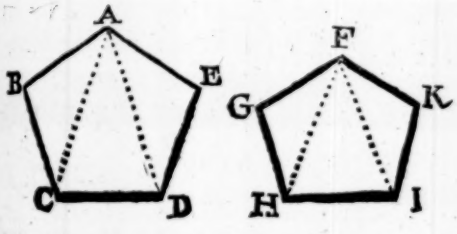
the means E F and F G; seeing that E F and F G are equal: Wherefore [a] the rectangle A C, conteined under the extreams A B and B C, is equal to the square E G; that is to say, to the rectangle conteined under the meanes E F and F G. Which was proposed. a) 16. 6.

But let the rectangle A C, and the square E G be equal; I say that as A B is to E F, so E F is to B C; for the rectangles A C and E G being equal; as A B shall be to E F, so F G to B C; but [b] as F G to B C, so E F [c] equal to the same F G, is to the same B C: Therefore as A B to E F, so E F to B C. Therefore, If three right lines, &c. Which was to be demonstrated. b) 16. 6. c) 7. 5.

CORLLARIE.

From the latter part of this Theorem it follows, that every right line is a mean proportional between two other, whatsoever right lines, which do contein a rectangle equal to the square of the same lines: For from that, that the right lines A B *and* B C, *do contein a rectangle equal to the square of the right line* E F, *it hath been shewn that as* A B *is to* E F, *so* E F *is to* B C. *Wherefore* E F *is a mean proportional between* A B *and* B C.

PROP. 18. PROBL. 6.



On a given right line C D, *to describe a rectiline figure* A B C D E, *alike, and alike posited to a given rectiline figure* F G H I K.

Construction FRom either of the angles, (as from F,) draw to each of the opposite angles, the right lines F H and F I, which lines do divide the rectiline figure into three triangles F G H, F H I, and F I K; then make the angle D C A equal to the angle I H F, and the angle C D A equal to H I F. Now A C and A D shall meet with one another at A, the angles A C D and A D C being lesse than two right angles, being made equal to F H I and F I H, ([a] which are lesse than two right angles,) and the other angle C A D shall be equal to the other angle H F I, and the triangle C A D equiangled to the triangle H F I. a) 17. 1. 32. 1.

Again, Let the angle A D E be made equal to F I K, and D A E equal to I F K; forasmuch [b] as the two angles K I F and K F I, are lesse than two right angles, the two angles E A D and E D A, equal unto them, shall be in like manner lesse than two right angles; and therefore A E and D E shall meet in the point E, and the triangle A E D shall be equiangled to the triangle F K I by the same reason. b) 17. 1.

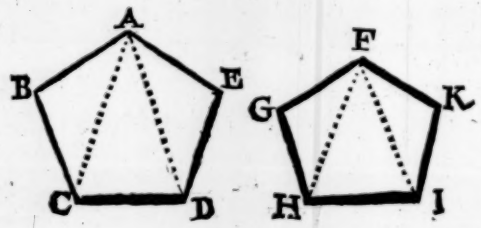
Moreover, Make the triangle A B C equiangled to the triangle F G H, by the same reason, and so of the others, if there be any more triangles; I say that the rectiline figure A B C D E is alike, and alike posited to the rectiline figure F G H I K.

Demonstration FOr seeing that the angle A C D is made equal to F H I, and A C B to F H G, the whole B C D shall be equal to G H I; and by the same reason C D E is equal to H I K, and the other angles

angles equal to the other angles, as appears by the Conſtruction; ſeeing that each part of the one is made equal to each of the parts of the other.

c) 4. 6. Therefore the rectiline [c] figure A B C D E ſhall be equiangled to F G H I K. Now foraſmuch as C D is to D A, as H I to I F, and as A D

d) 22. 5. to D E, ſo I F is to I K, [d] in equal reaſon, C D ſhall be to D E, as H I to I K; therefore the ſides about the equal angles C D E and H I K are proportional; even



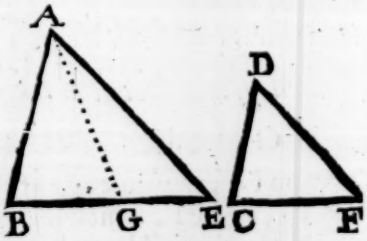
c) 4. 6. ſo [c] the ſides about the equal angles E and K are proportional; becauſe of the equiangled triangles A E D and F K I. Again, [f] E A is to A D, as F K

f) 4. 6. to F I, and A D to A C, as F I to F H, and A C to A B,

g) 22. 5. as H F to F G: Therefore [g] in equal reaſon, E A ſhall be to A B, as K F to F G: Wherefore the ſides about the equal angles E A B and K F G are proportional, and ſo of the others; therefore ſeeing that the rectiline figures are equiangled, and have the ſides about the equal angles proportional, they are alike, and alike deſcribed. Therefore, On a given, &c. Which was to be demonſtrated.

PROP. 19. THEOR. 13.

Like triangles A B E *and* D C F, *are to one another in a duplicate reaſon of their ſides of the ſame reaſon.*



Conſtruction LEt the triangles A B E and D C F, have the angles B and C equal; alſo E and F, and as A B to B E, ſo D C to C F: I ſay they are to one another in a duplicate reaſon of their ſides of like reaſon B E and C F, or A B and D C, or A E and D F; that is to ſay, if you find the third proportional B G, the triangle A B E ſhall be to the triangle D C F, as B E is to B G; ſeeing that by the tenth Definition of the Fifth Book, the reaſon double is ſuch.

Demonſtration LEt therefore B G be a third proportional, and let A G be joyned to it; ſeeing that as A B is to B E, ſo D C to C F; alternately, as A B ſhall be to D C, ſo B E to C F; but as B E to C F, ſo

a) 11. 5. C F to B G by Conſtruction: Therefore [a] as A B to D C, ſo C F to B G;

b) 15. 6. therefore ſeeing that the triangles A B G and D C F have the ſides about

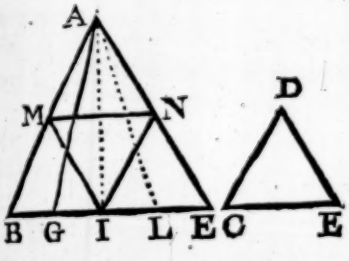
c) 7. 5. the equal angles B and C reciprocal, [b] they ſhall be equal to one another: Therefore as the triangle A B E ſhall be to the triangle D C F, ſo [c] the

d) 1. 6. ſame triangle A B E ſhall be to the triangle A B G. But as the triangle A B E is to the triangle A B G, ſo [d] the baſe B E to the baſe B G, they being of the ſame height. Therefore as the triangle A B E is to the triangle D C F, ſo B E is to B G.

Now ſeeing that the three lines B E, C F, and B G, are continually proportional, the reaſon of the firſt B E to the third B G, ſhall be ſaid to be

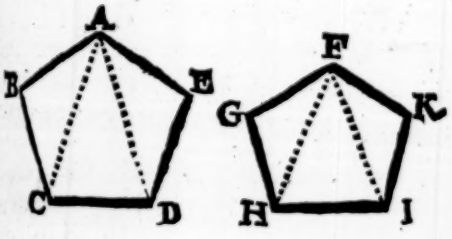
be double to the reason of B E the first, to C F the second; Therefore also the triangle A B E shall be to the triangle D C F, in reason double of the side B E to the side C F: Therefore, Like triangles, &c. Which was to be demonstrated.

COROLLARIE.



Whence it follows that if there be three lines proportional, as the first is to the third, so the triangle described on the first, shall be to the like triangle, and alike described on the second, &c. which is easie to be conceived by this last figure, where the third proportional line B G *being the quarter part of the base* B E; *or else* B E *being quadruple of the third proportional* B G, *the triangle* A B E *is also quadruple of the triangle* A B G, *as appeares by the eighteenth of the first,* G I, I L, *and* L E, *being equal to* B G; *and therefore also quadruple of the triangle* D C F, *as is shewn, and which is also manifest, the triangle* A B E *being divided into four other triangles* A M N, M B I, M I N, *and* I N E, *each of which is equal to the triangle* D C F, *which is easie to be understood by these figures which I thought good here to adde, to facilitate the sence and meaning of this Proposition, which had need be well understood, to the end the next following and the others after, may be the better comprehended.*

PROP. 20. THEOR. 14.



Like Polygons divide themselves into an equal number of like triangles, and proportional to their whole, and the Polygons are the one to the other in duplicate reason of their sides of the same reason.

LEt the Polygons be A B C D E and F G H I K, having the angles B A E and G F K equal, also the angles B and G, and so following, and having also the sides proportional about those angles; to wit, as A B to B C, so F G to G H, and as B C to C D, so G H to H I, &c. I say first of all that those Polygons may be divided into an equal number of like triangles.

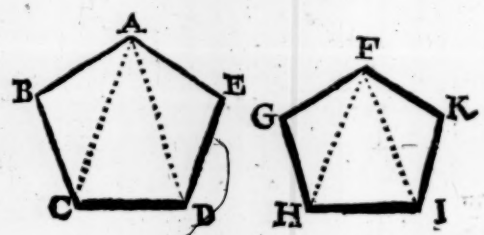
Demonstration FOr from the angles A and F, draw A C, A D, F H, and F I, to the opposite angles C, D, H, and I, they shall be divided into an equal number of triangles; and forasmuch as the angles B and G are put equal, and the sides about them proportional; [a] the triangles A B C and F G H shall be equiangled, having the angles B A C, G F H, B C A, and G H F, opposite to the equal Homologal sides; and [b] therefore shall have the sides about the equal angles proportioual, and so shall be alike; by the same reason, the triangles A E D and F K I shall

a) 6. 6.

b) 4. 6.

be alike; having the angles E A D, K F I, A D E, and F I K equal. And
c) 4. 6. furthermore, [c] seeing that as A C is to C B, so F H to H G, because of the similitude of the triangles A B C and F G H: But as B C to C D, so G
d) 22. 5. to H I, by supposition, by reason of the similitude of the Polygons [d] in equal
e) 6. 6. reason, as A C to C D, so F H to H I; and forasmuch as the angle B C D is put equal to G H I; but the angle cut off B C A, is shewn to be equal to the angle G H F, the remainder A C D shall be equal to the remainder F H I. Therefore seeing that the triangles A C D and F H I have the angles A C D and F H I equal, and the sides about them proportional; they shall be equiangled, and therefore alike; and so of the others, if there were more.

Secondly, I say that the triangles are proportional to their whole, that is to say, that each of the triangles in one of the Polygons, hath such reason to its triangle correspondent in the other Polygon, as the whole Polygon to the whole Polygon: Forasmuch as the triangles A B C and F G H are alike, they shall be to one another in double reason of their Homologal sides A C and F H, and by the same discourse the triangles A C D and F H I are in reason double of the same sides A C and F H. Wherefore as the triangle A B C is to the triangle F G H, so the triangle A C D shall be to the triangle F H I; seeing that the reason of the one and the other, is the duplicate reason of the side A C to F H. Likewise it may be concluded that the triangle A D E is to the triangle F I K, as the triangle A C D to the triangle F H I, and so following, if there were other triangles: Therefore the triangles of the one of the Polygons are proportional to the triangles of the other; in such



sort as the triangles of the one are antecedents of the proportions, and the triangles of the other consequents. Now as one of the antecedents is to one
f) 12. 5. of the consequents, [f] so all the antecedents are to all the consequents: Therefore as each of the triangles of one of the Polygons is to its correspondent triangle of the other Polygon, so the whole Polygon shall be to the whole Polygon: Therefore the triangles shall be proportional to their whole.

Lastly, I say that the Polygons are to one another in a duplicate reason of their sides, of the same reason, that is to say, if to the Homologal sides (for Example A B and F G,) there be found a third proportional, that the Polygon A B C D E shall be to the Polygon F G H I K, as the line A B the first, is to third proportional found. For seeing that as the triangle A B C is to the triangle F G H, so the Polygon A B C D E, to the Poligon F G H I K. But the triangle A B C is to the triangle F G H in a dupli-
g) 19. 6. cate reason [g] of the side A B to the side F G. Likewise [h] the Polygons shall
h) 11. 5. be the one to the other in duplicate reason of the same sides A B and F G. Therefore, The Polygons, &c. Which was to be demonstrated.

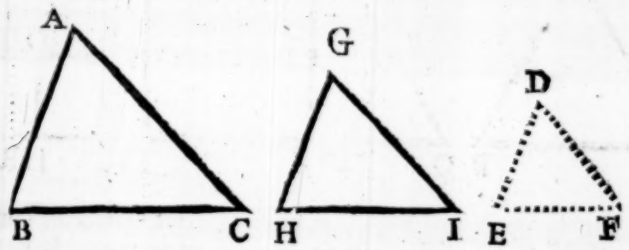
COROLLARIE.

From this Demonstration it is manifest that if there be three lines proportionals as the first shall be to the third, so the Polygon described on the first, shall be to the like

like Polygon, and alike described on the second, or else the Polygon described on the second, shall be to a like Polygon, and alike described on the third, as the first is to the third, which is evident, it having been demonstrated that the Polygon is to the Polygon, as the first A B *is to the third proportional*

PROP. 21. THEOR. 15.

Rectiline figures A B C *and* D E F, *alike to one and the same rectiline figure* G H I, *are also alike to one another.*



Demonstration FOr seeing that the angles of A B C, are equal to those of [a] G H I, because of their similitude, and those of D E F to those of G H I; for the same cause, [b] the angles of A B C, shall be equal to the angles of D E F.

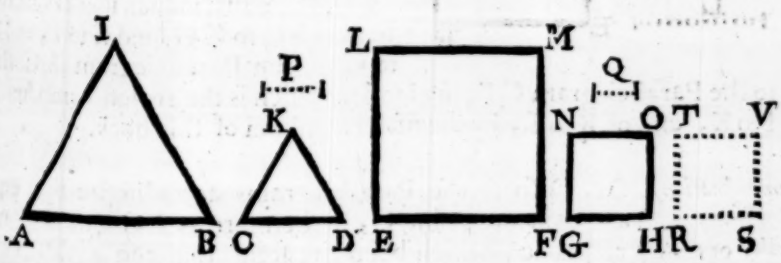
a) 1. def. 6.
b) 1. c. 5.

Again, because of the same similitude, the sides of A B C are proportional to those of G H I; to wit, those which are about the equal angles; also the sides of D E F are proportional to the sides of G H I; for the same cause, therefore [c] the sides of A B C shall be also proportional to the sides of D E F; to wit, those which environ equal angles. Therefore according to the Definition, A B C and D E F shall be alike: Therefore, Rectiline figures, &c. Which was to be demonstrated.

c) 11. 5.

PROP. 22. THEOR. 16.

If four right lines A B, C D, E F, *and* G H, *are proportional, the rectiline figures* A B I, C D K, E M *and* G O, *alike, and alike described on them shall be proportional, and if the rectiline figures alike, and alike described on such right lines are proportional, those right lines shall be also proportional.*

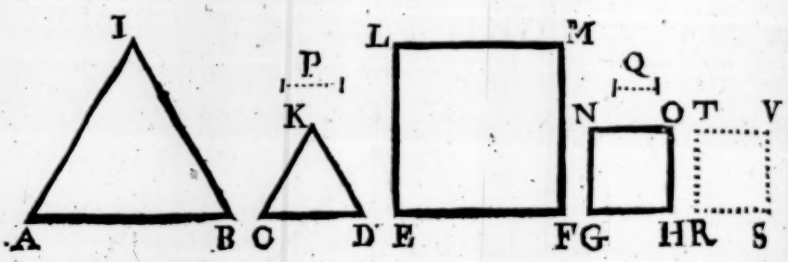


Demonstration FOr [a] to A B and C D, find the third proportional P, and to E F and G H, the third proportional Q: [b] In equal reason, as A B shall be to P, so E F to Q: But as A B is to P, so the recti-

a) 11. 6.
b) 22. 5.

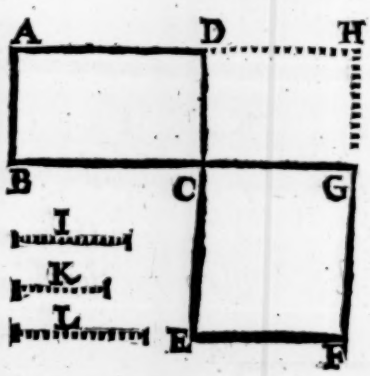
c) Co. 20. 6. rectiline figure c ABI is to the rectiline figure CDK alike, and alike described; by the same reason, as EF is to Q, so the rectiline figure EM;
d) 11. 5. is to the rectiline figure GO; therefore d as ABI is to CDK, so EM is to GO. Which was proposed.

Secondly, Let the rectiline figures ABI, CDK, EM, and GO, be proportional; I say that the four right lines AB, CD, EF, and GH, are proportional; to wit, as AB to CD, so EF to GH.



e) 12. 6. For to the three lines AB, CD, and EF, let e there be found a fourth proportional RS, on which describe the rectiline figure RV, alike, and
f) 21. 6. alike posited to the rectiline figure EM; and therefore f to the rectiline figure GO; forasmuch as AB is to CD, as EF is to RS: Likewise (as hath been already shewn,) as ABI shall be to CDK, so EM to RV;
g) 11. 5. but as ABI to CDK, so EM to GO: Therefore g as EM to RV, so EM
h) 9. 5. to GO; h therefore RV and GO are equal, which being alike, and alike posited, of necessity they shall be constituted on the equal right lines
i) 7. 5. RS and GH: Therefore i as EF shall be to RS, so EF to GH; but EF
k) 11. 5. is to RS, as AB to CD by supposition: Therefore k as AB to CD, so EF to GH: Therefore, If four lines, &c. Which was to be demonstrated.

PROP. 23. THEOR. 17.



Equiangled Parallelograms AC *and* CF, *are the one to the other in a reason compounded of that of their sides, (that is, of* BC *of* CG, *and of* DC *to* CE.

THat is to say, if you take three lines I, K, and L, in such sort as I may be to K, as BC to CG, and K to L, as DC to CE, the Parallelogram AC shall be to the Parallelogram CF, as I to L, which is the reason compounded of I to K, and of K to L, by the fifth Definition of this Book.

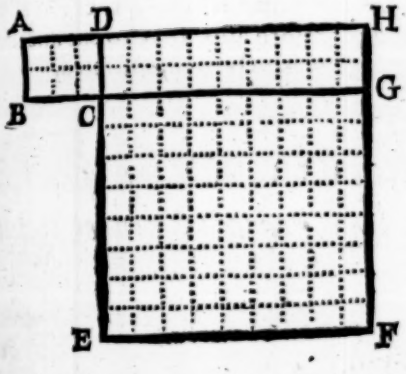
Demonstration FOr, Dispose the Parallelograms according to the equal angles in the point C, in such sort as BC and CG may make one only right line, which being so, seeing that the angles BCD and ECG are equal, and EC and CD shall also make one right line, as hath been shewn in the fourteenth Proposition; prolong AD and FG to the point H, making the Parallelogram CH, and assume any right line,

as

as I, and to BC, CG, and I, find a fourth proportional K; alſo to the three right lines DC, CE, and K, the fourth proportional L: Foraſmuch as [a] BC is to CG, as AC to CH: But as BC to CG, ſo I to K, as hath been ſhewn; likewiſe [b] as AC to CH, ſo I to K. a) 1.6. b) 11.5.

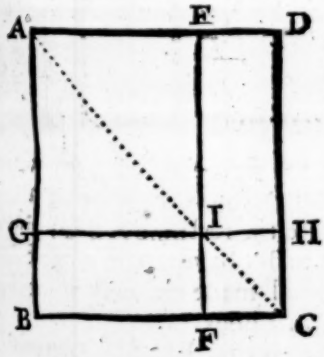
By the ſame reaſon it ſhall be ſhewn, that as HC is to CF, ſo K is to L; for as DC to CE, ſo [c] HC to CF: Therefore ſeeing that K is put to L, as DC to CE; ſo likewiſe HC ſhall be to CF, as K to L; Therefore [d] in equal reaſon, AC ſhall be to CF, as I to L; but the reaſon of I to L, is compounded of the reaſons of BC to CG, and DC to CE, by the fifth Definition of this Book: Therefore the reaſon of the Parallelogram AC to the Parallelogram CF, is compounded of the ſame reaſons. Therefore, The Parallelograms, &c. Which was to be demonſtrated. c) 1.6. d) 22.5.

SCHOLIUM.



This Propoſition will be eaſily underſtood by the annexed figure, thus divided into equal parts, where the ſides BC *is* 3, CG 9, *and* CE 8, *and in dividing* GC *by* BC, *to wit,* 9 *by* 3, *the quotient will give* 3, *for the denominator or quantity of the reaſon of* CG *to* CB; *and dividing alſo* CE 8 *by* DC 2, *the quotient will give* 4, *for the denominator or quantity of the reaſon of* CE *to* CD, *which denominators* 4 *and* 3, *multiplyed the one by the other, will produce* 12, *for the reaſon of the Parallelogram* CF, *to the Parallelogram* CA; *that is to ſay, that* CF *conteines* CA 12 *times, or elſe that* AC *is the twelfth part of* CF, *as is manifeſt, as well by the figure, as by the Demonſtration.*

PROP. 24. THEOR. 18.

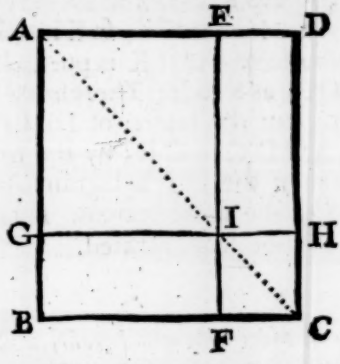


Parallelograms GE *and* FH, *which are about the Diameter* AC, *of every Parallelogram, are alike to their whole* BD, *and alike to one another.*

Demonſtration. FOr they are equiangled to the whole, foraſmuch as the angle A is common to the Parallelogram GE, and to the whole BD; and [a] the exterior angle AEI equal to the interior angle D, and the exterior angle AGI equal to the interior angle B, and EIG the exterior angle, equal to the interior angle IFB; that is to ſay, to BCD, equal to the ſaid IFB: Wherefore the Parallelogram EG is equiangled to BD, by the ſame reaſon FH is equiangled to the ſame BD. a) 29. 1.

Now let EG and FH have alſo the ſides above the equal angles proportional to the ſides of the whole BD: It ſhall be thus demonſtrated, ſeeing

b) 29. 1. or Cor. 4. 6. seeing [b] that the triangle A G I is equiangled to A B C, and the triangle A E I equiangled to the triangle A D C, as A B ſhall be to B C, ſo A G to G I, and ſo the ſides about the equal angles B and G ſhall be proportional; Again, [c] as B C to C A, ſo G I to I A: Likewiſe, as C A to C D, ſo I A to I E: Therefore [d] in equal reaſon, as B C to C D, ſo G I to I E; and therefore the ſides about the equal angles B C D and G I E are proportional.



c) 4. 6.

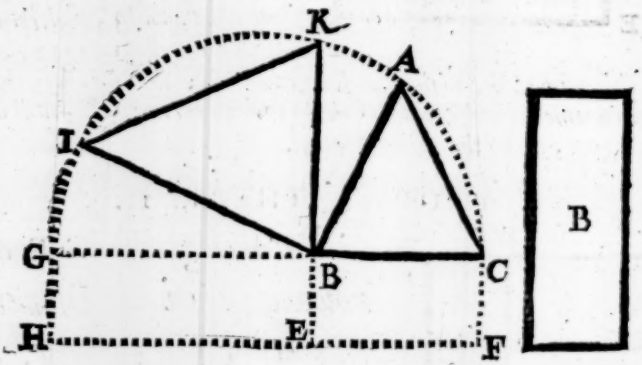
d) 22. 5.

It may be likewiſe ſhewn that the ſides about the other equal angles are proportional; Therefore by the firſt Definition of this Book, the Parallelogram E G ſhall be alike to the whole Parallelogram B D; and by the ſame reaſon, the Parallelogram F H alike to the ſame B D; and therefore alſo alike to one another: Therefore, Parallelograms, &c. Which was to be demonſtrated.

c) 21. 6.

PROP. 25. PROBL. 7.

To deſcribe a rectiline figure K I B, *alike to a given rectiline figure* A B C, *the which may be equal to another rectiline figure propoſed* B.



Conſtruction ON C B the ſide of the rectiline figure A B C, (to which another ought to be made alike,) conſtitute [a] the Parallelogram C E, in any angle whatſoever, equal to the rectiline A B C, and on B E make the Parallelogram B H, equal to B, having the angle E B G equal to B C F, and the lines C B and B G ſhall make one only right line, A B C, as by the 45th. Propoſition of the firſt Book.

a) 44, 45. 1.

b) 13. 6. Now [b] finde the mean proportional B K, between C B and B G, on which conſtitute the rectiline figure B I K, alike, and alike poſited to the rectiline I ſay that B I K is equal to the other rectiline figure B.

Demonſtration FOr ſeeing that C B, B K, and B G, are proportional, as C B [c] the firſt is to B G the third, ſo the rectiline figure on the firſt C B, to the rectiline figure B I K, on the ſecond B K; alike, and alike deſcribed; But [d] as C B to B G, ſo the Parallelogram C E, to the Parallelogram B H, of the ſame height: Therefore [e] as C E to B H, ſo A B C

c) Cor. 19, or 20. 6.

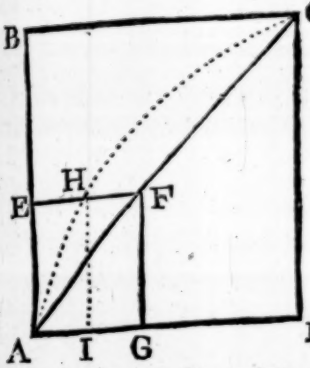
d) 1. 6.

e) 11. 5.

ABC to BIK; But [f] as CE to BH, so ABC to B: Forasmuch as ABC and CE are equal, and BH and B also equal: Therefore [g] as ABC to B, so ABC to BIK: Therefore [h] B and IBK shall be equal: But KIB is alike, and alike posited to ABC, by Construction: Therefore, We have described, &c. Which was to be done.

f) 7. 5.
g) 11. 5.
h) 9. 5.

PROP. 26. THEOR. 19.



If from a Parallelogram BD, *there be cut off a Parallelogram alike to the whole, and alike posited* EG, *having an angle* EAG *common with the whole; it doth consist about one and the same diameter* AC, *with the whole* BD.

Demonstration DRaw AF and CF, which lines if they make one right line, it is evident that EG shall be about one and the same diameter with BD. But if they make not a right line, draw AC, the diameter of the whole Parallelogram BD, cutting the side EF in the point H, by which point draw HI, parallel to FG. Now forasmuch as the Parallelograms BD and EI are about one and the same diameter AHC, [a] they shall be alike, and alike posited; Therefore [b] as BA to AD, so EA to AI; but as BA to AD, so EA is to AG; forasmuch as BD and EG are alike by supposition, and alike posited: Therefore [c] as EA to AI, so EA to AG; and therefore AI and AG shall be equal, the part to the whole, which is absurd: Therefore AF and FC do make one only right line; that is to say, the Parallelograms BD and EF do consist about one and the same diameter, Which was to be demonstrated.

a) 24. 6.
b) 1. def. 6.
c) 11. 5.

Now if it should be said that the right line AHC doth cut the side FG, there would happen the same absurdity.

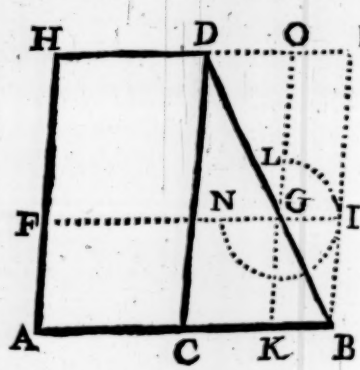
PROP. 27. THEOR. 20.

Of all Parallelograms applyed according to one and the same right line AB, *and being deficient by some Parallelogram figures, alike, and alike posited to that which is described on the halfe* BC; *the greatest is that which is applyed on the other half, and alike to the deficient.*

DIvide the line AB into two equal parts in C, and on the half CB, constitute any Parallelogram whatsoever, as CE, whose diameter is BD: Therefore if the Parallelogram ABEH be finished, the Parallelogram AD constituted on the half AC, shall be applyed according to AB, and wanting of the Parallelogram CE, and alike to the deficient I say that the Parallelogram AD applyed to the half AC, and being defici-

deficient of the Parallelogram C E, is the greateſt of all thoſe which are applyed according to the line A B, and deficient by like Parallelograms, and alike poſited to C E.

Demonſtration FOr having aſſumed the point contingent G, in the diameter B D, by which point having drawn the right lines F G I and K G parallel to A B and B E, the Parallelogram F K ſhall be applyed according to A B, and wanting by the Parallelogram K I, [a] which is alike to C E, and alike poſited; being about one and the ſame diameter with C E. But [b] forſmuch as the complements C G and G E are equal, if you adde the common Parallelogram K I, C I and K E ſhall be equal: But [c] C I is equal to C F, being on equal baſes A C and C B: Therefore C F and K E ſhall be alſo equal, and having added the common part C G, the Parallelogram A G, and the Gnomon L N ſhall be equal.



Therefore ſeeing that C E is greater than the Gnomon L N, (for it conteineth the Parallelogram G D more than the Gnomon,) A D [d] equal to C E becauſe of the equal baſes A C and C B, ſhall be alſo greater than A G, by the ſame Parallelogram D G; In like manner, it may be ſhewn that A D is greater than all other Parallelograms which are applyed according to A B, in ſuch ſort as the point G be between the points B and D; that is to ſay, which imployeth or occupieth a line greater than the half A C; but have leſſer altitude than A D, provided that the deficients be alike to C E.

a) 24. 6.
b) 43. 7.
c) 36. 1.
d) 36. 1.

SCHOLIUM.

Otherwiſe it may be demonſtrated that A D *is greater than* A G, *in this manner;* [e] *The Parallelograms* F D *and* D I *are equal, ſeeing that the baſes* H D *and* D E *are equal: But* D I *is greater than* G E, *that is to ſay, then the complement* C G, *(which is equal to* G E,*) by the Parallelogram* D G: *Therefore* F D *ſhall be alſo greater than* C G, *by the ſame parallelogram* D G; *and therefore adding the common* C F, A D *ſhall be greater than* A G, *by the ſame Parallelogram* D G.

e) 36. 1.

PROP. 28. PROBL. 8.

To a given right line A B, *to apply a parallelagram* A P, *equal to a rectilire figure given* C, *wanting by a parallelogram figure* P B, *which is alike to another given parallelogram* D, *but it behoveth that the given rectiline figure to which an equal rectiline figure ought to be applyed, be not greater than that which is applyed to the half* A E *of the given line, the deficients being alike to that which is applyed on the halfe, and to that which ought to be deficient by a like parallelogram.*

Con-

Conſtruction THe line AB being divided into two equal parts in the point E, on the half E B deſcribe the Parallelogram E G alike, and alike poſited to D, and ſo finiſh the whole Parallelogram B H; If then A F be equal to C, being applyed to A B, deficient by the Parallelogram E G, which is made alike to D, you have your deſire. But if A F be greater than C, (for it ought not to be leſſe, ſeeing that by the precedent Propoſition it is the greateſt of all the applyed, the Deficiences being alike, you cannot apply any one to A B, equal to C, but they ſhall all be leſſe; Wherefore *EUCLIDE* hath added [but it behoveth that the given Rectiline, &c.] E G equal to the ſame, ſhall be alſo greater than C; let it then be greater by I, and conſtitute K M alike, and alike poſited to D, or to E G, but equal to the exceſſe I, in ſuch ſort as E G be equal to C, and to K M together; and therefore greater than K M: Therefore ſeeing that as G F to F E, ſo N K to K L, becauſe of their reſemblance, the ſides G F and F E ſhall be alſo greater than N K and K L; for if they were equal to them, or leſſe, E G ſhould be likewiſe equal to N L, or leſſe, as appears. a) 18. 6.



Having therefore cut off F O and F Q equal to K N and K L, and having made the Parallelogram O Q, it ſhall be equal to K M; and alike, and alike poſited thereto; and therefore to E G, and [b] alſo about one and the ſame diameter with E G, which diameter is B F; and having prolonged Q P and O P, the Parallelogram A P ſhall be applyed to the line A B deficient of P B, [c] which is alike, and alike poſited to E G; and therefore alſo to D: I ſay that A P is equal to the rectiline C. b) 26. 6. c) 24. 6.

Demonſtration FOr [d] ſeeing that P G is equal to P E, the complement to the complement, if you adde the common P B, the Parallelogram B Q ſhall be equal to E R, that is to ſay, to E S, which is equal to E R, the baſes A E and E B being equal: Wherefore if to the equal parallelogram A O and B Q, you adde the common part E P, A P ſhall be equal to the Gnomon T V: But the Gnomon T V is equal to the rectiline C; (for ſeeing that the Parallelogram E G is equal to C and N L together, if you take away the equal Q O and L N, the Gnomon T V ſhall remaine equal to C:) Therefore A P ſhall be alſo equal to the ſame C: Therefore, To a given right line A B, &c. Which was to be done. d) 36. 1.

PROP. 29. PROBL. 9.

To a given right line A B, *to apply a parallelogram* A P, *equal to a given rectiline figure* C, *exceeding the ſame given line* A B, *by a parallelogram* R Q, *alike to another parallelogram given* D,

Conſtruction Divide A B into two equal parts in E, and on the halfe E B, deſcribe the Parallelogram E G alike to D, and alike poſited, then a make the ſquare H equal to the rectiline figure C, and Parallelogram E G, b to which make the Parallelogram I L equal, but alike to E G, and alike poſited: Wherefore I L ſhall be greater than E G, ſeeing that it is equal to the ſquare H, which is made equal to C, and to E G together; Therefore ſeeing that becauſe of the ſimilitude of M K and E G, as M I to I K, ſo E F to F G; the ſides M I and I K ſhall be alſo greater than E F and F G; for if I M and I K were equal to E F and F G, or leſſe, M K ſhould be equal to E G, or leſſe, as is manifeſt: Therefore having prolonged F E and F G, in ſuch ſort, as F O and F N may be equal to I M and I K, and having finiſhed the Parallelogram O N, the ſame ſhall be alike, and alike poſited to E G; ſeeing that it is equal to M K, and alike, and alike poſited: Wherefore c O N and E G are conſtituted about one and the ſame diameter F P; having prolonged A B and G B to Q and R, and P O until they meet with A V, parallel to F O, to the point V the Parallelogram A P ſhall be applyed to the line A B, exceeding it by the Parallelogram Q R, which d is alike to E G, and therefore to D.

a) 14. 2.
b) 25. 1.
c) 26. 6.
d) 24. 6.



Demonſtration I ſay then that A P is equal to the rectiline figure C, for e ſeeing that A O and E R are equal; and f E R is equal to the complement B N, A O ſhall be alſo equal to B N: Therefore adding the common part O Q, A P ſhall be equal to the Gnomon S T; but the ſaid Gnomon is equal to the rectiline C; (for ſeeing that M K, that is to ſay O N, is equal to C and E G together, if you take away the common part E G, there will remain the Gnomon S T, equal to C:) Therefore A P ſhall be alſo equal to the rectiline figure C: Therefore, To a given right line, &c. Which was to be done.

e) 36. 1.
f) 43. 1.

PROP. 30. PROBL. 10.

To divide a terminated right line propoſed A B, *according to extream and mean reaſon in* G.

Conſtruction Having deſcribed on the propoſed right line A B, the ſquare A C, a to the ſide D A apply the rectangle D H, equal to the ſquare A C, exceeding D A by the Parallelogram A H, alike to the ſame ſquare, in ſuch ſort as A H ſhall be likewiſe a ſquare, conſidering that a ſquare only is alike to a ſquare: But let I H divide A B in the point G, I ſay that A B is divided according to extream and mean reaſon.

a) 29. 6.

Demon-

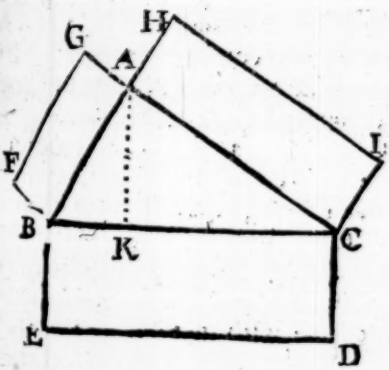
Demonſtration SEeing that the rectiline figure D H, and the ſquare A C, are equal, if you take away the common part A I, F G and G C will remain equal, the which having the angles A G H and B G I equal, to wit, right angles, [b] the ſide which are about thoſe equal angles ſhall be reciprocal, to wit I G, that is to ſay, A B its equal, is to G H, that is to ſay, to A G, equal to G H, as A G is to G B: Wherefore the whole A B being to the greateſt ſegment A G, as the ſaid ſegment A G is to the leſſer ſegment G B; A B is divided according to extream and mean reaſon, by the Definition. b) 14. 6.



Otherwiſe, we may demonſtrate that A B is divided in the point G, according to extream and mean reaſon; ſeeing that there are three lines given, A B, A G, and G B, and that the rectangle G C conteined under the firſt A B, and the third G B, is equal to the ſquare of the mean A G; [c] thoſe lines ſhall be proportional, to wit, as A B the firſt is to A G the ſecond, ſo A G the ſecond ſhall be to G B the third; Therefore by the Definition, A B is divided in the point G, according to extream and mean reaſon. c) 17. 6.

Yet otherwiſe, to finiſh the whole Problem; Let [d] A B be divided in the point G, in ſuch ſort as the rectangle conteined under the whole A B, and the ſegment G B be equal to the ſquare of the other ſegment A C: I ſay that A B is divided at G, according to mean and extream reaſon: For [e] again, as before, A B, A G, and G B, ſhall be proportional, and ſo the Propoſition manifeſt: Therefore, We have divided, &c. Which was to be done. d) 11. 2. e) 17. 6.

PROP. 31. THEOR. 21.

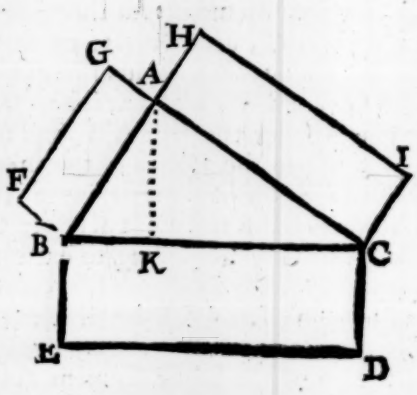


In rectangled triangles, as A B C, *the figure* B D, *deſcribed on the ſide* B C, *which ſubtendeth the right angle* B A C, *is equal to the two figures* A F *and* A I, *alike and alike deſcribed on the ſides* B A *and* A C, *which contein the right angle* B A C.

Demonſtration FOr having drawn the perpendicular A K, from the point A on B C, [a] as B C ſhall be to C A, ſo C A ſhall be to C K: Wherefore as B C the firſt, to C K the third, ſo the figure B D deſcribed on the firſt, ſhall be to the figure C H, alike, and alike deſcribed on the ſecond, [b] and by converſion of reaſon, a) Cor. 8. 6. b) Cor. 19, or 20. 6.

as C K to B C, ſo the figure C H to the figure B D: In like manner we ſhall ſhew that as B K is to B C, ſo the figure B G is to the figure B D, ſeeing that the three lines B C, B A, and B K are proportional.

Therefore ſeeing that as C K the firſt magnitude, is to B C the ſecond, ſo C H the third, is to B D the fourth; alſo as B K the fifth magnitude
c) 24. 5. is to B C the ſecond, ſo B G the ſixth, is to B D the fourth: [c] As the firſt C K, with the fifth B K, ſhall be to the ſecond B C; ſo the third C H, with the ſixth B G, ſhall be to B D the fourth; but C K the firſt, and B K the fifth together, are equal to the ſecond B C: Therefore the third C H, and the ſixth B G together, ſhall be likewiſe equal to the
d) 8. 6. fourth B D. Which was propoſed.



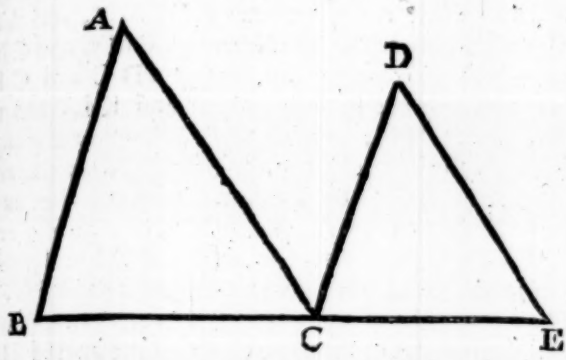
c) 19. 6. Otherwiſe, [d] ſeeing that the triangle K A C is alike to the triangle A B C, and that the ſides B C and C A are Homologal; (for as B C to C A, in the triangle A B C, ſo A C to C K, in the triangle A C K;) [e] the triangle K A C
f) 19, 20. 6. ſhall be to the triangle A B C in duplicate reaſon of C A to B C: But [f] the figure C H is to the figure B D, alſo in a duplicate reaſon of C A to B C; Therefore
g) 11. 5. [g] the triangle K A C ſhall be to the triangle A B C, as the figure C H to the figure B D, by the ſame reaſon it may be ſhewn that as the triangle K A B is to the triangle A B C, ſo the figure B G is to the figure B D.

Again, foraſmuch then as K A C the firſt magnitude, is to A B C the ſecond, ſo C H the third is to B D the fourth. Alſo as K B A the fifth, to
h) 24. 5. A B C the ſecond, ſo B G the ſixth, to B D the fourth; [h] as K A C the firſt, with K B A the fifth, ſhall be to the ſecond A B C; ſo C H the third, and B G the ſixth together, ſhall be to the fourth B D. But K A C and K B A, the firſt and fifth together, are equal to the ſecond A B C: Therefore C H and B G, the third and ſixth together, ſhall be likewiſe equal to B D the fourth. Which was propoſed.

Otherwiſe, As the ſquare of A C the firſt magnitude, is to the ſquare of B C the ſecond magnitude, ſo the figure C H the third magnitude,
i) 19, 20. 5. to the figure B D the fourth magnitude; ſeeing [i] that they are all in duplicate reaſon of their ſides A C and B C; Likewiſe, as the ſquare of A B the fifth magnitude, is to the ſquare of B C the ſecond magnitude, ſo the figure B G the ſixth magnitude, is to the figure B D the fourth magni-
k) 24. 5. tude: Therefore [k] as the firſt magnitude with the fifth, to wit, the ſquare of A C, with the ſquare of A B, ſhall be to the ſecond, that is to ſay, to the ſquare of B C, ſo the third magnitude with the ſixth, to wit, the figure C H, with the figure B G, is to the fourth, to wit, to the figure B D: But the ſquares of A C and A B together, are equal to the ſquare of B C: Therefore the figures C H and B G together, ſhall be alſo equal to the figure B D. Which was propoſed: Therefore, In rectangled triangles, &c. Which was to be demonſtrated.

PROP.

PROP. 32. THEOR. 22.



If two triangles A B C *and* D C E, *which have two sides* A B *and* A C *proportional to two sides* D C *and* D E, *are disposed (compounded) according to an angle* A C D, *in such sort as their Homologal sides* A B *and* D C, *and* A C *and* D E *may be also parallels, the other sides* B C *and* C E *of those triangles shall be placed in one and the same right line.*

Demonstration. FOr seeing that A B and D C are Parallels, [a] the angle A shall be equal to the alternate angle A C D, and by the same reason, the angle D shall be equal to the same angle A C D: And therefore A and D shall be equal to one another; forasmuch then as the triangles A B C and D C E have the sides about the equal angles A and D proportional, they shall be equiangled, and shall have the angles B and D C E equal: Therefore adding the equal angles A and A C D, the two angles B and A shall be equal to the two angles D C E and A C D, that is to say, to the whole angle A C E: Again, adding the common angle A C B, the two angles A C E and A C B shall be equal to the three angles of the triangle A B C: But [b] these three angles are equal to two right angles: Therefore the two angles A C E and A C B shall be equal to two right angles; and [c] therefore B C and C E shall make one right line: Therefore, If two triangles, &c. Which was to be demonstrated.

a) 29. 1.

b) 32. 1.

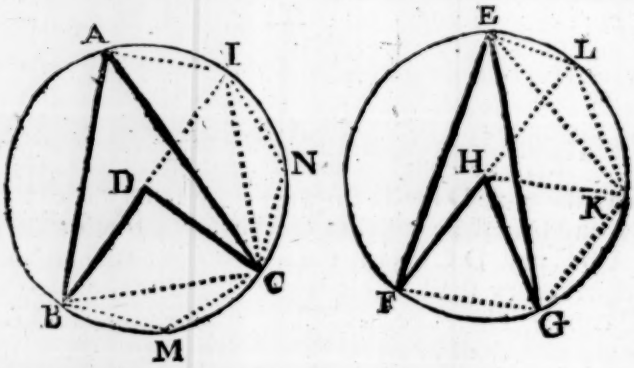
c) 14. 1.

PROP. 33. THEOR. 23.

In equal circles A B C *and* E F G, *the angles have the same reason to one another as the circumferenes* B C *and* F G, *on which they insist, whether they insist being constituted at the Centers* D *and* H, *or at the circumferences at* A *and* E; *And also the Sectors* B D C *and* F H G *are the same to one another, forasmuch as they are constituted at the Center.*

Demon-

Demonſtration FOr having drawn the right lines B C and F G, [a] apply in the circles the line C I, equal to B C, and G K and K L, each equal to F G, and draw the right lines I D, K H, and L H: Then foraſmuch as B C and C I are equal, [b] the arches B M C and C N I ſhall be likewiſe equal; and therefore [c] the angles B D C and C D I are equal; by the ſame reaſon, the arches F G, G K, and K L, and the angles F H G, G H K, and K H L, are equal: Therefore the angle B D I ſhall be as much Multiplex of the angle B D C, as the arch B C I is Multiplex of the arch B C; by the ſame reaſon, the angle F H L ſhall be as much Multiplex of the angle F H G, as the arch F G L is of the arch F G; ſeeing that the angles B D I and F H L are divided each into as many equal parts as the arches B I and F L, on which they inſiſt: Therefore, if the arch B C I be equal to the arch F G K L, [d] of neceſſity the angle B D I ſhall be equal to the angle F H L, and if the arch be greater than the arch, the angle ſhall be greater than the angle, and if leſſer, leſſer: Wherefore the arch B C I, and the angle B D I, equimultiplices of the firſt and third Magnitudes,



B C and B D C ſhall be together either greater, equal, or leſſer than the arch F G K L, and the angle F H L equimultiplices of the ſecond Magnitude F G, and of the fourth F H G; if they be taken ſo as to anſwer one another: Wherefore [e] as the arch B C the firſt Magnitude, is to the arch F G the ſecond Magnitude, ſo the angle B D C the third Magnitude, to the angle F H G the fourth Magnitude, the ſame may be ſhewn of the angles at the circumference.

Secondly, I ſay that as the circumference is to the circumference, ſo the ſector is to the ſector: Conſtitute in the ſegments B C and C I, the angles B M C and C N I, which [f] ſhall be equal, inſiſting on the equal arches B A C and C B A I: Wherefore the ſegments B M C and C N I ſhall be alike; [g] and therefore equal to one another, being conſtituted on the equal right lines B C and C I: Therefore adding the triangles B D C and C D I, [h] which are likewiſe equal, the ſectors B D C and C D I ſhall be made equal; wherefore the ſector B D I ſhall be as much Multiplex of the ſector B D C, as the arch B C I is of the arch B C: Likewiſe the ſector F H L ſhall be ſhewn to be as much Multiplex of the ſector F H G, as the arch F G K L is of the arch F G; but foraſmuch as if the arch B C I be equal to the arch F G K L, alſo the ſector B D I is equal to the ſector F H L, (as hath been ſhewn in the ſectors B D C and C D I,) and if greater, greater, and if leſſe, leſſe; therefore the arch B C I and the

a) 1. 4.
b) 28. 1.
c) 27. 1.
d) 27. 3.
e) 6. def. 5.
f) 27. 3.
g) 24. 3.
h) 4. 1.

the Sector B D I, equimultiplices of the first Magnitude B C, and of the third Magnitude B D C, shall be deficient together, of the arch F G K L, and of the sector F H L, equimultiplices of the second Magnitude F G, and of the fourth F H G; or together shall be equal, or shall exceed, if they be taken so as they answer to one another: Therefore [i] as the arch B C the first Magnitude, is to the arch F G, the second Magnitude; so the sector B D C, the third Magnitude, to the sector F H G the fourth Magnitude: Therefore, If in equal Circles, &c. Which was to be demonstrated. i) 6. def. 6.

COROLLARIE. I.

It is manifest from this, that the Sector is to the Sector, as the angle is to the angle; for the reason of the one to the other, to wit, of the angle B D C, *to the angle* F H G, *and of the Sector* B D C *to the Sector* F H G, *is the same as the reason of the arch* B C *to the arch* F G: *Wherefore* [k] *they shall be the same to one another.* k) 11 5.

COROLLARIE. II.

It is likewise manifest, that as the angle at the Center is to four right angles, so the arch which subtends the same angle, is to the whole Circumference; and contrarily, as four right angles are to the angle which is at the Center, so the whole Circumference is to the arch which subtends the said angle. l) 33. 6.

For as the angle at the Center is to a right angle at the Center, so the arch which subtends the same angle, is to a quarter of the Circumference of the Circle, or to the Quadrant which subtends the said right angle: Wherefore as the angle at the Center, shall be [m] *to the quadruple of the right angle; that is to say, to four right angles; so the arch subtended of the same angle, shall be to the quadruple of the Quadrant, that is to say, to the whole Circumference, by what hath been demonstrated in the two and twentieth Proposition of the Fifth Book. Which was first proposed.* m) 33. 6.

Forasmuch then as the angle at the Center is to four right angles, so the arch which subtends that angle, is to the whole Circumference; by conversion of reason, as four right angles to the angle at the Center, so the whole Circumference shall be to the arch which subtends the said angle at the Center. Which was in the second place proposed.

But it may also be thus demonstrated, [n] *seeing that as the right angle at the Center is to the angle at the Center, so the Quadrant or fourth part of the Circumference, which subtends the same right angle, is to the arch which subtends the same angle at the Center not being a right angle. Likewise as the quadruple of the right angle, that is to say, four right angles, shall be to the angle at the Center, by what hath been shewn in the two and twentieth Proposition of the Fifth Book; so the quadruple of the Quadrant, that is to say, the whole Circumference shall be to the arch which subtends the same angle. Which was proposed.* n) 33. 6.

The End of the Sixth Element of EUCLIDE.

THE SEVENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.



Hitherto, *Euclide* hath treated of the first part of *Geometry*, to wit, of that which concerneth *Planes*; there remaines the other part which treats of *Solids*, but first of all it hath been fonnd necessary to treat of *Lines commensurable* and *incommensurable*, for as much as the knowledge of these *Lines* is requisite for the explication and demonstration of the proprieties of divers *Solid Bodies*, and principally of those which are named *Regulars*; for that, without the knowledge of those *Lines* the tract of *Solids* would be imperfect; Moreover, without those *Lines*, divers sides, as well *Planes* as *Solids* (if you will reduce *Geometry* to use and practice) cannot be expressed or understood, for divers of the sides are those *Lines* which the Greeks call ἄλογοι, that is to say, *irrational*, if they be not irrational, they are incommensurable in length amongst themselves, and therefore are not expressed by *Numbers*.

Now seeing that the explication and understanding of those *Lines*, is as it were wrapped up, and joyned with *Numbers*, in such sort as without them they cannot be known, it is found needful in the first place to treat of *Numbers*; Where-

fore, in this ſeventh, and the two following Books, *Euclide* unfoldeth the affections and proprieties of *Numbers*, in as much as they may ſerve in *Geometry*, to the end that in the Tenth Book he may make the Demonſtrations of *Lines commenſurable* and *incommenſurable*, more amply, and with greater facility.

DEFINITIONS.

1 UNITY *is that according to which every thing, of thoſe which are, is ſaid to be one.*

BEginning then, according to his Cuſtome, by the Principles, he defineth firſt of all Unity: For according to unity we uſe to call one body, one body: one animal, one animal: one ſtone, one ſtone: and ſo of other things. And finally, Unity receiveth no diviſion in Numbers, even as a point in Magnitudes.

2 NUMBER, *is a Multitude compoſed of Unities.*

THat is to ſay, if you take as many unities as you pleaſe together, the collection of them ſhall be called number, from whence it is manifeſt, that in every number there are ſo many parts, as there are unites that make it: So as unity is part of each number named by the ſame number of which it is a part, as the number 8, compounded of 8 unites, divideth it ſelf into ſo many parts, to wit in 8 unites, each of them being ſaid to be the 8th. part of the number 8, &c. from whence it follows, that all the numbers are commenſurable amongſt themſelves, ſeeing they are all meaſured by one and the ſame meaſure, to wit, by unity, as is before ſaid, which cannot agree with all Magnitudes, as ſhall be ſhewn in the Tenth Book.

3 *A Number is a part of another number; the leſſer of the greater, when the leſſer doth meaſure the greater.*

THis Definition reſembles that of the 5th. Book, by which *Euclide* defineth the part of a quantity continued, for as there, ſo here, he defineth the aliquot part, which is ſaid exactly to meaſure his whole. As 6 is ſaid to be part of 18, foraſmuch as 6 meaſureth 18 exactly, and leaves no remainder, in like manner, each of theſe nnmbers 3, 4, 6, 8, is a part of 576. Seeing that each of them doe meaſure it exactly, for every part taketh his name from the number by which it meaſureth another. As 6 is ſaid to be the ſeventh part of 42, becauſe 6 meaſureth 42 by 7, that is to ſay, is conteined in 42, 7 times exactly, and ſo of the reſt.

4 *But*

4 *But a number is said to be parts of another greater number, when the lesser doth not measure the greater.*

AS 4 is not said to be part of 6, for as much as 4 the lesser number doth not measure 6 precisely, but shall be called the two parts of 6, to wit $\frac{2}{3}$, and 5 shall be called parts of 18, to wit $\frac{5}{18}$, for as much as unity which is their common measure, is conteined 5 times in 5, and 18 times in 18.

5 *A number is said to be Multiplex of another lesser number, when the lesser doth measure the greater.*

EVen as a number which doth precisely measure a greater number, is said to be part of that greater number; so also the greater number which is measured by the lesser number, is said to be Multiplex of the lesser number, and so also, as a number which doth not exactly measure a greater number, is not said to be a part but parts of the greater number, also the greater shall not be said to be Multiplex of the lesser number by which it is not measured, as 4 is a part of 24. Seeing that 4 doth measure 24 precisely, and in like manner, 24 is said to be Multiplex of 4, for that it is measured of 4, and 30 is Multiplex of 6, forasmuch as 6 doth measure 30, &c.

6 *An even number is that which may be divided in two equal parts.*

AS all the following numbers 4, 10, 40, 100, 1000, are said to be even numbers, forasmuch as each of them may be divided in two equal parts, whereof their halves are 2, 5, 20, 50, 500.

7 *But an odde number is that which cannot be divided into two equal parts, or which differs from an even number by an unite.*

AS all these numbers 5, 11, 15, 37, 101, are called odde numbers, because they cannot be divided in two equal parts, or because they differ by unity from the even numbers 4, 10, 14, 36, 100, &c.

8 *A number evenly even, is that which an even number doth measure by an even number.*

AS 32 is said to be evenly even, because 8 an even number doth measure it by 4, which is also an even number, 24 is also said to be evenly even; seeing that 4 an even number doth measure it by 6, also an even number, &c.

9 *A number evenly odde, is that which an even number doth measure by an odde number.*

THat is to say, that if an even number measure an even number by an odde number, the number measured shall be called evenly odde. As

30, which 2 an even number doth measure by 15 an odde number, and 6 an even number doth measure the same 30 by 5, an odde number, &c.

10 *But a number oddely odde, is that which an odde number doth measure by an odde number.*

AS 45, which is measured of 9 an odde number, by 5 also an odde number, is said to be a number oddely odde, and also 15 which is measured of 5 by 3, and these 9, 21, 25, 27, 33, 35, &c. are said to be oddely odde nnmbers.

11 *A Prime or First number, is that which only unity doth measure.*

THat is to say, that if a number cannot be measured by any other number, but onely by unity, so as it cannot be said to be evenly even, or evenly odde, or unevenly odde, that number shall be called a Prime number, as these numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, &c. for onely unity measureth them.

12 *Numbers, Primes to one another, are those which have only unity for their common measure.*

THat is to say, that if two or more numbers are proposed, and that there cannot be found any number which doth pretisely measure each of them, save onely unity, those numbers are said to be Primes amongst themselves, as 15 and 8, or 3 and 5, are Primes amongst themselves, for as much as there is onely unity that may be a common measure to them.

13 *A Compound number, is that which some number may measure.*

THe Geometricians do call that a Compound number which some other number doth measure besides unity, as 15, which is measured by 5 and 3. Now it is manifest that all even numbers excepting onely the Binary, are Compound numbers, being all measured by 2, from whence it follows, that all the Prime numbers save onely the Binary are odde, the Binary being the first of even numbers.

14 *But numbers compounded amongst themselves, are those which are measured by some number, as their common measure.*

TWo or more numbers, which are measured by some number besides unity, as by their common measure, are said to be compounded amongst themselves, although that each be not compounded of himself; As 15 and 24, are compounded amongst themselves, for as much as 3 doth measure both, as their common measure: also 7, 21, and 35 are compounded amongst themselves, for the first of them doth measure it self, and the two others, although it be a Prime of it self.

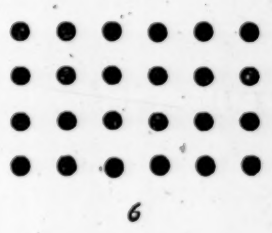
15 *A*

15 *A number is ſaid to multiply another number, when there is produced another number, which is compounded as many times of the Multiplicator, as there are unites in the Multiplicand.*

AS 6 ſhall be ſaid to multiply 8, when 8 ſhall be compounded as often times as there are unites in the Multiplicator 6, to wit, ſix times, and that there ſhall be produced another number, to wit 48, alſo by change, 8 ſhall be ſaid to multiply 6, if you take 6 eight times, to wit, as often as there are unites in 8, and that it make the ſame 48; even ſo a number ſhall be ſaid to be produced of two numbers, by multiplying the one in the other, as 63 of 7 and 9, &c.

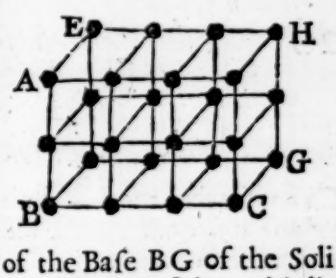
16 *But when two numbers multiplying one another doe produce a number, the number produced ſhall be called a Plaine, and the numbers which multiply one anonber ſhall be ſaid to be ſides of the Plaine.*

EVery number produced by the mutual Multiplication of two numbers is called a Plaine, foraſmuch as his unites being diſpoſed in length and breadth doe repreſent a rectangle Parallelogram, whoſe ſides are the two numbers multiplying one another, as hath been ſaid in the ſecond Book, as 24 the product of 4 by 6, is ſaid to be a Plaine number, whoſe ſides are 4 and 6; Seeing that their unites diſpoſed in length and breadth doe repreſent a rectangled Parallelogram.



17 *When three numbers multiplying one another doe produce a number, the number produced ſhall be called a Solid, and the numbers which multiply one another ſhall be called the ſides of the Solid.*

FOraſmuch as theſe three numbers 2, 3, 4, multiplying one another produce 24, for 2 by three produceth 6, and 6 by 4 produceth 24, otherwiſe 2 by 4 makes 8, and 8 by 3 makes 24, that number 24 ſhall be called a Solid; and the numbers 2, 3, 4, are the ſides; for as much as the unites diſpoſed in length, breadth and depth, doe repreſent a Solid figure, called Parallelepipedon, as ſhall be explained in the 11th. Book, whoſe three dimenſions are expreſſed by the three numbers which are multiplied in one another; For if you multiply 2 by 4, you ſhall have 8 for the number of the Baſe BG of the Solid AG, whoſe length is 4 unites, and breadth 2; and if this Baſe be multiplied by



by 3, the whole number of the Solid, to wit 24, will be produced, being three uuites high, &c.

18 *A Square number is that which is equally equal, or which is conteined under two equal uumbers.*

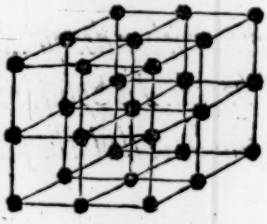
THat is to say, produced of two equal numbers multiplied by one another. This Plaine number, which is equally equal, that is to say, whose unites disposed in length and breadth, doe resemble a rectangle Parallelogram, whose Length and Breadth are equal, and in such sort as all the sides are equal, or that is produced of two equal numbers, and therefore is conteined under them, is called a square, as 25, conteined under 5 and 5, that is to say, is produced by the mutual multiplication of the two fives the one by the other, for his unites being ranged in form of a Plaine, doth represent a perfect Square, having five unites on each side, and therefore equally equal; But either of the sides of the Multiplication of which it is produced, is called the root of the Square.



19 *A Cube number is that which is equally equal equally, or which is conteined under three equal numbers.*

THat is to say, produced of three equal numbers multiplied by one another.

He calls this Solid a Cube, which is equally equal equally, that is to say, whose unites disposed in length, breadth, and depth, doe represent a Geometrical Cube, in such sort as that all his Dimensions, to wit, length, breadth and height, or depth, are equal; or which is produced of three equal numbers, as 27, which is conteined under these three numbers 3, 3, 3, for three times 3 makes 9, and three times 9 makes 27, for all his unites ranked in form of a Solid doe represent a Geometrical Cube, having 3 in every Dimension, and each of these three numbers is by the Geometricians called the Side of the Cube, and of divers Arithmeticians the Cubick Root.



20 *Numbers are proportional, when the first is as much Multiplex of the second, or the same part or parts thereof that the third is of the fourth.*

OR else when the first doth contein as many times the second, and over and above one and the same part or parts thereof, that the third doth contein of the fourth.

To the end you may the better comprehend all the numbers proportional are)

in all the kinds of proportion of inequality, (as all the unequal numbers are) we have added these words, as well because you may understand what *Euclide* saith hereon, as also to render this Definition more plain and intelligible; (or else, when the first conteineth as many times the second, and over and above one and the same part or parts thereof, as the third doth of the fourth.) For the vulgar definition of *Euclide*, (which *Clavius* thinks to be corrupted) comprehendeth onely the numbers proportional in the proportion multiplex and sub-multiplex, and the other proportions of lesser inequality. For in the proportion Multiplex, there are four numbers proportional, when the first is so much Multiplex of the second, as the third is of the fourth; and in the sub-multiplex when the first is the same part of the second, that the third is of the fourth; and in the other proportions of lesser inequality, when the first is the same parts of the second that the third is of the fourth, as the Definition of *Euclide* will have it, but by that it cannot be comprehended, which are the numbers proportional in reason *Super-particular*, *Super-partient*; Multiply *Super-particular* and Multiply *Super-partient*, for in all these the first number is not so Multiplex of the second, as the third is of the fourth, nor the same part or parts; but the first conteins the second, and the third the fourth equally, to wit, once or more, and over and above one and the same part thereof, or same parts; as is manifest by Definition the 4th. of the Fifth Book.

Wherefore these numbers 12, 4, 9, 3, are proportional, the first being so Multiplex of the second, as the third is of the fourth, to wit Triple; For 4 is such a part of 12, (to wit one third) as 3 is of 9: Againe, 6, 8, 9, 12, are proportional, for asmuch as 6 is the same parts of 8, as 9 is of 12, to wit, the three quarters; Lastly, 7, 6, 14, 12, and 7, 4, 14, 8, and 11, 5, 22, 10 and 12, 5, 24, 10, are numbers proportional; for as in the first example, the first number conteins once the second, and the third conteins once the fourth, and over and above, $\frac{1}{6}$ part, and in the second example, the first conteins once the second, and the third the fourth, and over and above $\frac{1}{4}$ parts, in the third example twice, and $\frac{1}{7}$ part, and in the last example the first conteins twice the second, and the third the fourth, and over and above $\frac{2}{7}$ parts, for otherwise the numbers should not be in any kind proportional: Therefore, whensoever 4 numbers are put proportional, you must accord that if the greater numbers are compared to the lesser, the first and the third shall be equi-multiplices of the second and the fourth; or else the first and the third shall equally contein the second and the fourth, and over and above the same part or parts, and contrariwise, if the first and the third be put equi-multiplices, or else that the first be said to contein so many times the second, as the third doth contein the fourth, and over and above the same part or parts, you may gather that the numbers are proportional, as if you compare the lesser with the greater, and that they are said to have the same proportion, you must confesse that the first is the same part of the second, as the third is of the fourth, or the same parts, and contrariwise, if the first be put the same part of the second as the third is of the fourth, or the same parts, you may conclude that those nnmbers are proportional.

But *Euclide* defineth onely those numbers proportional there, which have the same proportion of inequality: For if there be question of the proportion or reason of equality, it is manifest that the first ought to be equal to the second, and the third to the fourth, to the end they may be said to be proportional.

Now from this Definition may be gathered manifestly that the equal num-

numbers have the ſame reaſon to one and the ſame, and contrariwiſe one and the ſame to the equal, alſo that the numbers which have the ſame reaſon to one and the ſame, or to which one and the ſame number hath the ſame reaſon, are equal.

For ſeeing that equal numbers are equi-multiplices, or the ſame part, or the ſame parts; alſo ſeeing that one and the ſame number is equi-multiplex of the equal numbers, or the ſame part or parts, or conteineth them equally, and over & above the ſame part, or the ſame parts of them, it is manifeſt that equal numbers have the ſame reaſon to one and the ſame, or one and the ſame to equal numbers according to this Definition. Againe, forasmuch as the numbers which have the ſame reaſon to one and the ſame, are equi-multiplices thereof, or of the ſame part or parts, or doth equally contein it, and over and above the ſame part or parts thereof: And a number which hath the ſame reaſon to certain other numbers is equi-multiplex of them, or of the ſame part or parts, or conteineth it equally, and over and above the ſame part or parts thereof; according to this Definition, it is manifeſt that the numbers which have the ſame reaſon to one, or to which one and the ſame, hath the ſame reaſon, are equal amongſt themſelves.

In like manner, it may be ſaid that there is greater reaſon of a greater number to one and the ſame, then of a leſſer, and contrariwiſe, one and the ſame hath greater reaſon to a leſſer, then to a greater number, alſo that number that hath the greateſt reaſon to one and the ſame, is the greater number, and that number to which one and the ſame hath the greateſt reaſon is the leſſer number; which things are eaſie, if this Definition be well underſtood.

Clavius ſaith here, that this Definition agreeth alſo in broken numbers, whether there be whole numbers joyned with them or not: For Example, theſe four numbers are proportional, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{1}{4}$, the firſt being ſo Multiplex of the ſecond, as the third is of the fourth, to wit, double, which will appear, if you reduce the two firſt into the ſame Denomination, to wit to $\frac{6}{8}$ and $\frac{3}{8}$, and the two laſt to $\frac{2}{4}$ and $\frac{1}{4}$, in like manner theſe four are proportional $2\frac{1}{8}$, $4\frac{2}{12}$, $1\frac{1}{12}$, $2\frac{1}{6}$, ſeeing that the firſt is ſuch a part of the ſecond, as the third is of the fourth, to wit, the half, which is manifeſt, reducing them into one Denomination, to wit, to $\frac{17}{8}$, $\frac{34}{24}$, and $\frac{13}{12}$, $\frac{12}{24}$, in which *Clavius* is much miſtaken, in not conſidering that to ſpeak properly, there are no broken numbers or Fractions, Unity being indiviſible, according to *Euclide*, and the Fraction making it ſelf in the thing numbred, and not in the Numbers, to wit, when you divide any whole number into other equal parts, and of another name, which parts being applyed to numbers, or the numbers applyed to them, they are then abſolute numbers, that is to ſay, whole numbers, as in the Examples above, where he ſaith, that $\frac{6}{8}$ are double to $\frac{3}{4}$, you muſt underſtand that theſe numbers *6* and *3*, are taken abſolutely, and as whole numbers, without regarding the name, for you ought onely to compare the number to the number, without comparing the name to the name, that is to ſay, to the Denominator, and ſo you ought to underſtand as touching the reſt; for thoſe things thus reduced are no more Fractions; Seeing that Fractions are ſo called onely in compariſon to their whole, ſo divided into parts of another name: but theſe parts applyed to numbers, are whole numbers, and ſo *Euclide* doth underſtand.

21 *Plane*

21 *Like Plaine numbers and like Solid numbers, are such as have their sides proportional.*

TO the end that a Plaine number may resemble another Plaine number, it is not necessary that two sides of the one, (which you please,) be proportional to two sides of the other, (which you please also;) but it sufficeth that the one hath two sides, (which so ever they be) proportional to any two sides of the other, as the Plaine numbers 24 and 6 are alike; forasmuch as the sides of that of 6 and 4, are proportional to the sides of the other of 3 and 2, although that the other sides of the first 8 and 3, or 12 and 2, are not proportional to that of 3 and 2.

In like manner, it is not requisite that to the end that two Solid numbers be alike, that three of the sides of the one (which you please) be proportional to three of the other, (which you also please) but it sufficeth that three sides be proportional to any three sides, for so they are reduced into a Solid forme, according to their unites, and their Lengths, Breadths, and Heighths, shall be proportional, as the Solid numbers 192 and 24 are alike, the sides of the one 8, 6, 4, being proportional to the sides of the other, 4, 3, 2, although the other sides of the first, 12, 8, 2, or 16, 4, 3, are not proportional to the sides of the other, 4, 3, 2, &c.

22 *A perfect number, is that which is equal to all his aliquot parts.*

THat is to say, such a number to which his parts being put together are equal, (to wit all his aliquot parts,) and that number is called by the Mathematicians a perfect number, as 6, 28, 496, for the first conteineth these three aliquot parts 1, 2, 3, which together doe make 6, all these of the second are 1, 2, 4, 7, 14, whose summe do make 28, and those of the third 1, 2, 4, 8, 16, 31, 62, 124, 148, all which being added together do make 496, &c.

And if all the aliquot parts of a number taken together be greater then it, it shall be called abundant, if lesser, deficient.

Whence it appeares that this word part, is taken by *Euclide* onely for an aliquot part, for otherwise every number should be a perfect number, every number being equal to all his parts, if each lesser number may be said to be parts of a greater, whether it measure it or not.

To these Definitions of EUCLIDE we shall adde these which follow, according to some Interpreters.

23 *A number is said to measure a number by that number, which multiplying or being multiplied, produceth the same number.*

AS 4, shall be said to measure 12 by 3, forasmuch as 4 multiplied by 3 produceth 12; again, 3 shall be said to measure the same 12 by 4, seeing that 4 multiplying 3 produceth 12, &c.

24 *Proportion or Reaſon of numbers is an habitude of one number to another, according to what the one is Multiplex of the other, or part or parts, or elſe conteineth it once, or divers times, and over and above a cetaine part or parts thereof.*

AS if you compare 20, with 4 according to what 20 is Multiplex of 4, to wit, quintuple, this compariſon or habitude ſhall be called reaſon, as hath been ſaid in the third Definition of the 5th. Book, where *Euclide* hath ſpoken of Reaſon in general.

25 *Two numbers are ſaid to be terms or Roots of a proportion to which three cannot be found two leſſe numbers, in the ſame reaſon or proportion.*

26 *When three numbers are proportional, the firſt is ſaid to have to the third, the double reaſon of that of the firſt to the ſecond: and when there are four, the firſt is ſaid to be to the fourth in triple reaſon of the firſt to the ſecond, and ſo in one and the ſame order, one over and above, until the proportion be finiſhed.*

27 *If there be ſo many numbers as you pleaſe in order, the reaſon of the firſt to the laſt, is ſaid to be compoſed of the reaſon of the ſecond to the third, and of the third to the fourth, and ſo in order until the proportion be finiſhed.*

PETITIONS.

It is required to be granted.

1 *That it is poſſible to take numbers equal or Multiplices to any number given.*

2 *That it is poſſible to take a greater number then any given number.*

ALthough that number cannot be infinitely diminiſhed, but comes to unity indiviſible, it may nevertheleſſe be augmented infinitely by the

the continual addition of unity, therefore any number being propoſed, a greater may be found, to wit, that which is produced by the addition of a unite, or of divers unites to the number given.

COMMON SENTENCES.

1 *Numbers equi-multiplices of equal numbers, or of one and the ſame number are equal to one another.*

2 *Numbers of which one and the ſame number is equi-multiplex, or of which equal numbers are equi-multiplices, are equal one to another.*

3 *Numbers that are the ſame part, or the ſame parts of equal numbers, or of one and the ſame number are equal to one another.*

4 *Numbers, of which one and the ſame number or equal numbers, are the ſame part, or the ſame parts, are equal to one another.*

5 *Unity meaſureth every number, by the unites that are therein, that is to ſay, by the ſame number.*

FOr the unite being taken ſo many times as there are unites in the number propoſed, doth conſtitute the ſame number, and therefore it is meaſured by the unites that are therein, that is to ſay, by the ſame number made of his unites.

6 *Every number meaſureth it ſelfe by unity.*

SEeing that every number once taken is equal to it ſelf, it is manifeſt that every number meaſureth it ſelf by unity.

7 *If a number multiplying a number, produce another number, the Multiplicand ſhall meaſure the product by the Multiplicator: but the Multiplicator ſhall meaſure the ſame by the Multiplicand.*

FOr let B multiplying A produce C, I ſay, that A meaſureth C by B, and B meaſureth the ſame C by A, for (as by the 15 Defin.) ſeeing that B being

as many times compounded, as there are unites in A, doth make C: it is manifeſt that B doth meaſure C by A, and by the ſame reaſon, A doth meaſure the ſame by B; Seeing that B multiplying the ſame A doth produce the ſame C, as ſhall be demonſtrated in the 16 Propoſition of this Book.

A .... 4 B ... 3
C 12

8 *If a number meaſure another number, that number by which it meaſureth it, doth meaſure the ſame by the unites that are in the number meaſuring, that is to ſay, by the ſame number meaſuring.*

FOraſmuch as the number 6 doth meaſure 18 by 3, the ſame 3 ſhall meaſure 18, by 6, that is to ſay, by the unites that are in 6, for ſeeing that 6 doth meaſure 18 by 3, the number 18 ſhall be (according to the 23 Defin.) produced by the multiplication of 6 by 3, or of 3 by 6, therefore by the afore-going Axiome 3 ſhall meaſure 18 by 6.

9 *If a number meaſuring a number, multiply that number by which it meaſureth it, or is multiplied thereby, it will produce the number which it meaſureth.*

LEt the number A meaſure C by B, I ſay, that A multiplying B or multiplied of B produceth C, for (by the 25 Definition) A is ſaid to meaſure C by that number, the which multiplying or being multiplyed, produceth C, therefore A being put to meaſure C by B, it is manifeſt that A multiplying B, or being multiplyed thereby, produceth C.

A....4 B...3
C..... 12

10 *The number that meaſureth as many other numbers as you pleaſe, doth alſo meaſure the numbers compounded thereof.*

LEt the number A meaſure B C and C D; I ſay it doth alſo meaſure their compound B D; for ſeeing that A meaſureth B C and C D, as well

A....4
B....4 E....4 C....4 F....4 G....4 D 20

B C as C D ſhall be Multiplex of A, dividing therefore B C according to the parts B E and E C, equal to A and C D, according to C F, F G and G D, equal to A; B D compounded of all the parts B E, E C, C F, F G and G D, equal to A ſhall be Multiplex of A; therefore A meaſureth B, which was propoſed.

11 *That*

11 *That number that measureth some other number, doth also measure every number which that number measureth.*

LEt the number A measure B, and B measure C D. I say, that A doth also measure C D. For seeing that B doth measure C D, the same C D shall be Multiplex of B, dividing therefore C D into C E and E D, equal to B; A shall measure C E and E D, seeing that A is put to measure B, a number equal as well to C E, as to E D, (as by the 10th. Axiome) therefore A shall also measure C D, compounded of C E and E D, which was proposed.

A...3
B.....9
C.....9 E.....9 D

12 *That number that measureth the whole and the part cut off, measureth also the rest.*

LEt A measure the whole B C, and the part cut off B D; I say it will also measure D C, the rest; For seeing that A measureth B C and B D, as well B C as B D shall be Multiplex of A, or B D shall be equal to A; therefore having divided as well B C as B D, in parts equal to A, the rest D C, shall be one onely part equal to A, or Multiplex thereof, wherefore A shall measure D C, which was proposed.

A...3
B.....9 D...3 C
B.....9 D.....6 C
B...3 D.....6

All that hath been here above added, besides the Definitions of *Euclide*, is done to render the Demonstrations hereafter mentioned more clear and intelligible, they being all founded on what *Euclide* hath before set down in his former several Books.

PRO-

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

A.....10 E..2 G.1B
C...3 F..2D
H.....

If from two unequal numbers proposed A B, C D, *there be cut off ſtill alternately, the leaſt* C D, *from the greateſt* A B, *and that the remainder* E B *ſhall never meaſure his precedent* A B, *until the unite* G B *be taken; the numbers* A B, C D, *propoſed at the beginning, ſhall be Primes to one another.*

Demonſtration FOr if they be not Primes to one another, they ſhall be meaſured by ſome number, which ſuppoſe to be H, their common meaſure over and above unity; foraſmuch then as H meaſureth C D, and C D, A E, (C D being a part of A E, or equal thereto, ſeeing that it being taken from A B, there doth remaine E B) H [a] ſhall alſo meaſure A E. But H doth meaſure the whole A B, and [b] therefore ſhall meaſure the reſt E B. But E B doth meaſure C F, [c] therefore H doth meaſure the ſame C F: and therefore ſeeing that it meaſureth the whole C D, it [d] will alſo meaſure the reſt F D, and ſeeing F D meaſureth E G, H [e] ſhall alſo meaſure E G, but H meaſured the whole E B, therefore [f] H ſhall alſo meaſure the unite reſting G B, the whole his part, which is abſurd, therefore any number beſides unity, ſhall not meaſure A B and C D, and therefore they ſhall be Primes to one another which was to be demonſtrated.

a) 11. c. ſ.
b) 12. c. ſ.
c) 11. c. ſ.
d) 12. c. ſ.
e) 11. c. ſ.
f) 12. c. ſ.

PROP. 2. PROBL. 1.

A.....16 E.....6B
C.....6 F..D
G.....

To find the greateſt common meaſure F D, *of two given numbers* A B, C D, *that are not Primes to one another.*

Con-

Conſtruction LEt the leſſer number C D, be taken from A B the greater number, as often as may be, and there will reſt E B, which being ſubtracted from C D leaveth F D, and ſo following, alwayes taking the leſſer from the greater by an alternate ſubtraction, by which means you ſhall arrive to a number which will meaſure the precedent; For if you ſhould proceed to unity, [a] A B and C D, ſhould be Primes to one another, which is contrary to the Hypotheſis; come we then to F D, the which being ſubtracted from the precedent, as often as may be, there will remaine nothing; I ſay then, that F D is the greateſt common meaſure of A B and C D. a) 1. 7.

Demonſtration FOr that it meaſureth both of them, we ſhall thus demonſtrate, foraſmuch as F D meaſureth E B, and E B meaſureth C F, [b] F D, ſhall meaſure the ſame C F, and ſeeing it meaſureth it ſelf, [c] it ſhall alſo meaſure the whole C D, compounded of the two numbers C F and F D. But C D meaſureth A E, [d] therefore F D ſhall alſo meaſure A E, and ſeeing that F D doth alſo meaſure E B, it [e] ſhall meaſure the whole A B, compounded of A E and E B, therefore F D doth meaſure the two numbers A B and C D, and is alſo their greateſt common meaſure: For if it be not ſo, let it be another, greater then F D, ſuppoſe G; Foraſmuch then as G meaſureth A B and C D, being their common meaſure, and C D meaſureth A E, [f] the ſame G ſhall alſo meaſure A E, [g] and therefore E B: but E B meaſureth C F, [h] therefore G ſhall alſo meaſure C F, and [i] therefore it ſhall alſo meaſure the reſt, F D the greater, the leſſer, which is impoſſible; Therefore a greater then F D ſhall not meaſure A B and C D, and therefore F D is their greateſt common meaſure. But if the leſſer number C D doth meaſure the greater A B, in ſuch manner, as being ſubtracted there-from as often as ſhall be poſſible, there reſt nothing, C D ſhall be the greateſt common meaſure of the two; Seeing it meaſureth it ſelf, as appears by this figure. Therefore, &c. which was to be demonſtrated.

A.....10 B
C.....5 D

b) 11. c. ſ.
c) 10. c. ſ.
d) 11. c. ſ.
e) 10. c. ſ.
f) 11. c. ſ.
g) 12. c. ſ.
h) 11. c. ſ.
i) 11. c. ſ.

COROLARIE.

From this Demonſtration it is manifeſt that that number which meaſureth two others, meaſureth alſo their greateſt common meaſure; This is gathered from that part, by which it is ſhewn that F D *is the greateſt common meaſure of* A B *and* C D, *for it is ſhewn that if* G *meaſure* A B *and* C D, *it meaſureth alſo* F D, *their greateſt common meaſure.*

PROP. 3. PROBL. 2.

A.....16 D....4
B.....12 E..2
C.....6 F.....

To find the greateſt common meaſure D, *of three given numbers* A, B *and* C, *not Primes to one another.*

Con-

Conſtruction LEt D be found, the greateſt common meaſure of A and B, if D doth alſo meaſure C, it is manifeſt that D is the greateſt common meaſure of the three, A, B, and C.

Demonſtration a) Cor. 2. 7. FOr if a greater number then D be ſaid to meaſure the three A, B, C, the ſame [a] ſhall alſo meaſure D the greateſt common meaſure of A and B, the greater, the leſſer, which is impoſſible. But it D doth not meaſure C, at leaſt D and C ſhall be compounded between themſelves, for A, B and C, being compounded, any common meaſure of theirs ſhall alſo meaſure D, the greateſt common meaſure of A and B; therefore, ſeeing that that common meaſure doth alſo meaſure C, D and C ſhall be compounded between themſelves, and E be their greateſt common meaſure; b) 2. 7. [b] I ſay, that E is the greateſt common meaſure of the three, A, B and C, foraſmuch as E meaſureth C and D, but D meaſureth A and B; E meaſureth the ſame A and B, and therefore it ſhall meaſure the three A, B and C.

A.....16 D....4
B.....12 E..2
C.....6 F.....

Now that E is the greateſt common meaſure, it is evident: For ſuppoſe F to be their common meaſure, greater then E, foraſmuch as F meaſureth A and B, c) Cor. 2. 7. [c] it ſhall alſo meaſure D, their greateſt common meaſure: But F alſo meaſureth C, therefore F meaſuring D and C, ſhall alſo in like manner, meaſure E, their greateſt common meaſure, the greater, the leſſer, which is impoſſible: Therefore, a greater number then E will not meaſure A, B and C, and therefore E is their greateſt common meaſure; Therefore, &c. which was to be demonſtrated.

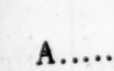
COROLARIE.

It is manifeſt from this Demonſtration, that one number that meaſureth three, will in like manner meaſure their greateſt common meaſure.

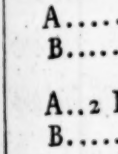
This is alſo gathered from the laſt part of the Demonſtration, where it is ſhewn that ſeeing that F *doth meaſure* A, B *and* C, *it meaſureth alſo* E, *their greateſt common meaſure: In like manner, if there be more then three numbers propoſed, not Primes to one another, this Probleme ſheweth how to find their greateſt common meaſure: For if there be four numbers, you muſt firſt find the greateſt common meaſure of three, and if five, of four, &c. and you muſt work, in the reſt, as hath been ſhewn of three numbers.*

PROP.

PROP. 4. THEOR. 2.



Every leſſer number A, *is a part or parts of every greater number* B.



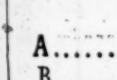
Demonſtration LEt A and B in the firſt place be Primes to one another, foraſmuch as each unite of A, is a part of B, it is manifeſt that A is parts of B, that is to ſay, ſo many parts as there are unites in A.

Secondly, let A and B not be Primes to one another, but compounded, and that A meaſureth B, that being [a] it appears that A is part of B.

But if A doth not meaſure B, [b] find C their greateſt common meaſure; and let A be divided according to the parts A D, D E, and E F, each equal to C [c], foraſmuch as C is part of B; ſeeing that it meaſureth it, A D ſhall be alſo part of B, in like manner D E and E F; and therefore all the number A is parts of B, to wit, as many parts as C is conteined times in A E. Therefor every Leſſer, &c. Which was to be demonſtrated.

a) 3. d. 7.
b) 2. 7.
c) 3 diff.

PROP. 5. THEOR. 3.



If one number be ſuch part of one number as another number is of another number, the one and the other together, ſhall be ſuch part of the one and the other together, as one ſingle number to a ſingle number.

THat is to ſay, if of four numbers, the firſt be ſuch a part of the ſecond, as the third is of the fourth, the firſt and third together, ſhall be ſuch part of the ſecond and the fourth together, as the firſt is of the ſecond.

Demonſtration FOr having divided the numbers B C and E F, according to the parts B G, G C, E H and H F, equal to A, to D, there will be in B C ſo many parts equal to A, as in E F, equal to D; ſeeing that A is ſuch part of B C, as D is of E F.

Foraſmuch then as A and B G are equal, if there be added to them the equal numbers D and E H, A and D together, ſhall be equal to B G and E H together. in like manner, A and D together, ſhall be equal to G C and H F together, and ſo on; if there were more parts in B C and E F: and therefore the one and the other, A and D together, ſhall be the ſame part of the one and the other, B C and E F together, as A is of B C, or D of E F; Therefore if one number, &c. Which was to be demonſtrated.

PROP. 6. THEOR. 4.



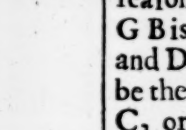
If one number A B, *be ſuch parts of one number* C, *as another number* D E, *is of another number* F, *the one and the other* A B *and* D E, *together, ſhall be ſuch parts of the one and the other* C *and* F *together, as one ſingle number* A B, *of* C, *is to one ſingle number* D E, *of* F.

THat is to ſay, if of four numbers, the firſt be ſuch part of the ſecond, as the third is of the fourth, the firſt and third together, ſhall be ſuch parts of the ſecond and fourth together, as the firſt is of the ſecond.

Demonſtration FOr having divided A B, and D E according to the parts A G, G B, D H, and H E, of the numbers C and F; A B ſhall contein as many parts of C, as D E of F; ſeeing that A B is ſuch parts of C, as D E is of F, then foraſmuch as A G is ſuch part of C, as D H is of F, the[a] one and the other A G, and D H, together, ſhall be ſuch part of the one and the other C and F together, as A G is of C, or D H of F; By the ſame reaſon G B, and H E, together, ſhall be the ſame part of C and F together, as G B is of C, or H E of F, and ſo following, if there be more parts in A B, and D E; Therefore, the one and the other, A B and D E together, ſhall be the ſame parts of the one and the other, C and F together, as A B is of C, or D E of F: Therefore if one number, &c. Which was to be demonſtrated.

a) 5. 7.

PROP. 7. THEOR. 5.



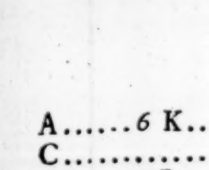
If one number A B, *be ſuch a part of one number* C D, *as the part cut off* A E, *is of the part cut off* C F, *the reſt* E B, *ſhall be alſo ſuch a part of the reſt* F D, *as the whole* A B *is of the whole* C D.

Demonſtration FOr let G C be taken, of which let E B be ſuch a part as A E is of C F, or the whole A B, of the whole C D, foraſmuch as A E is the ſame part of C F, as E B is of G C, [a] the one and the other A E and E B, together, ſhall be ſuch part of C F and G C, together, as A E is of C F, that is to ſay, as the whole A B, is to the whole C D: and therefore, A B being ſuch a part, as well of F G, as of C D, the ſame F G, and C D, ſhall be equal to one another; then taking away the common C F, G C, and F D, will remaine equal: therefore, E B ſhall be ſuch part of F D, as of G C, that is to ſay, as the whole A B, to the whole C D. Therefore, if one number, &c. Which was to be demonſtrated.

a) 5. 7.

PROP.

PROP. 8. THEOR. 6.



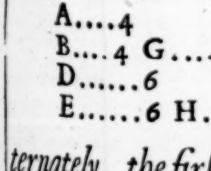
If one number A B *be ſuch parts of another number* C D, *as the part cut off* A E, *to the part cut off* C F, *the reſt* E B *ſhall be alſo ſuch parts of the reſt* F D, *as the whole* A B, *is of the whole* C D.

Demonſtration FOr let G H be taken equal to A B; G H ſhall be then alſo the ſame parts of C D, as A B is of C D, that is to ſay, as A E of C F, having then divided G H according to G I and I H, parts of C D, and A E, according to A K and K E parts of C F, the number of the parts G I, and I H, ſhall be equal to the number of the parts A K and K E, and as well G I as I H, ſhall be ſuch part of C D, as A K, or K E is of C F: and ſeeing that C D is greater then C F, as well G I as I H, part of C D, ſhall be greater then A K, or K E part of C F.

Having therefore taken G L and I M, equal to A K and K E; G L ſhall be the ſame part of C F, as A K, of the ſame C F, or as G I of C D, and therefore the whole G I, being the ſame part of the whole C D, that the part cut off G L is of the part cut off C F [a], the reſt L I, ſhall be ſuch part of the reſt F D, as the whole G I, is of the whole C D. So M H ſhall be ſhewn to be the ſame part of F D, as the whole G I, or I H, is of the whole C D, therefore, ſeeing that as well G I as I H, is the ſame part of C D, as L I and M H, of F D, the one and the other, G I and I H together, ſhall be ſuch parts of C D, as L I and M H together, of F D. But G H is the ſame parts of C D, as A B of the ſame C D, (A B, and G H, being equal,) therefore L I and M H together, ſhall be the ſame parts of F D, as A B of C D. a) 7. 7.

But foraſmuch as if from the equal A B and G H, there be taken the equal A K, K E and G L, I M, the reſt E B, ſhall be equal to the reſt L I and M H, together: and therefore the reſt E B, ſhall be the ſame parts of the reſt F D, as the whole A B of the whole C D: to wit, as L I and M H, together, of the ſame F D. Therefore if one number, &c. which was to be demonſtrated.

PROP. 9. THEOR. 7.



If one number A B, *be ſuch part of one number* B C, *as another number* D, *is part of another number* E F, *alſo alternately, the firſt* A, *ſhall be ſuch part or parts of the third* D, *as the ſecond* B C, *is of the fourth* E F.

THat is to say, If of four numbers, the first be such part of the second, as the third is of the fourth, also alternately, the first shall be such part or parts of the third, as the second is of the fourth.

Demonstration FOr having divided B C, and E F, according to the parts B G, and G C, each equal to A, and E H, and H F, each equal to D, the number of the parts of B G, shall be equal to the number of the parts of E F. But B G, and G C, being equal to one another, and lesse then E H, and H F, also equal to one another; forasmuch as the whole B C, is lesse then the whole E F, by supposition, B G shall be the same part of E H, or the same parts, as G C of H F [a], and therefore B G, and G C, together, to wit, B C the second, shall be the same part or parts of E H, and H F, together, that is to say, E F the fourth, as B G is of E H, that is to say, as A the first is of D the third: Therefore, if one number, &c. Which was to be demonstrated.

A....4
B....4 G....4 C
D.....6
E.....6 H.....6 F

a) 5, or 6. 7.

PROP. 10. THEOR. 8.

If a number A B, *be such parts of a number* C, *as another* D E, *is parts of another* F, *also alternately, the first* A B *shall be such part or parts of the third* D E, *as the second* G, *is of the fourth* F.

A..2 G..2 B
C.....6
D.....5 H.....E
F.....15

Demonstration FOr having divided A B, and D E, according to A G, and G B, parts of C; and D H, and H E, parts of F, there will be as many parts in A B, as in D E, and as well A G, as G B, shall be such part of C as D H, or H E, of F, [a] therefore, alternately A G shall be the same part or parts of D H, and G B of H E, as C is of F, and therefore, A G shall be the same part or parts of D H, as G B is of H E: [b] therefore A G, and G B, together, that is to say, A B the first, shall be the same part or parts of D H, & H E together, that is to say, of D E the third, as A G is of D H, that is to say, as C the second of F the fourth: Therefore, if one number, &c. Which was to be demonstrated.

a) 9. 7.

b) 5, 6. 7.

PROP. 11. THEOR. 9.

If as the whole A B, *is to the whole* C D, *so the part cut off* A E, *is to the part cut off* C F, *also the rest* E B, *shall be to the rest* F D, *as the whole is to the whole.*

A....4 E..2 B.
C.....8 F....4 D

Demonstration FOr seeing that A B is to C D, as A E is to C F, [a] A B being lesse then C D, shall be the same part, or the same parts, of C D, as A E is of C F; therefore the rest E B, shall be the same part, or parts

a) 20. d. 7.

parts of the reſt F D, as A B is of C D: [b] therefore, E B ſhall be to F D, as A B is to C D. Which was to be demonſtrated. b) 7, 8. 7.

PROP. 12. THEOR. 10.

A....4 C..2 E...3
B.....8 D....4 F.....6

If there be as many numbers as you pleaſe A, B, C, D, E, *and* F, *proportional, as one of the antecedents* A, *ſhall be to one of the conſequents* B: *ſo all the antecedents* A, C, *and* E, *ſhall be to all the conſequents* B, D, *and* F.

Demonſtration FOraſmuch then as becauſe of the ſame proportion, A is [a] ſuch a part, or ſuch parts (A, C, and E, being leſſe) of B, as C is of D, and E of F, [b] A and C together, ſhall be ſuch part, or parts of B and D together, as A is of B, or E of F. a) 20. d. b) 5, 6. 7.

Againe, foraſmuch as A and C, together as one, is the ſame part or parts of B and D together as one, as E is of F, the two A and C together, as one and F together, ſhall be the ſame part, or the ſame parts of the two B and D together, as one, and F together, as A is of B: [c] Therefore, there will be the ſame proportion of A, C and E together, to B, D and F together, as of A to B. c) 20. d. 7.

PROP. 13. THEOR. 11.

A..2
B....4
C.....6
D.....12

If four numbers A, B, C, *and* D, *be proportional, alſo alternately they ſhall be proportional.*

Demonſtration FOr ſeeing that A is to B, as C to D, [a] A is the ſame part, or parts of B, as C is of D, therefore, alternately, A ſhall be the ſame part or parts of C, as B is of D, [b] and therefore, as A ſhall be to C, ſo B ſhall be to D, which was to be demonſtrated. a) 20, 10. 7. b) 20. d.

PROP. 14. THEOR. 12.

A.....12 D..... 9
B.....8 E.....6
C....4 F..2

If there be as many numbers as you pleaſe of the one part; A, B, *and* C, *and as many of the other part,* D, E, *and* F, *the which being taken two and two, and in the ſame reaſon; alſo in equality they ſhall be in the ſame reaſon.*

Demon-

Demonſtration FOr ſeeing that as A is to B, ſo D is to E, [a] alternately, as A ſhall be to D, ſo B ſhall be to E; in like manner, ſeeing that as B is to C, ſo E is to F, alternately, as B ſhall be to E, ſo C ſhall be to F: Therefore as A is to D, ſo C ſhall be to F, (for ſeeing that the one and the other reaſon of A to D, and C to F, is the ſame to the reaſon of B to E, as hath been demonſtrated, they ſhall be alſo the ſame one to another) and [b] therefore, alternately, as A ſhall be to C, ſo D ſhall be to F. Which was to be demonſtrated.

A.....12 D.....9
B.....8 E.....6
C....4 F..2

a) 13. 7.
b) 13. 7.

PROP. 15. THEOR. 13.

If unity A, *meaſureth ſome number* BC, *and that another number* D *doth meaſure as oftentimes another number* EF, *alſo alternately, unity* A *ſhall meaſure the third number* D, *as oftentimes as the ſecond* BC *ſhall meaſure the fourth* EF.

A.1
B.1 G.1 H.1 C
D..2
E..2 I..2 K..2 F

Demonſtration FOr having divided BC, according to the unites BG, GH, HC and EF, according to the parts EI, IK and KF, each equal to D, there will be in EF as many parts equal to D, as there are unites in BC: therefore BG ſhall be the ſame part of EI, as GH of IK, and HC of KF: [a] therefore as BG, is to EI, ſo GH to IK, and HC to KF, [b] therefore, as BG to EI, ſo is BG, GH and HC, to EI, IK, and KF, all the antecedents to all the conſequents, that is to ſay, BC to EF, therefore BC ſhall be the ſame part of EF, as BG is of EI, that is to ſay, as A is to D: Therefore if unity, &c. Which was to be demonſtrated.

a) 20. d. 7.
b) 12. 7.

PROP. 16. THEOR. 14.

If two numbers A *and* B *multiplying one another, do produce other numbers* C *and* D, *the products* C *and* D, *ſhall be equal to one another.*

Unity. 1
A...3 B....4
C.....12
D.....12

Demonſtration FOraſmuch as A multiplying B produceth C [a], B ſhall be as often found in C, as unity in A, and therefore ſhall meaſure C, as often as the unite ſhall meaſure A [b], therefore, by permutation, A ſhall meaſure C, as often times as unity ſhall meaſure B: Again, ſeeing that B multiplying A produceth D, A ſhall be as often found in D as unity in

a) 15. d. 7.
b) 15. 7.
15. d.

in B, and therefore ſhall meaſure D, as oftentimes as unity ſhall meaſure B; But A ſhall meaſure C alſo, as oftentimes as unity ſhall meaſure B, therefore, A doth meaſure C and D equally, and therefore C and D are equal to one another. Therefore, if, &c. Which was to be demonſtrated.

PROP. 17. THEOR. 15.

Unity. 1
A...3
B..2 C....4
D.....6 E.....12

If one number A, *by multiplying two others* B *and* C, *doth produce other numbers* D *and* E, *the products of thoſe numbers ſhall have the ſame proportion to one another as the numbers multiplied.*

Demonſtration FOr ſeeing that A multiplying B, maketh D; B ſhall be as oftentimes conteined in D as unity in A: In like manner, C ſhall be as oftentimes conteined in E, as unity in the ſame A, and therefore B ſhall meaſure D as oftentimes as C ſhall meaſure E. Therefore B ſhall be ſuch part of D, as C of E [b], and therefore as B ſhall be to D, ſo C ſhall be to E [c], and alternately, as B is to C, ſo is D to E: Therefore if one number, &c. Which was to be demonſtrated. 15. d. b) 20. d. c) 13. 7.

PROP. 18. THEOR. 16.

A..2 B....4
C...3
D.....6 E..... 12

If two numbers A *and* B, *multiplying ſome other number* C, *ſhall produce other numbers* D *and* E, *their products* D *and* E *ſhall have the ſame proportion as the numbers multiplying* A *and* B.

Demonſtration FOr ſeeing that D is the product of C multiplyed by A [a], the ſame D ſhall be the product of A multiplyed by C: in like manner E being the product of C by B, the ſame E ſhall be the product of B by C: Foraſmuch then, as the ſame number C multiplying A and B, makes D and E, as A ſhall be to B, ſo D ſhall be to E: Therefore, if two numbers, &c. Which was to be demonſtrated. a) 16. 7. 17. 7.

PROP.

PROP. 19. THEOR. 17.

A...3 B..2
C.....6 D....4
E.....12
F.....12
G.....18

If four numbers A, B, C, *and* D, *are proportional, the product of the first* A, *and the fourth* D, *shall be equal to the product of the second* B, *and the third number* C: *and if the number produced of the first* A, *and the fourth* D, *be equal to the product of the second* B, *and the third* C, *those four numbers shall be proportional.*

Demonstration AS A is to B, so C is to D, and as A the first, multiplying D the fourth, maketh E, and B the second, multiplying C the third, makes F, I say that E and F are equal to one another; for again, let A multiplying C make G; forasmuch as A multiplying C and D, makes G and E, as C shall be to D: that is to say, as A is to B, so G shall be to E.

17. 7.

Again, forasmuch as A and B multiplying C, makes G and F, as [a] A shall be to B, so G shall be to F: therefore, as G shall be to E, so the same G shall be to F, therefore, E F shall be equal by what followes after the 20th. Definition.

a) 18. 7.

Now let E the product of A the first, by D the fourth, be equal to F, the product of B the second, by C the third: I say, the four numbers A, B, C, and D, are proportional, to wit, as A is to B, so C is to D.

Againe, let G be produced of A multiplyed by C; forasmuch as A multiplying C and D, makes G and E; as C shall be to D, so G to E, or to F, equal to E, for G hath the same proportion to E, as to F, as is shewn at the 20th. Definition.

Again, forasmuch A and B multiplying C, makes G and F, as A is to B, so the same G is to F, therefore, the proportions of A to B, and C to D, being the same with the proportion of G to F, they shall be so to one another, and so as A shall be to B, so C shall be to D: Therefore, if four numbers, &c. Which was to be demostrated.

PROP. 20. THEOR. 18.

A.....9
B.....6 D.....6
C....4

If three numbers A, B, *and* C, *are proprotional: to wit, as* A *to* B, *so* C *to* D, *the product of the extreames* A, *multiplyed by* C, *is equal to the product of the mean* B, *and if the product of the extreames* A *and* C, *be equal to the product of the mean, those three numbers shall be proportional.*

Demon-

Demonſtration FOr let D be aſſumed equal to B, as A ſhall be to B, ſo D ſhall be to C [a], and therefore the product of A by C, ſhall be equal to the product of B by D; that is to ſay, of B multiplyed by it ſelfe. a) 19. 7.

Now let the product of A the firſt, by C the third, be equal to the product of B the mean multiplyed by it ſelfe; I ſay, A, B and C, are proportional.

For again, let D be equal to B; as A ſhall be to B, ſo D to C, and the product of A by C ſhall be equal to the product of B by D, equal to B, that is to ſay, of B multiplyed by himſelfe: Therefore, the three numbers A, B, C, are proportional: Therefore, if three numbers, &c. Which was to be demonſtrated.

PROP. 21. THEOR. 19.

A...3 G..2 B
C..2 H. 1 D
E.....10
F.....6

The leaſt numbers A B, *and* C D, *of all thoſe which have the ſame proportion with them* E *and* F, *do equally meaſure the numbers which have the ſame proportion with them: to wit, the greateſt* A B, *the greateſt* E, *and* C D *the leſſer*, F *the leſſer.*

Demonſtration FOr ſeeing that A B is to C D, as E is to F [a], alternately, as A B to E, ſo C D to F, and therefore A B and C D, being leſſe then E and F, A B [b] ſhall be ſuch part or parts of E, as C D of F: But they cannot be parts, for (if poſſible) let A B, and C D, be divided in A G, and G B, and C H, and H D, parts of E and F; the parts A G, and G B, ſhall be equal in number to C H, and H D; and therefore, A G ſhall be the ſame part of E, as C H of F; therefore, as A G ſhall be to E, ſo C H ſhall be to F [c], and alternately, as A G ſhall be to C H, ſo E to F, or A B to C D: and therefore, A G, and G H leſſe then A B, and C D, ſhall have the ſame proportion as A B, and C D, which is impoſſible: ſeeing that A B, and C D, are the leaſt in their proportion by Suppoſition: Therefore, A B, and C D, ſhall not be the ſame parts of E and F, therefore, the ſame part. Therefore, A B, and C D, ſhall equally meaſure E and F. Therefore, the leaſt numbers, &c. Which was to be demonſtrated. a) 13. 7. b) 20. d. c) 20. d.

PROP. 22. THEOR. 20.

A....4 D.....12
B...3 E.....8
C..2 F.....6

If there be three numbers of the one part, A, B, *and* C, *and as many of the other part,* D, E, *and* F, *the which being taken from two to two, and in the ſame reaſon, and that their proportion be diſturbed: alſo by equality they ſhall be in the ſame reaſon.*

Demonſtration FOr ſeeing that as A is to B, ſo E is to F [a]; the product of A the firſt, by F the fourth, ſhall be equal to the product of B the ſecond, by E the third; and ſeeing that B is to C, as D to E, the product of C by D, ſhall be equal to the ſame product of B by E; and therefore the product of A the firſt, by F the fourth, ſhall be equal to the product of C the ſecond, by D the third [b]: Therefore, there ſhall be the ſame reaſon of A the firſt, to C the ſecond, as of D the third, to F the fourth: Therefore, if there be three, &c. Which was to be demonſtrated.

a) 19. 7.

b) 19. 7.

PROP. 23. THEOR. 21.

A.....7 B.....5
C.... D.... E..

The numbers that are primes to one another A *and* B, *are the leaſt of all thoſe that have the ſame reaſon with them.*

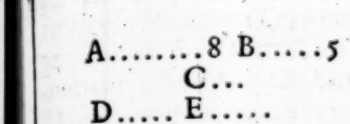
Demonſtration FOr if it be denied, ſuppoſe C and D to be leſſer in the ſame reaſon: Foraſmuch, as C and D are leſſer in the ſame reaſon of A and B [a], C ſhall meaſure A, and D ſhall meaſure B equally, and therefore according to one and the ſame number, which let be E, in ſuch manner as C meaſureth as often times A and D, as often times B as Unity is conteined in E: Therefore, ſeeing that Unity meaſureth E and C, meaſureth A equally [b], alternately, Unity ſhall meaſure C and E, ſhall meaſure A equally: Again, ſeeing that Unity meaſureth E and D, the number B equally: alſo alternately Unity ſhall meaſure B equally; therefore, ſeeing that E meaſureth A and B, the ſame E ſhall be their common meaſure: wherefore they ſhall not be primes to one another, but compounds, which is impoſſible, and againſt the Suppoſition; therefore, A and B are the leaſt in the ſame reaſon: Therefore, the numbers, &c. Which was to be demonſtrated.

a) 21. 7.

b) 15. 7.

PROP.

PROP. 24. THEOR. 22.

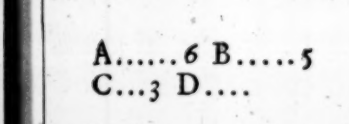


The leaſt numbers A *and* B *of all thoſe which have the ſame reaſon with them, are primes to one another.*

Demonſtration FOr if they be not Primes, they ſhall have one common meaſure, which ſuppoſe to be C; therefore, let C meaſure A as often times as Unity is in D, and B as many times as Unity is in E.

Foraſmuch then, as C is conteined as oftentimes in A, as Unity is in D, (Seeing that it meaſureth A according to D,) and as the ſame C is conteined as many times in B as Unity is in E [a]: D and E multiplying C, ſhall produce A and B: wherefore there ſhall be the ſame reaſon of A to B, as of D to E: therefore, ſeeing that D and E parts of A and B, are leſſer then the ſame A and B: A and B ſhall not be the leaſt of all thoſe which have the ſame reaſon with them, which is abſurd: Therefore, A and B are Primes to one another: and therefore the leaſt, &c. Which was to be demonſtrated. a) 9. c. ſ.

PROP. 25. THEOR. 23.

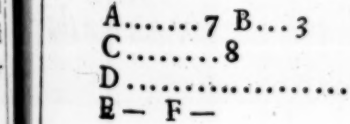


If there be two numbers A *and* B, *primes to one another, the number which meaſureth one of them, ſhall be prime to the other.*

Demonſtration FOr if B and C be not Primes to one another, they will have ſome number for their common meaſure, which ſuppoſe to be D.

Foraſmuch then, as D meaſureth C, and C meaſureth A, [a] D ſhall alſo meaſure A: But it alſo meaſures B: therefore, A and B are not Primes to one another, having D for common meaſure, which is impoſſible, and againſt the Suppoſition: Therefore, C ſhall be Prime to B: Therefore, If there be two numbers, &c. Which was to be demonſtrated. a) 11. ax.

PROP. 26. THEOR. 24.



If two numbers A *and* B, *are primes to ſome other number* C, *their product* D, *ſhall be alſo prime to that other* C.

Demonſtration FOr if C and D are not Primes to one another, let E be their common meaſure, meaſuring D as often times as there are unites in F, foraſmuch then, as E is as many times in D, as there are unites
a) 9. c. ſ. in F [a], F multiplying E ſhall produce D, and contrariwiſe, E multiplying
16. 7. F, ſhall produce the ſame D; But, D is the product of A by B; therefore, ſeeing that the number produced of E the firſt, by F the fourth, is the ſame as of A the ſecond, by B the third, there ſhall be the ſame reaſon of E the firſt, to A the ſecond, as of B the third, to F the fourth. But foraſmuch as A and C are Primes to one another, and E is ſuppoſed to meaſure C [b], E ſhall be Prime to A, and therefore E and A being Primes to one another, ſhall be the leaſt of their reaſon; Therefore, they ſhall equally meaſure B and F, which have the ſame reaſon: to wit, E ſhall meaſure B, and A ſhall meaſure F; Therefore, ſeeing that E meaſureth both the one and the other, B and C: B and C ſhall not be Primes to one another; which is abſurd, and againſt the ſuppoſition; Therefore, D ſhall be Prime to C, Therefore, if two numbers, &c. Which was to be demonſtrated.

A.....7 B...3
C.....8
D.....21
E— F—

19. 7.
b) 25. 7.
23. 7.
21. 7.

PROP. 27. THEOR. 25.

A....4 B.....5
C.....16
D....4

If two numbers A *and* B, *are Primes to one another, the product of the one of them* A, *ſhall be Prime to the other.*

Demonſtration FOr let D be made equal to A, it ſhall be alſo Prime to B, therefore, A and D, being Primes to B [a], the product of A,
a) 26. 7. multiplyed by D, this is to ſay, of A by himſelf; that is to ſay, C ſhall be Prime to the ſame B: ſo the product of B multiplyed by it ſelf, ſhall be ſhewn to be Prime to A; Therefore, if two numbers, &c. Which was to be demonſtrated.

PROP. 28. THEOR. 26.

A.....5 B...3
E.....15
C....4 D..2
F.....8

If two numbers A *and* B, *are Primes to two other numbers* C *and* D, *the one and the other, to the one and the other, their Products ſhall be alſo Primes to one another.*

a) 26. 7. *Demonſtration* FOr ſeeing that as well A as B is Prime to C, E [a] their product ſhall be Prime to the ſame C.

Again, A and B, being Primes to D, E their product ſhall be alſo Prime to

to the ſame D; foraſmuch then, as C and D are Primes to E [b]; F their product ſhall be alſo Prime to E; Therefore, if two numbers, &c. Which was to be demonſtrated. b) 26. 7.

PROP. 29. THEOR. 27.

A...3 B..2
C.....9 D....4
E 27 F 8
G 81 H 16

If two numbers A *and* B, *are Primes to one another, and the one and the other, multiplying it ſelfe, ſhall make a number, their products ſhall be Primes to one another, and if the numbers firſt propoſed, multiplying thoſe products do make a number, the ſame ſhall be alſo Primes to one another, and this will alwayes happen about the extreams.*

Demonſtration FOr ſeeing that A and B are Primes to one another [a], C the product of A by himſelfe, ſhall be Prime to the other B, and in like manner, B and C being Primes to one another, D the product of B by himſelfe, ſhall be alſo prime to C: and therefore the two products C and D, Primes to one another. Again, foraſmuch as A and B are Primes to one another [b], C product of A by it ſelfe, ſhall be Prime to B, and D the product of B by it ſelfe, prime to A: But C is alſo ſhewn to be Prime to B, therefore, the one and the other, A and C, ſhall be Prime to the one and the other, B and D [c], and therefore, E the product of A by C, ſhall be Prime to F, the product of B by D: And if again, G be the product of A by E, and H of B by F: Seeing that A and C are Primes to B [d]; E their product ſhall be alſo Prime to B: by the ſame reaſon F ſhall be Prime to A; Foraſmuch then, as the one and the other, A and E, is Prime to the one and the other, B and F, the [e] number G product of A by E, ſhall be Prime to H, product of B by F, and ſo following, if there were more: Therefore, &c. Which was to be demonſtrated.

a) 27. 7.
27. 7.
b) 27. 7.
c) 28. 7.
d) 26. 2.
e) 28. 7.

PROP. 30. THEOR. 28.

A.....8 B....5 C
D...

If two numbers A B, *and* B C, *are primes to one another, the one and the other together,* A C *ſhall be prime to each of them: and if the one and the other together, be prime to one of them, the numbers firſt propoſed, ſhall be alſo primes to one another.*

Demonſtration FOr if A C and A B be not Primes to one another, they ſhall be meaſured by ſome number, beſides Unity, which ſuppoſe to be D: Therefore, ſeeing that D meaſureth the whole A C, and the

the part cut off A B, it ſhall alſo meaſure the reſt B C; therefore A B and B C ſhall not be Primes to one another, ſeeing that D meaſureth them, which is abſurd, and contrary to the ſuppoſition; therefore A C ſhall be Prime to A B, and to B C.

10. c. ſ. Now let A B and B C together, be Prime to one of them; to wit, to A B, I ſay A B and B C are Primes to one another, otherwiſe they ſhould be meaſured by ſome number beſides Unity, which let be D, therefore D meaſuring A B, and B C, ſhall alſo meaſure their Compound A C, and therefore A C, and A B are not Primes to one another, which is abſurd, and againſt the ſuppoſition; therefore A B, and B C are Primes to one another, and in like manner, if A C be Prime to B C: Therefore, if two numbers, &c. Which was to be demonſtrated.

A.....8 B.....5 C
D...

COROLLARIE.

It follows hence, that the number compounded of two others, and is Prime to one of them, is alſo Prime to the other, for if A C *be Prime to* A B, A B *and* B C *ſhall be Primes to one another, by the ſecond part of this Propoſition; therefore* A C *ſhall be alſo Prime to* B C, *by the firſt part of the ſame Propoſition, which was propoſed.*

PROP. 31. THEOR. 29.

A.....5 B.....8
C—

Every prime number A, *is prime to every other number which it meaſureth not.*

Demonſtration FOr otherwiſe they ſhould be meaſured by ſome number beſides Unity, which ſuppoſe to be C; Therefore C ſhall not be the ſame as A, A being ſuppoſed not to meaſure B; therefore A being meaſured by another number C; A ſhall not be Prime, which is impoſſible, and contrary to the Suppoſition; Therefore, every firſt number, &c. Which was to be demonſtrated.

PROP. 32. THEOR. 30.

A....4 B.....6
C.....24
D...3 E.....8

If two numbers A *and* B, *multiplying one another, do make ſome other* C, *and that ſome prime number* D, *doth meaſure their product; It ſhall alſo meaſure one of the numbers firſt propoſed* A *or* B.

Demon-

Demonſtration FOr as D doth not meaſure A, but it doth meaſure C as many times as there are unites in E, in ſuch ſort, as C may be produced of D by E, the which C is produced of A by B.

Foraſmuch as C the product of D the firſt, by E the fourth, is equal to the product of A, the ſecond by B the third [a]; there will be the ſame reaſon of D the firſt, to A the ſecond, as of B the third to A the fourth [b]: But D the firſt being prime to A, for that it meaſures it not [c], D and A ſhall be the leaſt of their reaſon [d], therefore ſhall meaſure equally B and E: to wit, D the leaſt, ſhall meaſure B the leaſt, and A the greateſt, E the greateſt: Therefore, if D meaſure not A, it ſhall meaſure B, in like manner, it appears that if D meaſure not B, at leaſt, it ſhall meaſure A: Therefore, &c. Which was to be demonſtrated.

a) 19. 7.
b) 31. 7.
c) 23. 7.
d) 21. 7.

PROP. 33. THEOR. 31.

A..... 18
B.....6 C...

Every compound number A, *is meaſured by ſome prime number.*

Demonſtration FOr being compounded, it [a] ſhall be meaſured by ſome number, which let be B: and if B be a Prime number, the Propoſition is manifeſt: But if B be compounded, it ſhall be alſo meaſured by ſome number, which let be C, which ſhall be either prime or compounded: if prime, ſeeing that it meaſureth B, and B meaſureth A, the ſame [b] C prime, ſhall alſo meaſure A: therefore A ſhall be meaſured by a prime number: But if C be compounded, it ſhall be meaſured by another number.

Now foraſmuch as one number cannot infinitely diminiſh it ſelfe, we ſhall arrive at laſt to a number, that no other number may meaſure, and therefore to a prime number meaſuring all the precedents, it [c] ſhall alſo meaſure the compound number A.

a) 13. d.
b) 11. c. ſ.
c) 11. c. ſ.

PROP. 34. THEOR. 32.

A.....9

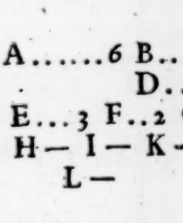
Every number A *is a prime number, or elſe is meaſured by ſome prime number.*

Demonſtration FOr ſeeing that every number is a prime or compound number, if A be a prime number, the Propoſition is manifeſt, but if it be a compound number, it [a] ſhall be meaſured by ſome prime number: Therefore, every number is prime, or, &c, Which was to be demonſtrated.

a) 33. 7.

PROP.

PROP. 35. PROBL. 3.



As many numbers as you pleaſe, A, B, *and* C, *being given, to find the leaſt numbers which have the ſame reaſon with them.*

Demonſtration FOr A, B, and C, are either primes to one another, or are not; if primes, a) they ſhall be the leaſt of all thoſe which have the ſame reaſon; but if they be compounded, let D be found their greateſt common meaſure, which ſhall meaſure the ſame A, B, and C, by E, F, and G,: I ſay, that E, F, and G, are the leaſt of all thoſe that have the ſame reaſon of A, B, and C: foraſmuch as D meaſureth A, B, and C, by E, F, and G b), D multiplying E, F, and G, ſhall produce A, B, and C: therefore E, F, and G, ſhall have the ſame reaſon as A, B, and C.

a) 23. 7.

b) 9. c. ſ.

I ſay alſo, that E, F, and G, are the leaſt: For if it be denied, let H, I, and K, be the leaſt in the ſame reaſon, the which ſhall meaſure A, B, and C, equally; Let them meaſure them then by L, that being ſo, L c) multiplying H, I, and K, will produce A, B, and A, and alternately, L ſhall meaſure A, B, and C, by H, I, and K; foraſmuch then, as E the firſt, multiplying D the fourth, and H the ſecond, multiplyiug L the third, doth produce the ſame A d), as E the firſt, to H the ſecond; ſo L the third ſhall be to D the fourth: But E is greater then H, therefore, L ſhall be greater then D: and therefore, ſeeing that L doth meaſure A, B, and C; D ſhall not be the greateſt common meaſure of the numbers A, B, and C, which is contrary to the Hypotheſis: Therefore E, F, and G, are the leaſt in the reaſon of A, B, and C: Therefore, if, &c. Which was to be done.

21. 7.

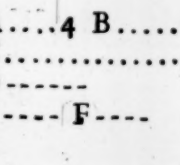
c) 8. c. ſ.

d) 19. 7.

COROLLARIE.

It is manifeſt from this, that the greateſt common meaſure of as many numbers as you pleaſe, doth meaſure them by the leaſt numbers of all thoſe which have the ſame reaſon with them, for it hath been ſhewn that E, F, *and* G, *by the which* D, *the greateſt common meaſure of* A, B, *and* C, *meaſureth the ſame* A, B, *and* C, *are the leaſt in the reaſon of* A, B, *and* C, *and ſo of all others.*

PROP. 36. PROBL. 4.



Two numbers A *and* B, *being given, to find the leaſt number that doth meaſure them.*

Con-

Conſtruction FIrſt of all, let A and B, be primes to one another, and multiplying one the other make C, I ſay, C is the number required.

Demonſtration NOw that it meaſureth them is manifeſt; For, C being the product of A by B, or of B by A, [a] A ſhall meaſure C by B, and B the ſame C by A; therefore, A and B do meaſure C, and if C be not the leaſt; let A and B meaſure another leſſer, to wit, D, (if poſſible) and as A meaſureth D by E, and B the ſame D by F [b]; ſo D ſhall be the product as well of A by E, as of B by F, and contrariwiſe; foraſmuch as the ſame D is produced of A the firſt, by E the fourth, and of D the ſecond, by F the third; as A the firſt, ſhall be to B the ſecond, ſo F the third, to E the fourth, therefore, A and B (being put primes to one another [c], and therefore the leaſt in their reaſon) they ſhall meaſure E and F the one as the other; to wit, A ſhall meaſure F: and B ſhall meaſure E.

a) 7. c. ſ. 9. c. ſ.

b) 19. 7.

c) 23. 7. 11. 7.

But, foraſmuch as A multiplying B and E, makes C and D; C ſhall be to D, as B to E: and therefore, ſeeing that B meaſureth E, as is ſhewn, C ſhall alſo meaſure D, the greateſt, the leaſt, which is impoſſible; therefore, C is the leaſt of all thoſe that do meaſure A and B.

Secondly, let A and B not be primes to one another; and [d] let C and D be found the leaſt in the ſame reaſon; to the end there be four numbers proportional; to wit, as A to B, ſo C to D, for ſo the product of [e] A the firſt, by D the fourth, ſhall be equal to the product of C the ſecond, by B the third, the which product let be E; I ſay, that E is the leaſt number which meaſureth A and B, and that it meaſureth them, is manifeſt; for E being the product as well of A in D, as B in C, [f] as well A as B, ſhall meaſure E, and if you deny it to be the leaſt, let A and B meaſure another, ſuppoſed leſſe then E; to wit F: Now as A meaſureth F by G, and B the ſame F by H, then [g] F ſhall be the product as well of A by G, as of B by H.

A.... B.....6
C..2 D...3
E.....12
F----
G---- H---

d) 35. 7.

e) 19. 7.

f) 7. c. ſ.

g) 9. c. ſ. 19. 7.

Foraſmuch then, as the ſame F is made of A the firſt, in G the fourth, and of B the ſecond, in H the third: As A the firſt, to B the ſecond, ſo H the third, to G the fourth: therefore, C and D being the leaſt in the reaſon of A to B, or of H to G, it [h] meaſureth equally H and G, to wit, C ſhall meaſure H, and D ſhall meaſure G. But, foraſmuch as A multiplying D and G, makes E and F [i], E ſhall be to F, as D to G, and therefore D meaſuring G as is ſhewn, E ſhall alſo meaſure F, the greateſt the leaſt, which is impoſſible: Therefore, E is the leaſt: Therefore, Two numbers, &c. Which was to be done.

h) 21. 7.

i) 17. 7.

COROLLARIE.

From hence it follows, that if two numbers do multiply, the leaſt, having the ſame reaſon, the greateſt, the leaſt; and the leaſt the greateſt, the product ſhall be the leaſt number they ſhall

measure: For C and D being ſuppoſed the leaſt in the reaſon of A to B, it hath been demonſtrated that E the product of A the leaſt, by D the greateſt, and of B the greateſt, by C the leaſt, is the leaſt, number meaſured by A and B.

PROP. 37. THEOR. 33.

A..2 B...3
C------ F----D
E.....6

If two numbers A and B, do meaſure ſome other number C and D, the leaſt it meaſureth E, ſhall alſo meaſure the ſame number.

Demonſtration FOr if E doth not meaſure C D, having taken E from C D, as often as may be, there will remaine a number leſſe then E; let there remaine F D, leſſe then E, (if poſſible) in ſuch ſort as E may meaſure the part cut off C F: Foraſmuch then, as A, as well as B, doth meaſure E, and E meaſureth C F, [a] as well A as B ſhall alſo meaſure C F. Therefore, ſeeing that A and B meaſureth the whole C D, and the part cut off C F, [b] it ſhall alſo meaſure the reſidue F D. Now F D is leſſe then E: therefore, E is not the leaſt number that A and B do meaſure, which is contrary to Suppoſition: Therefore, if two numbers, &c. Which was to be demonſtrated.

a) 11. c. ſ.
b) 12. c. ſ.

PROP. 38. PROBL. 5.

A...3 B....4 C.....6
D.....12
E------

Three numbers A, B, and C, being given, to find the leaſt number they meaſure.

A..2 B...3 C....4
D.....6
E.....12
F-----

Conſtruction LEt [a] D be the leaſt number that A and B doth meaſure, C doth meaſure it, or doth not meaſure it: if it doth meaſure it: I ſay, that it is the leaſt that the three A, B, and C do meaſure: otherwiſe, ſuppoſe E to be leſſe then D meaſured by A, B, and C: therefore A and B meaſuring E, leſſe then D: D ſhall not be the leaſt meaſured by A and B, which is againſt the Suppoſition.

If C meaſure not D, ſuppoſe [b] E to be the leaſt meaſured by C and D; I ſay, that it ſhall be the leaſt that A, B, and C, do meaſure, for A B meaſuring D, and D meaſuring E, [c] A and B ſhall alſo meaſure E, but C in like manner, meaſureth E; therefore, the three A, B, and C, meaſure E; and if you ſay E is not the leaſt, let F leſſe then E, be meaſured by A, B, and

a) 36. 7.
b) 36. 7.
c) 11. c. ſ.

and C, (if poſſible. Seeing A and B do meaſure F, D the leaſt, meaſured by them, [d] ſhall alſo meaſure F and C, D meaſuring F: (for A, B, and C, meaſures it,) E the leaſt number meaſured by C D, ſhall meaſure in like manner, F the greateſt, the leaſt, which is abſurd: Therefore E is the leaſt that A, B, and C, meaſureth. Therefore, &c. which was to be done. d) 37. 7.

COROLLARIE.

From hence follows, that if three numbers do meaſure a number that the leaſt number they meaſure, meaſureth alſo the ſame number: For in the laſt part of this Demonſtration, for that A, B, *and* C, *are put to meaſure* F, *it hath been demonſtrated that* E *which is the leaſt number meaſured by them, doth alſo meaſure* F.

PROP. 39. THEOR. 34.

A.....12
B....4 C...3

If one number B, *meaſure another number* A, *that which it meaſureth* A, *ſhall have a part denominated of the number meaſuring* B.

Demonſtration. FOr let B meaſure A as many times as there are unites in C, foraſmuch then, as unity meaſureth C, and B meaſureth A equally: [a] alternately, unity ſhall meaſure B, and C ſhall meaſure A equally, and therefore unity ſhall be the ſame part of B, as C is of A [b], but unity is a part of B denominated of the ſame B: therefore, C ſhall be a part of A denominated by B. Therefore, if a number &c. Which was to be demonſtrated. a) 15. 7. b) 2. d. 7.

PROP. 40. THEOR. 35.

A.....15
B...3 C.....5

If a number A, *hath a part* B, *the number denominated of that part* C, *meaſureth it.*

Demonſtration. FOr ſeeing that B part of A, is denominated of C, but unity is one part of C denominated of the ſame C, unity ſhall meaſure C, and B ſhall meaſure A equally [a]: Therefore, alternately, unity ſhall meaſure B, and C ſhall meaſure A equally. Therefore, if a number, &c. Which was to be demonſtrated. a) 15. 7.

PROP. 41. PROBL. 6.

To find the least number which hath the parts given A, B, *and* C.

D..2 A Second
E...3 B Third
F....4 C fourth
G.....12
H-----

Construction LEt D, E, and F, be denominated of the parts A, B, and C, and let G [a] be the least that D, E, and F, do measure; I say, that G is the least, having the parts given, A, B, and C.

a) 38. 7.

Demonstration FOr seeing that D, E, and F, do measure G, [b] G shall have the parts denominated by D, E, and F; that is to say, the parts A, B, and C, being denominated of D, E, and F; I say, that G is also the least, having such parts.

b) 39. 1.

40. 7. For if it be denied, let another lesse then G have the same parts A, B, and C, which suppose to be H; Forasmuch then as H hath the part A, B, and C, the numbers D, E, and F, denominated of the same parts A, B, and C, shall measure H; therefore, H being lesse than G; G shall not be the least number measured by D, E, and F, which is contrary to Supposition. Therefore, a lesse number then G, shall not have the parts given, A, B, and C, but G shall be the least: Therefore, &c. Which was to be demonstrated.

The End of the Seventh Element of EUCLIDE.

THE EIGHTH ELEMENT OF EUCLIDE.

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

A.....8 D—
B.....12 E—
C.....18 F—

If there be as many numbers as you please continually proportional: A, B, and C, and that the extreams A and C, be primes to one another, they shall be the least of all those which have the same rate with them.

Demonstration. FOr if it be denied, suppose D, E, and F, lesse then they in the same reason: Forasmuch as there are as many numbers as you please of the one part A, B, and C, and as many of the other, D, E, and F, which taken two and two, are in the same rate [a]; in equal rate, as A shall be to C, so D to F [b]: But A and C are the least of their rate, being primes to one another, by Supposition, [c] therefore as well A, as C, shall measure D and F equally, the greatest, the least, which is impossible: Therefore, A, B, and C, are the least of their rate. Therefore, if, &c. Which was to be demonstrated.

a) 14. 7.
b) 23. 7.
c) 21. 7.

PROP.

PROP. 2. PROBL. 1.

A 2 B 3
C 4 D 6 E 9
F 8 G 12 H 18 I 27

To find as many numbers as you please continually proportional, the least in a given rate.

Construction LEt A multiplying it self produce C, and multiplying B produce D, and B by it self produce E; I say, that C, D, and E, are the least in the rate of A to B, and are continually proportional.

Demonstration FOrasmuch as A multiplying A and B, makes C and D: as A is to B, so C is to D; again, seeing that B multiplying A, B makes D and E, [a] as A is to B, so D is to E: therefore, C, D, and E, are continually proportional in the rate of A to B, and are also the least.

a) 17. 7.

For seeing the extreams C and E, are products of A and B, multiplyed each by himself [b]; But A and B are primes to one another, being the least of their rate, [c] the extreams C and E shall be also primes to one another. Therefore C, D, and E, are the least in the rate of A, B, and C.

b) 24. 7.
c) 29. 7.

Secondly, let A multiplying the three numbers C, D, and E, produce F, G, and H, and B multiplying E make I; I say, that the four numbers F, G, H, and I, are the least in the same given rate of A to B, and continually proportional.

1. 8.

Demonstration FOr seeing that A multiplying C, D, and E, hath produced F, G, and H; F, G, and H, shall have the same rate as C, D, and E, that is to say, as A and B; again, seeing that A and B, multiplying E, have made H and I, [d] as A is to B, so H is to I, therefore F, G, H, and I, are continually proportional in the given rate of A and B, and are also the least.

d) 18. 7.

For A and B, being the least in their rate, [e] they shall be primes to one another: But C and E are products of A and B multiplyed by themselves, and F and I are also products of A and B, in C and E: to wit, F of A in C, and I of B in F, [f] F and I the extreams shall be also primes to one another: Therefore, F, G, H, and I, are the least of their rate, which is the rate of A to B, by the same reason we shall find 5 or 6, &c. Therefore, we have found, &c. which was to be done.

e) 24. 7.
f) 29. 7.
1. 8.

COROLLARIE. I.

It follows hence that, if three numbers are continually proportional, and the least of their rate, the extreams shall be squares: and if 4 be continually proportional, the extreames shall be Cubes, for C and E the extreams, are products of A and B, each by himself, and F and I, extreams of four proportionals are products of A and B, multiplyed each by his square C and D, &c.

CO-

COROLLARIE II.

It follows also that the extreams of the numbers continually proportional, found to be the least in the given rate, (by this Proposition) are Primes to one another, [a] *it having been demonstrated that* C E *and* F I, *are Primes to one another.* a) 24, 29. 7.

COROLLARIE III.

It appears also that two numbers, the least in the given rate, do measure all the means whatsoever of the least in the same rate, being products of the multiplication of them by some other numbers, as D *the mean, which is produced of* A *by* B, *and* G H *means of* A, *by* D *and* E, *or of* B *by* C *and* D, *&c.*

PROP. 3. THEOR. 2.

A 8 B 12 C 18 D 27
E 2 F 3
G 4 H 6 I 9
K 8 L 12 M 18 N 27

If there be as many numbers as you please continually proportional A, B, C, *and* D, *the least of all those that have the same rate with them, the extreams* A *and* D, *are primes to one another.*

Demonstration. FOr [a] let E and F be found the least in the rate of A to B, or B to C, or C to D, [b] E and F, shall be primes to one another: again, let the three G, H, and I, be found the least in the rate of E to F: and also the four K, L, M and N, and so following, until that K, L, M, and N, be equal in number to A, B, C, and D: For asmuch, as A, B, C, and D, are the least of their rate, and K, L, M, and N, equal in number to them, being also the least in the same rate, they shall be equal to them, each to his correspondent; to wit, A to K, and D to N, [c] now K and N are primes to one another: Therefore, A and D are also primes to one another. Therefore, if there be, &c. Which was to be demonstrated.

a) 35. 7. b) 2. 8. c) 2 Cor. 7. 8.

PROP. 4. PROBL. 2.

A 6 B 5 C 4 D 3
H 4 F 24 E 20 G 15
I — K — L —

A 6 B 5 C 4 D 3 E 5 F 7
H 24 G 20 I 15 K 21
L — D — N — O —

A 6 B 5 C 4 D 3 E 2 F 7
H 24 G 20 I 15
M 48 L 40 K 30 N 105
O — P — Q — R —

There being given as many rates as you please A *to* B, *and* C *to* D, *from their least numbers, to find as many numbers as you please continually proportional, the least according to the given rate.*

Con-

Conſtruction HAving [a] found E the leaſt number meaſured by B and C, 2 and 3; let F be found as oftentimes meaſured by A, as E by B, and G as oftentimes meaſured by D, as E by C: F, E, and G, ſhall be continually proportional, the leaſt according to the given rate.

Demonſtration FOraſmuch as A and B do equally meaſure F and E; that is to ſay, by one and the ſame number; let them meaſure it by H, that being, [b] A and B multiplying H, ſhall produce F and E; wherefore, as A is to B, ſo F is to E. In like manner, C and D, equally meaſuring E and G, as C to D, ſo E ſhall be to G; therefore, F, E, and G, are continually proportional, according to the rates of A to B, and C to D. I ſay, that they are alſo the leaſt. For if it be denied, ſuppoſe I, K, and L, to be leſſe in the ſame rates, each to his correſpondent. Foraſmuch then as A and B are the leaſt in their rate, [c] the ſame ſhall equally meaſure I and K; being in the ſame rate; to wit, B conſequent, ſhall meaſure K conſequent: by the ſame reaſon C and D ſhall equally meaſure K and L; to wit, C antecedent, K antecedent: Therefore, B and C, meaſuring K and E, [d] the leaſt, meaſured by B and C, ſhall alſo meaſure K, the greateſt, the leaſt, which is abſurd, therefore F, E, and G, are the leaſt.

A 6 B 5 C 4 D 3
H 4 F 24 E 20 G 15
I — K — L —

A 6 B 5 C 4 D 3 E 5 F 7
H 24 G 20 I 15 K 21
L — D — N — O —

A 6 B 5 C 4 D 3 E 2 F 7
H 24 G 20 I 15
M 48 L 40 K 30 N 105
O — P — Q — R —

Let there be three rates given to the leaſt numbers of A to B, C to D, and of E to F, to find four numbers, the leaſt continually proportional, according to the given rates, [e] having again found G the leaſt that meaſureth B, and C 2 and 3; let H be found as often times meaſured by A, as G by B, and I as oftentimes meaſured by D, as G by C, that being ſo, E ſhall meaſure I, or ſhall not meaſure it. Let it meaſure it in the firſt place, and as often times as I is meaſured by E, let K be as often times meaſured by F. I ſay, that H, G, I, and K, are continually proportional, and the leaſt, according to the given rates, as hath been ſhewn.

Now let E not meaſure I, and [f] having found K the leaſt that E and I do meaſure, let L be as often times meaſured by G, and M by H, as K by I; and let alſo N be as often times meaſured by F, as K by E; I ſay, that M, L, K, and N, are the leaſt according to the given rates.

Demonſtration FOr, H, G, and I, meaſuring M, L, and K, equally, as is before ſhewn, M ſhall be to L, as H to G; and L to K, as G to I: but by the ſame reaſon, H is to G, as A to B, and G to I, as C to D; (A and B meaſuring H and G, and C and D, G and I, equally,) therefore, as A to B, ſo is M to L, and as C to D, ſo is L to K. But as E is to F, K is alſo to N: (E and F equally meaſuring K and N,) therefore, M, L, K, and N, are continually proportional, according to the given rates.

They are alſo the leaſt; For if it be denied, let O, P, Q, and R, be ſuppoſed the leaſt according to the ſame rates. Foraſmuch as A and B are the leaſt of their rate, [g] they ſhall equally meaſure O and P in the ſame rate; to wit, B conſequent, ſhall meaſure P conſequent, and by the ſame reaſon, C and D, ſhall meaſure P and Q equally; to wit, C antecedent, P antecedent: wherefore, B and C meaſuring P, [h] G the leaſt number meaſured by B and C, ſhall meaſure the ſame P.

Now

Now having ſhewn, that as G is to I, ſo is L to K: that is to ſay, ſo is P to Q: alternately, as G to P, ſo I to Q: and therefore G meaſuring P, I ſhall meaſure Q: But E ſhall alſo meaſure Q, [i] ſeeing that E and F, the leaſt of their rate, do equally meaſure Q and R, in the ſame rate; to wit, the antecedent E, the antecedent Q: therefore, I and E, meaſuring Q and K, the leaſt number meaſured by I and E, ſhall alſo meaſure Q, the greateſt, the leaſt, which is abſurd: Wherefore M, L, K, and N, are the leaſt: and ſo following, if there be more rates given to the leaſt numbers, &c. Therefore, being given, &c. Which was to be demonſtrated. i) 21. 7.

PROP. 5. THEOR. 3.

A 24 B 48
G 18
C 4 D 6 E 3 F 19

Plain numbers A *and* B, *are the one to the other in a rate compounded of their ſides.*

Demonſtration FOr D multiplying E, produce G, foraſmuch as D multiplying C and E, hath produced A and G, A ſhall be to G, as C to E: and ſeeing that E multiplying D and F, hath produced G and B; [a] G ſhall be to B, as D to F: therefore, A, G, and B, are continually proportional, according to the rates of C to E, and of D to F the ſides. But [b] the rate of A to B, is compounded of the rates of A to G, and of G to B: therefore, the ſame rate of A to B, ſhall be compounded of the rates of C to E, and of D to F, the ſides. Therefore, the numbers, &c. Which was to be demonſtrated. 17. 7. a) 17. 7. b) 27. d. 7.

PROP. 6. THEOR. 4.

A 16 B 24 C 36 D 54
F 4 G 9 H 6

If there be as many numbers as you pleaſe, continually proportional, A, B, C, *and* D, *and that the firſt* A, *meaſure not the ſecond* B, *alſo neither of the other, ſhall meaſure any of the other.*

Demonſtration FOr that neither of them doth meaſure his next following, is manifeſt, foraſmuch as A, B, C, and D, are continually proportional, and as A meaſureth not B, [a] it ſhall be no part thereof, but parts: wherefore [b] B, which is the ſame parts of C, ſhall not alſo meaſure C, nor C and D: I ſay alſo, that neither of the other, ſhall meaſure any of the other, as A meaſureth not C. [c] For having found F, G, and H, the leaſt in the rate of A to B, or of A, B, and C, in equal rate, as A to C, ſo is F to H: but ſeeing that as A is to B, ſo F to G, & as A meaſureth not B, F ſhall not meaſure G, wherefore F ſhall not be unity, otherwiſe F ſhould meaſure G, unity meaſuring every number: therefore, [d] F and H, being primes to one another, & F being not unity, F ſhall not meaſure H, and therefore alſo A ſhall not meaſure a) 3, 4. d. 7. b) 20. d. 7. c) 2. 8. 35. 7. d) 3. 8.

C: for it hath been ſhewn that as A is to C, ſo F is to H; in like manner, is ſhewn that as B meaſureth not D, taking four numbers in the rate of A to B, and the leaſt, not any other ſhall meaſure any other. Therefore, &c. Which was to be demonſtrated.

PROP. 7. THEOR. 5.

A 3 B 6 C 12 D 24 E 48

If there be as many numbers as you pleaſe, A, B, C, D, *and* E, *continually proportional, and that the firſt* A, *meaſureth the laſt* E, *it ſhall alſo meaſure the ſecond* B.

Demonſtration. FOr if A be not ſaid to meaſure B the ſecond, [a] alſo neither of the other ſhall meaſure either of the other: therefore A the firſt ſhall not meaſure E the laſt, which is abſurd, A being put to meaſure E. Therefore, if there be, &c. Which was to be demonſtrated.

a) 6. 8.

PROP. 8. THEOR. 6.

A 24 C 36 D 54 B 81
G 8 H 12 I 18 K 27
E 32 L 48 M 72 F 108

If between two numbers A *and* B, *there fall mean proportionals* C *and* D, *in continual proportion: as many as ſhall fall between thoſe (of means continually proportional) there ſhall fall as many mean proportionals between two others that ſhall have the ſame rate.*

Demonſtration. FOr having taken G, H, I, and K, the leaſt in the rate of A to C, and equal in number to A, C, D, and B, in equal rate, as A ſhall be to B: and therefore, as E to F, ſo G to K [a]: wherefore, G and K, being primes to one another: ſeeing they are the extreams of the leaſt numbers, [b] and ſo the leaſt of their rate: [c] G ſhall meaſure E equally, and K ſhall meaſure F: therefore, as often times as G and K ſhall meaſure E and F, let there be two other numbers L and M, as many times meaſured by H and I, in ſuch ſort as G, H, I, and K, may meaſure E, L, M, and F, equally, each his correſpondent; [d] wherefore, G, H, I, and K, multiplying the number by the which they meaſure E, L, M, and F, they ſhall produce E, L, M, and F: therefore, E, L, M, and F, ſhall be in the ſame rate as G, H, I, and K, but G, H, I, and K, are continually proportional: therefore E, L, M, and F, ſhall be ſo alſo, and being equal in number to A, B, C, and D, there ſhall fall as many mean proportionals between E and F, as between A and B. Therefore if, &c. Which was to be demonſtrated.

2. 8.
a) 3. 8.
b) 23. 7.
c) 21. 7.
d) 9. c ſ.

PROP.

PROP. 9. THEOR. 7.

A 8 C 12 D 18 B 27
Unity 1
E 2 F 3
G 4 H 6 I 9
K 8 L 12 M 18 N 27

If two numbers A *and* B, *are primes to one another, and between them there fall mean numbers, continually proportional,* C, *and* D, *as many mean continual proportionals as ſhall fall between them, there ſhall alſo fall as many between each of them and unity.*

Demonſtration. FOr having propoſed unity, let E and F be found the leaſt in the rate of A to C, and let the three G, H, and I, be taken in the ſame rate, and then the four K, L, M, and N, and ſo on until the numbers taken be as many as A, C, D, and B. Now A and B the extreams, being primes to one another, A, C, D, and B, ſhall be the leaſt in the rate of E to F: But K, L, M, and N, which are as many as A, C, D, and B, are alſo the leaſt in the ſame rate, by the conſtruction, therefore, K, L, M, and N, are equal to A, C, D, and B, each to his correſpondent, that is to ſay, K to A, and N to B, to the end there be none leſſer than the leaſt. 2. 8. 1. 8.

But, (as is manifeſt by the demonſtration of the ſecond Prop. 8.) E multiplying it ſelfe, hath produced G, and multiplying G hath produced K; E ſhall [a] meaſure G by E, and G ſhall alſo meaſure K by E: But [b] unity meaſures E by E, therefore unity meaſures E, and E meaſures G, and G meaſures K equally; and therefore, unity is the ſame part of E, and E of G, as G of K; therefore, unity and E, G, and K, are continually proportional: in like manner, unity and F, I, and N, are continually proportional: Therefore, as well E, G, and K, as F, I, and N, with unity, being equal in number to K, L, M, and N, there ſhall fall as many numbers continually proportional between unity and K, or his equal A; and between unity and N, or his equal B, as between A and B. Therefore, if two, &c. Which was to be demonſtrated. a) 7. c. ſ. b) 5. c. ſ. 20. d.

PROP. 10. THEOR. 8.

A 8 I 12 K 18 B 27
E 4 H 6 G 9
D 2 F f 3
C 1

If between two numbers A *and* B, *and unity* C, *there fall numbers continually proportional, as many as ſhall fall of continual proportionals between each of them and unity, there ſhall as many continual proportionals fall between them* A *and* B.

Demonſtration. LEt A be the product of D in F, and I the product of D in H, and K the product of F in H, C being to D, as D to E, and

a) 5. c. ſ. D to E, as E to A: [a] and C meaſuring D by the unites that are in D, D
b) 9. c. ſ. ſhall meaſure E, alſo by the unites which are in D, [b] therefore, D by it ſelf ſhall produce E, and E by D ſhall produce A; in like manner, F by himſelf ſhall produce G, and G by F ſhall produce B.

c) 17. 7. A 8 I 12 K 18 B 27
E 4 H 6 G 9
D 2 F 3
C 1

Foraſmuch then as D multiplying D and F, produceth E and H, [c] as D is to F, ſo E is to H: by the ſame reaſon, ſeeing that F multiplying F and D, hath made H and G, as D is to F, ſo H is to G: therefore, E, H, and G, are continually proportional.

d) 17. 7. Again, D multiplying E and A, hath made A and I; therefore, as E [d] ſhall be to H, ſo A to I: and ſeeing that D and F multiplying H, makes I
e) 18. 7. and K, [e] as D ſhall be to F, ſo I to K: and in [f] like manner, K ſhall be to B,
f) 17. 7. as H to G: therefore A, I, K, and B, are continually proportional: therefore between A and B, there doth fall two meanes continually proportional: to wit, as many as between A and unity, or B and the ſame unity. Therefore, &c. Which was to be demonſtrated.

PROP. 11. THEOR. 9.

A 9 E 21 B 49
C 3 D 7

Between two ſquare numbers A, *and* B, *there is a mean proportional number: and the ſquare* A, *is to the ſquare* B, *in a double rate of the ſide* C, *to the ſide* D.

Demonſtration FOr let E be the product of C in D, or of D in C, foraſmuch as C by himſelf hath made A
a) 18. d. [a] a ſquare nnmber, and
b) 17. 7. by D hath made E, [b] as C is to D, ſo A is to E: again, ſeeing that D multiplying C hath made E, and multiplying it ſelf, hath made B a ſquare
18. d. number, as C ſhall be to D, ſo E to B: therefore, A, E, and B, ſhall be con-
17. 7. tinually proportional in the rate of their ſides C and D: Therefore, between A and B there falls a mean proportional number E.

c) 6. d. 7. 10. p. 5. Secondly, ſeeing that A, E, and B, are continually proportional, [c] A the firſt, ſhall be to B the third in a double rate of A the firſt, to E the ſecond, which is the ſame rate of the ſide C. Therefore, between, &c. Which was to be demonſtrated.

PROP. 12. THEOR. 10.

A 27 H 36 I 48 B 64
E 6 G 12 F 16
C 3 D 4

Between two Cube numbers A *and* B, *there are two numbers mean proportional,* H *and* I, *and the Cube* A, *is to the Cube* B, *in a Triple rate of the ſide* C, *to the ſide* D.

Demon-

Demonſtration FOr C multiplying it ſelf, makes E; and D multiplying it ſelf makes F; and C and D, multiplying one another makes G, and multiplying G makes H and I.

Foraſmuch as C multiplying C and D, hath made E and G, [a] as C ſhall be to D, ſo E to G: In like manner, ſeeing that D multiplying D and C, hath made G and F, [b] as C is to D, ſo G is to F: therefore, E, F, and G, are continually proportional in the rate of C to D. a) 17. 7. b) 17. 7.

Again, foraſmuch as C multiplying E, hath made A, a Cube number, and multiplying G hath made H, by the conſtruction, [c] A ſhall be to H, as E to G, that is to ſay, as C to D: in like manner, ſeeing that by the conſtruction, D multiplying G, hath made I, and multiplying F hath made the Cube B [d], I ſhall be to B, as G to F [e], that is to ſay, as C to D. But, [f] H is alſo to I, as C to D: foraſmuch as C and D multiplying G, have made H and I: Therefore, A, H, I, and B, are continually proportional in the rate of C to D: therefore, between the Cubes A and B, there do fall two mean proportionals H and I. 19. d. c) 17. 7. d) 19. d. e) 17. 7. f) 18. 7.

Secondly, foraſmuch as A, H, I, and B, are continually proportional, A the firſt ſhall be to B the fourth, in a Triple rate of A the firſt, to H the ſecond, that is to ſay, of the ſide C, to the ſide D: for C is to D, as A to H. Therefore, between two Cubes, &c. Which was to be demouſtrated.

PROP. 13. THEOR. 11.

If there be as many numbers as you pleaſe continually proportional, A, B, *and* C, *and that each multiplying it ſelf, make others, their products* D, E, *and* F, *ſhall be proportional, and if the numbers firſt taken* A, B, *and* C, *multiplying their products* D, E, *and* F, *do make others* G, H, *and* I, *thoſe ſhall alſo be proportional, and alwayes this happens about the extreams.*

A 2 B 4 C 8
D 4 N 8 E 16 O 32 F 64
G 8 P 16 Q 32 H 64 R 128 S 256 I 512

Demonſtration FOr let A and B multiplying one another make N, and B and C make O: then let A multiplying N and E, make P and Q, and B multiplying O and F, make R and S.

Foraſmuch as A multiplying A and B hath made D and N, [a] as A to B, ſo D to N, by the ſame reaſon, ſeeing that B multiplying A and B, hath made N and E, as A is to B, ſo N to E: therefore, D, N, and E, are continually proportional in the rate of A to B. a) 17. 7.

Again, ſeeing that B multiplying B and C, hath made E and O, [b] as B is C, ſo E is to O: in like manner, ſeeing that C multiplying B and C, hath made O and F, as B is to C, ſo O is to F; therefore, E, O, and F, are continually proportional in the rate of B to C, or of A to B: therefore, D, N, and b) 17. 7.

and E, being continually proportional in the ſame rate: in equal rate, as D to E, ſo E to F: therefore, D, E, and F, are continually proportional.

Moreover, ſeeing that A multiplying D, N, and E, hath made G, P, and Q; G, P, and Q ſhall be in the ſame rate as D, N, and E: that is to ſay, as A to B. And ſeeing that A and B multiplying E, have made Q and H,
c) 18. 7. [c] as A is to B, ſo Q is to H; therefore, G, P, Q, and H, are proportional in the rate of A to B: in like manner, ſeeing that B multiplying E, O, and F, hath made H, R, and S, H, R, and S, ſhall be proportionals in the rate of E, O, and F, that is to ſay, of B to C, or of A to B.

d) 18. 7. Laſtly, ſeeing that B and C, multiplying F, have made S and I, [d] S ſhall be to I, as B to C, or A to B: therefore, H, R, S, and I, are proportionals in the rate of A to B: therefore, ſeeing that G, P, Q, and H, are alſo in continual proportion in the rate of H, R, and I, in equal rate, as G ſhall be to H, ſo H to I: therefore G, H, and I, are continually proportional. Therefore, if there be, &c. Which was to be demonſtrated.

PROP. 14. THEOR. 12.

A 4 E 12 B 36
C 2 D 6

If a ſquare number A, *doth meaſure a ſquare number* B, *alſo the ſide* C, *ſhall meaſure the ſide* D, *and if the ſide* C *doth meaſure the ſide* D, *alſo the ſquare* A, *ſhall meaſure the ſquare* B.

Demonſtration FOr let C and D multiplying one another, make E, foraſmuch as C multiplying C and D, hath made A and E,
a) 17. 7. [a] as C ſhall be to D, ſo A to E: in like manner, ſeeing that D multiplying C
b) 17. 7. and D, hath made E and B, E [b] ſhall be to B, as C to D: that is to ſay, as
7. 8. A to E: therefore, A, E, and B, are continually proportional in the rate of C to D. But A the firſt, meaſureth B the laſt: therefore, A the firſt, ſhall alſo meaſure E the ſecond: therefore ſeeing that C is to D, as A to E, and A meaſureth E, therefore the ſide C, ſhall meaſure the ſide D.

Secondly, let the ſide C, meaſure the ſide D: I ſay, that the ſquare A,
c) 11. 8. meaſureth the ſquare B: [c] (for as hath been demonſtrated, A, E, and B, are continually proportional in the rate of C to D, but C meaſures D: there-
d) 11 c. ſ. fore A ſhall meaſure E, and E ſhall meaſure B: [d] therefore alſo A ſhall meaſure B. Therefore, if one number, &c. Which was to be demonſtrated.

PROP. 15. THEOR. 13.

A 8 H 24 I 22 B 216
E 4 G 12 F 36
C 2 D 6

If a cube number A, *meaſure a cube number* B, *alſo the ſide* C, *ſhall meaſure the ſide* D, *and if the ſide* C, *meaſure the ſide* D, *alſo the cube* A, *ſhall meaſure the cube* B.

Demon-

Demonſtration FOr let C and D, each multiplying it ſelf, make E and F, and multiplying one another make G, and laſtly, multiplying G, make H and I: Foraſmuch then, (as by the demonſtration of the 12 Prop. 8.) as well E, G, and F, as A, H, I, and B, are continually proportional in the rate of C to D: But A the firſt, meaſureth B the laſt, [a] A the firſt ſhall meaſure alſo H the ſecond; therefore A being to H, as C to D: C the ſide ſhall alſo meaſure D the ſide. a) 7. 8.

Now let the ſide C, meaſure the ſide D: I ſay, that the cube A ſhall meaſure the cube B: For by the reaſon above, as C is to D: ſo A is to H, A, H, I, and B, being continually proportional in the rate of C to D; as hath been ſhewn in the 12 Prop. 8. Wherefore C the ſide, meaſuring the ſide D, alſo A, ſhall meaſure H: but H meaſures I, and I meaſures the cube B: (being continually proportional:) [b] therefore A ſhall alſo meaſure the cube B. Therefore, if a number, &c. Which was to be demonſtrated. b) 11 c. 8.

PROP. 16. THEOR. 14.

A 16 B 81
C 4 D 9

If a ſquare number A, *doth not meaſure a ſquare number* B, *alſo the ſide* C, *ſhall not meaſure the ſide* D, *and if the ſide meaſure not the ſide alſo, the ſquare ſhall not meaſure the ſquare.*

Demonſtration FOr ſuppoſe the ſide C doth meaſure the ſide D: [a] therefore the ſquare A ſhall meaſure the ſquare B, which is abſurd: for it is propoſed not to meaſure it, wherefore C the ſide ſhall not meaſure the ſide D. a) 14. 8.

Now I ſay, that if the ſide C doth not meaſure the ſide D, that alſo the ſquare A ſhall not meaſure the ſquare B: for if A be ſaid to meaſure B, the ſide C ſhall alſo meaſure the ſide D, which is abſurd, for it is propoſed not to meaſure it: Therefore, the ſquare A ſhall not meaſure the ſquare B. Therefore, if a ſquare number, &c. Which was to be demonſtrated. 14. 8.

PROP. 17. THEOR. 15.

A 8 B 27
C 2 D 3

If a cube number A, *meaſure not a cube number* B, *alſo the ſide* C, *ſhall not meaſure the ſide* D, *and if the ſide meaſure not the ſide, neither ſhall the cube meaſure the cube.*

Demonſtration FOr ſuppoſe that the ſide C doth meaſure the ſide D, [a] therefore the cube A, ſhall meaſure the cube B, which is abſurd: for it is propoſed not to meaſure it: therefore, C ſhall not meaſure D. a) 15. 8.

Now

Now, as the ſide C meaſureth not the ſide D; I ſay, that the cube A, ſhall not meaſure the cube B. For if A be ſaid to meaſure B, the ſide C b) 15. 8. ſhall alſo meaſure the ſide D, which is abſurd: For it is propoſed not to meaſure it: Therefore the cube A, ſhall not meaſure the cube B. Wherefore if a cube number, &c. Which was to be demonſtrated.

PROP. 18. THEOR. 16.

A 13 G 18 B 27
C 6 D 2 E 9 F 3

Between two Plain numbers alike, A *and* B, *there is a mean proportional number* G, *and the Plain* A *is to the Plain* B, *in a double rate of the Homologal ſides (or ſides of like rate)* C E, *and* D F.

Demonſtration FOr let D and E multiplying one another, make G; foraſmuch then as C is to D, ſo E is to F, alternately, C ſhall be to E, as D to F: and ſeeing that D multiplying C and E, hath made A and G, a) 17. 7. a A ſhall be to G, as C to E: that is to ſay, as D to F: and ſeeing that E multiplying D and F hath made G and B, b) 17. 7. b G ſhall be to B, as D to F: therefore A, G, and B, are continually proportional in the rate of C to E: or of D to F: therefore between A and B, there is a mean proportional G. But foraſmuch as A, G, and B, are continually proportional, c) 26. d. 7. c A is to B, in a double rate of A to G; that is to ſay, of C to E, or of D to F: Therefore, the Plain A, is to the Plain B, in a double rate of C to E, or of D to F, ſides of one and the ſame rate: Therefore, between, &c. Which was to be demonſtrated.

PROP. 19. THEOR. 17.

Between two Solid numbers alike A *and* B, *there are two mean proportional numbers* M *and* N, *and the Solid* A, *is to the Solid* B, *in a triple rate of the Homologal ſide* C, *to the Homologal ſide* F.

A 30 M 60 N 120 B 240
I 6 L 12 K 24
C 2 D 3 E 5 F 4 G 6 H 10

Demonſtration FOr let C and D, multiplying one another, make I; and F and G, make K; and D and F, make L: Laſtly, E and H, multiplying L, make M and N: Foraſmuch then as C, D, and E, are proportional to F, G, and H: alternately, they ſhall be alſo proportional; to wit, as C to F, ſo D to G, and E to H: and ſeeiug that D multiplying C and F, hath made I and L, a) 17. 7. a as C is to F, ſo is I to L: in like manner, ſeeing that F multiplying D and G, hath made L and K; as D is to G, ſo is L to K; wherefore I, L, and K, are continually proportional in the rate of C to

to F, D to G, or of E to H: And foraſmuch as A ſolid, is produced from the mutual multiplication of the ſides C, D, and E: and I is made of C in D, or of D in C; E multiplying I, ſhall make A; in like manner, B ſolid, being made of the mutual multiplication of F, G, and H; and K of F in G, or of G in F, H multiplying K, ſhall make B.

Wherefore, ſeeing that E multiplying I and L, hath made A and M, [b] A ſhall be to M, as I to L: that is to ſay, as C to F, or D to G, or E to H, by the ſame reaſon, ſeeing that H multiplying L and K, hath made N and B, [c] N ſhall be to B, as L to K: that is to ſay, as C to F, or D to G, or E to H: But [d] M is to N, as E to H, ſeeing that E and H multiplying L, have made M and N: therefore, A, M, N, and B, are continually proportional in the rate of C to F, or of D to G, or E to H: therefore between A and B, like ſolids, there falls two mean numbers continually proportional.

Secondly, ſeeing that A, M, N, and B, are continually proportional, [e] A is to B, in a triple rate of A to M: But A is to M, as C is to F, or D to G, or E to H: therefore A ſhall be to B in a triple rate of C to F, or of D to G, or of E to H, ſides of like rate: Therefore, between two Solids, &c. Which was to be demonſtrated.

b) 17. 7.
c) 17. 7.
d) 18. 7.
e) 10. d. 5. 26. d. 7.

PROP. 20. THEOR. 18.

A 18 C 24 B 32
D 3 E 4 F 6 G 8

If between two numbers A *and* B, *there fall a mean proportional number* C, *thoſe numbers* A *and* B, *ſhall be like Plaines.*

Demonſtration FOr having taken D and E, the leaſt in the rate of A, C, and B; [a] D and E ſhall equally meaſure A and C: Let them meaſure them by F, they ſhall alſo equally meaſure C and B, in the ſame rate, which let be by G: therefore, [b] F multiplying D and E, ſhall produce A & C, and [c] G multiplying D & E, ſhall produce C and B: Foraſmuch then as E multiplying F and G hath produced C and B, [d] as C ſhall be to B, ſo F to G, but as C is to B, ſo was D to E: therefore, D ſhall be to E, as F to G, and by permutation, as D to F, ſo E to G: But foraſmuch as F multiplying D, hath made A, A ſhall be a Plain number, and the ſides ſhall be D and F: in like manner, ſeeing that G multiplying E, hath made B, B ſhall be alſo a Plain whoſe ſides ſhall be E and G, and thoſe ſides, being ſhewn to be proportional: to wit, as D to F; ſo E to G, [e] A and B ſhall be like Plaines: Therefore, if between, &c. Which was to be demonſtrated.

a) 21. 7.
b) 9. c. f.
c) 9. c. f.
d) 17. 7.
e) 21. d. 7.

PROP. 21. THEOR. 19.

A 8 C 12 D 18 B 27
E 4 F 6 G 9
H 2 I 2 M 2 K 3 L 3 N 3.

If between two numbers A *and* B, *there fall two mean proportionals* C *and* D, *they ſhall be like Solids* A *and* B.

Demonſtration LEt [a] E, F, and G, be the leaſt in the rate of A, C, D, and B, foraſmuch as between E and G, there falls a mean proportional F; [b] E and G ſhall be like Plaines.

a) 2. 8.

b) 20. 8.

Let H and I be ſides of E and K, and L the ſide of G, and foraſmuch as E, F, and G, doe meaſure A, C, and D, equally [c], being the leaſt in the ſame rate: Let them meaſure them by M, and [d] ſeeing that E, F, and G, do alſo meaſure C, D, and B; by the ſame reaſon, let it be by N: in ſuch manner, as M multiplying E, F, and G, may make A, C, and D; and N alſo multiplying E, F, and G, may make C, D, and B. Now E is produced of his ſides H and I, multiplyed by one another: and G is produced of the ſides K and L: therefore, A is the product of H, I, and M, multiplyed together, and B the product of K, L, and N: Therefore, [e] A is a ſolid number, having the ſides H, I, and M, and B alſo a ſolid number, having the ſides K, L, and N: But ſeeing that M and N, multiplying F, do make C and D, as is ſhewn, [f] C ſhall be to D, as M to N [g]; but C and D, are in the ſame rate as E and F: ſeeing that N multiplying E and F, hath made C and D, and alſo E and F, are in the ſame rate as H and K, or I and L: therefore, H ſhall be to K, and I to L, as M to N, and alternately, H ſhall be to I, as K to L, and I to M, as L to N: Therefore the ſides H, I, and M, are proportional to the ſides K, L, and M: [h] Therefore, the numbers A and B, are like Solids: Therefore, if between, &c. Which was to be demonſtrated.

c) 21. 7.

d) 21. 7.

e) 21. d. 7.

f) 18. 7.

g) 17. 7.

h) 21. d. 7.

PROP. 22. THEOR. 20.

A 9 B 54 C 324

If three numbers A, B, *and* C, *be continually proportional, and that the firſt* A, *be a ſquare, the third* C, *ſhall alſo be a ſquare.*

Demonſtration FOr ſeeing that between A and D [a], there fals a mean proportional, A and C ſhall be like Plaines; therefore A being a ſquare, C reſembling it, ſhall be alſo a ſquare: Therefore, if there be three numbers, &c. Which was to be demonſtrated.

a) 20. 8.

PROP.

PROP. 23. THEOR. 21.

A 27 B 45 C 75 D 125

If four numbers A, B, C, *and* D, *are proportional, and that the first* A, *be a Cube, the fourth* D, *ſhall alſo be a Cube.*

Demonſtration FOr ſeeing that between A and D, there falls two mean proportionals, B and C, [a] A and D ſhall be like Solids. Therefore, A being a cube, D reſembling him, ſhall be a cube alſo: Therefore, if four numbers, &c. Which was to be demonſtrated. a) 21. 8.

PROP. 24. THEOR. 22.

A 36 F 48 B 64
C 9 E 12 D 16

If two numbers A *and* B, *be in the ſame rate to one another, as a ſquare number* C, *to a ſquare number* D, *and if the firſt* A *be a ſquare, the ſecond* B, *ſhall alſo be a ſquare.*

Demonſtration FOraſmuch as C and D, are ſquares, [a] there ſhall fall between them a mean proportional: which let be E: [b] there will alſo fall one between A and B, which are in the ſame rate, which let be F: Foraſmuch as the three numbers A, F, and B, are continually proportional, and A the firſt is a ſquare, [c] B the third ſhall be alſo a ſquare. Therefore, if two numbers, &c. Which was to be demonſtrated. a) 11. 8. b) 8. 8. c) 22. 8.

COROLLARIE.

It is manifeſt from what is above demonſtrated that the proportion of every ſquare number, to any other number whatſoever that is not ſquare, cannot be exhibited in two ſquare numbers: for if they could, [d] *then the two firſt numbers which have the ſame rate as the ſquares of the proportion exhibited, ſhould be ſquares alſo: ſeeing that the firſt is propoſed a ſquare, which is abſurd, for the ſecond is propoſed no ſquare, whence it follows, that the numbers which are in a double rate, are not to one another as a ſquare number, to a ſqaare number: for all theſe double numbers here* 4, 8, 16, 32, 64, 128, *&c. ſhould be ſquares, for the firſt number* 4 *being a ſquare,* [c] 8 *ſhould be alſo a ſquare, and alſo* 16, *and* 32, *&c. which is impoſſible.* d) 24. 8. c) 24. 8.

For between 4 *and* 8, *between* 8 *and* 16, *between* 16 *and* 32, *&c. there would fall a mean proportional, if they were ſquares; ſeeing that it hath been formerly ſhewn, that between any numbers that bear a double proportion to one another, there cannot fall a mean proportional.*

In like manner, the numbers which are in a quintuple rate to one another, ſhall not have the ſame rate as a ſquare number to ſquare number, for if it ſhould, f *there would fall between them mean proportional.*

f) 11. 8. 18. 8.

Therefore, there would alſo fall a mean proportional between 5 *and* 1, *which are the leaſt nnmbers in a quintuple rate: but we have ſhewn the contrary.*

PROP. 25. THEOR. 23.

A 8 G 12 H 18 B 27
C 64 E 96 F 144 D 216

If two numbers A *and* B, *have the ſame rate the one to the other, as one cube number* C, *to another cube number* D, *and that the firſt* A, *be a cube, the ſecond* B, *ſhall be a cube alſo.*

Demonſtration FOr aſmuch as A is to B, as C is to D, but [a] between the cubes C and D, there falls two mean proportionals E and F: therefore between A and B, there ſhall alſo fall two mean proportionals G and H; therefore, foraſmuch as the four numbers A, G, H, and B, are continually proportional, and A the firſt is a cube, [b] B the fourth, ſhall be alſo a cube. Therefore, if two numbers, &c. Which was to be demonſtrated.

a) 12. 8. 8. 8.

b) 23. 8.

COROLLARIE.

It is manifeſt by what is above ſhewn, that the rate of any cube number whatſoever, to any other number whatſoever, not cube, cannot be found in two cube numbers: for if you ſuppoſe it could, the firſt numbers having the ſame rate as the cubes of the proportion found, ſhould be alſo cubes: ſeeing the firſt is propoſed a cube, which is abſurd, for the ſecond is propoſed no cube.

25. 8.

PROP.

PROP. 26. THEOR. 24.

A 20 C 30 B 45
D 4 E 6 F 9

Plain numbers A *and* B, *have the ſame rate to one another, as a ſquare number* D, *to a ſquare number* F.

Demonſtration FOr ſeeing that A and B are like Plaines, [a] their ſhall fall between them a mean proportional which ſhall be C. a) 18. 8.

Therefore, having taken the three numbers D, E, and F, the leaſt in the rate of A, C and B, the extreams D and F ſhall be ſquares: Therefore, ſeeing that in equal rate A is to B, as D to F: it is manifeſt, that A ſhall be to B, as a ſquare to a ſquare; to wit, as the ſquare number D, to the ſquare number F. Therefore the Plains, &c. Which was to be demonſtrated. 1 Cor. 2, 8

PROP. 27. THEOR. 25.

A 16 C 24 D 36 B 54
E 8 F 12 G 18 H 27

Like Solid numbers A *and* B, *have the ſame rate to one another, as a cube number* E, *to a cube number* H.

Demonſtration FOraſmuch as A and B are like Solids [a] there will fall beween them two mean proportionals C and D: And having taken the four numbers E, F, G, and H, the leaſt in the rate of A, C, D, and B, the extreams E and H ſhall be cubes: Therefore ſeeing that in equal rate A is to B, as E is to H, it appears that A ſhall be to B, as the cube number E, to another cube number; to wit, to H: Therefore the, &c. Which was to be demonſtrated. a) 19. 8. Cor. 1. 2, 8.

COROLLARIE. I.

If two numbers be in the ſame proportion the one to the other that a ſquare number is to a ſquare number: thoſe two numbers ſhall be like ſuperficial numbers. And if they be in the ſame proportion the one to the other, that a cube number is to a cube number, they ſhall be like ſolid numbers.

B 72 D 64
A 18 C 16

First, let the number A, have unto the number B, the ſame proportion, that the ſquare number C hath to the ſquare number D: Then I ſay, that A and B are like ſuperficial numbers. For, foraſmuch as between the ſquare numbers C and D, there falleth a mean proportional, [a] there ſhall alſo between A and B, (which have the ſame a) 11. 8.

same proportion with C and D) fall a mean proportional [b]. Wherefore A and B are like superficial numbers [c].

But if A be unto B, as the cube number C, is to the cube number D. Then are A and B like solid numbers. For, forasmuch as C and D are cube numbers, there falleth between them two mean proportional numbers [d]. And therefore, [e] between A and B, (which are in the same proportion that C is to D) there falleth also two mean proportional numbers. Wherefore [f] A and B, are like solid numbers.

B 16 C 8
A 56 D 27

b) 8. 8.
c) 20. 8.
d) 12. 8.
e) 8. 8.
f) 21. 8.

COROLLARIE. II.

If a number multiplying a square number, produce not a square number, the number multiplying shall be no square number.

For If it should be a square number, then should it and the number multiplyed, being like superficial numbers (by reason they are square numbers) have a mean proportional [a]. And the number produced of the said mean, should be equal to the number contained under the extreams, which are [b] square numbers. Wherefore, the number produced of the extreams, being equal to the square number produced of the mean, should be a square number. But the said number by supposition, is no square number. Wherefore neither is the number multiplying the square number, a square number.

a) 18. 8.
b) 20. 7.

The first part of the first Corollarie is the converse of the 26 Proposition of this book, and hath some use in the tenth book. The second part of the same also, is the converse of the 27 Proposition of the same.

The End of the Eighth Element of EUCLIDE.

THE NINTH ELEMENT OF EUCLIDE.

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

A 6 B 54
D 36 E 108 C 324

If two numbers being like Plains A, *and* B, *multiplying one another, do produce some one* C, *the product* C *shall be a square.*

Demonstration. LEt D be the product of A by it selfe, D shall be a square number: Therefore, forasmuch as A multiplying A and B, hath produced D and C; [a] as A shall be to B, so D shall be to C: But between A and B, like Plains, [b] there falls a mean proportional, therefore also [c] there shall fall one between D and C, which shall be E, to the end that D, C, and E, may be continually proportional: But D the first, is a Square by the construction; therefore [d] C the third, shall be also a square: Therefore if two plain numbers, &c. Which was to be demonstrated.

a) 17.7.
b) 18.8.
c) 8.8.
d) 22.8.

PROP. 2. THEOR. 2.

A 6 B 54
D 36 C 324

If two numbers multiplying one the other, A *and* B, *make a square* C, *they are like Plains.*

Demon-

Demonſtration FOr let D be the product of A by it ſelf, D ſhall be a ſquare, foraſmuch then as A multiplying A and B hath produced D and C, [a] as A is to B, ſo D is to C. But [b] between D and C ſquare numbers there fals a mean proportional, wherefore [c] between A and B there ſhall alſo fall a mean proportional, therefore [d] A and B are like Planes. Therefore, if two numbers &c. Which was to be demonſtrated.

A 6. B 54.
D 36. C 324.

a) 17. 7.
b) 11. 8.
c) 8. 8.
d) 20. 8.

PROP. 3. THEOR. 3.

If a Cube number A, *multiplying it ſelf, doth produce any one* B, *the product* B *ſhall be a Cube.*

A 8.
E 16. D 4.
F 32. C 2.
B 64. Unity.

Demonſtration LEt C be the ſide of the cube A, and let D be the product of C by it ſelf, and ſo the cube A ſhall be the product of C by D; foraſmuch then as C multiplying it ſelf hath made D [a] the ſame C, ſhall meaſure D by C: but [b] unity ſhall alſo meaſure C, by C, therefore, unity ſhall be the ſame part of C denominated of C, as C of D: and therefore, [c] as unity is to C, ſo is C to D.

Again, Foraſmuch as C, multiplying D, hath made A, [d] D ſhall meaſure A, by C: but C meaſures D by C, therefore C ſhall be the ſame part of D, as D is of A, therefore [e] as C is to D, ſo D is to A: but as C, is to D, ſo unity was to C, therefore as unity, is to C, ſo is, C to D, and D, to A, therefore between unity and A, there doth fall two mean proportionals C and D.

But [f] foraſmuch as A meaſure B by A, (for multiplying it ſelf makes B) and [g] unity meaſures A, by A, unity ſhall be the ſame part of A, as A of B, and therefore unity [h] ſhall be to A, as A to B, and therefore ſeeing that between unity and A, there falls two mean proportionals C and D, [i] there ſhall fall as many between A and B, which ſhall be E and F.

Therefore, ſeeing that the four numbers A, E, F, and B, are continually proportional, and A the firſt is a cube, B the fourth ſhall be in like manner a cube. Therefore, if a cube, &c. Which was to be demonſtrated.

a) 7. c. ſ.
b) 5. c. ſ.
c) 20. d.
d) 7. c. ſ.
e) 20 d.
f) 7. c. ſ.
g) 5. c. ſ.
h) 20 d.
i) 8. 8.
23. 8.

PROP. 4. THEOR. 4.

If a cube number A, *multiplying a cube, number* B, *makes ſome one* C, *the product* C *ſhall be a cube.*

A 8. B 27.
D 64. C 216.

Demonſtration LEt A multiplying it ſelf make D, D [a] ſhall be a cube, and foraſmuch as A multiplying A and B, hath made D and C, [b] as A ſhall be to B, ſo D ſhall be to C. But between the cubes A and B, there

a) 39.
b) 17. 7.

there fals two mean proportionals, c therefore, between D and C, there shall also fall two mean proportionals, therefore, d D being a cube, C shall be a cube also. Therefore if, &c. Which was to be demonstrated.

c) 12. 8. 8. 8.
d) 23. 8.

PROP. 5. THEOR. 5.



If a cube number A *multiplying some number* B *doth produce a cube* C, *the number multiplyed* B, *shall be a cube also.*

Demonstration FOr let D, be the product of A, multiplyed by it self: a D shall be a cube.

a) 3 9.

Forasmuch as A, multiplying A and B, hath made D and C, b as A, shall be to B, so shall D, be to C. c But between the cubes D and C, there falls 2 mean proportionals, d therefore, between A and B, there shall also fall 2 mean proportionals, e and therefore, A being a cube, B shall be also a cube. Therefore, if a cube number, &c. Which was to be demonstrated.

b) 17. 7.
c) 12. 8.
d) 8. 8.
e) 22. 8.

PROP. 6. PROBL. 6.

A 8 B 64 C 512 *If a number* A, *multiplying it self makes a cube* B, *the same* A, *shall be also a cube.*

Demonstration FOr let C, be the product of B by A, C shall be a cube number, as is manifest. Forasmuch then as B, cube multiplying another number, to wit, A hath produced C, a cube number, A shall be also a cube. Therefore, if a number, &c. Which was to be demonstrated.

19. d.
5. 9.

PROP. 7. THEOR. 7.

A 6 B 11 C 66 D 2 E 3 *If a compound number* A, *multiplying, some number* B, *makes some other* C, *the product* C, *shall be a Solid.*

Demonstration FOr seeing that A is compounded, some other number besides unity shall measure it: Therefore, let D measure A, by E, that being done, a D multiplying E, shall produce A. Therefore, seeing that B, multiplying the same A hath made C; C shall be produced by the mutual multiplication of the three numbers D, E, and B, and

13. d.
a) 9. c. f.

therefore, C ſhall be a Solid, whoſe ſides are D, E, and B. Therefore, if a compound number, &c. Which was to be demonſtrated.

17. d.

PROP. 8. THEOR. 8.

If from unity, as many numbers as you pleaſe A, B, C, D, E, F, G, H, I, K, L, *and* M, *are continually proportional, the third* B *from unity is a ſquare, and all the reſt which intermit, or leave out one* D, F, H, K, *and* M, *But the fourth is a cube* C, *and all the reſt in leaving two,* F, I, *and* M, *and the ſeventh,* F *is a cube, and a ſquare together, and all the reſt which leave five* M.

Unity. A 3 B 9 C 27 D 81 E 243 F 729.
G 2187 H 6561 I 19683 K 59049
L 177147 M 531441.

Demonſtration FOraſmuch as unity, is to A, as A is to B, unity ſhall meaſure A, and A ſhall meaſure B equally. [a] But unity meaſures A, by A, therefore, A ſhall alſo meaſure B, by A: [b] therefore, A multiplying A, ſhall produce B, therefore, B is a ſquare.

a) 5. c. 5.
b) 9. c. 5.

Now, foraſmuch [c] as B, C, and D, are continually proportional, and B is a ſquare D ſhall be alſo a ſquare: in like manner ſeeing that D, E, and F, are continually proportional, and as D is a ſquare, F ſhall be alſo a ſquare, and all the others leaving one.

c) 22. 8.

Again, Foraſmuch as unity is to A, as B to C, unity ſhall meaſure A and B ſhall meaſure C equally. But [d] unity meaſureth A by A: therefore, B ſhall alſo meaſure C, by A, [e] and therefore, B multiplying, A ſhall produce C, therefore, ſeeing that A multiplying it ſelf makes B, and multiplying B makes C, [f] C ſhall be a cube.

d) 5. c. 5.
e) 9. c. 5.
f) 17. d. 7.

But foraſmuch as C, D, E, and F, are continually proportion, and as C the firſt is a cube, F the fourth [g] ſhall be alſo a cube, by the ſame reaſon ſeeing that F, G, H, and I, are continually proportional, and that F is a cube, I, ſhall be alſo a cube, and all the reſt leaving or intermitting two. Laſtly, foraſmuch as F the ſeventh, from unity, is ſhewn to be a ſquare and a cube; the ſame ſhall be a cube, and a ſquare together, and in the ſame manner, M the ſeventh from F, leaving G, H, I, K, and L, ſhall be a cube, and a ſquare together, and alſo all the others, which leave 5. Therefore, if from unity, &c. Which was to be demonſtrated.

g) 23. 8.

PRO-

PROP. 9. THEOR. 9.

If from Unity there be as many numbers as you please A, B, C, D, E, *and* F, *continually proportional; and if that number which follows unity* A, *be a square, all the others also shall be squares, and if that which follows unity be a cube, also all the others shall be cubes.*

Unity A 4 B 16 C 64 D 256 E 1024 F 4096
Unity A 8 B 64 C 512 D 4096 E 32768 F 262144

Demonstration FOr seeing that A, B, C, D, E, and F, are continually proportional from unity, [a] B the third from unity, is a square, and all the rest leaving one: to wit, D and F, and forasmuch as A, B, and C, are continually proportional and that A is a square by the Supposition, [b] C the third shall be also a square: in like manner, taking C, D, and E continually proportional, seeing that C the first is a square, E the third, shall be also a square: therefore, all the numbers A, B, C, D, E, and F, are squares, and so of the rest.

a) 8. 9.
b) 22. 8.
22. 8.

Now let A next to unity be a Cube; I say, that the others are Cubes.

Forasmuch as from unity A, B, C, D, E, and F, are continually proportional, [c] C the fourth from unity is a cube, and all the others leaving two; to wit F, now that the others B, D, and E, are also cubes, we shall thus demonstrate.

c) 8. 9.

Forasmuch as unity is to A, as A to B, unity shall measure A, and A the number B, equally [d]; but unity shall measure A by the same A, therefore A shall measure B also by A [e]: therefore, A multiplying it self, shall produce B: But, [f] A is a cube, therefore B shall be also a cube; and forasmuch as A, B, C, and D, are continually proportional, and as A is a cube, so D shall be also a cube: Moreover, seeing that B, C, D, and E, are continually proportional, and as B is a cube, so shall E [h] also be a cube: therefore A, B, C, D, E, and F, are all cubes, and so on, if there were more. Therefore, if from unity, &c. Which was to be demonstrated.

d) 5. c. 7.
e) 9. c. 7.
f) 3. 9.
g) 23. 8.
h) 23. 8.

PROP. 10. THEOR. 10.

If from unity there be as many numbers as you please A, B, C, D, E, F, G, H, *and* I, *continually proportional; and if that number* A, *which follows after unity be no square; also not any one of the others shall be a square, except the third* B *from unity, and all the others who leave one* D, E, *and* F, *and if that which is after unity* A, *be not a Cube; also not any one of the others shall be a Cube besides the fourth from unity* C, *and all the others who leave two.*

Unity A 2 B 4 C 8 D 16 E 32 F 64
G 128 H 256 I 512

Demonſtration FOr if it be ſuppoſed that beſides thoſe there are other ſquares: Let E be ſuppoſed to be a ſquare; foraſmuch
a) 8. 9. then as [a] D is a ſquare, and that as D is to E, or E to F, ſo A is to B, and by converſion, as E is to D, or F to E, ſo B to A; B ſhall be to A,
8. 9. as the ſquare number E, to the ſquare number D, or as the ſquare F, to
b) 24. 8. the ſquare E: But B the third from unity, is a ſquare, therefore [b] A the firſt after unity, ſhall be a ſquare, which is abſurd, for by Suppoſition it is no ſquare: therefore E ſhall not be a ſquare, by the ſame reaſon we ſhall ſhew that not one other is a ſquare, beſides the numbers above mentioned.

Now let A next to unity be no cube; I ſay that not any other is a cube, beſides C the fourth from unity, and all thoſe that do leave two, to wit, F,
8. 9. and I, for if you ſay, that ſome other is a cube, ſuppoſe it to be D: therefore ſeeing that F is a cube, and as in equality as D is to F, ſo A is to C: ſeeing that D, E, F, and A, B, C, are in the ſame rate, and by converſion as F to D, ſo C to A; C ſhall be to A, as the cube number F, to the cube
c) 8. 9. number D; but [c] C the fourth from unity, is a cube, therefore [d] A the firſt
d) 25. 8. after unity, ſhall be alſo a cube, which is againſt the Suppoſition: Therefore D is no cube, by the ſame reaſons may be ſhewn that none but the fore-mentioned can be cubes. Therefore, &c. Which was to be demonſtrated.

PROP. 11. THEOR. 11.

If from Unity there be as many numbers as you pleaſe A, B, C, D, E, F, *and* G, *continually proportional, the leaſt doth meaſure the greateſt,* C *meaſures* G *by ſome one of thoſe which are between the proportional numbers* A, B, C, D, E, F, *and* G.

Unity A 3 B 9 C 27 D 81 E 243 F 729 G 2187

Demonſtration FOraſmuch as by equality, as C is to G, ſo unity is to D, (for C, D, E, F, and G, are in the ſame rate continued as unity,
a) 20. d. and A, B, C, and D) [a] unity ſhall meaſure D; and C the number G equal-
b) 5. c. f. ly: But [b] unity meaſureth D, by the ſame D, therefore C ſhall alſo meaſure G by D: by the ſame reaſon E ſhall meaſure F by A: ſeeing that as E is to F, ſo unity is to A, &c. ſo A ſhall meaſure G by F: ſeeing that by equality as A is to G, ſo unity is to F, &c. and ſo on: Therefore, if, &c. Which was to be demonſtrated.

PROP.

PROP. 12. THEOR. 12.

If from Unity there be as many numbers as you please A, B, C, *and* D, *all the prime numbers which measure the last number* D, *the same numbers shall also measure that which is next to Unity* A.

Unity A 6 B 36 C 216 D 1296 E 3

Demonstration FOr let E be a prime number, which shall measure the last D: I say, E shall also measure A: for if it shall not measure it, [a] the same E shall be prime to A: Forasmuch then as A and E are primes to one another, and A multiplying it self, makes B, as appears; [b] B shall be prime to E: therefore, seeing that A and B, are primes to E, and C is the product of A in B, [c] C shall be prime to E: again, seeing that A and C are alike primes to E, and that D is the product of A in C, [d] D shall be prime to E: therefore E measures not D, which is against the Supposition, therefore, E shall measure A: Therefore, &c. Which was to be demonstrated.

a) 31. 7.
b) 27. 7.
c) 26. 7.
d) 26. 7.

COROLLARIE.

Therefore; any prime number, greater then that next Unity, nor any lesser that measurtth not that which is next unity, shall not measure the last: for if it did measure it, it would also measure that which is next Unity, as hath been demonstrated, which is against the Supposition,

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PROP. 13. THEOR. 13.

Unity A 5 B 25 C 125 D 625
H---- G---- F---- E----

If from Unity there be as many numbers as you please continually proportional, A, B, C, *and* D, *and if that number next after Unity* A, *be prime, any other shall not measure the greatest* D, *beside those* A, B, *and* C, *that are between the proportionals.*

Demonstration FOr (if possible) let some other, as E, measure D, E shall be a prime, or a compound number; if a prime, and measuring D the extream, it shall also measure A, the prime next to Unity, which is absurd; therefore, E is not prime, but compounded, and [b] therefore measured by some prime number, which cannot be any other then A: for E measuring

12. 9.
b) 33. 7.

ſuring D, every prime number meaſuring E, ſhall alſo meaſure D. But
c) 12. 9. meaſuring D the greater, c it ſhall alſo meaſure A, the prime next to unity, which is abſurd: Therefore, any other prime number then A, ſhall not meaſures E, for as A meaſures E by F: therefore A multiplying F, makes

Unity A 5 B 25 C 125 D 625
H---- G---- F---- E----

d) 17. 7. E and D: and therefore d E ſhall be to D as F to C, wherefore E meaſuring D, F ſhall meaſure C; and in like manner, it ſhall be ſhewn, that F is no Prime, but a compound number, which ſhall be demonſtrated, as is before ſaid to be only meaſured by the prime A.

Let A then meaſure F by G, therefore A multiplying G and B, produ-
e) 17. 7. ceth F and C; wherefore e F ſhall be to C, as G to B; but F meaſureth C: therefore G doth alſo meaſure B.

f) 12. 9. Again, if G be a prime number meaſuring B, it ſhall f alſo meaſure the prime number A, which is abſurd: therefore G is compounded, which ſhall be only meaſured by A, as hath been demonſtrated of E: Let A then meaſure G by H; wherefore A multiplying A and H, ſhall make B and G: Therefore as G is to B, ſo H is to A; but G meaſures B; therefore H ſhall alſo meaſure A the firſt: wherefore H is equal to the ſame A.

g) 20. 7. But A is a mean proportional between H and G: ſeeing that g B the product of the extreams G and H, is equal to the ſame B, the ſquare of A the mean, which is impoſſible: Therefore any other number beſides A, B, and
h) 11. 9. C, ſhall not meaſure D the greateſt number, of which the leaſt h doth alwayes meaſure the greateſt: Therefore, if from unity, &c. Which was to be demonſtrated.

PROP. 14. THEOR. 14.

A 30 B 2 C 3 D 5
E---- F---

The leaſt number A, *meaſured by certaine prime numbers* B, C, *and* D, *ſhall not be meaſured by any other prime number then by thoſe* B, C, *and* D, *which meaſured it at firſt.*

Demonſtration FOr ſuppoſe that E a prime number differing from B, C, and D, doth meaſure A by F: foraſmuch as E meaſures A by F,
a) 9. c. ſ. E a multiplying F, ſhall make A, wherefore b each of the numbers B, C,
b) 32. 7. and D, ſhould meaſure one of the two E and F, they ſhall not meaſure E prime, other then by theſe; therefore, F which is leſſe then A, ſhall meaſure it, which is abſurd; for A is propoſed the leaſt, which may be meaſured by the prime numbers B, C, and D: Therefore, the leaſt, &c. Which was to be demonſtrated.

PROP.

PROP. 15. THEOR. 15.

A 9 B 12 C 16
D 3 E 4

If three numbers A, B, *and* C, *be continually proportional, and the least of all those which have the same rate with them the number* B, *compounded of any two of them shall be prime to the number resting.*

Demonstration FOr having taken D and E, the least in the same rate, it appears by the demonstration of 2 Prop. 8. that A is square to D: and C square to E, and B the product of D in E; and the number compounded of D and E multiplyed by D, shall produce a number equal to the number compounded of A and B: but [b] D and E, being primes to one another, [c] the number compounded of them shall be prime to E: and therefore [d] also prime to C, the square of E, and by the same reason D shall be prime to C, and the nnmber produced of the number compounded of D and E, multiplyed by D, shall be also prime to C, which product is shewn to be equal to the number compounded of A and B: therefore, the number compounded of A and B, is prime to C.

b) 24. 7.
c) 30. 7.
d) 27. 7. 26. 7.

Again, the number compounded of D and E, being prime as well to D, as to E, the product of the same compound number multiplyed by E, shall be also prime to D; but the same product is equal to C, the square of E, and to B together, the product of D in E: therefore, the numbers compounded of B and C, shall be also prime to D, and [e] therefore prime to A the square of D.

27. 7. 3. 9.
e) 27. 7.

Lastly, [f] seeing that D and E, are primes to one another, and [g] primes to their compound, B their product, [h] shall be also a prime to their compound; [i] and the product of the number compounded of D and E, multiplyed by it self, shall be prime to B: but the same product is equal to A and C, and to twice B together; therefore, the compound of A and C, and of twice B, is prime to B; then taking away twice B, the remainder of the compound of A and C, shall be prime to B: For if it were not so, they should be compounded to one another, and their common measure measuring the compound of A, C, and B, should also measure the compound of A, and C, and of twice B. Wherefore the compound of A, and C, and twice B, should not be prime to B, which is absurd, they having been demonstrated primes to one another: Therefore the compound of A, and C, is prime to B: Therefore, if three numbers, &c. Which was to be demonstrated.

f) 24. 7.
g) 30. 7.
h) 26. 7.
i) 27. 7.

10. c. ſ.

PROP. 16. THEOR. 16.

A 5 B 7 C----

If two numbers A *and* B, *are primes to one another: as the first* A, *is to the second* B, *so the second* B, *shall not be to any other.*

Demon-

Demonſtration FOr (if poſſible) as A is to B, ſo let B, be to another, to wit, C.

a) 23.7. b) 21.7. Foraſmuch then, as A and B, are primes to one another, and [a] the leaſt that are in the ſame rate, [b] they ſhall equally meaſure B and C being in the ſame rate: to wit, A ſhall meaſure B, and B ſhall meaſure C: But A meaſureth alſo it ſelf, therefore A ſhall meaſure A and B, primes to one another, which is abſurd: Therefore, as A is to B, ſo B is not to C.

In like manner, as B is to A, ſo A ſhall not be to another: Therefore, if two numbers, &c. Which was to be demonſtrated.

PROP. 17. THEOR. 17.

If there be as many numbers as you pleaſe A, B, C, *and* D, *continually proportional; and that the extreams* A, *and* D, *are primes to one another, as the firſt* A, *ſhall be to the ſecond* B, *ſo the laſt* D, *ſhall not be to ſome other.*

A 8 B 12 C 18 D 27 E----

Demonſtration FOr if it be denied, ſuppoſe as A is to B, ſo D may be to ſome other; to wit, to E: Foraſmuch then as A is to B, ſo D is to E, alternately, as A is to D, ſo B is to E: But A and D, being primes to one another, are [a] the leaſt in their rate; therefore [b] they ſhall meaſure B and E equally; to wit, A ſhall meaſure B, and D ſhall meaſure E: But as A is to B, ſo B is to C: therefore ſeeing that A meaſures B, alſo B ſhall meaſure C, and [c] therefore A ſhall meaſure C alſo, and ſeeing that as B is to C, ſo C is to D: But B meaſureth C, and C ſhall alſo meaſure D: wherefore A meaſuring C [d], ſhall in like manner meaſure D, and A meaſureth it ſelf; therefore A meaſureth A, and D, primes to one another, which is impoſſible, therefore as A is to B, ſo is not D, to ſome other number; to wit, to E: in like manner, as D is to C, ſo is not A to another: Therefore, if there be, &c. Which was to be demonſtrated.

a) 23.7. b) 21.7. c) 11.c.f. d) 11.c.f.

PROP. 18. PROBL. 1.

A 4 B 7
A 4 B 6 D 9 C 36
A 6 B 4 D 00 C 16

Two numbers A *and* B, *being given, to conſider if it be poſſible to find a third proportional to them.*

Demonſtration THe numbers A and B, are primes to one another, or not primes to one another, if primes, then [a] there can be no third proportional found; if they are not primes, let B multiplying it ſelf, make C. Now either A meaſureth C, or doth not meaſure it: if it meaſure it,

a) 16.9.

it, let it be by D. I say, there may be a third proportional found, which is D, for seeing that A, measureth C, by D, [b] C shall be the product of A, by D: but the same is the product of B, multiplyed in himself: therefore, C, contained under the extreams A and D, is equal to the product of the mean B, multiplyed by it self: therefore, [c] the three A, B, and C, are continually proportional, to wit, as A, is to B, So is B, to D: therefore, D shall be the third proportional.

b) 9. c. f.

c) 20. 7.

But if A doth not measure C, I say, that there cannot be found a third proportional, seeing that as A, is to B: so B, is to D, the number produced of A and D, the extreams, shall be equal to the product of B, the mean multiplyed in it self: that is to say, to C; therefore, seeing that C, is made of A, multiplyed by D, [d] A shall measure C, by D, which is absurd, being proposed not to measure it. Therefore, there cannot be found a third proportional to A and B, seeing that A, a Prime number measureth not C, the product of B, the second multiplyed in it self, &c. Therefore, two numbers, &c. Which was to be done.

d) 7. c. f.

PROP. 19. PROBL. 2.

Three numbers A, B, *and* C, *being given, to consider if it be possible to find a fourth proportional to them.*

A 8 B 12 C 18 E 27 D 216.
A 4 B 8 C 9 E 18 D 72.

A 4 B 6 C 9 E---- D 54
A 3 B 14 C 10 E---- D 40

Construction LEt B, multiplying C, make D, A shall measure D, or shall not measure it: suppose it to measure it by E, I say, that E, is the fourth proportional.

Demonstration SEing that A, measures D, by E, [a] D shall be the product of A, by E: but D is also the product of B by C, by construction, therefore, the product of the extreams A and E, is equal to the product of the means B, and C, therefore, [b] as A, is to B: So is C, to E.

a) 9. c. 5.

b) 19. 7.

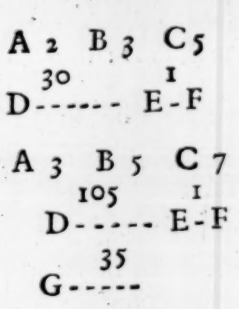
But if A, measures not D, I say, there cannot be found a fourth proportional, for otherwise suppose E, to be a fourth proportional. Forasmuch then as the four numbers A, B, C, and D, are continually proportional, [c] the number contained under the extreams A, and E, shall be equal to the number contained under the means B and C, to wit, to D. Therefore, seeing that D, is made of A, multiplyed by E, [d] the same A, shall measure D, by E, contrary to the supposition. In like manner we may examine, if as A, is to B, so C is to some other number. Therefore, three numbers, &c. Which was to be done.

c) 19. 7.

d) 7. c. f.

PROP. 20. THEOR. 18.

Prime numbers are in a greater multitude then any multitude of prime numbers propoſed. A, B, C.



THat is to ſay; That if you propoſe as many prime numbers as you pleaſe there will yet be found others.

Demonſtration FOr [a] having taken D, the leaſt that may meaſure A, B, and C, and adding unity E F, thereto, the whole D F, is a prime number, or not: if it be a prime number, then you have what is required, for A, B, C, and D F, are in greater number then the multitude propoſed A, B, and C.

a) 38. 7.

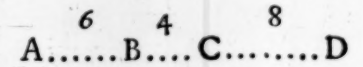
But if D F, be not a prime number, it [b] ſhall be meaſured by ſome, prime number, which let be G. I ſay, that G is not one of the numbers propoſed A, B, C, for if it were one of them, ſeeing that A, B, and C, meaſureth D E, G ſhould alſo meaſure D E; Therefore, meaſuring the whole D F, and the part cut off D E, it ſhould alſo meaſure the remainder E F; to wit, a number ſhould meaſure unity, which is impoſſible.

b) 33. 7.

11. c. ſ.

Therefore the prime number G, is none of the numbers propoſed A, B, C: Therefore, the prime numbers A, B, C, and G, are in a greater number then thoſe propoſed, A, B, and C, and ſo *ad infinitum* may be found other prime numbers: Therefore, The prime, &c. Which was to be demonſtrated.

PROP. 21. THEOR. 19.



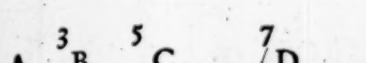
If as many even numbers as you pleaſe A B. B C. C D. *be added; the whole* A D. *ſhall be even.*

Demonſtration FOraſmuch as [a] every even number may be divided in two equal parts, the halves of A B, B C, and C D, ſhall make the halfe of A D: but all the halves together, are equal to all the other halves together; therefore A D is an even number, &c. Which was to be demonſtrated.

a) 6. 7.

PROP.

PROP. 22. THEOR. 20.

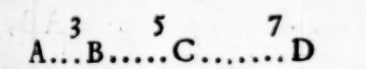


If as many odde numbers as you please A B, B C, C D, *and* D E, *be added, and that the number of them be even: the whole ſhall be even.*

Demonſtration FOr ſeeing that A B, B C, C D, and D E, are odde, each of them ſhall differ from an even number, by unity, according to the Definition: Wherefore if from each you cut off unity, each of the remainders ſhall be even: wherefore [a] the number compounded of them ſhall be even: Therefore [b] the whole A and E compounded ſhall be in like manner even: Therefore, If as, &c. Which was to be demonſtrated.

a) 21. 9.
b) 21. 9.

PROP. 23. THEOR. 21.

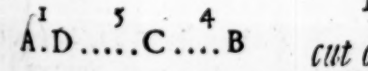


If as many odde numbers as you please A B, B C, *and* C D, *are added, and that the multitude of them be odde, the whole* A D, *ſhall be alſo odde.*

Demonſtration FOr ſeeing that an odde number, differeth from an even number by unity; by the Definition: having cut off unity E D, from the odde number C D, the reſt C E, ſhall be even: But [a] A C compounded of the odde number A B, and B C, even in multitude are even: therefore [b] alſo A E, compounded of the even numbers A C, and C E, ſhall be even, therefore if you adde unity E D, the whole A D, ſhall be odde; ſeeing that even and odde, differ by unity, by the Definition: Therefore, if as many, &c. Which was to be demonſtrated.

a) 21. 9.
b) 21. 9.

PROP. 24. THEOR. 22.



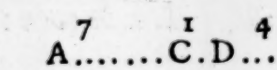
If from an even number A B, *be cut off an even number* C B, *the remainder* A C, *ſhall be alſo an even number.*

Demonſtration FOr if A C be not an even number, let there be cut off unity A D, the reſt D C will remaine even, ſeeing that an even and odde number differeth onely by unity) therefore [a] the componnded number D B, ſhall be an even number: Therefore adding unity A D, to the compounded number D B, an odde number, the whole A B, ſhall be un-

a) 21. 9.

even; which is impoſſible, for it is even by Suppoſition: therefore A C, ſhall not be odde: Therefore if, &c. Which was to be demonſtrated.

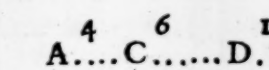
PROP. 25. THEOR. 23.



If from an even number A B, *there be cut off an odde number* C B, *the reſt* A C, *ſhall be alſo odde.*

Demonſtration FOr from C B, having cut off unity C D, the reſt D B, ſhall be even. Therefore, foraſmuch as the whole A B, is pure-
a) 24. 9. ven, the reſt [a] A D, ſhall be even; having therefore taken away unity C D, the reſt A C, ſhall be odde: Therefore, if from a number, &c. Which was to be demonſtrated.

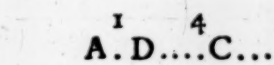
PROP. 26. THEOR. 24.



If from an odde number AB, *be cut off an odde number* C B, *the reſt* A C, *ſhall be even.*

Demonſtration FOr having cut off unity from the odde numbers A B, and C B, the remainders A D, and C D, ſhall be even numbers: foraſmuch then as from the even number A D, the even number C D, is cut
a) 24. 9. off, [a] the reſt A C, ſhall be an even number: Therefore if from, &c. Which was to be demonſtroted.

PROP. 27 PROP. 25.

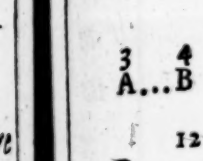


If from an odde number AB, *be cut off an even number* CB, *the reſt* A C, *ſhall be odde.*

Demonſtration FOr, from the odde number A B, let unity A D, be cut, the reſt D B ſhall be even: from which the even number CB,
a) 24. 9. being cut, the [a] reſt D C, ſhall be alſo even: Therefore adding unity A D, A C ſhall be odde: Therefore, if from an, &c. Which was to be demonſtrated.

PROP.

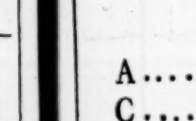
PROP. 28. THEOR. 26.



If an odde number A, *multiplying an even number* B, *produce ſome one* C, *the product* C, *ſhall be even.*

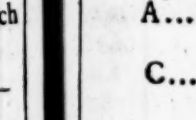
Demonſtration FOr ſeeing that C is the product of A multiplyed by B: the ſame C ſhall be compounded of ſo many numbers equal to B, as there are unites in A: but B is an even number, therefore C ſhall be compounded of as many even numbers equal to B, as there are unites in A; and therefore [a] C ſhall be an even number: Therefore, if an, &c. Which was to be demonſtrated. a) 21. 9.

COROLLARIE.



It follows alſo that an even number multiplyed by it ſelf, produceth an even number, as is manifeſt if you put the even numbers A *and* B, *equal.*

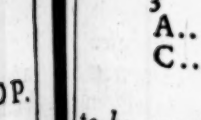
PROP. 29. THEOR. 27.



If an odde number A, *multiplying an odde number* B, *produce one* C, *the product* C, *ſhall be odde.*

Demonſtration FOr ſeeing that C is produced of A in B, the ſame C ſhall be compounded of ſo many numbers equal to B, as there are unites in A: Therefore ſeeing that as well A as B, is an odde number; C ſhall be compounded of as many odde numbers, equal to B, as there are unites in A, the odde number: Wherefore the multitude of them ſhall be odde, and therefore [a] C ſhall be an odde number. Therefore, if an, &c. Which was to be demonſtrated. a) 23. 9.

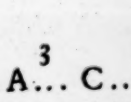
COROLLARIE.



It follows from this that an odde number multiplyed by it ſelf, produceth an odde number, as is manifeſt if the number A *and* B, *are propoſed to be equal.*

PROP.

PROP. 30. THEOR. 28.



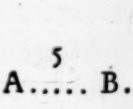
If an odde number A, *measureth an even number* B, *it shall also measure its halfe.*

Demonstration FOr let A measure B by C, the number C shall be even; and therefore shall have a half: Wherefore seeing that A measureth B by C, [a] C shall also measure B by A: and so C shall be a part of B, denominated of A, as is shewn in the third Definition of the Seventh Book, but seeing that as C is to his half, so is B to his half, and alternately, as C is to B, so the half of C, is to the half of B. But C is a part of B, denominated of A, as is shewn; also the half of C, shall be a part of the half of B, denominated of A: therefore [b] A shall measure the half of B: Therefore, if an, &c. Which was to be demonstrated.

a) 8. c. s.

b) 40. 7.

PROP. 31. THEOR. 29.



If an odde number A, *be prime to some number* B, *it shall be prime to its double* C.

Demonstration FOr if A and C, are not primes to one another, they shall be measured by some number, which let be D, which of necessity shall be odde: For if it be even, measuring the odde number A: [a] A the product of D an even number, multiplyed by the number by which it measures it, shall be an even number, which is absurd; for A is proposed odde: Therefore D an odde number, measuring C an even number, (for C is an even number, seeing it hath an half B,) [b] shall also measure B his half: But it shall also measure A, therefore D shall measure A and B, primes to one another, which is absurd; Therefore, A is a prime to C: Therefore, if a number, &c. Which was to be demonstrated.

a) 28. 9.

b) 30. 9.

PROP. 32. THEOR. 30.

Of all the numbers B, C, D, *and* E, *which follow the binary* A, *in a double Progression, each of them is onely evenly even.*

Unity A 2 B 4 C 8 D 16 E 32

Demonstration THat it is so, is manifest; For having proposed unity, seeing that A is a binary, and B, C, D, and E, doubles from the binary, they shall be continually proportional from unity; to wit, in double proportion: Therefore [a] A shall measure each of them, B, C, D, and E, and

a) 11. 9.

and each leſſer ſhall meaſure the greateſt following, by ſome, one of theſe, A, B, C, D, and E, which being all even numbers, to wit, double from the Binary, an even number ſhall meaſure each of theſe B, C, D, and E, by an even number; therefore each of them, ſhall be evenly even, acccording to the Definition.

Now that they are onely evenly even is manifeſt: For ſeeing that A, B, C, and D, are continually proportional from unity, and that A next to unity, is a prime number, to wit, a binary, [b] not any other number, ſhall meaſure each of them, except thoſe A, B, C, D, and E, which being all even, an even number ſhall meaſure each of them only by an even number: Therefore, of all, &c. Which was to be demonſtrated. b) 13. 9.

PROP. 33. THEOR. 31.

10
A.....
5 2
B..... C..
D---- E---

If a number A, *hath its half* B, *odde, the ſame* A, *is evenly odde.*

Demonſtration FOraſmuch as B an odde number, is the half of A, the binary C ſhall meaſure A an even number, by the ſame half B, an odde number; therefore [a] A is evenly odde: I ſay alſo, it is onely ſo: for let B be the half of A, and C a binary, if A be not evenly odde onely, it ſhall be alſo evenly even; therefore ſome even number ſhall meaſure it by an even number: Let D an even number, meaſure it by E, alſo an even number, and ſo [b] A ſhall be the product of D by E: but the ſame A is the product of C the binary, by B the half of A; therefore the number produced of C, the firſt, by B the fourth, is equal to that which is produced of D the ſecond, by E the third: therefore [c] as C is to D, ſo E to B, but C the binary, meaſures D an even number; therefore E an even number, ſhall meaſure alſo B an odde number, which is abſurd; therefore A is not evenly even, but is onely evenly odde. Therefore, if, &c. Which was to be demonſtrated. a) 9. d. b) 9. c. f. c) 19. 7.

PROP. 34. THEOR. 32.

20
A.....

If an even number A, *be none of thoſe that are double from unity, nor hath its half odde, it is evenly even, and evenly odde.*

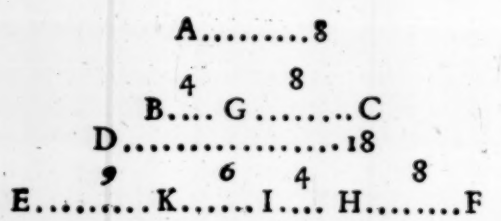
Demonſtration THat it is evenly even, is evident: For ſeeing that its half is even, the binary which is even, ſhall meaſure it by the ſame half, therefore it ſhall be evenly even. 9. d.

It is alſo evenly odde, for the ſame A being divided into two equal parts, and its half again divided in two, and ſo on, you ſhall at laſt come to an odde number (before you come to the binary, ſeeing that it is none of thoſe that are double from unity) which ſhall meaſure A by an even number. For if it ſhould meaſure it by an odde number, (ſeeing [a] that an odde number multiplying a) 29. 9.

plying an odde number, makes an odde number) A ſhould be odde, which is impoſſible, ſeeing that it is propoſed even: therefore ſeeing that an odde number meaſureth A by an even number, and that an even number doth alſo meaſure it by an even number, A ſhall be evenly even, and evenly odde. Therefore, if, &c. Which was to be demonſtrated.

PROP. 35. THEOR. 33.

If there be as many numbers as you pleaſe, continually proportional, A, B C D, *and* E F, *and if there be cut off a number* C G, *from the ſecond* B C, *as alſo from the laſt* EF, *a number* H F, *equal to the firſt, as the exceſſe* B G, *of the ſecond* B C, *ſhall be to the firſt* A, *ſo the exceſſe* E H, *of the laſt* E F, *ſhall be to all the antecedents* A, B C, *and* D.



Demonſtration FOr let F I be cut off equal to B C, and F K equal to D; foraſmuch as F I is equal to BC, and the part cut off F H, is equal to the part cut off C G, the reſt I H ſhall be equal to the reſt B G; but foraſmuch as A is to B C, ſo B C is to D, and D to E F: and by converſion, as E F is to D, ſo D is to B, and B C to A: But K F is equal to D C, & I F, to B C, and H F to A: in like manner, as E F ſhall be to K F, ſo K F to I F, and I F to H F; therefore by dividing, as E K, to K F, ſo K I, to I F, and I H to H F: and
a) 12. 7. a therefore alſo all the numbers E K, K I, and I H, ſhall be to all the numbers K F, I F, and H F, that is to ſay, the whole E H, to D, B C, and A, together, (to which K F, I F, and H F, have been taken equal) as I H, to H F; that is to ſay, as B G to A, for B G and A, are equal to I H and H F: Therefore, if, &c. Which was to be demonſtrated.

PROP. 36. THEOR. 34.

If from unity be taken as many numbers as you pleaſe, A, B, C, *and* D, *continually proportional in a double proportion, until the whole compound* E, *be made a prime number, and that whole* E *multiplyed by the laſt* D, *maketh ſome one* F, *the product* F, *ſhall be a perfect number.*

Demo.

Unity A..2 B....4 C....8 D.....16
E.31 G-----62 H---124 I---248 F----496
K 31 M 31 L 31

O---- P---- N 465

Demonſtration FOr, Let there be taken E, G, H, and I, in a double rate, from E, and equal in multitude to A, B, C, and D, foraſmuch then as A, B, C, and D, and E, G, H, and I, are equal in multitude, and being taken two and two, are in the ſame rate, being all in a double proportion in equal rate, as [a] A ſhall be to D, ſo ſhall E be to I, and therefore, [b] the product of A the firſt, by I, the fourth, ſhall be equal to the product of D, the ſecond, by E the third: But F is the product of D in E: therefore, F ſhall be alſo the product of A in I: therefore [c] I ſhall meaſure F by A the binary, wherefore F ſhall be double to I, and therefore E, G, H, I, and F, are continually proportional in a double rate. a) 14. 7. b) 19. 7. c) 7. c. ſ.

Let there be cut off from G the ſecond, and F the laſt, K and L, equal to E the firſt, whoſe exceſſes are M and N, as [d] M ſhall be to E, ſo N ſhall be to all the antecedents E, G, H, and I, together: But M is equal to E, being the half of G double to E: therefore N ſhall be equal to E, G, H, and I, together. But L is equal to unity, and A, B, C, and D, together, being propoſed equal to E or K, therefore the whole L N, that is to ſay, F is equal to unity, and to the numbers A, B, C, D, E, F, G, H, and I, together: wherefore [e] ſeeing that unity and each of the numbers A, B, C, D, E, F, G, H, and I, do meaſure F, (for ſeeing F is made of E in D, [f] D ſhall meaſure F, and therefore unity and A, B, and C, which meaſureth D, ſhall alſo meaſure F: and again, ſeeing that I, as hath been ſhewn, doth meaſure F, E, G, and H, which meaſureth I: [g] becauſe of the double proportion ſhall alſo meaſure F,) and no other ſhall meaſnre F, as we ſhall preſently ſhew: Unity, and A, B, C, D, E, G, H, and I, ſhall be all the parts that F can have, to which F being ſhewn to be equal, [h] the ſame F ſhall be a perfect number. d) 35. 9. e) 7. c. ſ. f) 11. c. ſ. g) 11. c. ſ.

Suppoſe ſome other number to meaſure F, beſides the aforegoing; to wit, O by P, [i] that being, F ſhall be the product of O by P: but the ſame F is alſo the product of E by D, therefore the number produced of E the firſt, by D the fourth, is the ſame, with the product of P the ſecond, by O the third, therefore [k] as E is to P, ſo O is to D. i) 9. c. ſ. k) 19. 7.

But, [l] ſeeing that A, B, C, and D, are continually proportional from Unity, and that A next to unity, is a prime number, no other number ſhall meaſure D the laſt, beſides thoſe of the proportion A, B, and C. Now O is different from A B and C, by the Suppoſition; therefore O ſhall not meaſure D; but as O is to D, ſo E is to P, therefore alſo neither ſhall E meaſure P: therefore [m] E being a Prime: E and P, ſhall be Primes to one another; and ſo [n] the leaſt of their rate Wherefore E ſhall meaſure O, and P ſhall meaſnre D equally; being in the ſame rate: But, [o] no number doth meaſure D, except A, B, and C, therefore P ſhall be the ſame with one of thoſe A, B, and C. m) 31. 7. n) 23. 7. 21. 7. o) 13. 9.

Suppoſe it the ſame as B, and let E, G, and H, be taken double from E, in ſuch multitude, as B, C, D, [p] in equal rate, as B to D, ſo E to H; [q] but the product of B in H, is equal to the product of D in B, and the product of D in E, hath been ſhewn equal to the product of P in O: therefore the p) 14. 7. q) 19. 7.

r) 19. 7.

number made of P by O, is the same as that which is produced of B by H, therefore r as P is to B, so is H to O; but P was the same with B, therefore H shall be the same with O, which is absurd; for O was put different from any of the numbers A, B, C, D, E, G, H, or I; Therefore, A, B, C, D, E, G, H, and I, do only measure the number F, the which they do compose: Therefore, if from Unity, &c. Which was to be demonstrated.

SCHOLIUM.

Euclide *in these three fore-going Books intends onely Arithmetical Numbers and not continued quantity in which there happens Fractions; but in numbers there happens none, but only in relation to the things numbered when they are divided into other parts, which are called Fractions, insomuch as they are compared to their whole divided into certain number of equal parts, the which are yet nevertheless explained by whole nmbers, that is to say, by the numbers of which* Euclide *speakes in these three foregoing Books; and then when such continued quantities are reduced to the same equal parts, they are no more Fractions, and numbers compared to such things, are compared the one to the other as whole numbers, and are taken absolutely, without referring them to any intire quantity, and all the demonstrations and operations, which may be performed by vulgar Fractions of common Arithmetick, are all performed by Arithmetical and whole numbers, concerning which* Euclide *speaks in these Seventh, Eighth, and Ninth Books.*

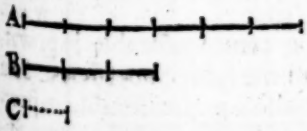
The End of the Ninth Element of EUCLIDE.

THE TENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

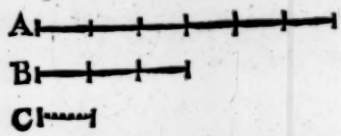
EUclide having finiſhed thoſe things which concern numbers in the three fore-going Books, ſo far forth as was needful for the underſtanding of Geometrical things: Now in this Tenth Book, he attempts the diſputation of commenſurable and incommenſurable Lines, for whoſe cauſe, as we have ſaid, thoſe Books of numbers were undertaken by him. For without the knowledge of theſe Lines, divers Magnitudes, as well Solids, as Plaines, can neither be perfectly underſtood, nor brought into practice when it is required, becauſe divers of their ſides are incommenſurable, which may be ſaid likewiſe of Plains, and Solids themſelves, for they alſo are often times incommenſurable, as we ſhall demonſtrate at the end of this Book.

DEFINITIONS.



1 *Commenſurable magnitudes* A *and* B, *are ſuch as are meaſured by one and the ſame common meaſure* C.

Euclide, according to his cuſtom, begins here by the explication of the terms which he ought to uſe: and declareth in the firſt place what Magnitudes are ſaid to be commenſurable, and which incommenſurable, defining that thoſe are ſaid to be commenſurable, which are meaſured by one and the ſame common meaſure, or Magnitude: As the Magnitudes

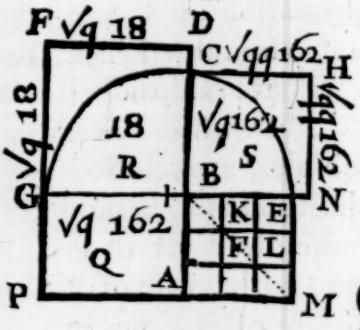


A and B; the which one and the ſame meaſure, as C doth meaſure: (for being repeated ſix times, it meaſureth A, and being repeated three times, it meaſureth B) ſhall be ſaid to be commenſurable.

After the ſame manner, a line of 20 palmes, and one of 13, are ſaid to be commenſurable; ſeeing that as well the line of one palme, as that of half a palme, &c. meaſureth both of them: In like manner, thoſe Superficies are ſaid to be commenſurable that are meaſured by one and the ſame Superficies, and ſolid bodies are alſo ſaid to be commenſurable; when they are meaſured by one and the ſame Solid.

2 *But Magnitudes incommenſurable, are thoſe which have no common meaſure.*

SUch Magnitudes are the Diameter of any ſquare, and the ſide of the ſame ſquare whereof it is the Diameter: foraſmuch as there is no common Magnitude that can meaſure them both, as ſhall be demonſtrated in the laſt Propoſition of this Book: There are alſo divers other lines incommenſurable, divers of which are unfolded in this Book; and the manner how to find them, is ſhewn: Again, Superficies and Solids, are ſaid to be incommenſurable, the which admit of no common meaſure.



3 *Right lines* AB, *and* BD, *are commenſurable in power, when their ſquares* BM, *and* BF, *are meaſured by one and the ſame Space or Superficies* E.

OF Right-lines, ſome are commenſurable in Length; to wit, thoſe which are meaſured by one and the ſame line, and are ſaid ſimply or abſolutely commenſurable, ſome others are ſaid to be incommenſurable; that is to ſay, ſuch as have no common meaſure, as hath been expounded in the firſt and ſecond Definitions.

Again, of Lines which are incommenſurable in Length, ſome of them are ſo, in ſuch manner as that their ſquares are commenſurable; as a line of two feet, and another, which ſhall be √ q. 8. (that is to ſay, of the which the ſquare ſhall contein 8 feet) are ſaid to be commenſurable in power, the ſquares 4, and 8, being meaſured by one and the ſame Superficies. So a line of three feet, and a line of 6 feet, ſhall be alſo commenſurable in power, ſeeing their ſquares 9, and 36, are meaſured by one and the ſame Superficies; to wit, by 3, or by 6.

Again,

Again, a line which ſhall be ✓qq of 20, and one that ſhall be ✓qq 125, are commenſurable in power: for their qq ✓ q 20, and ✓ q 125, are commenſurable, being as 5, to 2, and ✓ q 5, ſhall be in ✓ q 20, twice, and in ✓ q 125 5 times, which will be more eaſily underſtood by the figure, where the line AB is 3, and his q B M 9, whoſe Diameter is alſo B M, to which AB is incommenſurable, as hath been ſaid in the 2 Definition; and having drawn B D equal to the Diameter B M, and deſcribed the ſquare B F, the ſame ſhall be 18; it [a] being double the ſquare of B A, and his ſide ſhall be ✓q 18 commenſurable in power to B A: for B meaſureth both the one and the other ſquares, to wit, B M 9 times, and B F 18 times, E tripled meaſureth alſo B F 6 times, and B M 3 times. a) 47. 1.

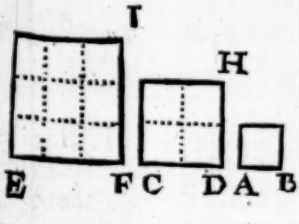
Now if you will have the ſquare E to be ✓ q 6, the q of the line A B, make ✓ q 486, and the q of B D, ſhall make ✓ q 1944, and ✓ q 6, ſhall meaſure both, & ✓ q 54, the triple of ✓q 6, ſhall alſo meaſure thoſe two quantities.

4 *But Right Lines are incommenſurable in power, when the ſquares made of them cannot be meaſured by any common meaſure.*

THe other lines incommenſurable, whoſe ſquares are alſo incommenſurable, are ſaid to be incommenſurable in power, as a line of 2 feet, and a line which ſhall be ✓ ✓6, ſhall be incommenſurable in power, as not having any common ſpace that may meaſure their ſquares 4, and ✓ 6: So ✓q 8, and ✓ qq 12, are alſo incommenſurable in power, for aſmuch as no Superficies can meaſure their Squares 8 and ✓ q 12.

Again, as in the figure of the third Definition, the rectangled Superficies P ↄ, is propoſed ✓q 162, to the which [a] let the ſquare Superficies B H, be made equal, the line G B, or B D, his equal; ſhall be incommenſurable in power to B C, or B N; for B C is a mean proportional between G B, and B A, or B E, equal to B A: therefore G B, B C, and B E, being continually proportional, [b] G D the ſquare of G B, is to B H the ſquare of B C, as G B the firſt, to B E the third: But G B is incommenſurable to B E, as hath been ſaid in the ſecond Definition: therefore [c] the ſquare of G B, which is G D, ſhall be incommenſurable to the ſquare of B C; that is to ſay, to B H: therefore G B, and B C, or B N, his equal, are incommenſurable in power; that is to ſay, their ſquares are incommenſurable: and this may not be thought ſtrange, that this Demonſtration be built on the Propoſitions hereafter mentioned; for that it is not neceſſary for the Demonſtration of any of them. a) 14. 1. b) 20. 6. c) 10.

Now all the right lines commenſurable in Length, are ſo alſo in power, ſeeing that the ſquare of their common meaſure, doth meaſure their ſquares, as appears by this figure: For as A B meaſureth C D, and E F, ſo the ſquare of A B, doth meaſure C H, and E I, not that all the right lines which are commenſurable in power, are ſo alſo in length, as appears by the figure of the third Definition, where the lines A B, and B D, are ſhewn to be incommenſurable in length, the which are ſhewn commenſurable in power, and as it ſhall be alſo demonſtrated in this Book.



5 *Theſe*

5 *These things being so, it is shewn, (that is to say, manifest,) that to every right line proposed, an infinite number of right lines are commensurable, and an infinite number are incommensurable, some in length and power, others onely in power; Now let that right line proposed, be called rational.*

IF there be proposed some right line, of a known, and determinated Magnitude, of all the lines compared with it, some shall be commensurable in length and power thereto, others in power onely, also some shall be incommensurable in length thereto, and some in length and power, as shall be shewn in this Book: But this line proposed, by reason of which, the others are said to be commensurable, or incommensurable, is named by the Greeks ῥητὴ, that is to say, rational; for that it is alwayes proposed of a known Magnitude, and that according thereto it is argued: But the others compared therewith, are not alwayes known: although according to the same lines taken apart, they may be known, each of them being divided into such equal parts as you please.

6 *And the right lines commensurable to this rational line, either in length and power, or in power alone, may be also called rationals.*

Euclide shews that the other lines commensurable, after what manner soever, to the line called *rational*, are also rationals, not by supposition, as the line it selfe: but as being compared therewith, they will be found commensurable thereto in length and power, or in power only.

7 *And the lines incommensurable to this rational line, may be called irrationals.*

SEeing that in the precedent Definition *Euclide* hath called the lines commensurable in length and power, or in power onely to the rational line proposed, Rationals, it is manifest by that Definition, that those lines which are incommensurable to the Rational proposed in length and power, and not in length onely, are Irrationals.

8 *And the square described of the rational line proposed, may be called rational.*

EVen as the line proposed to be known and determinated, is said to be rational: even so the square described thereof, is called rational, forasmuch as that also is known and certaine, and in comparison thereof, the other Superficies are said to be commensurables, or incommensurables.

9 *And*

9 *And the figures commenſurable to this ſqnare rational, may be called rationals.*

ALL the Plain Superficies commenſurable to the ſquare of the rational propoſed, are ſaid to be rationals, yet not by poſition as the ſame, but as being compared to the ſame ſquare, they are found to be commenſurable thereto, or incommenſurable, even ſo alſo as the lines in what manner ſoever commenſurable to the line propoſed rational, are called rationals.

10 *But the figures incommenſurable to the rational ſquare, may be called irrationals, or Surds.*

EVen ſo as the lines incommenſurable to the line propoſed rational, are ſaid to be irrational, ſo alſo the plain Superficies incommenſurable to the ſquare of the rational line, are called irrationals.

11 *And the right lines which are in power as the ſame irrational figures, are ſaid to be irrationals, and Surds; to wit, if they be ſquares, their ſides may be irrational; but if they be other rectilines whatſoever, the lines which are in power, as the ſquares equal to thoſe irrational figures may be irrational.*

HE calls the lines which are equal in power to the ſpaces, incommenſurable to the ſquare of the rational line, irrationals, in ſuch ſort, that if thoſe ſpaces irrational are ſquares, their ſides ſhall be ſaid to be irrational, if they be not ſquares, but are other rectilines; the right lines which deſcribe the ſquares, equal to theſe ſpaces incommenſurable, ſhall be ſaid to be irrational: for a right line is ſaid to be equal in power to a figure, when the ſquare deſcribed of the ſame line is equal to the ſame figure.

PETITIONS.

That any Magnitude whatſoever, may be multiplyed ſo many times, as that it may exceed any Magnitude whatſoever propoſed of the ſame kind.

FOr two Magnitudes unequal, but of the ſame kind being propoſed, the greateſt being not infinite, and the leaſt in power to be augmented infinitely, it is manifeſt, that the leaſt may be multiplyed ſo many times, as that it may exceed the greateſt, which appeares alſo by the fifth definition of the fifth Book.

COM-

COMMON SENTENCES.

1 *A magnitude measuring as many magnitudes as you please, doth also measure the magnitude compounded of them.*

2 *A magnitude which measureth any magnitude whatsoever, doth in like manner, measure every other magnitude, which that magnitude doth measure.*

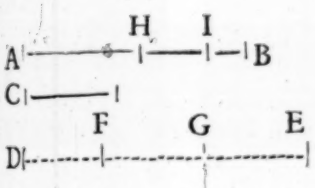
3 *A magnitude measuring a whole magnitude, and the part cut off there-from, doth also measure the rest.*

PRO-

PROPOSITIONS,

PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

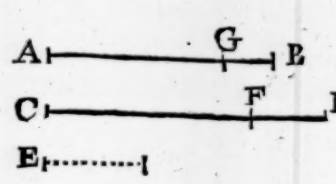


Two unequal magnitudes being proposed, A B, *and* C, *if from the greatest* A B; *there be cut off more then the halfe, and of the rest again, more than the halfe; and if that be done alwayes so, in continuing, there will remain in the end some magnitude lesse then the least magnitude* C *proposed.*

Demonstration FOr let C be multiplyed so many times, as that the product may exceed A B, and let D E be the product, so as that D E may be multiplex of C, and next greater then A B; and let D E be divided according to the parts D F, F G, G E, each equal to C, and from A B, let there be cut off more then the half A H, and of the rest H B, also more then the half H I, and so alwayes until the parts of A B, may be equal in number to those of D E; Let the parts then A H, H I, I B, be as many in number, as D F, F G, G E; Forasmuch as D E is greater then A B, and that from D E there be taken D F, lesse then the halfe, (or the halfe, if D E were onely double to C,) and from A B more then the halfe A H, the rest F E shall be greater then the rest H B.

Again, F E being greater then H B, if from F E be taken the halfe F G, (or lesse then the halfe, if F E were greater then the double of C:) but from H B, more then the half H I, the rest G E shall be in like manner greater then the rest I B; But G E is equal to C, therefore C shall be also greater then I B, the last part of A B: Therefore, of A B there will remaine a magnitude lesse then C. The same will happen, if you take away the halfe of A H, and from the rest H B, the halfe H I, for in like manner, I B the rest shall be lesse than G E, that is to say, then C. Therefore, Two, &c. Which was to be demonstrated.

PROP. 2. THEOR. 2.



Two magnitudes unequal A B *and* C D, *being propoſed, if you cut off ſtill alternately, the leaſt* A B, *from the greateſt* C D, *and that the magnitude remaining, ſhall never meaſure his precedent magnitude, thoſe magnitudes* A B *and* C D, *ſhall be incommenſurable.*

Demonſtration FOr otherwiſe they ſhould be meaſured by ſome common meaſure, which ſuppoſe to be E: Therefore, E ſhall be equal to A B, or leſſe, and having taken A B from C D, as many times as may be, let there reſt F D leſſe then A B, in ſuch ſort as A B meaſures C F.

Again, having taken F D from A B, let there be left G B, leſſe then the ſame F D, in ſuch ſort as that F D meaſure A G, and let this cutting off be ſtill continned, till at laſt [a] there ſhall remaine of C D or A B ſome magnitude leſſe then E.

a) 1. 10.

Foraſmuch as A B taken from C D leaves F D, leſſe then it ſelfe, C F cut off ſhall be greater then the half C D: for if C F were the halfe, or leſſe then the halfe of C D, A B might yet be cut off from C D: in like manner, A G, cut off from A B, ſhall be greater then the halfe of A B: and ſo it ſhall ſtill happen, cutting off ſtill therefore more then the halfe, there will remaine in the end ſome magnitude leſſe then E. Let then G B the reſt be leſſe then E.

b) 2. c. 5.
c) 3. c. 5.
d) 2. c. 5.
e) 3. c. 5.

Foraſmuch as E meaſures A B, and A B meaſures C F, [b] E ſhall alſo meaſure C F: but E meaſures the whole C D: therefore [c] E ſhall alſo meaſure the reſt F D, and F D ſhall meaſure A G: therefore [d] E ſhall alſo meaſure A G: but E meaſures the whole A B; therefore [e] E ſhall alſo meaſure the reſt G B the greateſt, the leaſt, which is abſurd: Therefore no magnitude ſhall meaſure A B and C D; Therefore they are incommenſurables: Therefore, If, &c. Which was to be demonſtrated.

PROP. 3. PROBL. 1.



Two magnitudes commenſurable A B, *and* C D, *being given, to find their greateſt common meaſure.*

Conſtruction FOraſmuch as A B and C D are commenſurable from the greateſt C D: Let there be taken A B the leaſt, as often as you can: and that there remaine E D, and from A B let there be taken E D alſo as often as you may, and there will reſt F B, and doing ſo ſtill

ſtill alternatively, there will at laſt remaine a magnitude, which ſhall meaſure his precedent, otherwiſe A B and C D ſhould be incommenſurable, contrary to the Suppoſition. Let F B then meaſure E D: I ſay, that F B is the greateſt common meaſure of A B and C D.

Demonſtration FOr ſeeing that F B meaſures E D, and E D meaſures A F; [a] F B ſhall meaſure A F: but F B meaſures it ſelfe: therefore, it ſhall alſo meaſure the whole A B: therefore [b] F B ſhall alſo meaſure C E, which is meaſured of A B: therefore ſeeing that F B meaſures alſo E D, [c] the ſame F B ſhall meaſure the whole C D, it ſhall therefore meaſure A B, and C D therefore is their common meaſure. a) 2. c. ſ. 1. c. ſ. b) 2. c. ſ. c) 1. c. ſ.

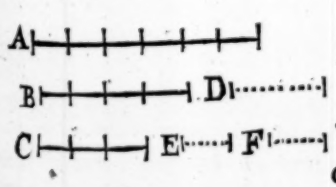
For if it be denied, ſuppoſe G to be the greateſt common meaſure: Foraſmuch then as G meaſures A B and C D, and that A B meaſures C E, [d] G ſhall alſo meaſure C E: but G meaſures C D, therefore [e] G ſhall alſo meaſure E D the reſt; (ſeeing that G meaſures the whole C D, and the part cut off C E:) but E D ſhall meaſure A F: therefore [f] G ſhall alſo meaſure A F, and meaſuring the whole A B, it ſhall alſo meaſure the reſt F B, leſſe then it ſelfe, which is abſurd; therefore F B is the greateſt common meaſure of A B and C D. d) 2. c. ſ. e) 3. c. ſ. f) 2. c. ſ. 3. c. ſ.

Now if the leaſt A B, meaſure the greateſt, in ſuch manner, as being cut off from C D, there ſhall remaine nothing, A B ſhall be the greateſt common meaſure of the two, ſeeing that A B meaſures it ſelfe: Therefore, Two magnitudes, &c. Which was to be done.

COROLLARIE.

From this Demonſtration it is manifeſt, that a magnitude which meaſureth two magnitudes, meaſureth alſo their greateſt common meaſure, as may be gathered from that part of the Demonſtration, by which it is ſhewn that F B *is the greateſt common meaſure of* A B, *and* C D, *for there it is ſhewn that if* G *meaſure* A B *and* C D, *it ſhall alſo meaſure their greateſt common meaſure* F B.

PROP. 4. PROBL. 2.



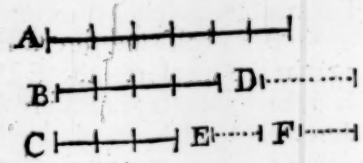
Three magnitudes commenſurable A, B, *and* C, *being given, to find their greateſt common meaſure.*

Conſtruction FInde [a] in the firſt place D, the greateſt common meaſure of A and B, if D alſo meaſure C, it appears that D is the greateſt common meaſure of the three A. B, and C. a) 3. 10.

Demonſtration IF a magnitude greater then D meaſure A, B, and C, the ſame [b] ſhall alſo meaſure D, the greateſt common meaſure b) Co. 3. 10.

of A and B, the greateſt, the leaſt, which is abſurd: but if D meaſure not C at leaſt, C and D ſhall be commenſurable.

For ſeeing that A, B, and C, are commenſurable, any whatſoever, their
c) Co. 3. 10. common meaſure, c ſhall meaſure D the greateſt common meaſure of A and B: therefore the ſame common meaſure meaſuring alſo C, D and C ſhall be commenſurable: Let
d) 3. 10. d E therefore be the greateſt common meaſure of C & D; I ſay, that E is the greateſt common meaſure of A, B, & C; Foraſmuch as E meaſures D and C,
e) 2. c. f. and D meaſures A and B, E e ſhall alſo meaſure A and B; But E alſo meaſures C: therefore E is the greateſt common meaſure of A, B, and C, I ſay, that E is alſo their greateſt oommon meaſure: For otherwiſe, ſuppoſe F a greater common meaſure then E.

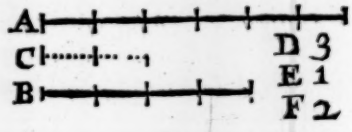


f) Co. 3. 10. Foraſmuch then, as F meaſures A and B, it f ſhall alſo meaſure D their
g) Co. 3. 10. greateſt common meaſure: But F alſo meaſures C, it g ſhall then meaſure alſo E the greateſt common meaſure of C and D, the greateſt, the leaſt, which is abſurd: Therefore, Three, &c. Which was to be done.

COROLLARIE.

It is manifeſt alſo that a magnitude which meaſureth three magnitudes, doth alſo meaſure their greateſt common meaſure. For it hath been demonſtrated that ſeeing that F *meaſures* A, B, *and* C, *it ſhall alſo meaſure their greateſt common meaſure* E: *In like manner, if there be more then three magnitudes commenſurable given, their greateſt common meaſure may be found, and if there were four, the queſtion will be firſt of three, if of five, the queſtion will be of four, &c. and this ſame Corollarie ſhall take place.*

PROP. 5. THEOR. 3.



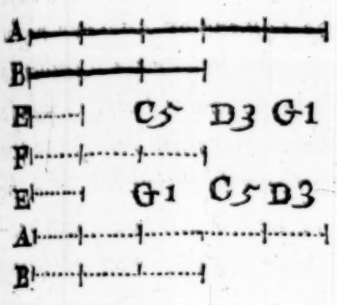
Magnitudes commenſurable A, *and* B, *have the ſame rate, the one to the other, as a number to a number.*

a) 3. 10. *Demonſtration* LEt a C be their greateſt common meaſure, and as often times as C meaſures A, let unity E meaſure as often times a number, as D, and as often times as C meaſures B, let unity alſo meaſure the number F.

Foraſmuch then as the magnitude C, and unity E, meaſures the magnitude A, and the number D equally, the magnitude A ſhall contein C, as many times as the number D conteineth unity: Therefore, as A is to C, ſo the

the number D is to unity; But as C is to B, ſo unity is to F, (ſeeing that C meaſures B; and unity meaſures F equally,) [b] therefore in equal rate A ſhall be to B, as the number D, to the number F: Therefore, The, &c. Which was to be demonſtrated. b) 22. 5.

PROP. 6. THEOR. 4.



If two magnitudes A, *and* B, *have the ſame rate to one another, as a number* G, *to a number* D; *they ſhall be commenſurable.*

Demonſtration FOr, Let the magnitude A be underſtood to be divided into as many equal parts as C conteins unites, and let one of the ſaid parts be equal to the magnitude E, and let there be another magnitude F meaſured as many times by E, as unity meaſures D.

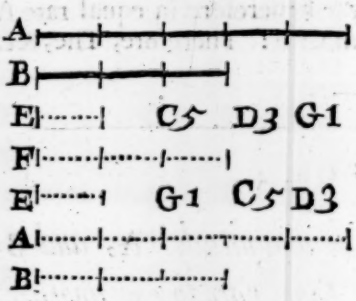
Foraſmuch as E is as many times in A, as unity in the number C; the magnitude A, and the number C, ſhall contein E, and unity equally; therefore A ſhall be to E, as C to unity. But E is to F as unity to D, (E meaſuring F, and unity meaſuring D eqnally,) therefore [a] in equal rate, A ſhall be to F, as the number C, to the number D: but A is alſo to B, as the number C to the number D, by Suppoſition; therefore, [b] as A to B, ſo is A to F: therefore, [c] B and F are equal: therefore E meaſuring F ſhall alſo meaſure B, equal to F: but E ſhould alſo meaſure A: therefore E meaſures A and B; therefore [d] A and B are commenſurable: Therefore, If Two, &c. Which was to be demonſtrated. a) 22. 5. b) 11. 5. c) 9. 5. d) 1. d.

Otherwiſe, Let the magnitude A be ſuppoſed to be divided into as many equal parts as there are unites in the number C, to one of which parts let the magnitude E be equal.

Foraſmuch then, as E is to A, ſo is unity to the number C, (E meaſuring A, and unity the number C equally,) but A is propoſed to be to B, as the number C to the number D; therefore in equal rate, as E is to [c] B, ſo is unity to D: but unity meaſures the number D: Therefore E ſhall meaſure B, and E alſo meaſure A: therefore, [f] A and B, having A for common meaſure, ſhall be commenſurable, which is propoſed on the ſecond part of the figure. c) 22. 5. f) 1 def.

COROLLARIE.

From the firſt part of this Theorem it appears how you may finde a right line, to which any other right line propoſed, may be as a number to a number: For, (ſee the firſt figure,) let there be found a line, to which the propoſed line A *may be as the number* C *to the number* D; *divide* A *into as many equal parts*

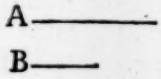


parts as there are unites in C, *and take* F, *conteining as many of the ſame parts of* A *as there are unites in* D, *and ſo* A *ſhall be to* F, *as the number* C, *to the number* D, *as is demonſtrated.*

It alſo appeares by what meanes may be found a right line, to whoſe ſquare, the ſquare of another right line given may be, as a number to a number: As for example, let there be found a line, to whoſe ſquare, let the ſquare of the line A *be as the number* C, *to the number* D, *; in the firſt place, let* F *be found, to which let* A *be as* C *to* D, *as is ſaid, then* [a] *let there be found the mean proportional between* A *and* F, [b] A *the line found, and* F, *being continually proportional; the ſquare of* A *ſhall be to the ſquare of the mean, as* A *to* F: *But* A *is to* F, *as the number* C, *to the number* D: *therefore the ſquare of* A, *ſhall be to the ſquare of the mean proportional, as the number* C *to the number* D.

a) 13. 6.
b) Co. 20. 6.

PROP. 7. THEOR. 5.

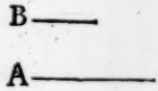


Incommenſurable magnitudes A, *and* B, *have not the ſame rate to one another, as a number to a number.*

Demonſtration FOr [a] if A and B, ſhould have the ſame rate to one another, as a number to a number, they ſhould be commenſurable, which is abſurd, being propoſed incommenſurable: therefore A and B are not to one another, as a number to a number: Therefore the Magnitudes, &c. Which was to be demonſtrated.

a) 6. 10.

PROP. 8. THEOR. 6.



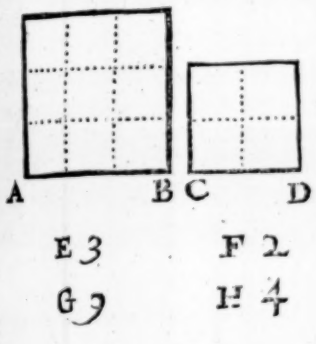
If two magnitudes A *and* B, *have not the ſame rate to one another, as a number to a number, thoſe magnitudes ſhall be incommenſurable.*

Demonſtration FOr if they were commenſurable, they [a] ſhould be to one another, as a number, to a number, which is abſurd, they being

a) 5. 10.

being not to one another, as a number to a number, by Supposition: Therefore A and B, are not commensurable: Therefore, If Two magnitudes, &c. Which was to be demonstrated.

PROP. 9. THEOR. 7.



The squares A B, *and* C D, *described on the right lines* A B, *and* C D, *commensurable in length, have the same rate to one another, as a square number to a square number, and the squares which have the same rate to one another, as a square number to a square number, shall have also their sides commensurable in length. But the squares described of the right lines incommensurable in length, have not the same rate to one another, as a square number to a square number: and the squares which have not the same rate to one another, as a square number to a square number, shall not have their sides commensurable in length.*

Demonstration. FOr A B and C D, being commensurable in length, [a] A B shall be to C D, as a number to a number, which let be as the number E to the number F, and let the squares of E and F, be G and H. a) 5. 10.

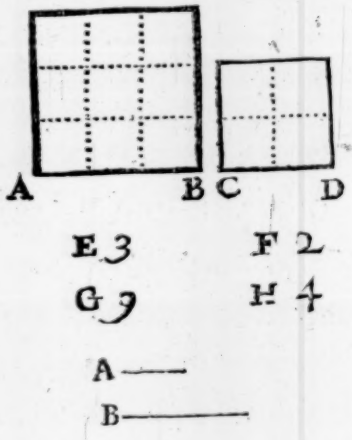
Therefore, seeing that A B. is to C D, as E to F: but [b] the square of C D, in a double rate of the side A B to the side C D, [c] and the square number G is to the square number H, also in a double rate of the side E to the side F, the square of A B shall be to the square of C D, as the square number G to the square number H, forasmuch as those two rates are double to the equal rates, which is proposed. b) 20. 6. c) 11. 8.

Secondly, Let the square of A B be to the square of C D, as the square number G to the square number H; I say, that A B and C D are commensurable in length: for let E and F be the sides of G and H.

Forasmuch then, as the square of A B, is to the square of C D, as the square number G, to the square number H: but [d] the square of A B is to the square of C D, in a double rate of the side A B, to the side C D, and [e] the square number G to the square number H, also in double rate of the side E to the side F: Therefore A B shall be to C D, as E to F, their double rates being equal, therefore A B and C D, are commensurable in length. d) 20. 6. e) 11. 8.

Thirdly,

Thirdly, Let A and B be incommenſurable in length; I ſay, that their ſquares are not to one another, as a ſquare number to a ſquare number: For if the ſquare of A were to the ſquare of B, as a ſquare number to a ſquare number, A and B ſhould be commenſurable in length, as hath been ſhewn, which is contrary to the Suppoſition; therefore the the ſquare of A is not to the ſquare of B, as a ſquare number to a ſquare number, which is propoſed.



Laſtly, Let not the ſquares of A and B be to one another, as a ſquare number; to a ſquare number; I ſay, that A and B are incommenſurable in length: For if it be not ſo, their ſquares are to one another, as a ſquare number to a ſquare number, as hath been ſhewn, which is contrary to Suppoſition: Therefore A and B are not commenſurable in length, which is propoſed: Therefore, The ſquares, &c. Which was to be demonſtrated.

COROLLARIE.

From theſe things thus demonſtrated, it is manifeſt, that right lines commenſurable in length, are alſo commenſurable in power: But that thoſe which are commenſurable in power, are not all ſo in length, and that thoſe which are incommenſurable in length are not all ſo in power: But thoſe that are incommenſurable in power, are all commenſurable in length.

a) 9. 10. *For the ſquares of lines* [a] *commenſurable in length, being to one another, as a ſquare number to a ſquare number; that is to ſay, as a number to a number ſimply;* b) 6. 10. [b] *they ſhall be commenſurable, and* c) 3. d. [c] *therefore their ſides commenſurable in power; therefore right lines commenſurable in length, are alſo all of them commenſurable in power.*

Again, ſeeing that lines whoſe ſquares are not to one another, as a ſquare number to a ſquare number, but ſimply, as a number to a number, d) 3. d. [d] *are commenſurable in power,* (e) 9. 10. [e] *their ſquares being commenſurable*) f) 9. 10. [f] *but not in length: It is manifeſt that the lines that are commenſurable in power, are not ſo alwayes in length, but onely the lines that are commenſurable in power, whoſe ſquares are to one another as a ſquare number to a ſquare number are alſo commenſurable in length, as appears by the ſecond part of this Theorem.*

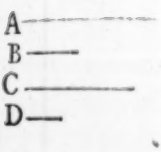
Again, foraſmuch as thoſe lines whoſe ſquares are not to one another, as a ſquare number to a ſquare number, but onely

as

as a number to a number, are incommenſurable in length, but commenſurable in power, as hath been ſaid; it appears that the lines incommenſurable in length, are not all ſo in power, but onely the lines incommenſurable in length, whoſe ſquares are not to one another, as a number to a number, are incommenſurable in power, their ſquares being incommenſurable.

Laſtly, it is manifeſt that the lines incommenſurable in power, are ſo alſo in length; for if they were commenſurable in length, they ſhould not be ſo in power, as is evident by the firſt part of this Corollarie; which is abſurd, being propoſed incommenſurable in power.

PROP. 10. THEOR. 8.

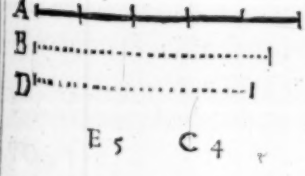


If four magnitudes A, B, C, *and* D, *be proportional, and that the firſt* A, *be commenſurable to the ſecond* B, *the third* C, *ſhall be alſo commenſurable to the fourth* D, *and if the firſt* A, *be incommenſurable to the ſecond* B, *the third* C, *ſhall be alſo incommenſurabe to the fourth* D.

Demonſtration. FOraſmuch as A and B, are commenſurable; [a] they ſhall be to one another, as a number to a number: Let then A be to B, as the number E, to the number F: But as A is to B, ſo C is to D, by Suppoſition: Therefore C ſhall be alſo to D, as the number E, to the number F: therefore [b] C and D, are commenſurable to one another. a) 5. 10. b) 6. 10.

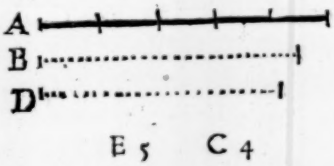
Now let A be incommenſurable to B; I ſay that, C is alſo incommenſurable to D: For ſeeing that A and B are incommenſurable: [c] A ſhall not be to B, as a number to a number, but as A is to B, ſo C is to D by Suppoſition: Therefore C ſhall not be to D, as a number to a number; [d] therefore C and D, are incommenſurable to one another: Therefore, &c. Which was to be demonſtrated. c) 7. 10. d) 8. 10.

PROP. 11. PROBL. 4.



To find two right lines B *and* D, *incommenſurable to a right line propoſed* A; *to wit, the one* D *onely in length, and the other* B *in length and power. That is to ſay, to the right line called Rational.*

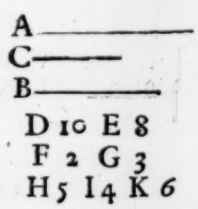
Conſtruction LEt there be found two numbers E and C being not to one another, as a ſquare number to a ſquare number: then a finde the line D, to whoſe ſquare let the ſquare of A be as the number E, to the number C: I ſay, that D is incommenſurable to A in length onely; For the ſquare of A is to the ſquare of D, as the number E to the number C: But E and C are not ſquare numbers: Therefore the ſquare of A ſhall not be to the ſquare of D; as a ſquare number to a ſquare number; therefore, b A and D ſhall be incommenſurable in length: and ſeeing that the ſquares of A and D are to one another, as a number to a number, they ſhall be commenſurable: therefore c A and D are commenſurable in power, and therefore they are onely incommenſurable in length.



Now to find a right line incommenſurable in length and power to the propoſed line A: Let d B be found, a mean proportional between A and D; I ſay, that B is the line required: For A, B, and D, being continually proportional, e the ſquare of A ſhall be to the ſquare of B, as A is to D, but A is incommenſurable in length to D; as is ſhewn: therefore f the ſquare of A ſhall be incommenſurable to the ſquare of B; therefore A and B are incommenſurable in power, by the Definition; and g by conſequence alſo in length; wherefore we have found two right lines, &c. Which was to be done.

a) Co. 6. 10.
b) 9. 10.
c) 6. 10.
d) 13. 6.
e) Co. 20. 6.
f) 10. 10.
g) Co. 9. 10.

PROP. 12. THEOR. 9.



The magnitudes A *and* B, *commenſurable to one and the ſame magnitude* C, *are alſo commenſurable to one another.*

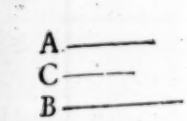
Demonſtration FOr ſeeing that A and C are commenſurable, a A ſhall be to C, as a number to a number, and let it be as the number D to the number E; and C and B being alſo commenſurable, C ſhall be to B, as a number to a number. Let it be then, as the number F is to the number G, and let b the three numbers H, I, and K, be found contiuually proportional, and the leaſt in the rate of D to E, and of F to G; and H ſhall be to I, as D to E, and I to K, as F to G: that is to ſay, as C to B.

Therefore A being to C, as H to I, and C to B, as I to K: in equal rate, A ſhall be to B, as H to K: therefore A and B, ſhall be commenſurable; c being to one another, as the number H to the number K: Therefore, &c. Which was to be demonſtrated.

a) 5. 10.
b) 4. 8.
c) 6. 10.

PROP.

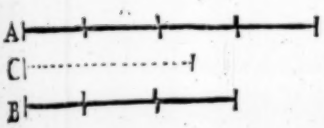
PROP. 13. THEOR. 10.



If there be two magnitudes A *and* B, *and that the one* A *be commenſurable, and the other* B *incommenſurable to one and the ſame magnitude* C, *thoſe magnitudes ſhall be incommenſurable to one another.*

Demonſtration. FOr if B be ſaid to be commenſurable to A, C being propoſed commenſurable to the ſame A; B and C ſhould be commenſurable to one another, which is abſurd, being propoſed incommenſurable to one another: Therefore A is not commenſurable to B: Therefore, If, &c. Which was to be demonſtrated. 12. 10.

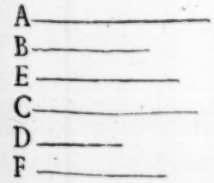
PROP. 14. THEOR. 11.



If there be two magnitudes commenſurable A *and* B, *and that one of them* A, *be incommenſurable to ſome magnitude* C, *the other* B *ſhall be alſo incommenſurable to the ſame* C.

Demonſtration. FOr if B be ſaid to be commenſurable to C, [a] A and C, ſhould be commenſurable to one another, which is abſurd, being propoſed incommenſurable: Therefore B and C are incommenſurable to one another: Therefore, If there be, &c. Which was to be demonſtrated. a) 12. 10.

PROP. 15. THEOR. 12.



If four right lines A, B, C, *and* D, *are proportional, and the firſt* A *be more in power then the ſecond* B, *by the ſquare of a line* E, *which ſhall be commenſurable in length thereto; the third* C *ſhall be alſo more in power then the fourth* D, *by the ſquare of a line* F, *which ſhall be commenſurable thereto in length; and if the firſt* A *be more in power then the ſecond* B, *by the ſquare of a line* E, *incommenſurable in length thereto, the third* C *ſhall be alſo more in power then the fourth* D, *by the ſquare of a line* F *incommenſurable in length thereto.*

Demonſtration FOr ſeeing that A is to B, as C to D: a alſo as the ſquare of A ſhall be to the ſquare of B, ſo the ſquare of C to the ſquare of D: But the ſquare of A is equal to the ſquares of B and E, by Suppoſition, and the ſquare of C, to the ſquares of D and F; therefore as the ſquares of B and E together, ſhall be to the ſquare of B, ſo the ſquares of D and F to the ſquare of D, and by dividing, as the ſquare of E to the ſquare of B, ſo the ſquare of F to the ſquare of D.

A
B
E
C
D
F

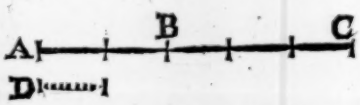
a) 22. 6.

Therefore b as E is to B, ſo is F to D, and alternately, as B is to E, ſo D is to F; in equal rate as A ſhall be to E, ſo C ſhall be to F: Therefore, c if A be commenſurable in length to E, alſo C ſhall be commenſurable in length to F, and if incommenſurable, incommenſurable: Therefore, if four lines, &c. Which was to be demonſtrated.

b) 22. 6.

c) 10. 10.

PROP. 16. THEOR. 13.



If two commenſurable magnitudes are compound, the whole A C *ſhall be alſo commenſurable to each of them: And if the whole* A C *be commenſurable to one of them, the magnitudes propoſed at firſt,* A B, *and* B C, *ſhall be commenſurable to one another.*

Demonſtration LEt a D be the common meaſure of A B and B C: Foraſmuch then as D meaſures A B and B C; it b ſhall alſo meaſure the whole A C: therefore ſeeing that D meaſures A C and A B, A C and A B ſhall be commenſurable: In like manner, ſeeing that D meaſures A C and B C: A C and B C ſhall be alſo commenſurable: Therefore A C ſhall be commenſurable to each of them A B and B C.

a) 3. 10.
b) 1. c. ſ.

Now let the whole A C be commenſurable to one of them A B or B C, of which it is compounded; to wit to A B: I ſay, that A B and B C are commenſurable to one another.

For let c D be the common meaſure of A C and A B: Therefore D meaſuring the whole A C, and the part cut off A B: it d ſhall alſo meaſure the reſt B C: Therefore A B and B C, are commenſurrble to one another, D meaſuring both the one and the other: Therefore if two magnitudes, &c. Which was to be demonſtrated.

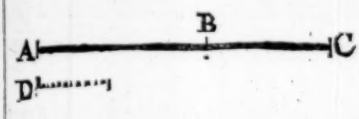
c) 3. 10.
d) 3. c. ſ.

COROLLARIE.

Hence it follows, that if a magnitude compounded of two, be commenſurable to one of them, it ſhall be alſo commenſurable to the other, as if A C *be commenſurable to* A B, *it ſhall be alſo commenſurable to* B C *the remainder, for as hath been ſhewn in the latter part of this Theorem:* D *common meaſure of the*

the whole A C, *and of* A B *the part cut off,* [c] *ſhall alſo meaſure the remainder* B C, *therefore* A C *and* B C *are commenſurable.* c) 3. c. 5.

PROP. 17. THEOR. 14.



If two magnitudes incommenſurable A B *and* B C *are compounded, the whole* A C *ſhall be alſo incommenſurable to each of them* A B *and* B C; *And if the whole* A C *be incommenſurable to one of them, the magnitudes propoſed at firſt,* A B *and* B C, *ſhall be incommenſurable.*

Demonſtration FOr if it be denied, it ſhould be commenſurable at leaſt to one of them: Suppoſe it then commenſurable to A B, and [a] let D be their common meaſure: Therefore ſeeing that D meaſures the whole A C, and the part cut off A B; [b] it ſhall alſo meaſure the reſt B C: Therefore A B and B C, ſhall be commenſurable, having D for their common meaſure, which is abſurd, for they are propoſed incommenſurable. a) 3. 10. b) 3. c. 5.

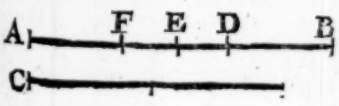
Now ſuppoſe the whole A C to be incommenſurable to one of them, to wit, to A B; I ſay, that A B and B C are incommenſurable.

For if you ſay it is not ſo: [c] the whole A C ſhall be commenſurable to each of them A B, and B C, which is abſurd, and contrary to the Suppoſition: Therefore A B, and B C, are incommenſurable, even ſo it might be demonſtrated, that A C and A B, are incommenſurable: Therefore, If, &c. Which was to be demonſtrated. c) 16. 10.

COROLLARIE.

It follows from theſe Demonſtrations, that if a magnitude compounded of two, be incommenſurable to one of them, it is ſo alſo to the other, as if A C *be incommenſurable to* A B, *it ſhall be ſo alſo to* B C: *For if* A C *were commenſurable to* B C: [d] A C *ſhould be alſo commenſurable to the reſt* A B, *which is abſurd, for that they are propoſed incommenſurable: Therefore* A C *is not commenſurable to* B C, *which is propoſed.* d) C. 16. 10.

PROP. 18. THEOR. 15.



If there be two unequal right lines A B, *and* C, *and that to the greateſt* A B, *be applyed a Paralellogram equal*

equal to the fourth part of the ſquare of the leaſt C, *wanting of* A B, *by a ſquare figure, & that it divide the ſame greater, in parts commenſurable in length, the greateſt* A B *ſhall be more in power then the leaſt* C, *by the ſquare of a line* F D, *which ſhall be commenſurable in length thereto; And if the greateſt* A B *be more in power then the leaſt* C, *by the ſquare of a line which ſhall be commenſurable in length thereto, and that there be applyed a Paralellogram upon the greateſt of them* A B, *equal to the fourth part of the ſquare of the leaſt* C, *wanting of* A B *by a ſquare figure; it ſhall divide the ſame in parts commenſurable in length.*



Demonſtration FOraſmuch as A B is divided equally by E, and unequally by D, the Rectangle conteined under A D and B D, with the ſquare of E D, ſhall be equal to the ſquare of E B: And therefore four times the Rectangle under A D, and D B, and four times the ſquare of E D ſhall be equal to four times the ſquare of E B: But the ſquare of C is equal to four times the Rectangle under A D and D B, (the Rectangle under A D and D B, being propoſed equal to a quarter of the ſquare of C,) and the ſquare of F D is equal to four times the ſquare of E D, the halfe of F D, and the ſquare of A B equal to four times the ſquare of E B, the half of A B: Therefore the ſquares of C and F D, are equal to the ſquare of A B: Wherefore A B is more in power then C, by the ſquare of F D: I ſay alſo that F D is commenſurable in length to A B: For A D and D B being propoſed commenſurable in length,

a) 16. 10. [a] the whole A B ſhall be alſo commenſurable in length to his part D B, but the compounded of D B and A F, is alſo commenſurable in length to D B, (the compounded of A F and D B equals, being double to B D,) therefore A B, and the compounded of A F and D B, being commenſurable in length

b) 12. 10. each to D B, [b] A B and the compounded of A F and D B, ſhall be commenſurable in length to one another.

Therefore, A B compounded of A F and D B, as of one, and of F D, being commenſurable in length, to the compounded of A F and D B his

c) C. 16. 10. part, the ſame A B [c] ſhall be commenſurable in length to the remainder F D, therefore A B is more in power then C, by the ſquare of F D commenſurable in length to A B.

Now let A B be more in power then C by the ſqnare of a line commenſurable in length thereto, and let there be applyed thereto a Paralellogram, as is ſaid before, making the ſegments A D and D B: I ſay, that A D and D B are commenſurable in length to one another: For the ſame conſtruction ſubſiſting, it may be demonſtrated in like manner, that A B is more in power then C, by the ſquare of F D: Wherefore A B being propoſed to

be

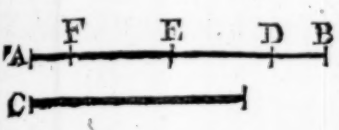
be more in power then C, by the ſquare of a Right-line commenſurable in length thereto, A B ſhall be commenſurable in length to F D.

Therefore the whole A B, compounded of F D, and A F and D B as of one, being commenſurable in length to F D, the ſame A B [d] ſhall be alſo commenſurable in length to the remainder, compounded of A F and D B; but the compounded of A F and D B, is alſo commenſurable in length to D B, (being the double thereof,) [e] therefore A B and D B being commenſurable in length to the compounded of A F and D B, they ſhall be commenſurable to one another: Therefore the whole A B compounded of A D and D B, being commenſurable in length to D B: A D and D B, ſhall be commenſurable in length to one another: Therefore, If there be, &c. Which was to be demonſtrated.

d) C. 16. 10.

e) 12. 10.

PROP. 19. THEOR. 16.



If there be two unequal right lines A B, *and* C, *and that to the greateſt* A B *there be applyed a Rectangle, equal to the quarter part of the ſquare of the leaſt* C, *wanting by a ſquare figure, and that it divide the ſame in parts incommenſurable in length* A D, *and* D B, *the greateſt* A B, *ſhall be more in power then the leaſt* C, *by the ſquare of a line, which ſhall be incommenſurable in length thereto: And if the greateſt* A B, *be more in power then the leaſt* C, *by the ſquare of a line incommenſurable in length thereto: And that there be applyed to the greateſt a Paralellogram equal to the quarter part of the ſquare of the leaſt, wanting a ſquare figure: It ſhall divide the ſame into parts incommenſurable in length.*

Demonſtration. FOr having made the Conſtruction as in the former, we ſhall ſhew as there is done, that A B is more in power then C by the ſquare of F D. And I ſay that F D is incommenſurable in length to A B, for A D and D B, being propoſed incommenſurable in length, [a] the whole A B ſhall be incommenſurable in length to his part D B: But D B is commenſurable in length to the compounded of A F and D B, the one being double to the other,) therefore ſeeing that of two lines commenſurable D B, and the compounded of A F and D B, the one, to wit, D B, is commenſurable in length to A B; [b] the remaining, compounded of A F and D B, ſhall be alſo incommenſurable in length to A B, therefore A B compounded of A F and D B, as of one, and of F D, being incommenſurable in length to the compounded of A F and D B, the ſame A B [c] ſhall be alſo incommenſurable in length to the remainder F D: Therefore A B is more

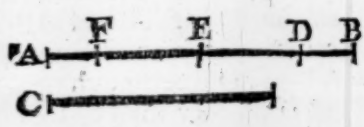
a) 17. 10.

b) 14. 10.

c) C. 17. 10.

more in power then C, by the ſquare of F D, which is incommenſurable in length thereto.

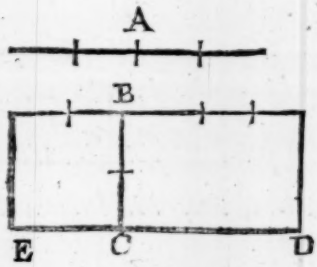
Now let A B be more in power then C, by the ſquare of a line which ſhall be incommenſurable in length thereto; and let there be applyed to A B a Parallelogram equal to the quarter part of the ſquare of C, wanting a ſquare figure, and making in A B, the parts A D and D B; I ſay, that A D and D B are incommenſurable in length.



For obſerving the ſame conſtruction, we ſhall ſhew, as before, that A B is more in power then C, by the ſquare of F D, wherefore ſeeing that A B is propoſed to be more in power then C, by the ſquare of a line incommenſurable in length thereto: A B ſhall be incommenſurable in length to F D; therefore the whole A B, compounded of F D, and of A F and D B, as of one, being incommenſurable in length to F D: [d] the ſame A B ſhall be incommenſurable in length to the remainder, compounded of A F and D B: but the compounded of A F and D B, is commenſurable in length to D B (the one being double to the other,) therefore ſeeing that the two lines commenſurable; to wit, the compounded of A F and D B, and D B: the ſame compounded of A F and D B is incommenſurable in length to A B, [e] the remainder D B ſhall be incommenſurable in length to the ſame A B: Therefore the whole A B, compounded of A D and D B, being commenſurable in length to D B: [f] the ſame A D and D B, ſhall be incommenſurable in length to one another: Therefore, If there be, &c. Which was to be demonſtrated.

d) C. 17. 10.
e) 14. 10.
f) C. 17. 10.

PROP. 20. THEOR. 17.



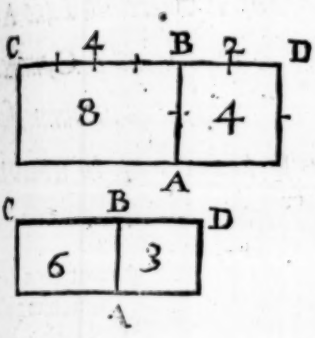
The Rectangle B D, *conteined under two Rational Lines* B C *and* C D, *commenſurable in length, according to ſome of the kinds aforeſaid, is Rational.*

Demonſtration FOr of the one of them, as of B C: Let there be deſcribed the ſquare B E: Foraſmuch as B C is Rational, commenſurable to the Rational propoſed A, either in length and power, or in power onely, [a] his ſquare B E ſhall be commenſurable to the ſquare of A: But the ſquare of A is rational; [b] therefore B E commenſurable thereto, ſhall be Rational: but B C, that is to ſay, E C and C D, being commenſurable in length, (for they are propoſed Rational, commenſurable in length to one another,) and as [c] E C is to C D: ſo E B to B D: [d] E B and B D ſhall be alſo commenſurable in length: wherefore B D commenſurable to the ſpace Rational B E, is alſo Rational: Therefore, &c. Which was to be demonſtrated.

a) 3. d.
b) 9. d.
c) 1. 6.
d) 10. 10.

PROP.

PROP. 21. THEOR. 18.



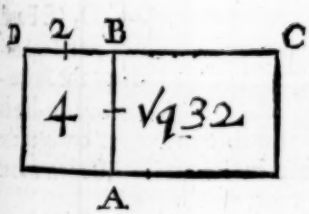
If a Rational Rectangle A C, *be applyed to a Rational line* A B, *it makes the breadth* B C *Rational, commensurable in length to the line* A B, *to which it is applyed.*

THat is to say, if a rational Rectangle have one side Rational, it will also have the other side rational, and those sides shall be commensurable in length to one another.

Demonstration FOr on A B let there be described the square A D, [a] which shall be rational: But as the rectangle C A is to the square A D, so [b] C B is to B D, or to B A his equal. Now the rectangle A C and the square A D, are rational: and therefore commensurable: Therefore, [c] C B and B D, or B A, shall be commensurable in length: But A B is rational; therefore B C shall be also rational: Therefore, If, &c. Which was to be demonstrated.

a) 8. d.
b) 1. 6.
c) 10. 10.

PROP. 22. THEOR. 19.

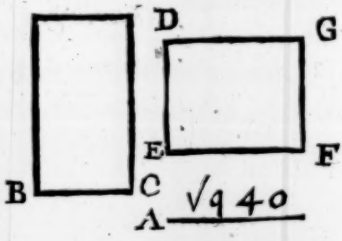


The Rectangle A C, *conteined under two rational right lines* A B *and* B C, *commensurable in power onely, is irrational, and the right line equal in power to the said irrational, may be called Medial.*

Demonstration FOr on A B rational, let be described the square A D, therefore [a] A D shall be rational; and forasmuch as A B, or D B, his equal, is incommensurable in length to B C; and that as D B is to B C, so [b] D A is to A C: [c] A D shall be also incommensurable to A C, but the square A D is rational: therefore [d] the Rectangle A C shall be irrational: Therefore the right line which is equal in power to the Rectangle A C that is to say, whose square is equal to A C is irrational: Let that line be called Medial, or mean proportional, between A B and B C: Forasmuch as his square [e] is equal to the Rectangle A C conteined under A B and B C: Therefore the Rectangle, &c. Which was to be demonstrated.

a) 8. d.
b) 1. 6.
c) 10. 10.
d) 10. d.
e) 17. 6.

PROP. 23. THEOR. 20.



The ſquare of a Medial Line A, *applyed to a rational Line* B C, *makes the breadth rational, and incommenſurable in length to the Line to which it is applyed.*

Demonſtration FOraſmuch as A is a Medial, it [a] is equal in power to a Rectangle conteined under two rational lines commenſurable in power onely: the which let be E G, under E F and F G rationals: But it is equal alſo in power to B D: therefore B D and E G are equal: and being [b] equi-angled, they will have their ſides reciprocal about the equal angles, to wit, as B C to E F, ſo F G to C D: Therefore as the ſquare of B C to the ſquare of E F, ſo the ſquare of F G to the ſquare of C G, but [c] the ſquare of B C is commenſurable to the ſquare of E F; B C and E F being rationals: therefore [d] the ſquare of F G, is alſo commenſurable to the ſquare of C D: therefore F G and C D are at leaſt commenſurable in power: But F G is propoſed rational; therefore C D ſhall be alſo rational.

a) 22. 10.
b) 14. 6.
c) 22. 6.
d) 10. 10.

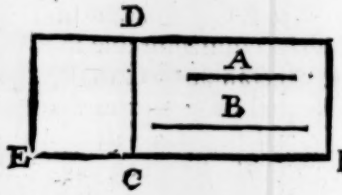
I ſay alſo, that it is incommenſurable in length to B C: Foraſmuch as E F and F G, are rational, commenſurable in power onely, and as E F to F G; ſo the ſquare of E F to the Rectangle E G; [e] the ſquare of E F ſhall be incommenſurable to the rectangle E G: and therefore alſo to the Rectangle B D.

e) 10. 10.

But the ſquare of E F, is commenſurable to the ſquare of C D, (E F and C D, being ſhewn rationals,) therefore [f] the ſquare of C D, and the Rectangle B D, are incommenſurable: but the ſquare of C D is to the Rectangle B D, as C D to B C; therefore C D and B C are incommenſurable in length: therefore [g] C D is rational, commenſurable in power onely to the rational B C; that is to ſay, incommenſurable in length: Which was to be demonſtrated.

f) 13. 10.
g) 10. 10.

PROP. 24. THEOR. 21.



The Right Line B *commenſurable to a Medial Line* A, *is alſo Medial.*

Demonſtration FOr let C D a rational, be propoſed, to which let there [a] be applyed the Rectangle D E equal to the ſquare of the Medial A, making the breadth E C: Therefore [b] E C is rationa, lincommenſurable in length to C D. Again, to C D let there be applyed the Rectangle D F, equal to the ſquare of B, making the breadth C F: Then foraſmuch as A and B are propoſed commenſurable; their ſquares, that is to ſay, the Rectangles D E and D F, equal

a) 45. 1.
b) 13. 10.

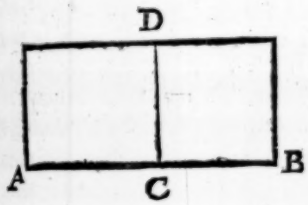
equal to them, ſhall be commenſurable: But [c] as E D to D F, ſo the line E C is to C F: Therefore [d] E C and C F are commenſurable in length: But E C is demonſtrated rational, and incommenſurable in length to C D; [e] therefore C F is alſo rational, and incommenſurable in length to C D: Therefore D C, and C F, are rational, commenſurable in power onely; and [f] therefore the right line B, equal in power to the Rectangle D F, conteined under the ſame C D and C F, is Medial: Which was to be demonſtrated.

[c] 1. 6.
[d] 10. 10.
[e] 14. 10.
[f] 22. 10.

COROLLARIE.

From this Demonſtration it is manifeſt that a ſpace Medial, commenſurable to a ſpace Medial, is alſo Medial: For after it hath been demonſtrated that D F *is commenſurable to the medial Rectangle* D E, *from thence hath been ſhewn that* D F *is alſo a medial, conteined under the rationals* C D *and* C F, *commenſurable in power onely.*

PROP. 25. THEOR. 22.

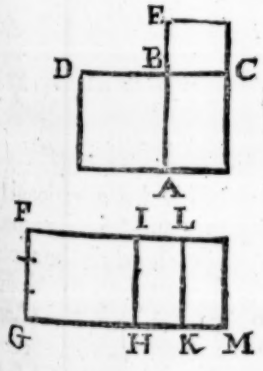


The Rectangle A D, *conteined under two medial right lines* A C *and* C D, *commenſurable in length, is medial.*

Demonſtration FOr on the medial C D; let there be deſcribed the ſquare B D, which ſhall be medial: And ſeeing that as A C is to C B; ſo A D is to D B: But A C and C B, that is to ſay, A C and C D, are commenſurable in length; A D and D B, ſhall be alſo commenſurable: Therefore the ſpace A D, commenſurable to the medial ſpace D B, [a] is medial: Therefore the Rectangle, &c. Which was to be demonſtrated.

1. 6.
a) C. 24 10.

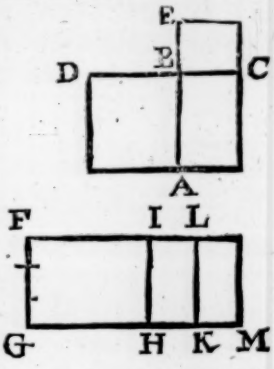
PROP. 26. THEOR. 23.



The Rectangle A C, *conteined under two medial right lines* A B *and* B C, *commenſurable in power onely, is Rational, or medial.*

Demonſtration FOr of A B and B C Medials, let there be deſcribed the ſquares A D and C E, which ſhall be Medials; and let the rational F G, be expoſed, [a] to which let there be applyed the Rectangles F H, H L, L M, equal to A D, A C, and C E: F M ſhall be a Rectangle

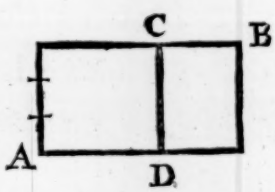
a) 45. 1.



gle alone; and A D and C E, being Medials, their equal Rectangles F H, and L M, shall be
b) 34. 1. also Medials, which applyed to [b] F G & K L
c) 23. 10. rationals, (for K L is equal to F G,) & making the breadths G H & K M, [c] the lines G H & K M, shall be rational, incommensurable in length to F G, or K L; and forasmuch as A B, and B C, are commensurable in power to one another, their squares A D and C E, and therefore their equal Rectangles F H, and L M, shall be commensurable:
d) 1. 6. But as [d] F H is to L M, so G H is to K M;
e) 10. 10. therefore [e] G H and K M, shall be commensurable in length.

Therefore, seeing that it is shewn that G H and K M, are rationals, and commensurable in length to one another, and commensurable in power onely to the rational proposed; being shewn incommensurable in length
f) 20. 10. thereto; [f] the Rectangle under G H and K M, shall be rational. and [g] seeing
g) 1. 6. that as D B is to B C, so A D is to A C, and as A B to B E, so A C to C E: (D B and B C, being equal to A B and B E, each to his correspondent,) as A D shall be to A C, so A C to C E: therefore A D, A C, and C E, shall be proportional: Wherefore F H, H L, and L M, their equal
h) 1. 6. Rectangles shall be continually proportional: But [h] G H, H K, and K M, are in the same rate, as the Rectangles, F H, H L, and L M, they shall be then
i) 17. 6. proportional, [i] and the Rectangle conteined under G H and K M, is equal to the square of H K: but the Rectangle under G H and K M, is shewn rational, therefore the square of H K, shall be also rational, therefore H K
k) 6. d. shall be rational: and [k] therefore commensurable to F G, exposed rational, or to his equal H I, to wit, in length and power, or in power alone. And if H K be commensurable in length to H I; that is to say, to his equal
l) 20. 10. F G, [l] the Rectangle I K, or his equal, shall be rational, and if in power
m) 22. 10. onely, [m] the Rectangle I K shall be a Medial, or his equal A C: therefore the Rectangle A C conteined under A B and A C Medials, commensurable in power onely, is rational or medial: Therefore, &c. Which was to be demonstrated.

PROP. 27. THEOR. 24.



A Space or figure medial A B, *exceedeth not a figure medial* A C, *by a Rational figure* D B.



45. 1. *Demonstration.* FOr let D B be a rational (if possible,) and let the rational E F be exposed, on which let there be applyed the Rectangle E G, equal to the Medial A B, being equal to the Medials A B, A C, and H I, rational, and the Rectangle E H equal to the Medial A C, and so the rest H I, shall be equal to the rational D B, and E G, and E H, shall be Medials, being equal

equal to the rational figure D B, and E G and E H being applyed on the rational E F; they [a] shall have the other sides F G, and F H rational, incommensurable in length to E F. [a] 23. 10.

Again, seeing that H I rational, is applyed to the rational E F: [b] that is to say, to H K, his equal, the line [c] H G shall be rational, commensurable in length to the same E F, or H K; but F H is rational, incommensurable in length to E F: Therefore F H is incommensurable in length to H G, [c] but as F H is to H G, so the square of F H is to the Rectangle conteined under F H and H G; therefore the square of F H is incommensurable to the Rectangle conteined under F H and H G: But the squares of F H and H G, are commensurable to one another: (For the one and the other, are described on the rationals F H and H G;) and [f] therefore the two squares F H, and H G, together, are commensurable to the square of F H; and twice the Rectangle under F H, and H G, is commensurable to the Rectangle F H, and H G, the one being double to the other: Therefore the two squares of F H, and H G, together, shall be incommensurable to twice the Rectangle conteined under F H, and H G. Therefore the compounded of the squares F G, and H G, and of the Rectangle twice under F H, and H G; that is to say, the square of F G, [g] equal to those squares and Rectangle, [h] is incommensurable to the compounded of the squares F H, and H G, which is rational, being commensurable to the rational square of F H. [b] 34. 1. [c] 21. 10. [c] 14. 10. [f] 16. 10. [g] 4. 2. [h] 17. 10.

Therefore seeing that the square of F G is incommensurable to the compounded of the two squares F H, and H G, which is rational, [i] it shall be also irrational, and [k] F G irrational, which is absurd, for it hath been shewn to be rational, commensurable in length to E F: Therefore H I, or his equal D B, is not rational: Therefore, a figure, &c. Which was to be demonstrated. [i] 10 d. [k] 11. d.

PROP. 28. PROBL. 4.

A
B
C
D

To find two medial lines A *and* B, *commensurable in power onely, conteining a rational Rectangle.*

Construction LEt the two Rationals A, and B, be proposed, commensurable in power onely; and [a] let there be found C, a mean proportional between A and B, and as A is to B, so let [b] C be to D: I say, that C and D are Medials; commensurable in power onely, which do contein a rational Rectangle. [a] 13. 6. [b] 12. 6.

Demonstration FOr seeing that A and B are proposed rational, commensurable in power onely; [c] the Rectangle conteined under A and B, shall be irrational, called medial: But [d] C is equal in power to that Rectangle, therefore it shall be medial: and seeing that as A is to B, so C is to D, and A & B, are commensurable in power onely, C and D [e] shall be also commensurable in power onely; and therefore D commensurable to the medial C, shall be also medial: Therefore we have found C and D medials, commensurable in power onely: I say, they do contein a rational Rectangle: For seeing that A is to B, as C to D; alternately, A shall be to C, as B to D: But as A is to C, so is C to B: therefore C shall be also to B, as B to D: Therefore B a mean proportional between C and D, is equal in power to the [c] 22. 10. [d] 17. 6. [e] 10. 10. 24. 10.

17. 6. the Rectangle conteined under C and D: But the ſquare of B a rational line, is rational: Therefore the Rectangle under C and D, ſhall be alſo rational: Therefore we have found C and D medials, commenſurable in power onely, which conteins a rational Rectangle: Which was to be done.

PROP. 29. PROBL. 5.

A
D
B
C
E

To find two medial lines commenſurable in power onely, which doe contein a medial Rectangle.

Conſtruction LEt there be taken the three rational lines A, B, and C, commenſurable in power onely; and a between A and B, let there be found a mean proportional D; then let it be made as B is to C, ſo D to E: I ſay that D and E are medials, commenſurable in power onely, and do contein a medial Rectangle.

a) 13. 6. 12. 6.

Demonſtration FOr ſeeing that A and B are rationals, commenſurable in power onely; the Rectangle conteined under A and B, and therefore the ſquare of D equal b thereto, ſhall be irrational, called medial, and the line D equal in power to it medial: And ſeeing that B is to C as D to E, and B and C are rational, commenſurable in power onely, D and E ſhall be alſo commenſurable in power onely: Therefore ſeeing that D is a medial, c D and E commenſurable in power onely to D, ſhall be alſo medial: I ſay that D and E do contein a medial Rectangle.

b) 17. 6. 22. 10.

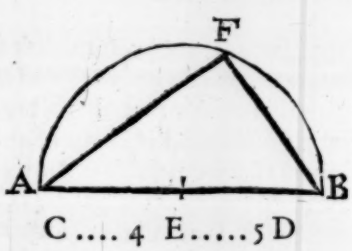
c) 24. 10.

Foraſmuch as B is to C, as D to E, alternately, as B ſhall be to D: ſo C to E; But as B to D, ſo D to A: Therefore as D ſhall be to A, ſo C ſhall be to E: Therefore d the Rectangle conteined under D and E, is equal to the Rectangle conteined under A and C: But the Rectangle conteined under A and C rationals commenſurable in power onely, is irrational, and medial: therefore e the Rectangle under D and E, is alſo medial, therefore D and E are medials required: Which was to be done.

d) 16. 6.

e) 22. 10.

PROP. 30. PROBL. 6.



C .... 4 E ..... 5 D

To find two rational Lines commenſurable in power onely, in ſuch ſort as the greateſt may be more in power then the leaſt, by the ſquare of a right line commenſurable in length thereto.

Conſtruction LEt the rational AB be expoſed, and let there be found two ſquare nnmbers C D and C E whoſe exceſſe E D may not be a ſquare; ſeeing a as the number C D, is to the number D E: ſo the ſquare of

a) Co. 6. 10.

of AB, to the ſquare of ſome line A F: Let A F be fitted in the Semicircle deſcribed on A B, and let F B be joyned; foraſmuch as the angle F [b] is a right angle in the Semicircle, [c] the ſquare of A B ſhall be equal to the two ſquares of A F and F B; that is to ſay, that A B ſhall be more in power then A F, by the ſquare of F B: and the ſquare of A B being to the ſquare of A F as C D to D E, a number to a number; [d] the ſaid ſquares of A B and A F ſhall be commenſurable, and ſeeing that the ſquare of A B is rational, being deſcribed on the rational A B, the ſquare of A F ſhall be alſo rational: and therefore A F rational; therefore A B and A F are rational, and commenſurable at leaſt in power; and the ſquares of A B and A F, being not to one another, as a ſquare number to a ſquare number, the ſame A B and A F, ſhall be incommenſurable in length: But they are ſhewn to be commenſurable in power: Therefore A B and A F are rational, commenſurable in power onely.

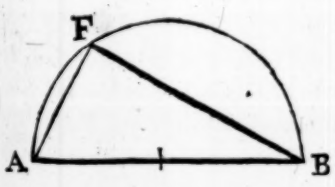
b) 31. 3.
c) 47. 1.
d) 6. 10.

Now ſeeing that as C D is to D E, ſo the ſquare of A B, is to the ſquare of A F, by converſion of reaſon, as the ſquare number C D ſhall be to the ſquare number C E: ſo the ſquare of A B to the ſquare of B F: (For as C D exceedeth E D, by C E a ſquare number, ſo the ſquare A B exceeds the ſquare of A F, by the ſquare of F B:) Therefore the right lines A B and B F ſhall be commenſurable in length: therefore we have found A B and A F rationals, commenſurable in power onely, ſuch as A B the greateſt is more in power then A F by the ſquare of B F, commenſurable in length thereto: Which was to be done.

9. 10.

PROP. 31. PROBL. 7.

To find two rational lines commenſurable in power onely, in ſuch ſort as the greateſt be more in power then the leaſt, by the ſquare of a line incommenſurable in length thereto.



C 144 E 16 D C 160 D
C..... 6... E... 3 D

Conſtruction LEt the rational A B beexpoſed, and find the two ſquare numbers C E and E D, ſuch as that C D compounded of them be not a ſquare number: Or let C D a ſquare number be divided into two numbers not ſquare numbers, C E and E D, ſo the whole ſhall not be to any of them, as a ſquare number to a ſquare number; then [a] let there be found A F, to whoſe ſquare let the ſquare of A B be as C D is to D E, and let A F be fitted in the Semicircle deſcribed on A B, and let F B be joyned: Therefore [b] A B ſhall be more in power then A F, by the ſquare of F B and A B, A F ſhall be rational, commenſurable in power onely, as appears by the fore-going Demonſtration.

a) Co. 6. 10.
b) 31. 3. 47. 1.

But foraſmuch as by converſion of reaſon, (as before) as C D is to C E: ſo the ſquare of A B is to the ſquare of B F: But C D and C E are not to one another, as a ſquare number to a ſquare number: in like manner, the ſquares of A B and B F, ſhall not be to one another, as a ſquare number to a ſquare number: Therefore [c] the lines A B and F B, are incommenſurable in length:

c) 9. 10.

length: Therefore we have found two rationual lines AB and A F, commensurable in power onely, such as A B the greatest, is more in power then A F the least, by the square of F B incommensurable in length thereto: Which was to be done

PROP. 32. PROBL. 8.

A
C
B
D

To find two medials commensurable in power onely, which contein a rational rectangle, in such manner, as the greatest may be more in power then the least, by the square of a line commensurable in length thereto.

Construction LEt [a] there be found A and B rationals, commensurable in power onely, such as A the greatest may be more in power then B the least, by the square of a line commensurable in length thereto; [b] and let C be a mean proportional between A and B, and as A is to B, so let C be to D.

a) 30. 10.
b) 13. 6. 12. 6.

Demonstration FOrasmuch as A and B are rationals, commensurable in power onely: the [c] Rectangle conteined under A and B, is irrational, and the line C equal in power to the same medial; and A being to B as C to D, and A and B commensurable in power onely: [d] C and D shall be also commensurable in power onely; therefore [e] D commensurable to the medial C, is a medial; therefore we have found two medials, C and D commensurable in power onely: I say, that they comprehend a rational rectangle.

c) 12. 10.
d) 10. 10.
e) 24. 10.

For seeing that A is to B, as C to D, also alternately A shall be to C, as B to D: But as A is to C, so C is to B: C shall be in like manner to B, as B to D: Therefore the rectangle under C and D, shall be equal to the square of B: wherefore the square of B rational being rational; the rectangle under C and D, his equal shall be also rational: therefore C and D do contein a rational rectangle.

17. 6.

But forasmuch as A is to B, as C to D; and A is more in power then B, by the square of a line commensurable in length to the same; by the Construction, [f] C shall be also more in power then D, by the square of a line commensurable thereto in length; therefore we have found two medials C and D, commensurable in power onely, which do contein a rational rectangle, and the greatest C is more in power then the least D, by the square of a line commensurable in length thereto, which was to be done.

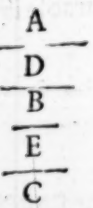
f) 15. 10.

But if A and B [g] are found rational, commensurable in power onely; in such sort as A may be more in power then B, by the square of a line incommensurable in length thereto, and that the rest be done as before, it will be shewn, in like manner, that C and D are medials, commensurable in power onely, which conteineth a rational rectangle: in such sort as C is more in power then B, by the square of a line, which is incommensurable in length thereto.

g) 31. 10.

PROP.

PROP. 33. PROBL. 9.



To find two medial lines commensurable in power onely, the which doe contein a medial Rectangle: in such sort, as that the greatest may be more in power then the least, by the square of a line which shall be commensurable in length thereto.

Construction. LEt there be found the three rationals A, B, and C, commensurable in power onely, in such sort, as A may be more in power then C, by the square of a line which shall be commensurable in length to it: Having found two rationals A and C, commensurable in power onely; in such sort as the greatest may be more in power then the least, by the square of a line which shall be commensurable thereto in length; let B be found commensurable in power onely, to the one and the other: then let D be taken a mean proportional between A and B, and as D is to B, so let C be to E.

Demonstration. FOrasmuch as A is to C, as the Rectangle under A and B, is to the Rectangle under B and C: but the square of D is equal to the Rectangle under A and B: and the Rectangle under D and E, is equal to the Rectangle under B and C: Seeing that D and B, C and E are proportional; in like manner, as A is to C, so the square of D, shall be to the Rectangle under D and E: But as the square of D is to the Rectangle of D and E, so D is to E: Therefore as A is to C, so D is to E: but A and C are commensurable in power onely: therefore D and E shall be so also: But 22. 10.
forasmuch as D is equal in power to the Rectangle of A and B rationals, commensurable in power onely; is rational and medial, E commensurable 24. 10.
to D, shall be also medial: We have therefore found two medials D and E, commensurable in power onely.

And forasmuch as the Rectangle under D and E is shewn to be equal to the Rectangle of B and C medial; (seeing that B and C are rational, commensurable in power onely,) the Rectangle under D and E shall be also 22. 10.
medial.

Lastly, seeing that we have shewn that as A is to C, so D is to E: But A is more in power then C, by the square of a line commensurable in length 15. 10.
to it, by the Construction, D shall also be more in power then E by the square of a line which shall be commensurable in length thereto.

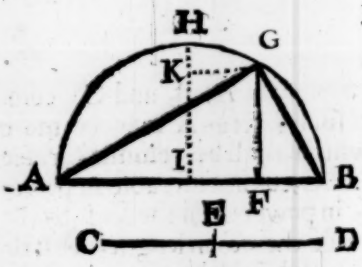
We have therefore found two medials D and E, commensurable in power onely; which do contein a medial Rectangle; and the greatest D is more in power then the least E, by the square of a line which is commensurable in length thereto: Which ought to be done.

But if the Rationals before mentioned had been found, in such sort as A might be more in power then C, by the square of a line incommensurable in length thereto: the medial D would be more in power then E, by the square of a line incommensurable in length thereto; and having finished

the Conſtruction, as before is ſaid, it might be demonſtrated in like manner, that D and E are found medials, commenſurable in power onely, conteining a medial Rectangle, in ſuch ſort as the greateſt may be more in power then the leaſt, by the ſquare of a line incommenſurable in length thereto, &c.

PROP. 34. PROBL. 10.

To find two right lines incommenſurable in power, which do make ſome compound of their rational, ſquares: But the Rectangle conteined under them a medial.



a) 31. 10. *Conſtruction* LEt [a] there be found two Rationals A B and C D, commenſurable in power onely, ſuch as the greateſt A B may be more in power then the leaſt C D; by the ſquare of a line which ſhall be incommenſurable in length thereto; and having divided C D in two equal parts in E: let there be applyed to A B a Rectangle equal to the ſquare of C E, wanting by a ſquare figure; and let it be that which is conteined under A F and F B, and having deſcribed the Semicircle A G B: let there be drawn F G at right angles, and let A G and G B, be joyned together.

Demonſtration FOraſmuch as A and B is more in power then C D by the ſquare of a line incommenſurable in length thereto; and that there is applyed to A B a Rectangle conteined under A F and F B, equal to the quarter part of the ſquare of C D wanting by a ſquare figure;

b) 19. 10. [b] A F ſhall be incommenſurable in length to F B.

But as A F is to F B, ſo the Rectangle under A B and A F, to the Rectangle under A B and F B: but the Rectangle under A B and A F, is equal to the ſquare of A G, and the Rectangle under A B and F B, equal to the ſquare of B G: Seeing that the angle A G B, is right

c) 31. 3. [c] being in the Semicircle: Therefore as A F to F B: ſo the ſquare of A G to the ſquare of G B: and

d) 10. 10. therefore ſeeing that A F and F B, are incommenſurable in length, [d] the

e) 4. d. ſquare of A G and G B, ſhall be incommenſurable: Therefore [e] the lines A G and G B, are incommenſurable in power. And foraſmuch as the ſquare

f) 47. 1. of the rational A B is rational; and [f] equal to the ſquares of A G and G B, the compounded of the ſquares of A G and G B, is alſo rational.

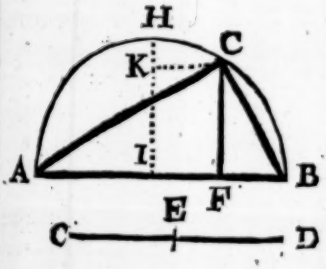
Again, Foraſmuch as the Rectangle under A F and F B, which is made

g) 17. 6. equal to the ſquare of C E, [g] is equal to the ſquare of F G: (Seeing that the angle A G B is right, and F G perpendicular to A B,) the ſquares of F G and C E, ſhall be equal; and therefore the lines F G and C E equal: wherefore C D the double of C E, ſhall be alſo double to F G: therefore the Rectangle under A B and C D, ſhall be double, the Rectangle under A B and F G: but the Rectangle conteined under the Rationals A B and C D, commenſurable in power onely, is medial: therefore the Rectangle under

under A B and F G commenſurable thereto, being its halfe, is h alſo medial: But the Rectangle conteined under A G and G B is equal to the Rectangle conteined under A B and F G: Therefore the Rectangle conteined under A G and G B, is alſo medial, but the compounded of the ſquares of A G and G B, is ſhewn rational: Therefore we have found two right lines A G and G B, incommenſurable in power, which make the compound of their ſquares Rational, and the Rectangle conteined under them medial: Which was to be done. h) C. 24. 10.

PROP. 35. PROBL. 11.

To find two right lines incommenſurable in power, which do make the Compound of their ſquares medial, and the Rectangle conteined under them Rational.



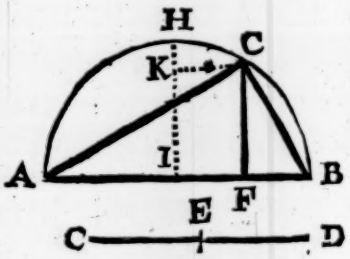
Conſtruction LEt a there be found two medials A B and C D, commenſurable in power onely; conteining a Rectangle rational; in ſuch ſort as A B the greateſt, may be more in power then C D the leſſer, by the ſquare of a line incommenſurable thereto in length; and do for the reſt as in the 34 Propoſition is ſhewn, ſo as in the ſame 34 Prop. that A C and G B, are incommenſurable in power, and the ſquare of the medial A B, (equal b to the ſquares of A G and G B,) being medial, the compounded of them, the ſquares of A G and G B, ſhall be alſo medial. a) 32. 10. b) 47. 1.

Again, as in the 34 Propoſition, the Rectangle under A B and C D is ſhewn double, the Rectangle under A B and F G, and therefore commenſurable: Wherefore the Rectangle under A B and C D, being Rational, by the Conſtruction, the Rectangle under A B and F G, commenſurable thereto, ſhall be alſo rational; but the Rectangle under A G and G B, is equal to the Rectangle uuder A B and F G: Therefore the Rectangle under A G and G B, is alſo Rational.

Now the Compound of the ſquares of A G and G B, is ſhewn to be medial; therefore A G and G B are found incommenſurable in power, making the the Compound of their ſquares medial: But the Rectangle under them Rational: Which was to be done.

PROP. 36. PROBL. 12.

To find two right lines incommenſurable in power, making the Compound of their ſquares medial, and the Rectangle conteined under them medial, and incommenſurable to the Compound of their ſquares.



Conſtruction LEt [a] there be found A B and C D medials, commenſurable in power onely; conteining a rectangle medial, in ſuch ſort as A B the greateſt may be more in power then C D the leaſt, by the ſquare of a line incommenſurable thereto in length: and having done the reſt, as in the 34 Propoſition: I ſay, that A G and G B are the lines required.

a) 33. 10.

Demonſtration FOr as in the 34 Propoſition is ſhewn that A G, and G B, are incommenſurable in power; and foraſmuch as the ſquare of A B medial is medial, (being [b] equal to the ſquares of A G and G B,) the Compound of the ſquares A G and G B, ſhall be alſo medial: Again, the Rectangle under A B and C D, is double the Rectangle under A B and F G, (as [c] is ſhewn,) and therefore commenſurable: therefore by the Conſtruction, the Rectangle under A B and C D, being medial, the Rectangle under A B and F G his equal; (that is to ſay, the Rectangle under A G and G B, his equal,) is alſo medial; but A B being propoſed incommenſurable in length to C D, and C E commenſurable in length to the ſame C D, C D being double to C E: [d] A B and C E ſhall be incommenſurable in length.

b) 47. 1.

c) 34. 10.

d) 13. 10.

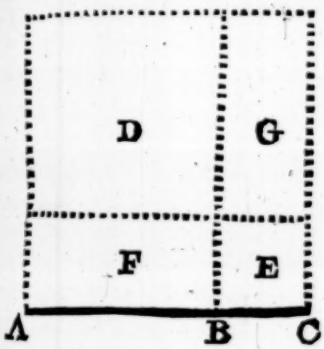
But as A B to C E, ſo the ſquare of A B to the Rectangle under A B and C E: that is to ſay, to the Rectangle under A B and F G, that is to ſay, under A G and G B, (for the Rectangles under A B and F G, and under A G and G B, are equal:) Therefore [e] the ſquare of A B is incommenſurable to the Rectangle under A G and G B: Therefore A G, and G B, are found incommenſurable in power, making the Compound of their ſquares medial, and the Rectangle under them medial, and incommenſurable to the Compound of their ſquares: Which was to be done.

e) 10. 10.

Here

Here begins the Senaries of Irrational Lines by Compoſition.

PROP. 37. THEOR. 25.



If two rational Lines A B, *and* B C, *commenſurable in power onely, are compounded, the whole* A C *ſhall be irrational: and is called a Binomium, or a Binomial line.*

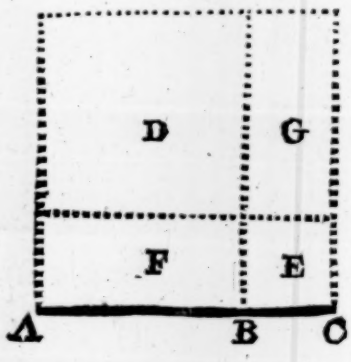
Demonſtration FOr A B being incommenſurable in length to B C; and as A B to B C, ſo the Rectangle under AB and B C to the ſquare of BC; to wit [a] F to E, the Rectangle under AB and BC, *ſhall* be incommenſurable to the ſquare of BC. But twice the Rectangle under A B and B C is commenſurable to the ſame Rectangle under A B and BC: to wit, F and G together, to G or F, and the ſquare of A B, is commenſurable to the ſquare of B C; to wit, D to E, (being rationals,) wherefore [b] the Compound of the ſquares of A B and B C, is commenſurable to the ſquare of B C. a) 16. 10. b) 16. 10.

Therefore the Compound of twice the Rectangle under A B and B C, and the Compound of the ſquares of A B, and B C, are incommenſurable; wherefore the ſquare of A C, compounded of the ſquares of A B and B C, and of twice the Rectangle under A B and B C; (for [c] the ſquare of A C is equal to the two ſquares of A B and B C, and to the two Rectangles under A B and B C, as appears,) is [d] incommenſurable to the Compound of the ſquares of A B and B C: but the Compound of the ſquares of A B and B C, is rational, being ſhewn to be commenſurable to the ſquare of the Rational B C: therefore the ſquare of A C incommenſurable to the ſquare of the Rational B C, is [e] irrational: Therefore A C ſhall be irrational: Now let that line be called by two names, or as others expreſſe it *Binomial:* Foraſmuch as it is compounded of two names, that is to ſay, of two rational lines A B and B C, commenſurable in power onely: Therefore if two lines, &c. Which was to be demonſtrated. c) 4. 2. d) 17. 10. e) 10. d.

A——B——C

PROP.

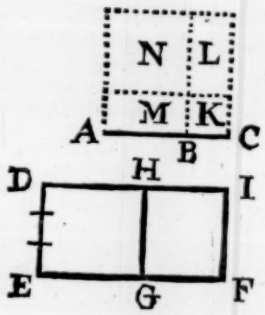
PROP. 38. THEOR. 26.



If two medial lines A B, *and* C D, *commensurable in power onely, containing a rational Rectangle, are compounded, the whole* A C *shall be irrational: And is called a first Bimedial line.*

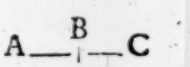
Demonstration. FOr A B being to B C, as the Rectangle conteined
a) 10. 10. under A B and B C, is to the square of B C, and [a] A B and B C incommensurable in length, the Rectangle under A B and B C shall be incommensurable to the square of B C: to wit, as F to E: But twice the Rectangle under A B and B C is commeusurable to the Rectangle under A B and B C, and the Compound of the squares of A B and B C is commensurable to the
b) 16. 10. square of B C, (for A B and B C, being commensurable in power, their squares shall be commensurable; and [b] therefore the Compound of them shall be commensnrable to the square of B C:) Therefore twice the Rectangle under A B and B C, and the Compound of the squares A B and
c) 4. 2. B C, are commensurable: Therefore the Compound of the squares of A B, and B C and of twice the Rectangle under A B & B C, (which is [c] the square
d) 17. 10. of A C) is [d] incommensurable to the Compound of twice the Rectangle under A B and B C: but the Rectangle under A B and B C is commensurable to twice the Rectangle under A B and B C, being the half thereof: Therefore
e) 13. 1. [e] the square of A C is incommensurable to the Rectangle under A B and B C: but the Rectangle under A B and B C, is proposed rational: therefore the square of A C is irrational; and therefore
f) 10. d. 11. d. [f] A C shall be irrational: Then let the line be called first of two medials, or a Prime *Bimedial*: Therefore, &c. Which was to be demonstrated.

PROP. 39. THEOR. 27.



If two medial lines A B *and* B C, *commensurable in power onely, containing a medial Rectangle, are compounded, the whole* A C *shall be irrational: And is called a second Bimedial line.*

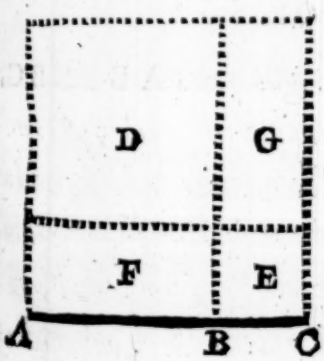
Demonstration. FOr let the Rational D E be exposed, to which let there be
a) 45. 1. [a] applyed the Rectangle D F, equal to the square



ſquare AC: and DG equal to the Compound of the ſquares of AB and BC; to wit NK, and the ſquare of AC, (or DF, his equal Rectangle (being [b] equal to the two ſquares of AB and BC, and twice the Rectangle under AB and BC, the reſt HF ſhall be equal to the Rectangle under AB and BC, twice, to wit, to ML: But the Rectangle under AB and BC, being propoſed medial, the double of that which is commenſurable thereto, to wit, [c] HF ſhall be alſo medial. b) 4. 2. c) C. 24. 10.

Again, ſeeing that the ſquares of AB and BC medials, commenſurable in power, are commenſurable; [d] their Compound, to wit, DG, ſhall be commenſurable to each of them, and being medials, [e] DG ſhall be a medial; therefore ſeeing that the medials DG and HF, are applyed to the Rational DE; and HG [f] his equal, [g] the breadth EG and GF ſhall be Rational, incommenſurable in length to DE: Again, AB and BC, being incommenſurable in length, and as AB to BC, ſo the ſquare of AB to the Rectangle under AB and BC: to wit, as N to L, the ſquare of AB ſhall be incommenſurable to the Rectangle under AB and BC: but the Compound of the ſquares of AB and BC is ſhewn to be commenſurable to the ſquare of AB, and twice the Rectangle under AB and BC, being the double thereof: Therefore the Compound of the ſquares of AB and BC, and twice the Rectangle under AB and BC, or their equal Rectangle DG and HF, are incommenſurable: Wherefore [h] DG being to HF as EG to GF; [i] EG and GF ſhall be incommenſurable in length, and being ſhewn Rationals, they ſhall be rational, commenſurable in power onely; therefore [k] the whole EF ſhall be irrational: Wherefore the Rectangle DF, under DE Rational, and EF Irrational, ſhall be Irrational; and the ſquare of AC equal to the ſame DF, is alſo Irrational: Wherefore AC is Irrational, which let be called ſecond of two Medials, or a ſecond Bimedial line: Therefore, If two, &c. Which was to be demonſtrated. d) 16. 10. e) C. 24. 10. f) 34. 1. g) 23. 10. h) 1. 6. i) 10. 10. k) 17. 10.

PROP. 40. THEOR. 28.

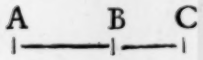


If two right lines AB *and* BC, *incommenſurable in power, making the Compound of their Squares rational, and the Rectangle conteined under them Medial, are compounded, the whole ſhall be irrational: And is called the Greater, or Major.*

Demonſtration FOraſmuch as the Rectangle conteined under AB and BC, is propoſed Medial, twice the Rectangle conteined under AB and BC, commenſurable thereto, [a] ſhall be alſo Medial, and therefore Irrational: Now the Compound of the ſquares of AB and BC, is propoſed rational: Therefore [b] twice the Rectangle under AB and BC, ſhall be incommenſurable thereto: Therefore the ſquare of AC compounded of the ſquares of AB and BC, and twice the Rectangle under AB and BC, ſhall a) C. 10. 10. b) 10. d. 17. 10.

shall be incommensurable to the Compound of the squares of A B and B C: and this Compound of the squares of A B and B C being proposed rational; [c] the square of A C incommensurable thereto, shall be Irrational, [d] and A C Irrational: Let this line be called Major: Therefore, If, &c. Which was to be demonstrated.

PROP. 41. THEOR. 29.

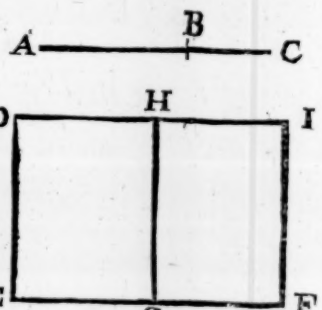


If two right lines A B *and* B C, *incommensurable in power, are compounded, the which do make the Compound of their Squares medial; and the Rectangle conteined under them rational; the whole shall be Irrational: And is called a line equal in power to a rational, and a medial Superficies.*

Demonstration FOr the Compound of the squares of A B and B C, being medial: but twice the Rectangle under A B and B C rational, his half, to wit, the Rectangle once, under A B and B C, being proposed rational, [a] the Compound of the squares of A B and B C, is incommensurable to twice the Rectangle under A B and B C: therefore [b] the Compound of the squares of A B and B C, and twice the rectangle under A B and B C; that is to say, the square of A C is incommensurable to twice the rectangle under A B and B C, which being rational, as before said, the square of A C incommensurable thereto, is Irrational, and therefore A C Irrational: And is equal in power to a rational and a medial: Therefore, &c. Which was to be demonstrated.

a) 10. d.
b) 17. 10.
10. d.
11. d.

PROP. 42. THEOR. 30.



If two right lines A B *and* B C, *incommensurable in power, are compounded, which make the Compound of their squares medial, and the Rectangle conteined under them medial, and incommensurable to the Compound of their squares; the whole shall be irrational: And is called a line equal in power to two Medials.*

Demonstration FOr let the rational D E be exposed, [a] to the which let there be applyed the rectangle D F, equal to the square of A C, and D G equal to the Compound of the squares of A B and B C, and

a) 45. 1.

and the ſquare of A C [b] being equal to the Compound of the ſquares of A B and B C, and to twice the Rectangle under A B and B C, the reſt H F ſhall be equal to twice the reſt Rectangle under A B and B C.

But ſeeing that as well the Compound of the ſquares of A B and B C; that is to ſay, the Rectangle D G, as the Rectangle under A B and B C: [c] therefore the Compound of twice the Rectangle under A B and B C: that is to ſay, H F (being commenſurable thereto,) is propoſed medial, D G and H F, applyed to the Rational D E, ſhall make the breadths E G and G F Rational, incommenſurable in length to D E.

Again, the Compound of the ſquares of A B and B C; that is to ſay, the Rectangle D G being propoſed incommenſurable to the Rectangle under A B and B C, & twice the Rectangle under A B & B C; to wit H F, being commenſurable to the ſame Rectangle under A B and B C, (being double thereto,) [d] D G and H F ſhall be incommenſurable: Wherefore [e] E G being to G F, as D G to H F: [f] E G and G F ſhall be incommenſurable in length, and being ſhewn rational, the ſame E G and G F ſhall be rational, commenſurable in power onely: Therefore [g] the whole E F compounded of them is irrational: Therefore the Rectangle D F, conteined under D E rational, and E F irrational, is irrational; and therefore the ſquare of A C equal thereto, is alſo irrational, and the line A C equal in power to the ſame is irrational: And is called a line equal in power to two Medials: Therefore, If, &c. Which was to be demonſtrated.

b) 4. 2.

c) C. 24. 10. 23. 10.

d) 13. 10.
e) 1. 6.
f) 10. 10.
g) 37. 10.

PROP. 43. THEOR. 31.

A Binomial line A B, *may be divided in its names onely in one point* C.



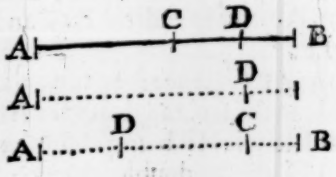
Conſtruction LEt A B a *Binomial* be divided in the point C, in its names, in ſuch ſort as A C and C B may be rationals, commenſurable in power onely, according to the 37 Propoſition of the Tenth: I ſay the line A B cannot be divided into other names, at another point: that is to ſay, in other lines then A C and C B, which may be rational commenſurable in power onely.

Demonſtration FOr (if poſſible) let it be againe divided in his names at D: It is manifeſt that A B is not divided at C in two equal parts, for A C and C B ſhould then be commenſurable in length, which is contrary to the Suppoſition, nor in like manner at D: Therefore A C is divided as well in C as in D unequally: Therefore A C and C B, are parts leſſe unequal then A D and D B, or more unequal: Wherefore the ſquares of A C and C B, ſhall be leſſe then the ſquares of A D and D B, or greater, (for the parts A D and D B, ſhall not be equal to the parts A C and C B, each to his correſpondeut; to wit, the greateſt to the greateſt, and the leſſer to the leſſer, from what part ſoever the point D ſhall fall, (otherwiſe A B ſhould be divided at the ſecond Diviſion, at the ſame point as in the firſt.)

But foraſmuch as, if from equal things there be taken unequal; the exceſſe [a] of the remainders, is equal to the exceſſe of the parts cut off; but the ſquares of A C and C B, with twice the Rectangle under A C and C B, are equal to the ſquares of A D and D B, with twice the rectangle under A D and D B: [b] Seeing that as well the one as the other, are equal to the ſquare of A B: It follows that the exceſſe of the Compound of the ſquares of A C and C B, and of the Compound of the ſquares of A D and D B, ſhall be equal, or the ſame with the Compound of twice the rectangle under A C and C B, and of twice the rectangle under A D and D B.

a) Cor. 2. 1.

b) 4. 2.



But the exceſſe of the Compounds of the ſquares of A C and C B, and A D and D B, is rational, (for A C and C B being rational, and therefore their ſquares rational, and their compound [c] being commenſurable to each of them, ſhall be rational; by the ſame reaſon the compound of the ſquares of A D and D B ſhall be rational: therefore ſeeing that one rational exceeds another rational by a rational: It is manifeſt that the exceſſe of the compounds of the ſquares of A C and C B, and of A D and D B, is rational: Therefore the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B, is alſo rational: But [d] the Rectangle under A C and C B being medial, twice the Rectangle under A C and C B commenſurable thereto [e], is alſo medial: In like manner, twice the rectangle under AD & DB, is alſo medial: Wherefore [f] a medial that exceeds not a medial by a rational, the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B, ſhall not be a ſpace rational: But we have ſhewn that it is rational, which is abſurd: Therefore A B cannot be divided in its names in any other point then C: Therefore it is dided in its names onely in one point. Which was to be demonſtrated.

c) 16. 10. 10. d.

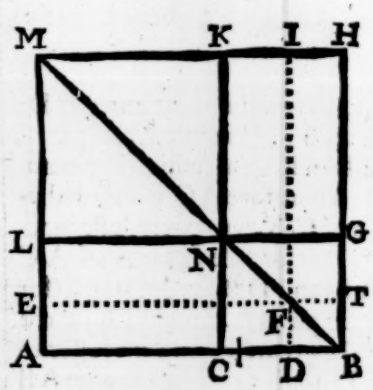
d) 22. 10.

e) C. 24. 10.

f) 27. 10.

¶ This Demonſtration is alſo eaſie to be underſtood by the Figure hereafter expreſſed.

PROP. 44. THEOR. 32.



A firſt Bimedial line A B, *is divided in his names onely in one point* C.

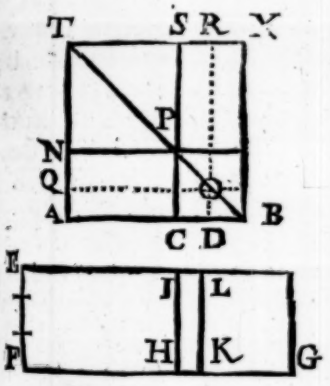
Demonſtration FOr (if poſſible,) let the ſame be divided in D alſo, in ſuch ſort as A D and D B may be medials. commenſurable in power onely, and conteining a rational Rectangle, from what part ſoever the point D ſhall fall, it will be ſhewn as in the precedent, that the exceſſe of twice the Rectangle under A C and C B, and twice the rectangle under A D and D B, is the ſame as the

the exceſſe of the Compounds of the ſquares of A C and C B, and of A D and D B: But the exceſſe of the ſaid Rectangle is rational; (for the Rectangle under A C and C B, being rational, twice the Rectangle under A C and C B being commenſurable thereto, is alſo rational, and in like manner, twice the Rectangle under A D and D B ſhall be rational, and a figure rational, exceeding a figure rational, by a figure rational; the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B, is rational:) Therefore the exceſſe of the Compounds of the ſquares of A C and C B, and of the ſquares of A D and D B, is rational: A C C B being medials commenſurable in power, their ſquares L K and C G ſhall be medials, and commenſurable: Wherefore [a] their Compound ſhall be commenſurable to each of them, and therefore medial; by the ſame reaſon, the Compound of the ſquares of A D and D B, which are E I and D I, is medial; and [b] a ſpace medial not exceeding a medial by a rational; the exceſſe of the Compounds of the ſquares of A C and C B, and of A D and D B, ſhall not be rational, but we have ſhewn it rational; which is abſurd: Therefore A B the firſt of two Medials, or a Prime Bimedial, cannot be divided in his names at any other point then C: And therefore is divided in his names onely in one point: Which was to be demonſtrated.

a) 16. 10. C. 24. 10.

b) 27. 10.

PROP. 45. THEOR. 33.



A ſecond Bimedial line A B, *is divided in his names, onely in one point* C.

Demonſtration FOr if poſſible, let it be divided in D, in ſuch ſort as A D and D B, may be alſo medials commenſurable in power onely, and conteining a medial rectangle: We ſhall ſhew as in the 43 Prop. that the ſquares of A C and C B, which are N S and C V, are greater or leſſer then the ſquares of A D and D B, to wit, Q R, and O B.

Let the rational E F be expoſed, to which let there be [a] applyed the Rectangle E G equal to the ſquare of A B, and E H, equal to the ſquares of A C and C B: Therefore the reſt I G ſhall be equal to twice the Rectangle under A C and C B: Seeing [b] that the ſquare of A B is equal to the ſaid ſquares of A C and C B, and twice the Rectangle under A C and C B, as appeares by the figure.

a) 45. 1.

b) 4. 2.

In like manner, if to E F there be applyed E K, equal to the ſquare of A D and D B, the reſt L G, ſhall be equal to twice the Rectangle under A D and D B; and ſeeing that the ſquares of A C and C B, are unequal to the ſquares of A D and D B, the Rectangles E H and E K, which are equal unto them, ſhall be alſo unequal: and therefore F H and F K, ſhall be unequal.

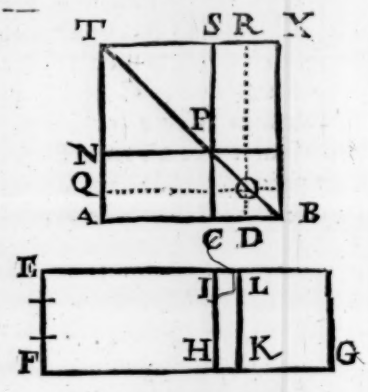
Again, the ſquares of A C and C B, being greater then twice the Rectangle conteined under A C and C B, E H ſhall be greater then I G, and therefore E H greater then the half of E G, and the line F H greater then

the halfe of F G, by the ſame reaſon, F K ſhall be ſhewn greater then the halfe of F G: Therefore the parts F H and H G, are unequal to the parts of F K and K G, each to his correſpondent: But foraſmuch as A C and C B, are medials, commenſurable in power, their ſquares are medials, and commenſurable: Therefore [c] the compound of them E H ſhall be commenſurable to every one of them; and [d] therefore medial: by the ſame reaſon, E K ſhall be medial; therefore [e] E H and E K applyed to the rational E F, make the breadths F H and F K rational, incommenſurable in length to E F.

c) 16. 10.
d) C. 24. 10.
e) 23. 10.

In like manner, ſeeing that the Rectangle under A C and C B is propoſed medial, twice the Rectangle under A C and C B commenſurable thereto; that is to ſay, [f] I G is alſo medial: Therefore being applyed to the rational H I, H G ſhall be rational, incommenſurable [g] in length to H I. In like manner, we ſhall ſhew L G to be a medial, and K G rational, incommenſurable in length to K L.

f) C. 24. 10.
g) 23. 10.



Again, ſeeing that A C and C B are incommenſurable in length, and that as A C is to C B: ſo the ſquare of A C to the Rectangle under A C and C B, the ſquare of A C ſhall be incommenſurable to the Rectangles under A C and C B: But the Compound of the ſquares of A C and C B is commenſurable to the ſquare of A C: (for A C and C B being commenſurable in power; that is to ſay, their ſquares being commenſurable; the [h] Compound of them ſhall be commenſurable to each of them; to wit, to the ſquare of A C: But twice the Rectangle under A C and C B, is commenſurable to the Rectangle under A C and C B: Therefore the Compound of the ſquares of A C and C B; that is to ſay, the Rectangle E H is incommenſurable to twice the Rectangle under A C and C B, that is to ſay, to I G.

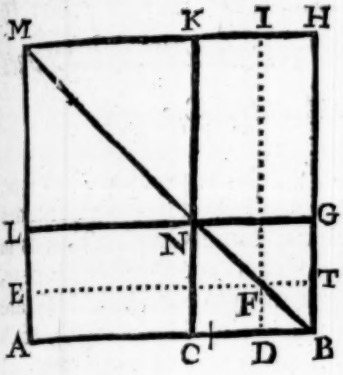
10. 10.
h) 16. 10.

Therefore ſeeing that E H [i] is to I G, as F H to H G: [k] F H and H G ſhall be incommenſurable in length: But they are ſhewn rationals, and therefore rationals commenſurable in power onely: [l] Therefore the whole F G is irrational, which is called by two names, and divided in his names in the point H: In like manner, we ſhall alſo demonſtrate that F G of two names, is divided into other names at another point K, [m] which is impoſſible: Therefore A B a Second Bimedial, is not divided in his names in any other point then C: And therefore is divided in his names at a point onely. Which was to be demonſtrated.

i) 1. 6.
k) 10. 10.
l) 37. 10.
m) 43. 10.

PROP.

PROP. 46. THEOR. 36.



A major Line A B *is divided in his names in one onely point* C.

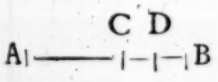
Demonſtration FOr (if poſſible) let it be divided into other names qy D, [a] as it hath been demonſtrated, the exceſſe of the compound of the ſquares of A D and D B, is the ſame as that of twice Rectangle under A D and D B, but the exceſſe of the compounds of the ſaid ſquares is rational: (for ſeeing both the one and the other compound is propoſed rational, their exceſſe ſhall be alſo rational;) therefore the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B is rational: But foraſmuch as the Rectangle under A C and C B, is propoſed medial, twice the Rectangle of [b] A C and C B is alſo medial, being commenſurable: In like manner, the Rectangle under A D and D B twice, is medial: [c] Therefore ſeeing that a medial exceedeth not a medial by a rational, the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B ſhall not be rational: But we have ſhewn that it is alſo rational, which is abſurd: Therefore A B cannot be divided in his names in any other point then C: Therefore it is divided in his names in one onely point. Which was to be demonſtrated.

a) 43. 10.

b) C. 24. 10.

c) 27. 10.

PROP. 47. THEOR. 35.



The line equal in power to a Rational and a Medial is divided its names onely in a point C.

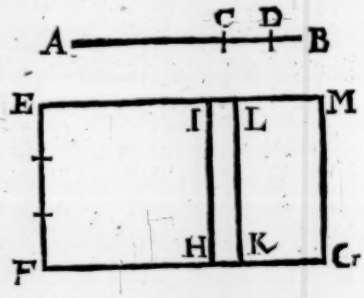
Demonſtration FOr, if poſſible, Let it be divided into other names at D, as we have ſhewn in the 43 Propoſition, that the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B, is the ſame as the exceſſe of the compound of the ſquares of A C and C B, and the compound of the ſquares of A D and D B: but the exceſſe of the ſaid Rectangles is rational, [a] as hath been ſhewn: Therefore the exceſſe of the compound of the ſquares of A C and C B, and of A D and D B, is alſo rational: But the compounds of the ſaid ſquares are propoſed Medial; [b] therefore their exceſſe ſhall not be rational: But it is ſhewn rational, which is abſurd; Therefore A B is not divided in his names in any other point then C: And therefore onely in one point: Which was to be demonſtrated.

a) 44. 10.

b) 27. 10.

PROP.

PROP. 48. THEOR. 36.



The line A B *equal in power to two medials, is divided in his names in one point onely* C.

Demonstration FOr, if possible, Let it be divided into other names by D; using the same Construction as in the 45 Proposition, we shall shew here, (as there) that the parts F H and H G, are unequal to the parts F K and K G, each to his correspondent: Forasmuch then as the compound of the squares of A C and C B is proposed Medial, E H his equal is also medial: and seeing that the Rectangle under A C and C B, is proposed medial also, I G equal
a) C. 24. 10. to the double thereof, [a] is medial, being commensurable: Therefore E H and I G medials, applyed to the rational E F, make F H and H G, rationals, incommensurable in length to E F; (for H I is equal to E F,) so we shall prove that F K, and K G, are rational, incommensurable in length to E F, but seeing that the compound of the squares of A C and C B; that is to say, E H is incommensurable to twice the rectangle under
b) 1. 6. A C and C B, by the Supposition; that is to say, I G: and as [b] E H to I G,
c) 10. 10. so F H to H G, F H and H G are [c] incommensurable in length: But they are shewn rational; therefore F H and H G are rational, commensurable in power onely: Therefore the whole F G is rational, called by two names; and divided in his names in H, but we shall shew that F G is also divided in other names at another point K, which is demonstrated to be impossible:
d) 43. 10. [d] Therefore A B equal in power to two Medials, is not divided in his names at any other point then C: And therefore in one onely point: Which was to be demonstrated.

SECOND DEFINITIONS.

A Rational line being exposed, and a line of two names, (which is called Binomial,) divided in his names, of which the greatest name is more in power than the least, by the square of a right line which is commensurable in length thereto.

1 *If the greatest name be commensurable in length to the Rational exposed: the whole shall be called first of two names, or* A Prime Binomial.

2 *But*

2 *But if the leaſt name be commenſurable in length to the Rational expoſed: Let it be called the ſecond of two names, or* A Second Binomial.

3 *That if neither the one or the other of thoſe names be commenſurable in length to the Rational expoſed: Let it be called the third of two names, or* A Third Binomial.

Again if the greateſt name be more in power then the leaſt, by the ſquare of a right line which may be incommenſurable in length thereto.
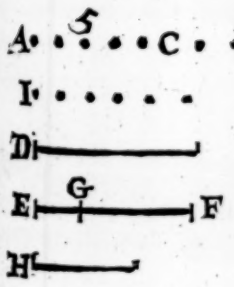
4 *If the greateſt name be commenſurable in length to the Rational expoſed: Let it be called the fourth of two names, or* A Fourth Binomial.

5 *But if the leſſer name be commenſurable in length to the Rational expoſed: Let it be called the fifth of two names, or* A Fifth Binomial.

6 *That if neither of thoſe names be commenſurable in length the Rational expoſed: Let it be called the Sixth of two names, or* A Sixth Binomial.

PROP. 49. PROBL. 13.

To find out a firſt Binomial line.



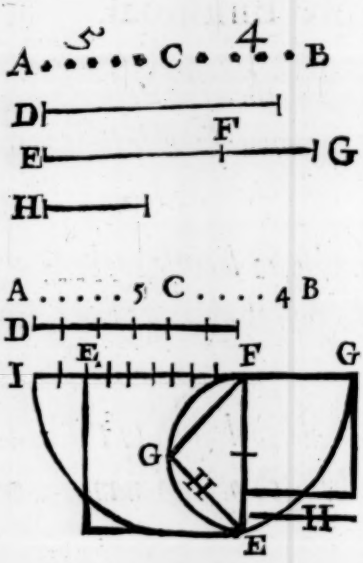
Conſtruction HAving found two ſquare numbers A B and C B, whoſe exceſſe A C may be no ſquare number, in ſuch ſort as A B and C B may be to one another as a ſquare number to a ſquare number; but A B and A C may not be to one another as a ſquare number to a ſquare number; and let there be expoſed a Rational line D, to the which let there be taken E F, commenſurable in length; E F commenſurable to D rational, ſhall be alſo a rational: then let there be drawn F G, to whoſe ſquare a)Co.6.10.

ſquare let the ſquare of E F be as the number A B to the number A C, (which will be eaſily done, by dividing E F into as many equal parts as there are unites in A B;) and taking F I of five ſuch parts, and F G a mean proportional between them, as appeares by the Figure; I ſay, that E G the total, is a firſt Binomial.

Demonſtration FOr ſeeing that the ſquares of E F and F G being to one another as the numbers A B and A C,[b] are commenſurable at leaſt in power: But E F is ſhewn rational: Therefore F G is rational alſo: and A B being not to A C, as a ſquare number to a ſquare number, the ſquares of E F and F G ſhall not be to one another as a ſquare number to a ſquare number: Therefore E F and F G are incommenſurable in length; and therefore Rational, commenſurable in power onely; and the whole E G is irrational, called a Binomial: I ſay that it is alſo firſt, for the ſquare of E F being to the ſquare of F G as A B to A C, a number to a number; and A B being greater then A C, the ſquare of E F ſhall be greater then the ſquare of F G: Let it then be greater by the ſquare of H: and as appeares by the Figure. For aſmuch as the number A B is to the number A C, as the ſquare of E F to the ſquare of F G; by converſion of reaſon, as A B ſhall be to C B; (to wit, to the exceſſe, therefore A B the antecedent, exceeds A C the conſequent:) ſo the ſquare of E F is to the ſquare of H; to wit, to the exceſſe; therefore the antecedent ſquare of E F exceeds the conſequent ſquare of F G: But A B and C B are ſquare numbers; therefore the ſquares of E F and H, are to one another as a ſquare number to a ſquare number therefore E F and H are commenſurable in length: therefore ſeeing that the greateſt number E F is more in power then the leaſt F G, by the ſquare of H, commenſurable in length thereto: and the ſame E F is alſo commenſurable in length to D a rational expoſed: E G ſhall be firſt of two names, or the firſt Binomial, according to the Definition: Therefore we have found, &c. Which was to be done.

b) 6. 10.

9. 10.



PROP. 50. PROBL. 14.

To find out a ſecond Binomial line.

Conſtruction HAving found two ſquare numbers A B and C B, as in the fore-going Propoſition, & the rational D being expoſed; let F G be taken commenſurable in length thereto; F G ſhall be then alſo rational: and as the number A C is to the number A B: So [a] let the ſquare of F G be to the ſquare of F; which will be eaſie, as in the precedent Propoſition, in taking a line as F I, of nine ſuch parts as F G is five thereof, and F E a mean proportional between them, as appeares by the Figure; I ſay that E G is a ſecond Binomial.

a) C. 6. 10.

Demon-

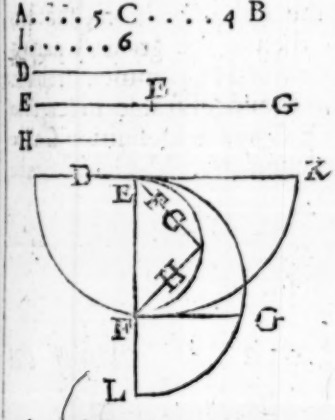
Demonſtration FOr the ſquares of F G and E F being to one another as the numbers A C and A B; [b] they ſhall be commenſurable; and the lines F G and E F, commenſurable at leaſt in power, and F G is ſhewn rational: therefore E F is alſo rational, But ſeeing that neither A C nor A B are to one another as a ſquare number to a ſquare number, the ſquares of F G and E F alſo ſhall not be as a ſquare number to a ſquare number; and [c] therefore E F and F G incommenſurable in length, and ſo are rationals commenſurable in power onely; and the whole E G irrational, which is called a Binomial: I ſay alſo that it is the ſecond Binomial.

b) 6. 10.

c) 9. 10. 37. 10.

For ſeeing that the ſquare of F G is to the ſquare of E F, as the number A C to the number A B, and by converſion of reaſon, the ſquare of E F to the ſquare of F G as A B to A C: and as A B is greater then A C, the ſquare of E F ſhall be greater then the ſquare of F G: Let it then be greater by the ſquare of H, we ſhall ſhew as in the fore-going that H is commenſurable in length to E F: Now ſeeing that the greateſt name E F is more in power then the leaſt F G, by the ſquare of H commenſurable in length thereto, and the leaſt name F G is commenſurable in length to the Rational expoſed D, E G by the Definition, ſhall be the ſecond Binomial: Therefore we have found, &c. Which was to be done.

PROP. 51. PROBL. 15.



To find a third line of two names, or a third Binomial.

Conſtruction HAving taken two numbers A B and C B, as in the 49 Propoſition: Let there be taken another number I, which may not be to A B or to A C, as a ſquare number to a ſquare number: which may be done in taking I, the number not ſquare, the next greater then A C: that being done, it ſhall not be to A B, as a ſquare number to a ſquare number.

Again, there being no neer greater then A C, it ſhall differ from A C by one or two: Therefore between I and A C there will not fall a mean proportional; therefore they ſhall not be like Plaines, nor ſhall be to one another as a ſquare number to a ſquare number: Now let the Rational D be propoſed, and as I is to A B, ſo the ſquare of D may [a] be to the ſquare of E F, which will alſo be eaſie in taking E K, of nine ſuch parts as D is ſix of them, and E F a mean proportional between the two D and E K: therfore the ſquares of D and E F ſhall be commenſurable, and D and E F commenſurable at leaſt in power, and D being a Rational, E F ſhall be ſo alſo; and foraſmuch as I is not to A B; that is to ſay, the ſquare of D is not to the ſquare of E F as a ſquare number to a ſquare number [b] D, and E F ſhall be uncommenſurable in length.

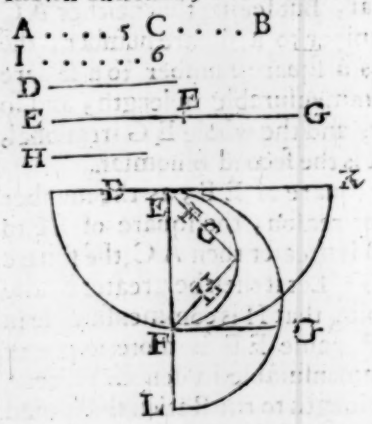
a) Co 6. 10.

b) Co. 9. 10.

Again, let F G, to whoſe ſquare let the ſquare of E F be as A B to A C; [c] which will be done by dividing E F in nine equal parts, and making F L of five

c) 6. 10.

five such parts; then taking F G a mean proportional between E F and F L, as appeares by the figure, those squares shall be commensurable, and E F and F G, at least commensurable in power: Therefore E F being shewn rational, F G shall be also rational; and forasmuch as A B is not to A C; that is to say, the square of E F is not to the square of F G as a square number to a square number, E F and F G d shall be incommensurable in length: Therefore E F and F G are rational, commensurable in power onely: Therefore e the whole E G is irrational, called a Binomial: I say, that it is the third.



d) 9. 10.

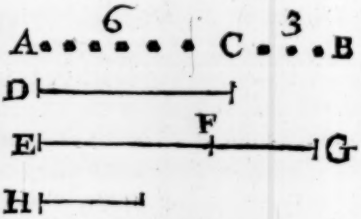
e) 37. 10.

Demonstration FOr seeing that as I to A B so the square of D to the square of E F, and as A B to A C; so the square of E F to the square of F G, in equal reason, as I is to A C, so the square of D to the square of F G: But I and A C are not to one another as a square number to a square number; therefore the squares of D and F G also, shall not be as a square number to a square number; therefore f D and F G are incommensurable in length: But forasmuch as A B is to A C as the square of E F is to the square of F G: But A B is greater then A C, therefore the square of E F shall be greater then the square of F G: Let it then be greater by the square of H, we shall shew, as in the 49 Proposition; that H is commensurable in length to E F: Forasmuch then as the greatest name is more in power then the least F G, by the square of H, commensurable in length thereto; and that neither E F nor F G is commensurable in length to the Rational proposed D, as is demonstrated: E G by the Definition, shall be the third of two names: Therefore we have found, &c. Which was to be done.

f) 9. 10.

PROP. 52. PROBL. 16.

To find a line fourth of two names, or a fourth Binomial.



Construction HAving found two numbers A C and C B, such as the Compound A B be not to either of them as a square number to a square number: Let the rational D be proposed, and E F taken commensurable in length thereto; therefore the same E F shall be also Rational; and having made the rest as in the 49 Proposition; we shall shew as there that the whole E G is a Binomial: I say, it is the fourth.

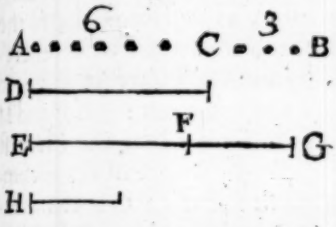
Demonstration FOr the square of E F, (as in the 49,) shall be greater then the square of F G: Let it be so then, by the square of H, and by conversion of reason, as in the 49, as A B is to C B: so the square of E F is to the square of H: But A B being not to C B as a square number to a square

ſquare number, the ſquare of E F ſhall not be to the ſquare of H, as a ſquare number to a ſquare number; therefore [a] E F and H are incommenſurable in length: and ſeeing that the greateſt name E F is more in power then the leaſt F G, by the ſquare of H incommenſurable in length thereto; and the greateſt name E F is commenſurable in length to the Rational D, E G by the Definition, ſhall be the fourth Binomial: Therefore, &c. Which was to be done.

a) 9. 10.

PROP. 53. PROBL. 17.

To find a fifth line of two names, or a fifth Binomial.



Conſtruction HAving found two numbers A C and C B, as in the precedent; Let the Conſtruction be made as in the 50th. that is to ſay; let F G be taken commenſurable in length to the expoſed rational D, it ſhall be ſhewn, as there, that E G is a Binomial: I ſay alſo that it is the fifth; for it ſhall be ſhewn as in the 50th. that the ſquare of E F ſhall be greater then the ſqnare of F G: Let it then be greater by the ſquare of H.

Demonſtration BUt ſeeing by converſion of reaſon (as is demonſtrated in the 49, as A B is to C B, ſo the ſquare of E F, is to the ſquare of H,) it ſhall be ſhewn as in the precedent, that E F and H, are incommenſurable in length: therefore ſeeing that the greateſt name E F is more in power then the leſſer F G, by the ſquare of H, incommenſurable in length thereto; and the leaſt name F G is commenſurable in length to the Rational expoſed D, by the Definition E G ſhall be a fifth Binomial: Therefore we have found, &c. Which was to be done.

PROP. 54. PROBL. 18.



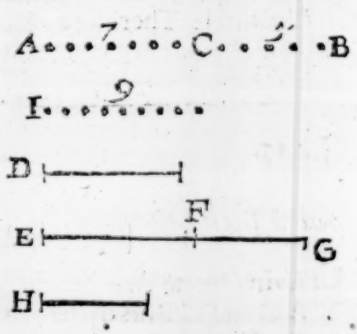
To find a line ſixth of two names, or a ſixth Binomial.

Conſtruction HAving found two numbers A C and C B Plaines, not alike, neither of them being a ſquare, and the Compound of them A B, alſo not a ſquare, nor being to one another as a ſquare number to a ſquare number; (which may be done by dividing ſome number not ſquare, into two numbers,) Primes to one another; [a] for ſo the whole ſhall be Prime to each of them; and therefore ſhall not be to either of them, as a ſquare number to a ſquare

a) 30. 7.

number,) and let there be taken some other number I, that may not be to AB nor to AC, as a square number to a square number, and a rational D be exposed, and as I is to AB; so let the square of D be made [b] to the square of EF, and let the rest be done as in the 51th. We shall shew (as there,) that D and EF, are incommensurable in length, and that the whole EG is a Binomial: I say also that it is the sixth.

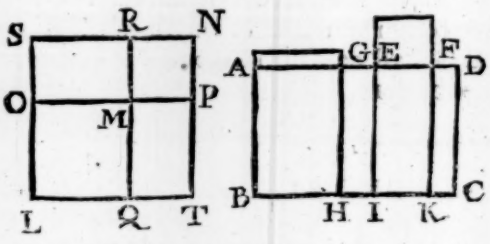
b) Co. 6. 10



Demonstration FOr it shall be demonstrated as in the 51 Proposition, that D and FG are incommensurable in length, and that the square of EF is greater then the square of FG: Let it then be greater by the square of H, we shall demonstrate also, as in the 49th. that by conversion of reason, as AB is to CB, so the square of EF is to the square of H: therefore seeing that AB is not to CB, and therefore the square of EF; to the square of H, as a square nnmber to a square number: [c] EF and H shall be incommensurable in length: Therefore seeing that the greatest name EF, is more in power then the least FG, by the square of H, incommensurable in length thereto; and neither of them EF nor FG is commensurable in length to the exposed Rational D, according to the Definition, EG shall be the Sixth Binomial: Therefore we have found, &c. Which was to be done.

c) 9. 10.

PROP. 55. THEOR. 37.



If a Superficies AC *be conteined under a Rational* AB, *and a first Binomial line* AD, *the right line being equal in power to the said Superficies, is irrational; and is called a Binomial line.*

Demonstration FOr let AE be the greatest names of AD; therefore AE and ED, shall be rational, commensurable in power onely, according to the Definition; and AE shall be more in power then ED, by the square of a line commensurable in length thereto: and AE shall be commensurable to the Rational exposed AB: Let ED be divided in two equal parts in F: Forasmuch then as AE is more in power then ED, by the square of a line commensurable in length thereto: if there be applyed at AG a Rectangle equal to a quarter of the square of ED; that is to say, to the square of EF, conteined under AG and GE: and wanting by a square figure, it [a] shall divide the same in parts commensurable in length: Therefore AG and GE

a) 18. 10.

GE are commensurable in length to one another: Now let there be drawn GH, EI, and FK, parallel to AB and DC; and [b] let the square LM be equal to the Parallelogram AH, and the square MN equal to the Parallelogram GI, and let those squares be joyned together in the point M, in such sort as MO and MP may make one right line OP: Therefore QMR shall also make a right line QR, as [c] appeares: and having finished the Rectangle LN, OM and MP, being equal to QM and MR; (because of the squares LM and MN:) and therefore the whole OP, to the whole QR, and OP [d] being also equal; as well to SN as to LT, and QR to LS and TN, the Rectangle LN shall be equilateral, and therefore a square.

b) 14. 2.

c) 15, 13, & 14. 1.

d) 34. 1.

But the Rectangle under AG and GE; being made equal to the square of EF; as [e] AG to EF: so EF is to GE; and [f] therefore as AH to EK, so EK to GI: Therefore EK is a mean proportional between AH and GI; that is to say, between their equal squares LM and MN: But TM is a mean proportional between LM and MN: Therefore TM is equal to EK: Therefore MS being equal to TM, and [g] FC equal to EK, MS shall be also equal to FC, wherefore the whole square LN shall be equal to the whole Rectangle AC: Therefore the whole OP is equal in power to the Superficies AC conteined under the Rational AB and AD the first Binomial: I say, that OP is irrational, called a Binomial: For seeing that AG and GE are shewn to be commensurable in length, the whole AE shall be commensurable in length to each of them: But AE (being the greatest name of AD the first Binomial,) is commensurable in length to the Rational AB; Therefore AG and GE are commensurable in length to the same AB, [h] as appeares, therefore AB being Rational, AG and GE shall be Rational: Therefore [i] the Rectangles AH and GI conteined under the Rational commensurable in length, are Rationals, as also their equal squares LM and MN: and therefore OM and MP shall be Rational.

e) 17. 6.

f) 1. 6.

g) 43. 1. 36. 1.

16. 10.

h) 12. 10.

i) 20. 10.

And AE being incommensurable in length to ED, and AG is shewn commensurable in length to AE, and EF commensurable in length to ED, being half thereof, AG and EF shall be commensurable in length to one another: Wherefore AH and EK being in the same rate as AG and EF; that [k] is to say, their equals LM and MT are incommensurable: Therefore OM and MP are incommensurable in length, ([l] having the same reason as LM to MT:) but OM and MP are shewn rationals; commensurable in power onely: Wherefore the whole OP equal in power to the Superficies AC is irrational called a Binomial: Therefore, If a Superficies, &c. Which was to be demonstrated.

k) 10. 10.

l) 37. 10.

PROP. 56. THEOR. 38.

If a Superficies AC *be conteined under a rational line* AB, *and a second Binomial line* AD, *the right line equal in power to the said Superficies is irrational, and is called a first Bimedial line.*

Demonstration FOr let AE be the greater name of AD; therefore AE and ED, are rational,, commensurable in power onely, according to the Definition; and AE is more in power then ED, by the square of

of a line commensurable in length thereto; and E D shall be commensurable in length to the Rational exposed A B, Let E D be divided in two equal parts in F; and having done the rest as in the precedent, it shall be demonstrated (as in that Proposition,) that O P is equal in power to the Superficies conteined under A B Rational, and A D a second Binomial: I say, that O P is irrational, and is called a first Bimedial.

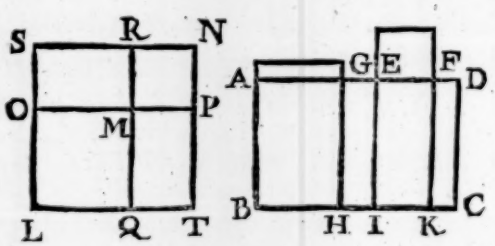
For A E being incommensurable in length to E D, the lesser name of A D a second Binomial, commensurable in length to A B rational, [a] A E and A B are incommensurable in length, and A G and G E being shewn [b] commensurable in length, [c] the whole A E shall be commensurable in length to each of them: Wherefore A E the greatest name of A D being rational, A G and G E shall be also rational: Therefore seeing that as well A G as G E, is commensurable in length to A E; but A E incommensurable in length to A B rational; [d] both A G and G E are incommensurable in length to the same A B: Therefore as well A B and A G as A B and G E are rational, commensurable in power onely: Therefore [e] the Rectangles A H and G I, or their equal squares L M and M N are medials: and therefore O M and M P medials: But A G and G E being commensurable in length, [f] A H and G I being in the same rate, and therefore their equal squares L M and M N shall be commensurable: Therefore O M and M P are commensurable at least in power.

And seeing that A E and E D are incommensurable in length, and A G is shewn commensurable in length to A E: and E F is commensurable in length to E D; being the halfe thereof, A G and E F shall be incommensurable in length: Therefore A H and E K having the same rate, [g] their equal Rectangles L M and M T are incommensurable; and [h] therefore O M and M P are incommensurable in length, having the same rate; therefore O M and M P being shewn medials, and commensurable, they shall be medial, commensurable in power onely.

Lastly, E D the least name being commensurable in length to A B; that is to say[e] to E I: but E F is also commensurable in length to E D, [i] E I and and E F shall be commensurable in length; wherefore E I being rational, E F shall be also rational: Therefore [k] E K is a rational Rectangle: But M T conteined under O M and M P, is equal to E K, therefore M T is rational. Therefore O M and M P are medials, commensurable in power onely, conteining a Rectangle rational; and so [l] O P is irrational, and is called the first Bimedial: Therefore if a Superficies, &c. Which was to be demonstrated.

a) 13. 10.
b) 15. 10.
c) 6. 10.
d) 14. 10.
e) 22. 10.
f) 10. 10.
g) 10. 10.
h) 1. 6.
i) 12. 10.
k) 20. 10.
l) 38. 10.

PROP. 57. THEOR. 39.



If a Superficies A C *be conteined under a Rational line* A B, *and a third Binomial line* A D, *the right line equal in power to the said Superficies* A C, *is irrational, and is called a second Bimedial.*

Demon-

Demonſtration FOr let A E be the greateſt name of A D; therefore A E and E D ſhall be rational, commenſurable in power onely; according to the Definition; and A E ſhall be more in power then E D, by the ſquare of a line commenſurable in length thereto, and neither A E nor E D ſhall be commenſurable in length to the rational expoſed A B.

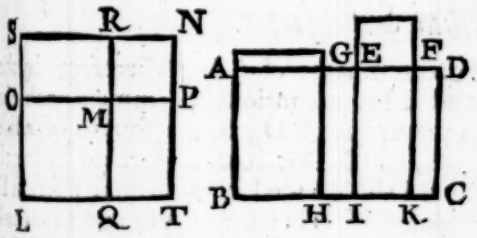
Let E D be divided in two equal parts in F, and do the reſt as is ſhewn in the 55 Propoſition, it ſhall be demonſtrated (as there,) that O P is equal in power to the Superficies A C, and as in the precedent Propoſition 56, that O M and M P are medials, commenſurable in power onely: Seeing that A E is propoſed incommenſurable in length to A B rational; as the ſame A E was there incommenſurable in length to A B, but E D and E F being commenſurable in length, and E D incommenſurable in length to the rational A B; that is to ſay E I: [a] E F ſhall be alſo incommenſurable in length to E I, but E F and E I are rationals, E F being the halfe of E D rational, and E I equal to the rational A B: Therefore E F and E I are rational, commenſurable in power onely, therefore [b] E K is a medial: Therefore M T his equal is alſo medial, conteined under O M and M P medials: Therefore O M and M P being medials, commenſurable in power onely, conteining a Superficies medial, [c] O P ſhall be irrational, and is called a ſecond Bimedial: Therefore, &c, Which was to be demonſtrated.

a) 14. 10.

b) 22. 10.

c) 22. 2.

PROP. 58. THEOR. 40.



If a Superficies A C *be conteinod under a Rational line* A B, *and a fourth Binomial line* A D, *the right line equal in power to the Superficies* A C, *is irrational, and is called a Major line.*

Demonſtration FOr let A E be the greateſt name of A D: Therefore A E and E D ſhall be rational, commenſurable in power onely, according o the Definition, and A E ſhall be more in power then E D; by the ſquare of a line incommenſurable in length thereto, and A E ſhall be commenſurable in length to A B: For having divided E D in two equal parts in F; Let the reſt be done as by the 55 Propoſition: Therefore [a] A G and G E ſhall be incommenſurable in length: Now we ſhall ſhew (as there,) that O P is equal in power to the Superficies A C: I ſay, that O P is irrational, called Major: For A G and G E being incommenſurable in length: [b] A H and G I having the ſame rate, are incommenſerable in length, and therefore their equal ſquares L M and M N incommenſurable: Wherefore O M and M P are incommenſurable in power: But A E the greateſt name being commenſurable in length to A B rational, ſhall be alſo rational: [c] and the Rectangle A I under them rational: which being equal to the Compound of

a) 19. 10.

b) 10. 10.

c) 20. 10.

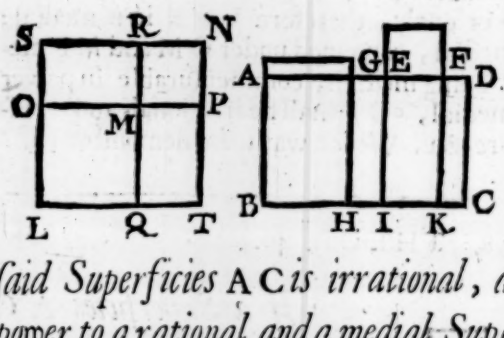
of the ſquares L M and M N, the ſame Compound of L M and M N is Rational.

And foraſmuch as E D the leaſt name, is incommenſurable in length to the Rational A B; E F its halfe, ſhall be alſo incommenſurable in length to A B: and E F is rational, being commenſurable to E D rational: Therefore E F and A B are rational commenſurable in power onely; d Wherefore E K conteined under them, and therefore M T his equal, conteined under O M and M P, is a medial; wherefore O P and M P being incommenſurable in power, and making the Compound of their ſquares L M and M N rational, and the Rectangle M T under the ſame medial, the whole c O P ſhall be irrational, and is called Major: Therefore, &c. Which was to be demonſtrated.

d) 22. 10.

c) 40. 10.

PROP. 59. THEOR. 41.



If a Superficies A C *be conteined under a Rational line* A B, *and a fifth Binomial line, the right line equal in power to the ſaid Superficies* A C *is irrational, and is called a line equal in power to a rational and a medial Superficies.*

Demonſtration. FOr let A E be the greateſt name of A D: Therefore by the Definition A E and E D, are rational, commenſurable in power onely, and A E is more in power then E D, by the ſquare of a line incommenſurable thereto in length, and E D is commenſurable in length to A B the rational expoſed: Let E D be divided in two equal parts at F, and let it be done as in the 55 Propoſition: Therefore a A G and G E ſhall be incommenſurable in length, and we ſhall ſhew (as there) that O P is equal in power to the Superficies A C: I ſay, that O P is irrational, and is called a line equal in power to a Rational and a Medial: For as in the precedent Propoſition, O M and M P ſhall be incommenſurable in power, and A E the greateſt name being rational, and incommenſurable in length to the rational A B: A E and A B ſhall be rational, commenſurable in power onely; b wherefore the Rectangle A I under them is medial: But A I is equal to the Compound of the ſquares L M and M N; therefore the ſame Compound ſhall be medial.

a) 19. 10.

b) 22. 10.

Again, E D being propoſed rational, commenſurable in length to the rational A B, being the leaſt name of A D a fifth Binomial, E F the halfe of E D ſhall be alſo commenſurable in length to A B, and rational: therefore c E K conteined under E I and E F rationals, commenſurable in length, and therefore his equal Rectangle M T under O M and M P is rational; therefore O M and M P being incommenſurable in length, and making the Compound of their ſquares L M and M N medial: but the Rectangle M T under them rational; the d whole O P ſhall be irrational, and is called a line equal in power to a Rational, and a medial: Therefore, If a Superficies, &c. Which was to be demonſtrated.

c) 20. 10.

d) 40. 10.

PROP.

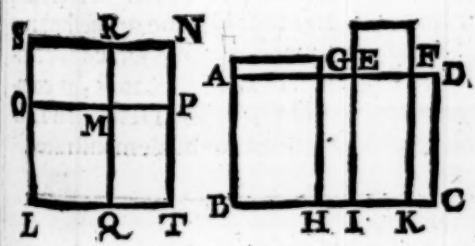
PROP. 60. THEOR. 42.

If a Superficies A C *be conteined under a rational line* A B, *and a ſixth Binomial line* A D, *the right line equal in power to the ſame Superficies* A C, *is irrational, and is called a line equal in power to two medials.*

Demonſtration. FOr let A E be the greateſt name: Therefore A E and E D are rationals, commenſurable in power onely, according to the Definition, and A E ſhall be more in power then E D, by the ſquare of a line which ſhall be incommenſurable thereto in length, and neither the one or the other of them AE and ED ſhall be commenſurable in length to the Rational A B: Having done as in the precedent Propoſition, A G [a] and G E ſhall be incommenſurable in length, and it ſhall be ſhewn as in the 58 Propoſition, that O P is equal in power to the Superficies A C, and that O M and M P are incommenſurable in power. a) 19. 10.

Again, as in the precedent Propoſition, the Compound of the ſquares L M and M N ſhall be medial; but as in the 58 Propoſition, M T under O M and M P ſhall be alſo medial: And foraſmuch as of A E and E F, the one A E is incommenſurable in length to E D, and the other E F is commenſurable to E D; [b] A E and E F ſhall be incommenſurable in length; b) 13. 10.
therefore [c] A I and E K are incommenſurable; having [d] the ſame rate as A E c) 10. 10.
and E F; wherefore the Compound of the ſquares L M and M N, equal d) 1. 6.
to A I, and M T equal to E K, are incommenſurable: Therefore O M and M P being commenſurable in power, and making the Compound of their ſquares L M and M N medial, and M T the Rectangle under them medial, and incommenſurable to the Compound of their ſquares, the [e] whole O P ſhall be Irrational, and is called a line equal in power to two Medials: Therefore, If, &c. Which was to be demonſtrated. e) 42. 10.

PROP. 61. THEOR. 43.



The Square of a Binomial line A B, *applyed unto a Rational line* D E, *makes the breadth* D G, *a firſt Binomial.*

Demonſtration. FOr to D E Rational let there be applyed D H, equal to the ſquare of A C: and to H I let there be applyed I K, equal to the ſquare of C B: Therefore the reſt L F ſhall be equal to twice the Rectangle under A C and C B, [a] the ſquares of A C and C B, and twice a) 4. 2.
the Rectangle under A C and C B; being equal to the ſquare of A B, even as the Rectangle D F is equal to the ſame ſquare of A B, by the Conſtruction.

Let L G be divided in two equal parts at M, and let M N be drawn parallel to L K and G F, both the one and the other Rectangles L N and
b) 37.10. M F, shall be equal to the Rectangle under A C and C B: And [b] A C and C B being Rationals, commensurable in power onely, the whole A B being a Binomial, the squares of A C & C B shall be rational, & therefore commensu-
c) 16.10. rable, [c] and their compounds being commensurable to each of them, the said compound shall be also rational, to which D K being equal, by the cõstruction D K shall be a rational: & seeing that the same D K is applyed to the rational, it shall make the breadth D L rational, & commensurable in length to D E.

Again, seeing that A C and C B are rationals commensurable in power
d) 22.10. onely; the [d] Rectangle under them shall be medial, and therefore twice the Rectangle under A C and C B, commensurable thereto; that is to say,
e) C.24.10. L E [e] shall be medial; therefore L F applyed to L K Rational, [f] makes the
f) 23.10. breadth L G rational, incommensurabe in length to L K, that is to say, to
g) 13.10. D E: but D L is shewn commensurable in length to D E; therefore [g] D L and L G are incommensurable in length: Therefore they are rational, com-
h) 37.10. mensurable in power onely; and [h] therefore D G is a Binomial, or of two names: I say that it is the first.

For the Rectangle under A C and C B, being a mean proportional between the squares of A C and C B, L N shall be also a mean proportional between D H and I K; therefore D I, L M, and I L having the same rate as D H, L N, I K, and L M, shall be also mean proportionals between D I
i) 17.7. and I L: [i] Therefore the Rectangle under D I and I L shall be equal to the square of L M, and the squares of A C and C B being commensurable, (seeing that A C and C B are proposed commensurable in power,) D H
k) 10.10. and I K equal to them, shall be commensurable: Therefore [k] D I and I L having the same rate, shall be commensurable in length.

But seeing that D K is greater then L F, the squares of A C and C B being greater then twice the Rectangle under A C and C B; D L shall be greater
l) 1.6. then L G: ([l] D L and L G having the same rate as D K and L F;) therefore seeing that D L is greater then L G, and at D L is applyed the Rectangle under D I and I L, equal to the square of L M; that is to say, to a quarter of the square of L G, the lesser wanting a square figure, (for the Rectangle under D I and I L, is shewn equal to the square of L M,) and D L is divided at I, in D I and I L, commensurable in length, as hath been demonstrated: Therefore
m) 18. 10. [m] D L the greatest, shall be more in power then L G the lesser, by the square of a line commensurable in length thereto; wherefore D G being shewn to be a Binomial, and that D L the greatest name is more in power then the lesser L G, by the square of a line which is commensurable in length thereto, and that the same D L the greatest, is commensurable in length to the Rational proposed D E, by the Definition D G shall be first Binomial: Therefore, &c. Which was to be demonstrated.

PROP. 62. THEOR. 44.

The square of a first Bimedial line A B, *applyed to a Rational line* D E, *makes the breadth* D C *a second Binomial line.*

Demonstration LEt there be done as in the precedent Proposition, in such sort as that D H and I K may again be equal to the squares of

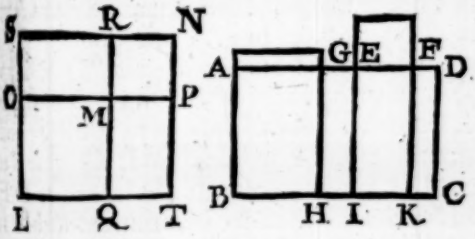
of A C and C B, and L N and M F, equal each of them to the Rectangle under A C and C B, forasmuch as A C and C B compounding A B a first Bimedial, are [a] medials, commensurable in power onely; conteining a rationa Rectangle: the squares of A C and C B, and therefore their equal Rectangles D H and I K are commensurable and medials; and [b] therefore the whole D K being commensurable to each of them, D H and I K [c] shall be also medial, the which being applyed to the Rational D E, [d] its breadth D L shall be Rational, incommensurable in length to D E.

a) 38. 10.
b) 16. 10.
c) C. 24. 10.
d) 23. 10.

Again, the Rectangle under A C and C B being Rational, the double thereof, to wit, L F shall be also Rational, which being applyed to the rational L K [e] its latitude L G shall be rational, commensurable in length to L K; that is to say, to D E; therefore seeing that L G is commensurable in length to D E, and D L incommensurable in length to D E, [f] D L and L G shall be incommensurable in length, and being shewn rational, they shall be rational, commensurable in power onely; and [g] therefore the whole D G is a Binomial, and as in the precedent we shall shew that D L is the greatest name, and is more in power then the least L G; by the square of a line commensurable thereto in length, and the least name L G being shewn commensurable in length to the rational exposed D E, according to the Definition, D G shall be second of two names, or a second Binomial: Therefore, &c. Which was to be demonstrated.

e) 21. 10.
f) 13. 10.
g) 37. 10.

PROP. 63. THEOR. 45.



The Square of a second Bimedial line A B, *applyed to a Rational line* D E, *makes the breadth* D G, *a third Binomial line.*

Demonstration. FOr having made the Construction, as in the 61 Proposition; Forasmuch as A C and C B, making the whole A B a second Bimedial, [a] are medials, commensurable in power onely; conteining a medial; the squares of A C and C B; and therefore their equal Rectangles D H and I K, are commensurable, and medials: Wherefore the whole D K being commensurable to each of them, [b] shall be also medial, and being applyed to the Rational D E; [c] shall make the breadth L D rational, commensurable in length to D E.

a) 39. 10.
b) C. 24. 10.
c) 23. 10.

Again, the Rectangle under A C and C B being medial; the double thereof, to wit L F, shall be also medial: which being applyed to the Rational L K, [d] the other side or breadth L G shall be Rational, commensurable in length to L K; that is to say, to D E; and seeing that A C is incommensurable in length to C B; and as A C to C B, so the square of A C to the Rectangle under A C and C B: [e] the square of A C is also incommensurable to the Rectangle under A C and C B. [f] But the Compound of the squares of A C and C B, is commensurable to the square of A C, (those squares

d) 23. 10.
e) 10. 10.
f) 16. 10.

squares being commensurable, described of the lines proposed commensurable in power,) and twice the Rectangle under A C and C B is commensurable to the Rectangle under A C and C B, being double thereto: Therefore the Compound of the squares of A C and C B; that is to say, the Rectangle D K is commensurable to twice the Rectangle under A C and C B; that is to say, to the Rectangle L F; therefore D L and L G having the same rate as D K and L F, g are incommensurabre in length: But they are shewn rationals; Therefore D L and L G are rationals, commensurable in power onely: Therefore h the whole D G is a Binomial, and as in the 61 Proposition, we shall shew, that D L is the greatest name, and is more in power then L G the lesser, by the square of a line which shall be incommensurable thereto in length, and neither the one nor the other, D L nor L G, being shewn not to be commensurable in length to the Rational proposed, by the Definition, D G shall be a third Binomial: Therefore, &c. Which was to be demonstrated.

g) 10. 10.

h) 37. 10.

PROP. 64. THEOR. 46.

The Square of a Major line A B, *applyed to a Rational line* D E, *makes the breadth* D G *a fourth Binomial line.*

Demonstration FOr let the Construction be made as in the 61 Proposition: Foraſmuch as A C and C B, making the Major A B, a are incommensurable in power, and do make the Compound of their squares rational, and the Rectangle under them medial: D K equal to the Compound of those squares, shall be also rational: But L F the double of the Rectangle under A C and C B medial: Foraſmuch then as D K is applyed to the Rational D E: b D L shall be rational, commensurable in length to D E; in like manner, L F medial being applyed to the Rational L K, c L G shall be rational, incommensurable in length to L K; that is to say, to D E: Therefore seeing that D L is commensurable in length to D E, and L G incommensurable: D L and L G shall be incommensurable in length: but they are shewn Rational: Therefore D L and L G are rational, commensurable in power onely; therefore d D G is a Binomial.

a) 45. 10.

b) 21. 10.

c) 23. 10. 13. 10.

d) 37. 10.

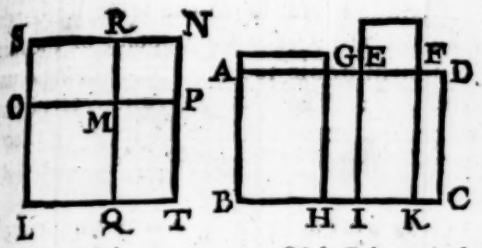
Now, as is shewn in the 62 Proposition, the Rectangle under D I and I L is equal to the Square of L M: but foraſmuch as the squares of A C and C B are incommensurable, (A C and C B being incommensurable in power,) their equal Rectangles D H and I K, shall be also incommensurable; and e therefore D I and I L being in the same reason, are incommensurable in length: But as is shewn in the 61 Proposition, D L is greater then L G, and to D L there hath been applyed a Rectangle conteined under D I and I L, equal to the square of L M: that is to say, to the quarter part of the square of L G, wanting a square figure, which divideth D L at I, in parts incommensurable in length; f D L shall be more in power then L G, by the square of a line which is incommensurable thereto in length: Wherefore D L (the greatest name of D G a Binomial,) being shewn commensurable in length to D E the rational proposed: D G by the Definition, shall be a fourth Binomial: Therefore, &c. Which was to be demonstrated.

e) 10. 10.

f) 19. 10.

PROP.

PROP. 65. THEOR. 47.



The Square of a line A B, *equal in power to a Rational* A C, *and a medial* C B, *applyed to a Rational line* D E, *makes the breadth* D G, *a fifth Binomial line.*

Demonſtration FOraſmuch as A C and C B, which make A B equal in power to a rational and a medial, are incommenſurable in power, making the Compound of their ſquares medial, and the Rectangle under them rational, D K equal to the Compound of the ſquares of A C and C B, ſhall be medial: and L N equal to the Rectangle under A C and C B: and therefore the double thereof to wit L F, ſhall be rational: Therefore D K a medial, applyed to D E a rational, makes [a] D L the breadth rational, incommenſurable in length to D E rational: but L F rational applyed to the Rational L K, [b] makes the breadth L G rational, commenſurable in length to L K, that is to ſay, to D E. 41.10. a) 23.10. b) 21.10.

Foraſmuch [c] then as L G is commenſurable in length to D E rational, and D L incommenſurable in length, D L and L G ſhall be incommenſurable in length; wherefore D L and L G are rational, commenſurable in power onely; and therefore [d] D G is a Binomial. c) 13.10. d) 37.10.

Now we ſhall ſhew as in the 61 Propoſition, that the Rectangle under D I and I L is equal to the ſquare of L M: and as in the precedent, D L is the greateſt name, and is more in power then L G, by the ſquare of a line which ſhall be incommenſurable in length thereto; wherefore the leaſt name L G being ſhewn commenſurable in length to the rational expoſed D E, by the Definition, D G ſhall be a fifth Binomial: Therefore, &c. Which was to be demonſtrated.

PROP. 66. THEOR. 48.

The ſquare of a line A B, *equal in power to two medials, applyed to a Rational line* D E, *makes the breadth* D G, *a ſixth Binomial.*

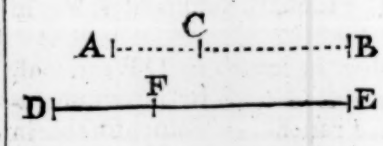
Demonſtration FOr having made the Conſtruction as in the 61 Propoſition: Foraſmuch as A C and C B, making A B equal in power to two medials, are incommenſurable in power, and make the Compound of their ſquares medial, and the Rectangle under them medial, and incommenſurable to the Compound of their ſquares: as well D K equal to the ſame Compound, as L F equal to twice the Rectangle under A C and C B, is medial: and D K and L F medials, being applyed to the Rational D E, 42.10.

a) 23.10. DE, [a] their breadths DL and LG ſhall be rational, incommenſurable in length to DE rational.

b) 42.10. And [b] foraſmuch as the Compound of the ſquares of AC and CB is incommenſurable to the Rectangle under AC and CB, and twice the Rectangle under AC and CB, is commenſurable to the ſame Rectangle under AC and CB, being double thereto; c) 13.10. [c] the Compound of the ſaid ſquares, that is to ſay, DK, is commenſurable to twice the Rectangle under AC and CB; that is to ſay, to LF: Wherefore DL and LG, having the ſame rate as DK and LF, d) 10.10. [d] are incommenſurable in length; therefore DL to LG are rational, commenſurable in power onely e) 37.10. [e]; Therefore DG is a Binomial.

And we ſhall demonſtrate as in the precedent, that DL is the greateſt name, and more in power then LG, by the ſquare of a line which is incommenſurable in length thereto; and being ſhewn that neither DL nor LG is commenſurable in length to DE the rational expoſed, by the Definition, DG ſhall be a ſixth Binomial: Therefore, &c. Which was to be demonſtrated.

PROP. 67. THEOR. 49.



The line DE *commenſurable in length to the line of two names* AB, *or Binomial, is alſo a line of two names, and of the ſame order.*

Demonſtration. a) 12.6. FOr as the whole AB is to the whole DE; ſo [a] the part cut off AC, to the part cut off DF; b) 19.5. therefore [b] the reſt CB ſhall be to the reſt FE, as the whole AB to the whole DE: And foraſmuch as DE is propoſed commenſurable in length to AB; c) 10.10. [c] DF ſhall be alſo commenſurable in length to AC, and FE to CB: But d) 37.10. [d] AC and CB names of AB Binomial, are rational; therefore DF and FE are alſo rational.

Again, ſeeing that as AC is to DF, ſo CB to FE; and alternately, as AC to CB, ſo DF to FE: but AC and CB are commenſurable in power onely: e) 10.10. therefore [e] DF and FE are alſo commenſurable in power onely: Therefore DF and FE are rational, commenſurable in power onely; f) 37.10. and [f] therefore DE is a Binomial: I ſay, that DE is alſo of the ſame order as AB, for AC is more in power then CB; by the ſquare of a line which is commenſurable or incommenſurable thereto; and if AC be more in power than CB, by the ſquare of a line that may be commenſurable in length thereto; g) 15.10. [g] DF ſhall be more in power than EF, by the ſquare of a line which ſhall be commenſurable in length thereto.

And if AC be commenſurable in length to the Rational expoſed, in ſuch ſort as AB may be a firſt Binomial, h) 12.10. [h] DF ſhall be alſo commenſurable in length to the ſame rational, both the one and the other, being rational; And DF being alſo commenſurable in length to AC: Wherefore DE according to the Definition, is a firſt Binomial; that is to ſay, of the ſame order.

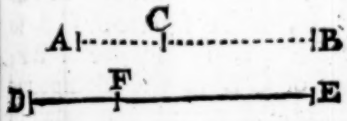
But if CB be commenſurable in length to the Rational expoſed: in ſuch ſort

ſort as A B is a ſecond Binomial: In like manner, F E ſhall be commenſurable in length to the Rational expoſed: wherefore D E by the Definition, is a ſecond Binomial.

Laſtly, if neither A C nor A B be commenſurable in length to the rational expoſed: in ſuch ſort, as A B may be a third Binomial; neither the one nor the other D F nor F E, is commenſurable in length to the Rational expoſed: Therefore D E by the Definition, is a third Binomial.

But if A C be more in power then C B, by the ſquare of a line which ſhall be incommenſurable thereto in length, D F ſhall be more in power than F E, by the ſquare of a line incommenſurable in length thereto: wherefore we ſhall ſhew (as before,) that D E is a fourth, fifth, or ſixth Binomial, according as A B is a fourth, fifth, or ſixth Binomial: Therefore, &c. Which was to be demonſtrated.

PROP. 68. THEOR. 50.



The right line D E *commenſurable to a Bimedial line* A B, *the ſame is alſo Bimedial, and of the ſame order.*

Demonſtration. FOr as [a] the whole A B is to the whole D E, ſo the part cut off A C, to the part cut off D F: Therefore [b] the reſt C B ſhall be to the reſt F E, as the whole A B to the whole D E; wherefore D E being propoſed commenſurable in length to A B, D F [c] ſhall be commenſurable in length to A C, and F E to C B: But A C and C B are medials: therefore [d] D F and F E commenſurable unto them, are alſo medials.

Again, A C being to D F as C B to F E; alternately, as A C is to C B, D F ſhall be to F E: but [e] A C and C B are commenſurable in power onely: Therefore [f] D F and F E ſhall be alſo commenſurable in power onely: and being ſhewn medials, the ſame D F and F E ſhall be medials, commenſurable in power onely: wherefore [g] D E ſhall be a Bimedial.

I ſay, that it is of the ſame order as A B: for ſeeing that as A C to C B, ſo is D F to F E: but as A C to C B, ſo the ſquare of A C to the Rectangle under A C and C B, and as D F to F E, ſo the ſquare of D F to the Rectangle under D F and F E; as the ſquare of A C ſhall be to the Rectangle under A C and C B: ſo the ſquare of D F ſhall be to the Rectangle under D E and E F: and alternately, as the ſquare of A C to the ſquare of D F: ſo the Rectangle under A C and C B, to the Rectangle under D F and F E: but the ſquare of A C is commenſurable to the ſquare of D F: A C and D F being ſhewn commenſurable in length: [h] Therefore the Rectangle under A C and C B ſhall be commenſurable to the Rectangle under D F and F E: Wherefore if the Rectangle under A C and C B be rational; in ſuch ſort as A B may be a firſt Bimedial: the Rectangle under D F and F E, commenſurable thereto, ſhall be alſo rational; and [i] therefore D E ſhall be a firſt Bimedial.

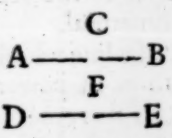
But if the Rectangle under A C and C B were medial; in ſuch ſort as A B were a ſecond Bimedial, the Rectangle under D F and F E, commenſurable

a) 12. 6.
b) 19. 5.
c) 10. 10.
d) 24. 10.
e) 38. 10.
f) 10. 10.
g) 39. 10.
h) 10. 10.
i) 38. 10.

surable thereto, should [k] be also medial: Therefore D E should be a second Bimedial: Therefore, &c. Which was to be demonstrated.

k) 39. 10.

PROP. 69. THEOR. 51.



The right line D E *commensurable to a Major line* A B, *is also a Major line.*

Demonstration. LEt there be done as before, in such sort as A C and C B may bear the same rate to D F and F E, as the whole A B to the whole D E: Forasmuch then as A B and D E are commensurable either in length and power, or in power alone, as well [a] A C and D F, as as C B and F E shall be commensurable after the same manner.

a) 10. 10.

Again, forasmuch as A C is to D F, as C B is to F E; and alternately, as A C to C B, so D F to F E: as [b] the square of A C shall be to the square of C B, so the square of D F to the square of E F: And in compounding, as the Compound of the squares of A C and C B, to the square of C B, so the Compound of the squares of D F and F E, to the square of E F; and by conversion of reason, as the square of C B to the Compound of the squares of A C and C B, so the square of F E to the compound of the squares of D F and F E; and alternately, as the square of C B to the square of F E, so the compound of the squares of A C and C B to the compound of the squares of D F and F E: but the square of C B is commensurable to the square of F E: C B and F E, being shewn commensurable, be it in length and power, or in power onely: Therefore [c] the compound of the squares of A C and C B, is commensurable to the compound of the squares of D F and F E; but the compound of the squares of [d] A C and C B is Rational: seeing that A C and C B compound the Major A B: therefore the compound of the squares of D F and F E is also Rational.

b) 22. 6.

c) 10. 10.

d) 40. 10. 9 Def.

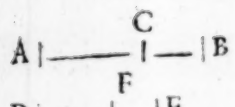
Again, seeing that as A C is to C B so D F to F E; but as A C to C B, so the square of A C to the rectangle under A C and C B; and as D F to F E, so the square of D F to the rectangle under D F and F E, as the square of A C shall be to the Rectangle under A C and C B, so the square of D F shall be to the rectangle under D F & F E; and alternately, as the square of A C to the square of D F, so the rectangle under A C & C B, to the Rectangle under D F and F E.

But the square of A C is commensurable to the square of D F, A C and D F being shewn commensurable either in length and power, or in power alone; therefore [e] the Rectangle under A C and C B, is also commensurable to the Rectangle under D F and F E: but [f] the Rectangle under A C and C B is medial; therefore the Rectangle under D F and F E, commensurable thereto, is also medial: But A C being to C B as D F to F E; and [g] A C and C B, are incommensurable in power, (compounding the Major A B,) D F and F E shall be also incommensurable in power: Therefore D F and F E being incommensurable in power, and making the compound of their squares Rational, and the Rectangle under them Medial, as hath been shewn: [h] the whole D E shall be a Major line: Therefore, &c. Which was to be demonstrated.

c) 40. 10.
f) C. 24. 10.
g) 40. 10. 10. 10.
h) 40. 10.

PROP.

PROP. 70. THEOR. 52.



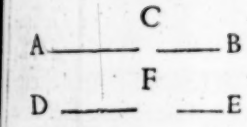
The line DE *commensurable to the line* AB, *equal in power to a Rational and a Medial, is also a line equal in power to a Rational and a Medial.*

Demonstration. FOr having made the same Construction as before, we shall shew as in the precedent, that the compound of the squares of AC and CB is commensurable to the compound of the squares of DF and FE: but the compound of the squares of AC and CB is a Medial; seeing that AC and CB do compound AB equal in power to a Rational and a Medial; therefore the compound of the squares of DF and FE, commensurable thereto, is Medial.

In like manner, as in the precedent Proposition, the Rectangle under AC and CB, which [a] is rational, shall be commensurable to the Rectangle under DF and FE; and therefore the Rectangle under DF and FE shall be Rational. a) 41.10. 9.d.

Lastly, as in the precedent Proposition, DF and FE shall be incommensurable in power; therefore DF and FE being incommensurable in power, and making the compound of their squares Medial, and the Rectangle under them Rational, as is shewn, [b] the whole DE shall be a line equal in power to a Rational and a Medial: Therefore, &c. Which was to be demonstrated. b) 41.10.

PROP. 71. THEOR. 53.



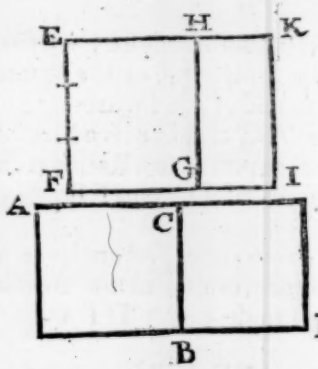
The line DE *commensurable to the line* AB, *equal in power to two medials, is also a line equal in power to two medials.*

Demonstration. FOr, the Construction made, as before, it shall be shewn as in the 69 Proposition, that the compound of the squares of AC and CB is commensurable to the compound of the squares of DF and FE, and the Rectangle under AC and CB commensurable to the Rectangle under DF and FE: wherefore [a] as well the compound of the squares of AC and CB, or the Rectangle under AC and CB, being medial: also [b] as well the compound of the squares of DF and FE, as the Rectangle under DF and FE, shall be medial: But DF and FE are incommensurable in power, as before is said. a) 42.10. b) C.24.10.

Lastly, [c] the compound of the squares of AC and CB, and the Rectangle under AC and CB, being incommensurable, by Supposition, seeing that AB is equal in power to two medials: But the compound of the squares of DF and FE, is shewn commensurable to the compound of the squares of AC and CB, and the Rectangle under DF and FE shewn com- c) 42.10.

commensurable to the Rectangle under A C and C B, the compound of the squares of D F and F E, and the Rectangle under D F and F E shall be incommensurable: Therefore D F and F E being incommensurable in power, and making the compound of their squares medial, and the Rectangle under them medial, and incommensurable to the compound of their squares,
d) 42.10. [d] D E shall be a line equal in power to two medials: Therefore, &c. Which was to be demonstrated.

PROP. 72. THEOR. 54.



If a rational Superficies A B, *be compounded with a medial* C D, *there will be made four Irrationals, to wit, that which is called a Binomial, or a first Bimedial, or a line major, or a line equal in power to a rational and a Medial.*

Demonstration. FOr the Space A B, shall be greater then C D, or lesse, for it cannot be equal: For A B being rational, if C D were equal thereto, it should be also rational, contrary to the Supposition.

Let first of all A B be greater then C D, and to the rational proposed E F, let there be applyed the Rectangle E G equal to A B, and to H G let there be applyed H I equal to C D, insuch sort as the whole E I be equal to the whole A D, and A B being a rational, and C D medial; E G shall be also rational, and H I medial, which applyed to the rational E F: E H
a) 21.10. [a] shall be rational, commensurable in length to E F: [b] but H K rational,
b) 23.10. incommensurable in length to E F.

Again, E G and H I being incommensurable, as appeares by the Definion, E G being rational, and H I irrational, to wit, medial; [c] E H and H K, having the same rate, [d] shall be incommensurable in length, and therefore are rational, commensurable in power only: [e] Therefore E K is a
c) 1.6. d) 10.10. e) 37.10. binomial, and A B being proposed greater then C D; that is to say, E G greater then H I: E H shall be also greater then H K, E H and H K having the same rate as E G and H I: Now E H the greatest name, shall be more in power then H K the lesser, by a square of a line commensurable in length or incommensurable thereto.

If E H be more in power then H K, by the square of a line commensurable in length thereto, E K shall be a first Binomial, by the Definition: E H being the greater name, shewn commensurable in length to the rational E F: Wherefore the line equal in power to the Space E I, conteined under the rational E F, and E K is a first Binomial; and therefore the Space A B compounded of the rational A B, and of the Medial C D [f] is irrational,
f) 55.10. called a Binomial.

But if E H be more in power then H K, by the square of a line which may be incommensurable in length thereto: E K shall be a fourth Binomial, by the Definition, E H being the greatest name, shewn commensurable in length

length to the rational E F: Therefore the right line equal in power to the ſquare E I, conteined under E F rational, and E K a fourth binomial, and therefore alſo the ſquare A D g is irrational, called Major. g) 58. 10.

Now let A B be the greateſt irrational, and medial, and C D the leaſt rational: having made the Conſtruction as before, E H ſhall be incommenſurable in length to E F, and H K commenſurable, and therefore E H and H K rational, commenſurable in power only, and the whole E K a Binomial; therefore E H the greateſt name, ſhall be more in power then H K, by the ſquare of a line which ſhall be commenſurable thereto in length, or incommenſurable: if it be commenſurable, and E H is ſhewn incommenſurable in length to the rational propoſed, E K by the Definition, ſhall be a ſecond Binomial: Therefore the line equal in power to the Superficies E I, a ſecond Binomial, is irrational, h called a firſt Bimedial, if incommenſurable, E K ſhall be a fifth Binomial; therefore the right line equal in power to the Superficies E I, (which is A B,) conteined under E F rational, and E K a fifth Binomial, i is irrational, called a line equal in power to a rational, and a medial: Therefore, &c. Which was to be demonſtrated. h) 56. 10. i) 59. 10.

PROP. 73. THEOR. 55.

If two Superficies medials A B *and* C D, *incommenſurable to one another, are compounded, two other irrational lines are made; to wit, either the ſecond Bimedial, or the line equal in power to two medials.*

Demonſtration. FOr A B ſhall be greater then C D, or leſſe, and not equal; otherwiſe A B and C D ſhould be commenſurable, which is contrary to the Suppoſition.

Let it in the firſt place be ſuppoſed greater, and let it be done as in the precedent Propoſition, and A B and C D being propoſed medials, and incommenſurable, their equals E G and H I, ſhall be alſo medials, and incommenſurable: Therefore a E H and H K having the ſame rate, ſhall b be incommenſurable in length: (and E G and H I medials,) being applyed to the rational E F, both c E H and H K are rational, incommenſurable in length to the rational E F: Therefore E H and H K are rational, commenſurable in power onely, therefore the whole E K is a Binomial. a) 1. 6. b) 10. 10. c) 23. 10.

But ſeeing that A B is propoſed greater then C D, as in the precedent Propoſition, E H ſhall be the greateſt name of E K: and therefore ſhall be more in power then H K, by the ſquare of a line which is commenſurable in length thereto, or incommenſurable: If E H be more in power then H K by the ſquare of a line which may be commenſurable in length thereto, as well E H as H K being ſhewn incommenſurable in length to E F rationals, by the Definition, E K ſhall be a third Binomial: Therefore the right line equal in power to the Superficies E I, conteined under E F a rational, and E K a third Binomial, and therefore the line equal in power to the Superficies A D is irrational, called a ſecond Bimedial.

If E H be more in power then H K, by the ſquare of a line incommenſurable in length thereto, as well E H as H K, being incommenſurable in length to E F rational, E K ſhall be a ſixth Binomial, by the Definition,

Therefore the line equal in power to E I, or A D is irrational, called a line equal in power to two Medials: and if A B were leſſe then C D, it may be demonſtrated in like manner: Therefore, &c. Which was to be demonſtrated.

COROLLARIE.

From all theſe things may be gathered eaſily that the line of two names, or Binomial, and the other irrational lines following it are different among themſelves; and alſo from the medial line.

For the ſquare of a medial line applyed to a rational line,
a) 23. 10. [a] *makes the breadth rational, incommenſurable in length to the ſaid rational to which it is applyed.*

But the ſquare of a line of two names applyed to a rational
b) 61. 10. *line,* [b] *makes the breadth the firſt of two names, or firſt Binomial.*

And the ſquare of a firſt Bimedial applyed to a Rational line
c) 62. 10. [c] *makes the breadth a ſecond Binomial.*

And the ſquare of a ſecond Bimedial, applyed to a rational
d) 63. 10. *line,* [d] *makes the breadth a third Binomial.*

c) 64. 10. *But the ſquare of a Major line applyed to a Rational,* [e] *makes the breadth a fourth Binomial.*

And the ſquare of a line equal in power to a Rational and a
f) 65. 10. *Medial, applyed to a Rational line,* [f] *makes the breadth a fifth Binomial.*

Laſtly, the ſquare of a line equal in power to two Medials,
g) 66. 10. *applyed to a Rational line,* [g] *makes the breadth a ſixth Binomial.*

Now ſeeing that all theſe breadths do differ from the breadth of the Medial, and among themſelves (to wit, from the breadth of the Medial, the one being rational, to wit, the medial, and the others irrational:) But foraſmuch as they are not Binomials, of the ſame order they differ from one another; It is manifeſt that all the irrational lines whereof we have ſpoken hitherto, are different among themſelves.

Here begins the Senaries of Irrational lines by interſection.

PROP. 74. THEOR. 56.

A — C — B

If from a Rational line A B *be cut off a Rational line* A C, *commenſurable in power only to the whole* A B, *the remminder* B C *is irrational: and may be called an* Apotome, *or a* Reſidual. Demon-

Demonstration FOr as A B is to AC, ſo the ſquare of A B to the Rectangle under AB & A C, and A B and A C, being incommenſurable in length; the [a] ſquare of A B ſhall be incommenſurable to the Rectangle under A B and A C: but the compound of the ſquares of A B and A C is commenſurable to the ſquare of AB, (for AB and AC being propoſed commenſurable in power, the ſquares of A B & A C ſhall be commenſurable: [b] and therefore their Compound commenſurable to the ſquare of A B:) but twice the Rectangle under A B and A C is commenſurable to the Rectangle under A B and AC: therefore the Compound of the ſquares of A B and A C, and twice the Rectangle under A B and A C are incommenſurable: Now [c] the Compound of the ſquares of A B and A C is equal to twice the Rectangle under A B and A C, and to the ſquare of B C: therefore [d] the Compound of the ſquares of A B and A C is incommenſurable to the remaining ſquare of BC: wherefore the Compound of the ſquares of A B and A C being rational: (as being commenſurable to the ſquare of the rational A B;) the ſquare of B C ſhall be irrational, and therefore B C irrational: And let it be called an *Apotome*, which others call a *Reſidual*: Therefore, &c. Which was to be demonſtrated.

a) 10. 10.
b) 16. 10.
c) 7. 2.
d) C. 17. 10.

PROP. 75. THEOR. 57.

A C B

If from a medial line A B, *be cut off a medial line* A C, *commenſurable only in power to the whole* A B, *which doth contein a Rational Rectangle with the whole* A B, *the reſt* C B *is irrational: And let it be called a firſt Reſidual Medial.*

Demonſtration FOr A B and AC being commenſurable in power, the ſquares of AB and AC ſhall be commenſurable; and [a] therefore their compound ſhall be commenſurable to the ſquare of A C: but the ſquare of A C medial is irrational, and medial: therefore [b] the compound of the ſquares of A B and A C, commenſurable thereto, is irrational, and medial, and the Rectangle under A B and A C being propoſed Rational, twice the Rectangle under AB and A C ſhall be Rational; wherefore the compound of the ſquares of AB and A C ſhall be incommenſurable to twice the Rectangle of A B and A C: Therefore [c] ſeeing that the compound of the ſquares of A B and A C, is equal to twice the Rectangle under A B and A C, and to the ſquare of B C; twice the Rectangle under A B and A C, with the ſquare of B C, ſhall be alſo incommenſurable to twice the Rectangle under A B and A C: Wherefore [d] twice the Rectangle under A B and A C, and the ſquare of B C are incommenſurable, and twice the Rectangle nnder A B and A C, being rational, the ſquare of B C ſhall be irrational; and therefore BC irrational: And let it be called an *Apotome*, or firſt *Reſidual*: Therefore, &c. Which was to be demonſtrated.

a) 16. 10.
b) C. 24. 10.
c) 7. 2.
d) 17. 10.

PROP.

PROP. 76. THEOR. 58.



If from a medial line A B, *be cut off a medial line* A C, *commensurable in power onely to the whole* A B, *which conteins with the whole* A B *a medial* Rectangle, *the remainder* B C *is irrational: And is called a Second* Apotome, *or* Residual *of a medial.*

Demonstration FOr asmuch as the squares of A B and A C commensurable in power, are commensurable, their [a] compound shall be commensurable to each of them: but each of those squares is medial; (A B and A C being proposed medial:) therefore [b] their compound is medial, being commensurable to each of them.

a)16.10.

b)C.24.10.

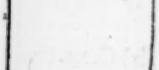
Again, seeing that the Rectangle under A B and A C is proposed medial: [c] the double thereof, to wit, twice the Rectangle under A B and A C, shall be also medial: Therefore [d] seeing that the compound of the squares of A B and A C is equal to twice the Rectangle under A B and A C, and to the square of B C, the compound of the squares of A B and A C which is medial, shall exceed twice the Rectangle under A B and A C, (which is also medial,) by the square of B C: But [e] a medial doth not exceed a medial by a Rational: Therefore the square of B C shall not be rational: therefore irrational, and B C irrational: And is called a second *Apotome* of a medial line: Therefore, &c. Which was to be demonstrated.

c)C. 24.10.

d)7. 2.

e)27. 10.

PROP. 77. THEOR. 59.



If from a right line A B, *be cut off a right line* A C, *incommensurable in power to the whole* A B, *making with the whole* A B, *the Compound of their squares rational, and the* Rectangle *under them medial, the remainder* B C *is irrational: And is called a Minor.*

Demonstration FOr the Compound of the squares of A B and A C, being rational: But the Rectangle under A B and A C: and [a] therefore the double thereof which is commensurable thereto; that is to say, twice the Rectangle under A B and A C medial, that is to say, irrational: [b] the compound of the squares of A B and A C, shall be incommensurable to twice the Rectangle under A B and A C; But the compound of the squares of A B and A C is equal to twice the Rectangle under A B and A C, and to the square of B C: therefore the compound of the squares of A B and A C [c] shall be incommensurable to the remaining square of B C: but the compound of the squares of A B and A C is proposed rational: there

a)C.24. 10.

b) 10.10.

c) C.17.10.

therefore [d] the ſquare of BC incommenſurable thereto, is irrational, and the line BC irrational: And is called Minor: Therefore, &c. Which was to be demonſtrated. d) 10. d.

PROP. 78. THEOR. 60.

A C B

If from a right line AB, *be cut off a right line* CD, *incommenſurable in power to the Whole* AB, *making with the Whole* AB *the Compound of their ſquares medial: But the Rectangle conteined under them rational, the remainder* CB *is irrational: And is called a line making with a rational Space, a whole medial.*

Demonſtration. FOr the compound of the ſquares of AB and AC being medial; that is to ſay irrational, and the Rectangle under AB and AC: and therefore the double thereof, to wit, twice the Rectangle under AB and AC rational: the [a] compound of the ſquares of AB and AC ſhall be incommenſurable to twice the Rectangle under AB and AC: But [b] the compound of the ſquares of AB and AC is equal to twice the Rectangle under AB and AC, and to the ſquare of BC: therefore twice the Rectangle under AB and AC, with the ſquare of BC, is incommenſurable to twice the Rectangle under AB and AC; therefore [c] twice the rectangle under AB and AC, and the ſquare of BC are incommenſurable: Therefore twice the rectangle under AB and AC being rational, the ſquare of BC ſhall be irrational, and the line BC irrational: And is called a line making with a rational Space a whole medial: Foraſmuch as its ſquare added to twice the Rectangle under AB and AC, which is rational, makes the whole medial: Therefore, If, &c. Which was to be demonſtrated. a) 10. d. b) 7. 2. c) 17. 10.

PROP. 79. THEOR. 61.

A C B

If from a right line AB, *be cut off a right line* AC, *incommenſurable in power to the whole* AB, *making with the whole* AB, *the Compound of their ſquares medial, and the Rectangle contained under them medial, and incommenſurable to the compound of their ſquares: the remainder* CB *is irrational; and let it be called a line making a whole medial with a rectangle medial.*

Demonſtration. FOraſmuch as the compound of the ſquares of AB and AC is equal to twice the Rectangle under [a] AB and AC, and to the ſquare of BC, the compound of the ſquares of AB and AC, propoſed medial: a) 7. 2.

b) C. 24. 10. medial, ſhall exceed twice the rectangle under A B and A C, [b] which is medial, (being commenſurable to the medial: conteined under A B and A C:) of the ſquare of B C: But a medial doth not exceed a medial by a rational, wherefore the ſquare of B C is not rational, therefore irrational, and B C irrational: And is called a line making a whole medial with a medial: Foraſmuch as the ſquare thereof with the Rectangle under A B and A C, which is medial, makes the whole compound of the ſquares of A B and A C, which is alſo medial: Therefore, If, &c. Which was to be demonſtrated.

PROP. 80. THEOR. 62.



There belongs only to a Reſidual *or* Apotome A B, *one only right line* B C *rational, commenſurable in power only to the whole* A D.

Demonſtration FOr (if poſſible,) let another rational B D agree to it, commenſurable in power only to A D, and B C being rational, A C commenſurable in power only thereto, ſhall be rational: Therefore A C and B C are rational, commenſurable in power only, In like manner A D aud B D ſhall be rationals, commenſurable in power onely.

But there being the ſame exceſſe between the compound of the ſquares of A C and B C, and twice the Rectangle under A C and B C, as between the compounds of the ſquares of A D and B D, and twice the Rectangle under A D and C B; (for the exceſſe between the compound of the ſquares of A C and B C, and twice the Rectangle under A C and B C is the ſquare of A B: the [a] compound of the ſquares of A C and B C, being equal to twice the Rectangle under A C and B C, and the ſquare of A B.)

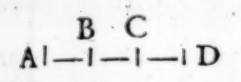
a) 7. 2.

In like manner the ſquare of A B is the exceſſe between the compound of the ſquares of A D and B D, and twice the Rectangle under A B and B D, (the compound of the ſquares of A D and B D, being equal to twice the Rectangle of A D and B D, and the ſquare of A B;) alſo alternately, there will be the ſame exceſſe between the compound of the ſquares of A C and B C, and the compound of the ſquares of A D and B D, as between twice the Rectangle of A C and B C, and twice the Rectangle of A D and B D: but the exceſſe between thoſe compounds is rational, the ſaid compounds being rational: (for A C and C B being rational, commenſurable in power onely, their ſquares ſhall be rational and commenſurable: Therefore their compound ſhall be commenſurable to each of them; and therefore Rational: In like manner, the compound of the ſquares of A D and B D ſhall be ſhewn rational;) therefore the exceſſe between twice the Rectangle under A C and B C, and twice the Rectangle under A D and B D is a ſpace rational: And ſeeing that [b] the Rectangle under A C and B C rationals, commenſurable in power onely, is medial, [c] the double thereof, to wit, twice the Rectangle under A C and B C, ſhall be medial: by the ſame reaſon, the

b) 22. 10.
c) C. 24. 10.

the Rectangle under A D and B D is medial; and [d] a medial exceeds not a medial by a rational; therefore the excesse between twice the Rectangle under A C and B C, and twice the Rectangle under A D and B D, is not a space rational: but it hath been shewn rational, which is absurd: Therefore any other rational agrees not with A B, commensurable in power onely to the whole, besides B C: Therefore, &c. Which was to be demonstrated. d) 27. 10.

PROP. 81. THEOR. 63.

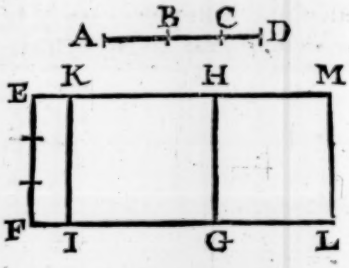


To a medial first Residual A B, *there agrees onely a medial line* B C, *commensurable in power only to the whole* A D, *conteining with the whole* A D *a rational Rectangle.*

Demonstration. FOr, (if possible,) let another medial B D agree therewith, commensurable in power onely to the whole: and let the Rectangle under A D and B D, be rational, and B C being Medial, commensurable in power only to A C, [a] A C shall be medial; therefore A C and B C are medials commensurable in power only. a) 24. 10.

And seeing that there is the same excesse between the compound of the squares of A D and B D, as between twice the Rectangle under A C and B C, and twice the Rectangle under A D and B D, as is shewn in the precedent, but the excesse between those Rectangles is rational, the one and the other being rational: (for the Rectangle under A C and B C being proposed Rational, twice the Rectangle under A C and B C, commensurable thereto is rational, and by the same reason, twice the Rectangle under A D and B D is rational:) Therefore the excesse between the compounds of the squares of A C and B C; and that of the squares of A D and B D is rational, and A C and B C being medials, commensurable in power only, their squares shall be medials, and commensurable: Therefore [b] their compound is commensurable to each of them, and [c] therefore medial; by the same reason, the compound of the squares of A D and B D shall be medial: But a medial not exceeding a medial by a rational, the excesse between the compounds of the squares of A C and B C, and that of the squares of A D and B D is not rational: But it hath been shewn rational, which is absurd: Therefore another medial agrees not to A B, besides B C, which may be commensurable in power onely to the whole, and contein a rational Rectangle with the whole: Therefore, &c. Which was to be demonstrated. b) 16. 10. c) C. 24. 10.

PROP. 82. THEOR. 64.



To the medial ſecond Reſidual AB, *there can agree but one only right line* BC *medial, commenſurable in power only to the whole* AC, *and conteining a medial Rectangle under* AC *and* BC, *with the whole.*

Demonſtration FOr, (if poſſible,) let BD medial agree therewith, commenſurable in power only to the whole AD; and let the Rectang'e under AD and BD be medial, and let EF the rational be expoſed, to
a) 45. 1. which [a] let there be applyed the Rectangle EG, equal to that which is compounded of the ſquares of AC and BC, and to EF let there be applyed the Rectangle EI, equal to the ſquare of AB; then the Rectangle EL equal to the Rectangle which is compounded of the ſquares of AD and BD: Foraſmuch
b) 7. 2. [b] then as the compound of the ſquares of AC and BC, is equal to twice the Rectangle under AC and BC, and to the ſquare of AB, KG ſhall be equal to twice the Rectangle under AC and BC: ſo KL ſhall be equal to twice the Rectangle under AD and BD, and AC and BC being medials, commenſurable in power only, (for BC being propoſed medial, AC
c) 16. 10. [c] commenſurable in power thereto, ſhall be medial alſo,) their ſquares
d) C. 24. 10. ſhall be medials, and commenſurable: Therefore [d] their compound is commenſurable to each of them, and therefore medial: Therefore EG equal to the ſame compound is alſo medial, and EG being applyed to EF rational, EH ſhall be rational, incommenſurable in length to EF.

Again, the Rectangle under AC and BC being medial, the double thereof, to wit, KG ſhall be medial, and being applyed to KI rational, KH ſhall be rational, incommenſurable in length to KI; that is to ſay, to EF: and ſeeing that AC and BC are incommenſurable in length, and as AC to BC, ſo the ſquare of AC to the Rectangle under AC and BC, the ſquare of AC ſhall be incommenſurable to the Rectangle under AC and BC: but the compound of the ſquares of AC and BC, that is to ſay, the Rectangle EG is commenſurable to the ſquare of AC; (for AC and BC being propoſed commenſurable in power, their ſquares ſhall be commenſurable to each of them; to wit, to the ſquare of AC:) and the double of the Rectangle under AC and BC, to wit KG, is commenſurable to the ſaid Rectangle under AC and BC: Therefore EG is incommenſurable to KG, therefore EH and KH bearing the ſame rate as EG and KG, are incommenſurable in length, and being ſhewn rational, they ſhall be ratio-rational, commenſurable in power only: Wherefore from EH rational, having cut off the rational KH, commenſurable in power only to EH,
e) 74. 10. [e] EK ſhall be reſidual, and KH agreeing therewith: So it may be ſhewn that EK is a Reſidual to KM, agreeing therewith: Therefore to the Reſidual there agrees only one rational line, commenſurable in power only to the whole, which is abſurd, as hath been demonſtrated: Therefore, to a medial Reſidual, &c. Which was to be demonſtrated.

PROP.

PROP. 83. THEOR. 65.



To a minor line A B, *there agrees only one right line* B C *incommenſurable in power to the Whole* A C, *and making with the Whole* A C *the Compound of their ſquares rational, and the Rectangle under them medial.*

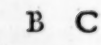
Demonſtration. FOr otherwiſe let another B D agree with A B incommenſurable in power to the whole A D, making the compound of the ſquares of A D and B D rational, and the Rectangle under A D and B D medial.

Foraſmuch then as hath been demonſtrated in the 80 Propoſition, the exceſſe of the compound of the ſquares of A C & B C, and the compound of the ſquares of A D & B D is the ſame as that between twice the rectangle under A C & B C, and of twice the rectangle under A D & B D: but the exceſſe between thoſe compounds is rational, both the one and the other, being propoſed rational: Therefore the exceſſe between twice the rectangle under A C and B C, and twice the rectangle under A D & B D is a rational Space: But it is alſo irrational, (for the rectangle under A C & B C being propoſed medial, twice the rectangle under A C and B C double thereto,[a] is alſo medial: In like manner, twice the rectangle under A D and D B ſhall be medial; therefore[b] the one ſhall not exceed the other by a rational, which is abſurd: Therefore another line agrees not to A B, beſides B C incommenſurable in power to the whole: Therefore, &c. Which was to be demonſtrated.

a) C. 24. 10.

b) 27. 10.

PROP. 84. THEOR. 66.

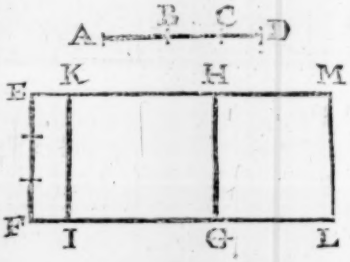


To a line A B, *making with a Space rational the whole medial, there agrees one only right line* B C, *incommenſurable in power to the whole* A C, *and making with the Whole the Compound of their ſquares medial: But the Rectangle conteined under them Rational.*

Demonſtration. FOr (if poſſible) let another line B D agree therewith incommenſurable in power to the whole A D, and making the compound of the ſquares A D and B D medial, and the Rectangle under A D and B D rational: Foraſmuch as it is ſhewn in the 80th. Propoſition, that there is the ſame exceſſe between the compound of the ſquares of A C and B C, and the compound of the ſquares of A D and B D, as between twice the Rectangle under A C and B C, and twice the Rectangle under

AD and BD: But the exceſſe between thoſe Rectangles is a rational Space, as hath been ſhewn in the 81 Propoſition; therefore the exceſſe between thoſe compounds of the ſquares is alſo a rational Space: but it is not
a) 27.10. alſo rational, both the one and the other, being medial, [a] one medial exceeding not a medial by a rational, which is abſurd: Therefore any other line then BC agrees not with AB, incommenſurable in power to the whole: Therefore the line, &c. Which was to be demonſtrated.

PROP. 85. THEOR. 67.



To a line AB, *making with a Space medial a whole medial, there agrees one only right line* BC, *incommenſurable in power to the whole* AC, *and making with the whole* AC, *the Compound of their ſquares medial, and the Rectangle conteined under them medial, and incommenſurable to the compound of their ſquares.*

Demonſtration FOr (if poſſible, let BD agree thereto, alſo incommenſurable in power to the whole AD, making what is before ſaid; and let there be done as in the 82 Propoſition: Foraſmuch as the compound of the ſquares of AC and BC is medial, EG his equal is alſo me-
a) 23.10. dial; therefore [a] EH is rational, incommenſurable in length to EF: Again foraſmuch as the Rectangle under AC and BC is medial, the double
b) 23.10. thereof KG is alſo medial; therefore [b] KH is rational, incommenſurable in length to EF; and foraſmuch as KG is commenſurable to the Rectangle under AC and BC, being double to the ſaid Rectangle: But the Rectangle under AC and BC is propoſed incommenſurable to the compound of
c) 14.10. the ſquares of AC and BC: [c] KG ſhall be alſo incommenſurable to the compound to the ſaid Compound, that is to ſay to EG; and therefore EH
d) 10.10. and HK being in the ſame rate as EG and KG, are [d] incommenſurable in length; but they are ſhewn rational; therefore are rational, commenſurable in power only.

Taking then from the rational EH, the rational KH, commenſurable in
c) 74. 10. power only to EH, [e] EK ſhall be a Reſidual, and thereto agrees KH: in like manner we ſhall ſhew that EK is a Reſidual, to the which KM agrees. Therefore one only line doth not agree to a Reſidual, &c. Which is ab-
f) 80. 10. ſurd, being [f] ſhewn the contrary: Therefore, &c. Which was to be demonſtrated.

THIRD

THIRD DEFINITIONS.

A rational line being propoſed and a Reſidual, when the whole compounded of the Reſidual and of the agreeing or added line, be more in power then the agreeing line, by the ſquare of a line which is commenſurable thereto in length.

1 *If the whole be commenſurable in length to the rational propoſed, let the reſidual be called a* First Reſidual.

2 *But if the agreeing or added line, be commenſurable in length to the rational propoſed, let it be called a* Second Reſidual.

3 *And if neither the whole nor the agreeing, be commenſurable in length to the rational propoſed, let it be called a* Third Reſidual.

Again, if the whole be more in power then the agreeing, by the ſquare of a right line incommenſurable in length thereto.

4 *If the whole be commenſurable in length to the rational propoſed, let it be called a* Fourth Reſidual.

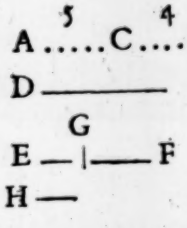
5 *But if the agreeing be commenſurable in length to the rational propoſed, let it be called a* Fifth Reſidual.

6 *And if neither the whole, nor the agreeing, be commenſurable in length to the rational propoſed, let it be called a* Sixth Reſidual.

PROP.

PROP. 86. PROBL. 19.

To finde an Apotome, *or a* First Residual.



Construction HAving found two square numbers, A B and C B, whose excesse A C may be no square; let the rational D be proposed, and let E F be commensurable in length thereto: E F shall
a) C.6. 10. be also rational, and [a] as the number A B to A C: so let the square of E F be made to the square of G F: I say that E G is a first Residual.

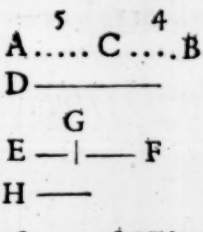
Demonstration FOrasmuch as the squares of E F and G F, being to one another as the number A B to the number A C are commensurable, E F and G F shall be commensurable at least in power: Therefore seeing that E F is shewn rational, G F shall be rational also.

And seeing that A B and A C are not to one another as square numbers, the squares of E F and G F shall not be to one another as a square number to
b) 9. 10. a square number; therefore [b] E F and G F are incommensurable in length,
c) 74. 10. they shall be then rational, commensurable in power only: therefore [c] the remainder E G is a Residual: I say, it is a first Residual: For let E F the greatest, be more in power then G F by the square of H; and forasmuch as the number A B is to the number A C, so the square of E F to the square of G F; by conversion, as A B to C B, so the square of E F to the square of H: but A B and C B are square numbers; therefore the square of E F shal be to the square of H, as a square number to a square number: therefore E F and H are commensurable in length.

Forasmuch then as the whole E F is more in power then G F, agreeing with the square of H, commensurable in length thereto, and the whole E F is commensurable in length to the rational exposed D, by the Definition E G shall be a first Residual: Therefore, &c. Which was to be done.

PROP. 87. PROBL. 20.

To finde a Second Residual.



Construction HAving found two numbers A B and C B, as in the precedent, and proposed the rational D, let G F be taken commensurable in length thereto, G F shall be also rational, and as
a) Co. 6. 10. [a] the number A C to the number A B, so let the square of G F be made to the square of E F: I say, that E G is a Second Apotome.

Demonstration FOr the squares of G F and E F being to one another as the numbers A C and A B are commensurable, and G F and E F commen-
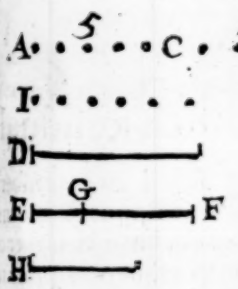
commensurable at least in power, and seeing that G F is shewn rational, E F shall be also rational: But forasmuch as the numbers A C and A B, and by the same meanes the squares of G F and E F, are not to one another as a square number to a square number, [b] G F and E F shall be incommensuarable in length, therefore G F and E F are rational, commensurable in power only, and therefore [c] E G remainder, is a Residual: I say also, it is a second Residual. For let E F be more in power then G F, by the square of H.

b) 9. 10.

c) 74. 10.

Forasmuch then as A C is to A B, so the square of G F is to the square of E F, and by conversion of reason, as A B to A C, so the square of E F to the square of G F. Now we shall shew as in the preeedent, that the line H is commensurable in length to E F: wherefore the whole E F being more in power then the agreeing G F, by the square of H, commensurable in length therero; and that the same agreeing G F is commensurable in length to the rational exposed D, by the Definition, E G shall be an *Apotome*, or *Second Residual*: Therefore, &c. Which was to be done.

PROP. 88. PROBL. 21.



To finde an Apotome, *or* Third Residual.

Construction HAving found two numbers A B and C B, as in the 86 Proposition; Let there be taken another number I, as is shewn in the 51 Proposition, which may not be to either of them as a square number to a square number, and having proposed D rational, let it be made as I to A B: so [a] is the square of D to the square of E F, and those squares of D and E F shall be to one another as the number I and A B, and therefore commensurable: and therefore D and E F commensurable in power, and D being rational, E F shall be also rational; and forasmuch as the numbers I, and A B, and therefore the squares of D and E F are not to one another as a square number to a square number, [b] D and E F shall be incommensurable in length.

a) C. 6. 10.

b) 9. 10.

Again, [c] as A B is to A C, so the square of E F to the square of G F: I say, that E G is a third Residual: For, [d] seeing that the squares of E F and G F being to one another as the numbers A B and A C, are commensurable, E F and G F shall be commensurable at least in power, and E F being shewn rational, G F shall be also rational; and forasmuch as A B and A C, and so likewise the squares of E F and G F, are not to one another as square numbers, E F and G F shall be incommensurable in length: Therefore E F and G F are rational, commensurable in power only; and therefore G F being cut off from E F, commensurable thereto in power only, E G shall be *Residual*, I say, that it is a *Third Residual*.
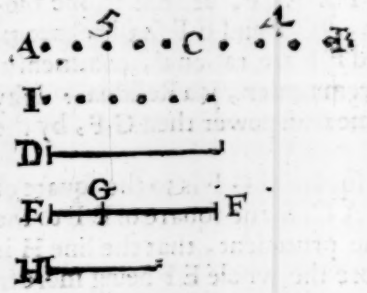
c) C. 9. 10.

d) 6. 10.

Demonstration FOrasmuch as I is to A B, as the square of D to the square of E F, and as A B to A C, so the square of E F to the square of G F, in equal rate, as I to A C, so the square of D to the square of G F: Now

Now I and A C are not to one another as a ſquare number to a ſquare number: therefore c) the ſquares of D and G F ſhall not be to one another as a ſquare number to a ſquare number: therefore D and G F are incommenſurable in length: therefore neither the one nor the other, E F nor G F, is commenſarable in length to the rational propoſed D.

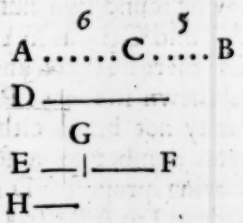
c) 9. 10.



Now let E F be more in power then G F, by the ſquare of H, we ſhall ſhew as in the 86 Propoſition, that H is commenſurable in length to E F: Wherefore ſeeing that E F the whole is more in power then the agreeing G F, by the ſquare of H, commenſurable in length thereto, and that neither the one nor the other, E F nor G F, is commenſurable in length to the rational propoſed D, by the Definition, E G ſhall be a Third Reſidual: Therefore, &c. Which was to be done.

PROP. 89. PROBL. 22.

To finde a Fourth Reſidual.



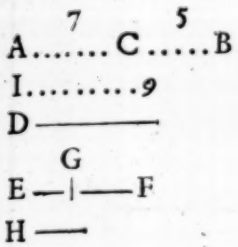
Conſtruction HAving found two numbers A C and C B, ſo as A B compounded of them may not have to either of them ſuch proportion as a ſquare number to a ſquare number: Let the rational D be expoſed, to the which let E F be commenſurable in length; E F ſhall be alſo rational, and having finiſhed the reſt as in the 86 Propoſition, we ſhall ſhew (as there) that E G is a Reſidual; I ſay, that it is alſo a fourth Reſidual.

Demonſtration FOr let E F be more in power then G F, by the ſquare of H: and ſeeing that as A B to A C, ſo the ſquare of E F to the ſquare of G F; by converſion of reaſon, as A B ſhall be to C B, ſo the ſquare of E F to the ſquare of H. Therefore ſeeing that A B and C B are not to one another as ſquare numbers, E F and H ſhall be incommenſurable in length. Foraſmuch then as the whole E F, is more in power then the agreeing G F, by the ſquare of H, incommenſurable in length thereto, and the ſame whole E F is commenſurable in length to the rational D; E G by the Definition, ſhall be a fourth Reſidual: Therefore, &c. Which was to be done.

PROP.

PROP. 90. PROBL. 23.

To finde a Fifth Residual.



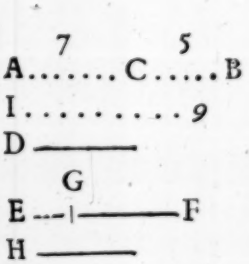
Construction HAving found two numbers A C and C B, as in the precedent Proposition: Let the same Construction be made as in the 87 Proposition; that is to say, let G F be taken commensurable in length to the rational D, &c. Therefore as in the 87 Proposition, we shall shew that E G is a Residual; I say also, that it is a Fifth Residual.

Demonstration FOr let E F be more in power then G F, by the square of H, and forasmuch as we shall shew as in the 86 Proposition, that by conversion of reason, as A B is to C B, so the square of E F is to the square of H.

Again, as in the precedent Proposition, E F and H, are incommensurable in length, and the agreeing G F is commensurable in length to the rational proposed; by the Definition, E G shall be a Fifth Residual: Therefore, &c. Which was to be done.

PROP. 91. PROBL. 24.

To finde a Sixth Residual.

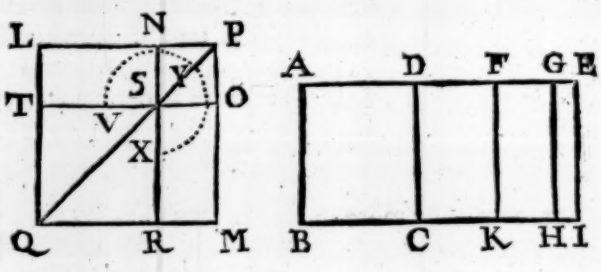


Construction HAving found three numbers A C, C B, and I, as in the 54 Proposition in such sort as A B be not to A C, nor to C B, and I be not to A B, nor to A C, as a square number to a square number: Let the rational D be exposed, and let the rest be done as in the 88 Proposition, we shall shew as there, that D and E F are incommensurable in length, and that E G is a Sixth Residual.

Demonstration FOr as in the 88 Proposition, D and G F, shall be also incommensurable in length; and therefore neither the one nor the other, E F nor G F, is commensurable in length to the rational proposed D. Now let E F be more in power then G F, by the square of H, which we shall shew to be incommensurable in length to E F, as in the 89 Proposition: Therefore seeing that the whole E F is more in power then the agreeing G F, by the square of H incommensurable in length thereto; and neither E F nor G F, is commensurable in length to the rational exposed D, by the Definition, E G shall be a Sixth Residual: Therefore, &c. Which was to be done.

PROP. 92. THEOR. 68.

If a Space A C, *be conteined under a rational line* A B, *and a firſt Reſidual* A D, *the right line equal in power to the Space* A C, *is a Reſidual.*



Demonſtration FOr let D E be the agreeing line to A D; therefore A E and D E by the Definition of the firſt Reſidual, ſhall be rational, commenſurable in power only: and the whole A E commenſurable in length to the rational A B, and the whole A E ſhall be more in power then the agreeing D E, by the ſquare of a line incommenſurable in length thereto.

Let D E be divided in two equal parts in F, and let there be applyed to A E a Rectangle, equal to the ſquare of E F; that is to ſay, equal to the quarter of the ſquare of D E, wanting by a ſquare figure, to wit, under A G and G E; and foraſmuch as A E is more in power then D E, by the ſquare of a line commenſurable thereto in length, A G and G E [a] ſhall be commenſurable in length; wherefore [b] the one and the other ſhall be commenſurable in length to the whole A E: but the whole A E is commenſurable in length to the rational A B: therefore [c] as well A G as G E, is commenſurable in length to A B, and therefore [d] rational: therefore having drawn G H and E I parallels to A B, meeting B C prolonged, in H and I, [e] the one and the other Rectangle A H and G I conteined under two rational lines commenſurable in length, ſhall be rational.

a) 18. 10.
b) 16. 10.
c) 12. 10.
d) 6. d.
e) 20. 10.

Again, foraſmuch as the one and the other, D F and F E, is commenſurable in length to D E, and D E incommenſurable in length to the rational A B, (for if it were commenſurable in length, A E being alſo commenſurable in length to A B: [f] A E and D E, ſhould be alſo commenſurable in length, which is abſurd, for they are propoſed commenſurable in power only,) [g] the one and and the other of them, D E and F E, ſhall be alſo incommenſurable in length to the rational A B.

f) 12. 10.
g) 14. 10.

And foraſmuch as the one and the other D F and F E, is commenſurable to the rational D E, they are rational; therefore as well A B and D F, as A B and F E, ſhall be rational, commenſurable in power only: Therefore having drawn F K parallel to A B, [h] the one and the other Rectangles D K and F I, conteined under the rational, commenſurable in power only, is Medial.

h) 22 10.

Now [i] let there be made the ſquare L M equal to A H, and N O equal to G I, having one angle common with L M; to wit, L P M; therefore [k] the ſquares L M and N O, ſhall be about one and the ſame Diameter; which let be P Q, and let the figure be finiſhed as you ſee.

i) 14. 10.
k) 26. 6.

For-

Foraſmuch as the Rectangle under A G and G E is equal to the ſquare of E F, by Conſtruction, the three lines A G, F E, and G E, ſhall be proportional; and therefore A H, F I, and G I, having the ſame rate, F I ſhall be a mean proportional between A H and G I; that is to ſay, between the ſquares L M and N O, equal to thoſe Rectangles: But L O is alſo a mean proportional between the ſame L M and N O: therefore the Rectangles F I and L O are equal: But [l] D K is equal to F I, and M N to L O; therefore the whole D I is equal to all the Gnomon R O, O N, N P, with the ſquare N O: But the whole A I is equal to the ſquares L M and N O, by Conſtruction, therefore the remainder A C ſhall be equal to the remaining ſquare T R; therefore T S is equal in power to the Space A C: I ſay, that T S is a Reſidual. l) 36. 10.

For ſeeing that the ſpaces A H and G I are ſhewn rational, and to them are equal L M and N O, the ſame ſquares L M and N O ſhall be rational, and therefore T O and S O rational.

Again, ſeeing that F I is ſhewn medial, L O equal thereto ſhall be alſo medial: Therefore L O and N O, are incommenſurable, the one being rational, and the other irrational; therefore [m] the lines T O and S O, having the ſame rate as L O and N O, are [n] incommenſurable in length; therefore the remainder T S is a Reſidual: Therefore, &c. Which was to be demonſtrated. m) 1. 6. n) 10. 10.

PROP. 93. THEOR. 69.

If a Space A C, *be conteined under a rational line* A B, *and a ſecond Reſidual* A D, *the right line equal in power to the ſame Space* A C, *is a Reſidual firſt Medial.*

Demonſtration. FOr let D E be the agreeing line to A D; therefore A E and D E, by the Definition of the ſecond Reſidual, ſhall be rational, commenſurable in power only, and the agreeing D E commenſurable in length to the rational A B; and laſtly, A E ſhall be more in power then D B, by the ſquare of a line commenſurable in length thereto: Let D E be divided in two equal parts in F, and for the reſt let it be done as in the precedent Propoſition, [a] A G and G E ſhall be commenſurable in length; and therefore [b] each ſhall be commenſurable to the whole A E; but A E is incommenſurable in length to the rational A B; (for if it were commenſurable in length to A B, D E being propoſed commenſurable in length to the ſame A B: [c] A E and D E ſhould be commenſurable in length to one another, which is abſurd, being propoſed commenſurable in power only:) therefore [d] as well A G as G E is incommenſurable in length to A B: But A G and G E commenſurable to the rational A E are rational: Therefore as well A B and A G, as A B and G E, are rational, commenſurable in power onely: Therefore [e] as well the Rectangle A H as G I, conteined under the rational commenſurable in power only, is Medial. a) 18. 10. b) 15. 10. c) 12. 10. d) 14. 10. e) 22. 10.

Again, as well D F as F E, being commenſurable in length to D E, propoſed commenſurable in length to the rational A B, D F and F E, ſhall be commenſurable in length to A B rational, as appears by the 12 Propoſition, and therefore rational: therefore [f] as well the Rectangle D K as F) 20. 10.

F I, conteined under the rational commenſurable in length is rational. And it ſhall be ſhewn as in the 92 Propoſition, that T S is equal in power to the Space A C. I ſay that T S is alſo a Reſidual firſt Medial.

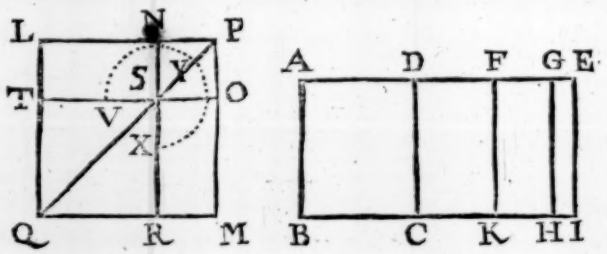
g) 10. 10. For A G and G E, being commenſurable in length, [g] A H and G I having the ſame rate as they ſhall be commenſurable, and therefore alſo their equal ſquares L M and N O commenſurable: therefore the ſides T O and S O, ſhall be at leaſt commenſurable in power. But T O and S O are Medials, being the ſquares L M and N O equal to A H and G I (ſhewn medials,) are Medials. And foraſmuch as F I, and therefore its equal L O,

h) 10. 10. is rational, and therefore incommenſurable to the Medial N O; [h] T O and S O having the ſame rate as they ſhall be incommenſurable in length, and being ſhewn Medials, and commenſurable, the ſame T O and S O ſhall be Medials, commenſurable in power only: and conteining the Re-

i) 73. 10. ctangle L O, ſhewn rational, [i] T S ſhall be a firſt Reſidual: Therefore, &c, Which was to be demonſtrated.

PROP. 94. THEOR. 70.

If a Space A C, *be conteined under a rational line* A B, *and a third Reſidual* A D, *the right line equal in power to the ſame Space is a Reſidual ſecond Medial.*



Demonſtration FOr let D E be the agreeing line to A D; therefore A E and D E according to the Definition of the third Reſidual ſhall be rational, commenſurable in power only; and neither A E nor D E ſhall be comenſurable in length to the rational A B, and A E ſhall be more in power then D E, by the ſquare of a line commenſurable in length thereto: Let D E be divided in two equal parts in F, and let the reſt of the Conſtruction be finiſhed, as in the 92 Propoſition.

a) 16. 10. Again, as in the 92 Propoſition, A G and G E, ſhall be commenſurable

b) 14. 10. in length to one another [a]: and therefore [b] as well A G as G E, commenſurable in length to the whole A E: but A E is propoſed incommenſurable to the rational A B: therefore as well A G as G E, ſhall be incommenſurable in length to A B: Therefore as well A G as G E, commenſurable to A B rational, being rational: as well A B and A G, as A E and G E, are ratio-

c) 22. 10. nal, commenſurable in power only: therefore [c] the one and the other Rectangle A H and G I, ſhall be Medial.

Again, D E being propoſed incommenſurable in length to the rational

d) 14. 10. A B, D F and E F commenſurable in length to D E, [d] ſhall be alſo incommenſurable in length to A B: therefore D F and F E, being commenſurable

to

to D E rational, as well A B and D F, as A B and F E ſhall be rational, commenſurable in power onely; and therefore [e] as well the Rectangle D K as F I, ſhall be Medial: and as in the 92 Propoſition, it ſhall be ſhewn that T S is equal in power to the Space A C: I ſay, that T S is a Reſidual ſecond Medial. [c] 22. 10.

For A H and G I being ſhewn Medials, their equal ſquares L M & N O ſhall be alſo Medial; and therefore T O and S O Medials; and [f] ſeeing that A H and G I (having the ſame rate as A G and G E ſhewn commenſurable,) are commenſurable, the ſquares L M and N O ſhall be alſo commenſurable; therefore T O and S O are commenſurable at leaſt in power. [f] 10. 10.

But foraſmuch as A E and D E are commenſurable in power only; that is to ſay, incommenſurable in length; and G E is ſhewn commenſurable in length to A E, and F E commenſurable in length to D E, G E and F E ſhall be incommenſurable in length; and therefore [g] G I and F I, having the ſame rate as G E and F E; that is to ſay, their equals N O and L O are incommenſurable. Wherefore T O and S O having the ſame rate as L O and N O are incommenſurable in length, and being ſhewn Medials, and commenſurable, they ſhall be Medials, commenſurable in power only; and conteining L O a Medial: (for ſeeing that L O is equal to F I, as appears by the 92 Propoſition, which is ſhewn medial, L O ſhall be alſo a medial;) T S ſhall be a Reſidual ſecond Medial: Therefore, &c. Which was to be demonſtrated. [g] 10. 10.

PROP. 55. THEOR. 71.

If a Space A C *be conteined under a rational line* A B, *and a fourth Reſidual* A D, *the right line equal in power to the ſaid Space* A C, *is a line Minor.*

Demonſtration FOr let D E be the agreeing line to A D; therefore A E and D E ſhall be rational, commenſurable in power only, by the Definition of the fourth Reſidual: and A E commenſurable in length to A B rational: and the ſame A E ſhall be more in power then D E, by the ſquare of a line incommenſurable in length thereto: Let D E be divided in two equal parts in F, and the reſt done as is before ſhewn: Therefore A G and G E [a] ſhall be incommenſurable in length; ſeeing that A E is more in power then D E, by the ſquare of a line incommenſurable in length thereto: And to A E is applyed a Rectangle under A G and G F, equal to a quarter of the ſquare of D E, wanting a ſquare figure, and A E being rational, and commenſurable in length to the rational A B, [b] the Rectangle A I ſhall be rational. [a] 19. 10. [b] 20. 10.

Again, D E being rational, and incommenſurable in length to A B rational: D I [c] and therefore its half F I, ſhall be medial. And moreover A G and G E being incommenſurable in length, A H and G L [d] having the ſame rate, ſhall be incommenſurable; and as in the 92 Propoſition, ſo here it ſhall be ſhewn that T S is equal in power to the Space A C: I ſay, that T S is a Minor. [c] 22. 10. [d] 10. 10.

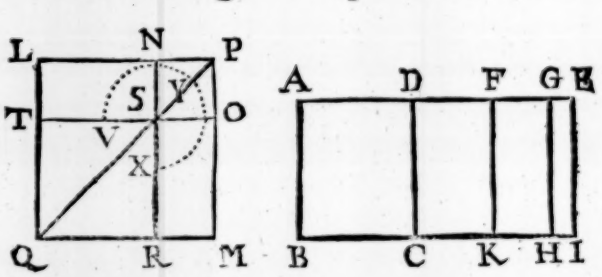
For the compound of the ſquares L M and N O, deſcribed of T O and S O, is equal to the Rectangle A I, by Conſtruction: but the ſame is ſhewn

shewn rational; therefore the compound of the squares T O and S O shall be rational: In like manner F I being shewn Medial, L O conteined under T O and S O, equal thereto, shall be Medial.

Lastly, A H and G I, being shewn incommensurable, their equal squares L M and N O, shall be incommensurable, and therefore T O and S O incommensurable in power: Therefore T O and S O being incommensurable in power, and the compound of their squares being rational, and the Rectangle under them medial, the remainder T S shall be a Minor: Therefore, &c. Which was to be demonstrated.

PROP. 96. THEOR. 72.

If a Space A C, *be conteined under a rational line* A B, *and a fifth* Residual A D, *the right line equal in power to the said Space* A C, *is a line making with a Space rational, a whole medial.*



Demonstration LEt D E agree to A D, by the Definition of the fifth Residual, A E and D E shall be rational, commensurable in power only, and D E commensurable in length to the rational A B, so A E shall be more in power then D E, by the square of a line incommensurable in length thereto.

Let D E be divided in two equal parts in F, and do the rest as is before shewn. Therefore as in the precedent Proposition, A G and G E shall be incommensurable in length; and forasmuch as A E is rational, incommensurable in length to A B rational, as is said in the 93 Proposition, A I [a] shall be Medial.

a) 22. 10.

Also, seeing that D E is rational, commensurable in length to A B rational, [b] the Rectangle D I, and therefore its half F I, is rational.

b) 20. 10.

Again, as in the precedent Proposition, A H and G I are incommensurable, and as is shewn in the 92 Proposition, T S equal in power to the Space A C, I say that T S is the line with which a Space rational makes a whole Medial.

Forasmuch as it hath been shewn that A I is a Medial, the compound of the squares L M and N O, of the lines T O and S O, equal thereto is medial, and F I being a rational, as we have shewn, the Rectangle L O conteined under T O and S O, equal thereto, is also rational, and T O and S O are incommensurable in power, as is shewn in the precedent: therefore seeing that the compound of their squares is medial, and the Rectangle under them rational, the remainder [c] T S is the line with which a rational makes a whole Medial: Therefore, &c. Which was to be demonstrated.

c) 78. 10.

PROP.

PROP. 97. THEOR. 73.

If a Space A C *be conteined under a rational line* A B, *and a ſixth Reſidual* A D, *the right line* T S, *equal in power to the ſaid Space* A C, *is that which with a Rectangle Medial makes a whole Medial.*

Demonſtration FOr let D E agree to the ſame A D: therefore A E and D E by the Definition of the Sixth Reſidual, are rational, commenſurable in power only, and neither one nor the other is commenſurable in length to the rational A B; and finally, A E ſhall be more in power then D E, by the ſquare of a line incommenſurable in length thereto.

Let D F be divided in two equal parts in F, and let the reſt be done as before is ſhewn: Therefore as in the 95 Propoſition, A G and G E ſhall be incommenſurable in length.

And ſeeing that as well A E as D E is rational, incommenſurable in length to the rational A B: as well [a] A I as D I, and therefore F I the halfe of D I, ſhall be medial, and A H aud G I as in the 95 Propoſition, ſhall be incommenſurable. a) 22. 10.

And foraſmuch as A E and D E are commenſurable in power only, [b] A I and D I which are in the ſame rate, are incommenſurable; and D I and F I being commenſurable, F I [c] ſhall be incommenſurable to A I: Now we ſhall ſhew as in the 92 Propoſition, that T S is equal in power to A C: and I ſay that T S is that, which with a Rectangle Medial makes a whole Medial. b) 10. 10. c) 14. 10.

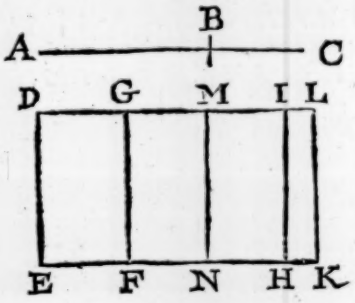
Foraſmuch as A I is ſhewn medial, the compound of the ſquares L M and N O, of the right lines T O and S O, equal thereto, is Medial, alſo F I being ſhewn Medial, S O his equal conteined under T O and S O, is a Medial.

Again, F I being incommenſurable to A I, as is ſhewn L O conteined under T O and S O, ſhall be incommenſurable to the compound of the ſquares of T O and S O: ſeeing that L O is equal to F I, and the compound of the ſquares of T O and S O, equal to A I. Laſtly, T O and S O are incommenſurable in power, as is ſhewn in the 95 Propoſition: Therefore ſeeing that the compound of their ſquares is medial, and the Rectangle under them Medial, and incommenſurable to the compound of their ſquares, the [d] remainder T S is that which with a Medial makes a whole Medial: Therefore, &c. Which was to be demonſtrated. d) 79. 10.

PROP.

PROP. 98. THEOR. 74.

The ſquare of a Reſidual AB, *applyed to a rational line* DE, *makes the breadth* D G *a firſt Reſidual.*



Demonſtration. FOr to D E let there be applyed the Rectangle D H, equal to the ſquare of A C, and to H I let there be applyed I K, equal to the ſquare of B C, in ſuch ſort as the whole D K may be equal to the compound of the ſquares of A C and C B.

a) 7. 2. And [a] foraſmuch as the compound of the ſquares of A C and B C, is equal to twice the Rectangle under A C and B C, and to the ſquare of A B, if the ſquare of A B and the Rectangle D F be taken away, the Rectangle G K ſhall remaine equal to twice the Rectangle under A C and B C: therefore G L being divided into two equal parts in M, and M N being drawn parallel to D E; M K ſhall be equal to the Rectangle under A C and B C.

b) 16. 10. And foraſmuch as A C and B C are rationals; their ſquares ſhall be rational, and therefore commenſurable: Therefore [b] ſeeing that the compound of the ſquares of A C and B C is commenſurable to each of them, the ſaid compound, that is to ſay, his equal Rectangle D K ſhall be rational;

c) 21. 10. which being applyed to the rational D E, [c] D L ſhall be rational, commenſurable in length to D E.

Again, ſeeing that A C and B C are rational, commenſurable in power only, the Rectangle under them, and his double G K is Medial, which

d) 23. 10. being applyed to the rational G F, G L [d] ſhall be rational, incommenſurable in length to C F; that is to ſay, to D E: And foraſmuch as D K rational, and G K Medial, that is to ſay irrational, are incommenſurable, D L

c) 10. 10. and G L [e] having the ſame rate as D K and G K, ſhall be incommenſurable in length: Therefore being ſhewn rational, they ſhall be rational,

f) 74. 10. commenſurable in power only, and therefore [f] the remainder D G ſhall be a Reſidual. I ſay it is a firſt Reſidual.

Foraſmuch as the Rectangle under A C and B C, that is to ſay M K, is a mean proportional between the ſquares of A C and B C, that is to ſay, between D K and I K: D H, M K, and I L, are continually proportional, and the lines D I, M L, and I L, continually proportional, being in the ſame

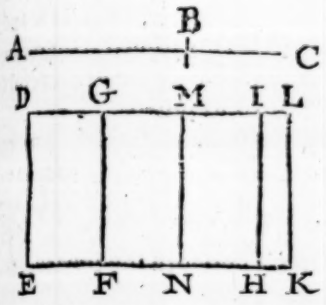
g) 17. 6. rate. Wherefore [g] the Rectangle under D I and I L, is equal to the ſquare of M L; that is to ſay, a quarter of the ſquare of G L.

h) 16. 10. And foraſmuch as the ſquares of A C and B C, or their equals D H and I K are commenſurable: [h] D I and I L, being in the ſame rate, ſhall be commenſurable in length; therefore D L and G L being unequal, and to the greateſt there be applyed a Rectangle under D I and I L, equal to the quarter of the ſquare of G L the leaſt, wanting a ſquare figure, and D I being ſhewn commenſurable in length to I L, D L ſhall be more in power then G L, by the ſquare of a line commenſurable thereto in length: Wherefore

i) 18. 10. D G being ſhewn Reſidual, and that the whole D L [i] is more in power then G L, his agreeing line by the ſquare of a line commenſurable in length thereto;

thereto; and that the ſame whole D L is commenſurable in length to the rational D E, by the Definition, D G ſhall be a firſt Reſidual: Therefore the ſquare, &c. Which was to be demonſtrated.

PROP. 99. THEOR. 75.



The ſquare of a Reſidual firſt medial A B, *applyed to a rational line* D E, *makes the breadth* D G, *a ſecond Reſidual.*

Demonſtration FOr let it be done as in the precedent Propoſition, in ſuch ſort as D H and I K may be equal to the ſquares of A C and B C, and G K equal to twice the Rectangle under A C and B C, and therefore M K equal to once the ſame Rectangle; foraſmuch then as A C and B C are Medials, commenſurable in power only, and their ſquares, that is to ſay, their equals D H and I K medials, and commenſurable: therefore [a] the whole D K ſhall be commenſurable to each of them, [b] therefore medial; And D K being applyed to the rational D E, [c] D L ſhall be rational incommenſurable in length to D E. a) 16. 10. b) C. 24. 10. c) 22. 10.

Again, foraſmuch as the Rectangle under A C and B C is propoſed rational, the double thereof G K ſhall be alſo rational, which being applyed to the rational D E, [d] G L ſhall be rational, commenſurable in length to D E. d) 21. 10.

But foraſmuch as D K Medial, that is to ſay irrational, and G K rational, are incommenſurable in length, and being ſhewn rational, [e] D L and L G ſhall be rational, commenſurable in power only; and therefore [f] the remainder D G a Reſidual: I ſay that it is a ſecond Reſidual: For (as in the precedent Propoſition) we ſhall ſhew that the whole D L is more in power then the agreeing line G L, by the ſquare of a line commenſurable thereto in length: therefore G L being ſhewn commenſurable in length to the rational D E, the ſame D G by the Definition, ſhall be a ſecond Reſidual: Therefore, &c. Which was to be demonſtrated. e) 10. 10. f) 74. 10.

PROP. 100. THEOR. 76.

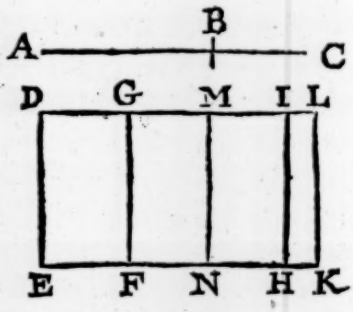
The ſquare of a Reſidual ſecond medial A B, *applyed to a rational line* D E, *makes the breadth* D G *a third Reſidual.*

Demonſtration LEt there be done as is before ſhewn, and it ſhall be ſhewn as in the precedent Propoſition, that D K is a Medial, and therefore D L rational incommenſurable in length to D E: and foraſmuch as the Rectangle under A C and B C is medial, and therefore the double thereof, to wit G K, G L [a] ſhall be alſo rational, incommenſurable in length to D E. a) 23. 10.

But forasmuch as A C and B C are incommensurable in length, and as A C to B C, so the square of A C to the Rectangle under A C and B C; b) 10. 10. b the square of A C shall be incommensurable to the Rectangle under A C and B C: But c) 16. 10. c the compound of the squares of A C and B C is commensurable to the square of A C, those squares being commensurable, described of lines commensurable in power, and twice the Rectangle under A C and B C is commensurable to the said Rectangle under A C and B C: Therefore the compound of the squares of A C and B C, that is to say D K, is incommensurable to twice the Rectangle under A C and B C, that is to say G K: and therefore d) 10. 10. d D L and G L being in the same rate as D K and G K, are incommensurable in length, and are shewn rational; therefore D L and G L are rational, commensurable in power only; and therefore e) 74. 10. e D G the remainder is a Residual: I say it is a third Residual.

For we shall shew as in the 98 Proposition, that the whole D L is more in power then G L, its agreeing line, by the square of a line commensurable thereto in length: Therefore seeing that neither D L nor G L are commensurable in length to the rational D E, as hath been shewn, D G shall be a third Residual, by the Definition: Therefore the square, &c. Which was to be demonstrated.

PROP. 101. THEOR. 77.



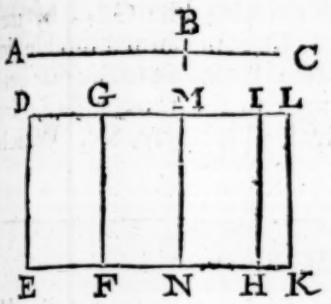
The square of a line minor A B, *applyed to a rational line* D E, *makes the breadth* D G *a fourth Residual.*

Demonstration. FOrasmuch as the Compound of the squares of A C and B C, that is to say D K, their equal, is rational; D L a) 21. 10. a shall be rational, commensurable in length to D E; and seeing that the Rectangle under A C and B C, and therefore the double thereof G K is medial: G L b) 23. 10. b shall be rational, incommensurable in length to D E.

Again, forasmuch as D K rational, and G K irrational; to wit Medial, are incommensurable; D L and G L c) 10. 10. c being in the same rate, are incommensurable in length; and being shewn rational, D L and G L are rational, commensurable in power only: therefore d) 74. 10. d D G the remainder is a Residual: I say that it is a fourth Residual. For A C and B C being commensurable in power, their squares or their equal Rectangles D H and I K, shall be incommensurable; and therefore e) 10. 10. e D I and I L incommensurable in length: And seeing that the Rectangle under D I and I L is equal to the square of M L, that is to say, to a quarter of the square of G L, as is demonstrated in the 98 Proposition, D L f) 19. 10. f shall be more in power then G L, by the square of a line incommensurable in length thereto, seeing that to D L the greatest, there is applyed the Rectangle under D I and I L, equal to a quarter of the square of G L the least, wanting a square figure, and dividing D L in

in parts incommenſurable in length D I and I L: Therefore ſeeing that D L the whole is more in power then the agreeing line GL, by the ſquare of a line incommenſurable in length thereto, and the whole D L is ſhewn commenſurable in length to the rational D E: by the Definition D G ſhall be a fourth Reſidual: Therefore, &c. Which was to be demonſtrated.

PROP. 102. THEOR. 78.



The ſquare of a line A B, *which with a Space rational* G K, *makes the whole Medial, applyed to a rational line* D E, *makes the breadth* D G, *a fifth Reſidual.*

Demonſtration LEt it be done as is before ſhewn; foraſmuch as the compound of the ſquares of A C and B C, or his equal D K is Medial: D L [a] ſhall be rational, incommenſurable in length to D E; and foraſmuch as the Rectangle under A C and B C, or the double thereof G K is rational, G L [b] ſhall be rational, commenſurable in length to D E. a) 23.10. b) 21.10.

And foraſmuch as D K Medial, or irrational, and G K rational, are incommenſurable; [c] D L and G L having the ſame rate, ſhall be incommenſurable in length: But they are ſhewn rational: Therefore D L and G L are rational, commenſurable in power only: therefore [d] the remainder D G is a Reſidual: I ſay it is a fifth Reſidual. c) 10.16. d) 74. 10.

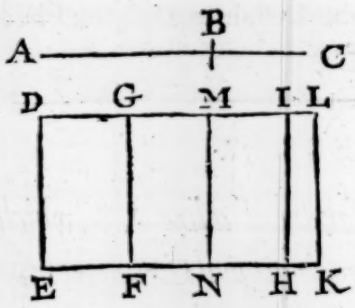
For as in the precedent, it ſhall be ſhewn that D L the whole is more in power then G L, the agreeing line, by the ſquare of a line incommenſurable in length thereto: Therefore ſeeing that the agreeing line G L, is ſhewn commenſurable in length to the rational D E, by the Definition, D G ſhall be a fifth Reſidual: Therefore, &c. Which was to be demonſtrated.

PROP. 103. THEOR. 79.

The ſquare of a line A B, *making with a Space medial, the whole Medial, applyed to a rational line* D E, *makes the breadth* D G, *a ſixth Reſidual.*

Demonſtration LEt the ſame be done as in the former; foraſmuch as, as well the compound of the ſquares of A C and B C, or D K his equal, as the Rectangle under A C and B C: and therefore the double thereof G K is medial, as well [a] D L as G L, ſhall be rational, incommenſurable in length to D E, and the Rectangle under A C and B C, a) 23.10.

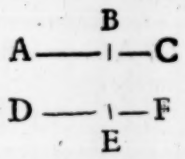
and the double thereof G K commenſurable; G K [b] ſhall be incommenſurable to the ſaid compound, that is to ſay, to D K. And therefore [c] D L and G L having the ſame rate as D K and G K, ſhall be incommenſurable in length. And they are ſhewn rational: Therefore D L and G L are rational, commenſurable in power only; therefore [d] D G is a Reſidual: I ſay it is a ſixth Reſidual.



For as in the 101 Propoſition, we ſhall ſhew that the whole D L is more in power then the agreeing line G L, by the ſquare of a line incommenſurable in length thereto: Therefore ſeeing that neither D L nor G L is commenſurable in length to the rational D E; by the Definition, D G ſhall be a ſixth Reſidual. Therefore, &c. Which was to be demonſtrated.

b) 14. 10.
c) 10. 10.
d) 74. 10.

PROP. 104. THEOR. 80.



The right line D E *commenſurable in length to a Reſidual* A B, *is alſo a Reſidual, and of the ſame order as* A B.

Demonſtration. FOr let it be as A B to D E, [a] ſo B C to E F; therefore [b] the whole A C ſhall be to the whole D F, as A B to D E; or B C to E F: foraſmuch then as A B and D E are propoſed commenſurable in length; [c] A C and D F, and B C and E F, ſhall be alſo commenſurable in length; and foraſmuch as A C and B C are rational, their commenſurables D F and E F ſhall be rational: Again, foraſmuch as A C is to D F, as B C is to E F; and alternately, as A C to B C, ſo D F to E F: and A C and B C are commenſurable in power only: Therefore D F and E F ſhall be alſo commenſurable in power only, and being ſhewn rational, the remainder D E is a Reſidual: I ſay that it is of the ſame order as A B. For firſt let A C be more in power then B C, by the ſquare of a line commenſurable in length thereto; that being [d] D F ſhall be more in power then E F, by the ſquare of a line commenſurable in length thereto: Therefore if A C be commenſurable in length to the rational expoſed, to the end that A B may be a firſt Reſidual, D F ſhall be alſo commenſurable in length to the rational expoſed, ſeeing that as well the rational expoſed, as D F, is commenſurable in length to the ſame A C; Therefore, by the Definition, D E ſhall be a firſt Reſidual, to wit, of the ſame order as A B. But if B C be commenſurable in length to the rational, in the ſame manner E F ſhall be commenſurable in length to the rational; therefore the one and the other A B and D E, by the Definition ſhall be a ſecond Reſidual.

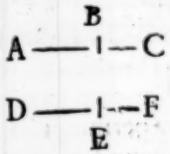
Laſtly, if neither the one or the other A C nor B C, be commenſurable in length to the rational expoſed; [e] alſo neither the one nor the other D F nor E F ſhall be commenſurable in length to the ſame rational: Therefore as

a) 12. 6.
b) 12. 5.
c) 10. 10.
d) 15. 10.
e) 14. 10.

as well the one as the other A B or D E, ſhall be a third Reſidual by the Definition.

Now let A C be more in power then B C, by the ſquare of a line incommenſurable in length thereto, that being [f] D F ſhall be more in power then E F, by the ſquare of a line which ſhall be incommenſurable in length thereto. Wherefore we ſhall ſhew as before, that D E is a fourth, fifth, or ſixth Reſidual. Therefore, &c. Which was to be demonſtrated. f) 15. 10.

PROP. 105. THEOR. 81.



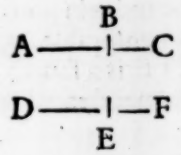
The right line D E, *commenſurable to a Medial Reſidual* A B, *is alſo a Reſidual Medial, and of the ſame order as* A B.

Demonſtration. FOr as A B is to D E, [a] ſo let B C be to E F: Therefore the whole A C ſhall be to the whole D F, as A B to D E, and B C to E F: foraſmuch then as A B and D E are propoſed commenſurable, [b] B C and E F, and A C and D F ſhall be alſo commenſurable, and A C and B C being Medials D F and E F [c] commenſurable to them, ſhall be alſo Medials. a) 12. 6. b) 10. 10. c) 24. 10.

Again, A C being to D F as B C to E F, and alternately, as A C to B C, ſo D F to E F; but A C and B C are commenſurable in power only; Therefore [d] D F and E F ſhall be commenſurable in power only: Wherefore D F and E F being ſhewn Medials, the remainder [e] D E ſhall be a Reſidual Medial: I ſay that it is of the ſame order as A B, for ſeeing that as A C to B C, ſo D F to E F: and as A C to B C, ſo the ſquare of A C to the Rectangle under A C and B C, and as D F to E F, ſo the ſquare of D F to the Rectangle under D F and E F: Therefore as the ſquare of A C to the Rectangle under A C and B C, ſo the ſquare of D F to the Rectangle under D F and E F: and alternately, as the ſquare of A C to the ſquare of D F: ſo the Rectangle under A C and B C, to the Rectangle under D F and E F: therefore the ſquare of A C being commenſurable to the ſquare of D F, A C and D F being ſhewn commenſurable, [f] the Rectangle under A C and B C ſhall be commenſurable to the Rectangle under D F and E F: therefore if the Rectangle under A C and B C be rational, in ſuch ſort as A B may be a firſt Reſidual Medial, the Rectangle under D F and E F, commenſurable thereto, ſhall be alſo rational; and therefore [g] D E ſhall be a Reſidual firſt Medial: but if the Rectangle under A C and B C be Medial, in ſuch ſort as A B may be a ſecond Reſidual Medial: the Rectangle under D F and E F, commenſurable thereto, [h] ſhall be alſo Medial: Therefore [i] D E ſhall be a ſecond Reſidual Medial: Therefore, &c. Which was to be demonſtrated. d) 10. 10. e) 75, 76. 10. f) 10. 10. g) 75. 10. h) 24. 10. i) 76. 10.

PROP.

PROP. 106. THEOR. 82.

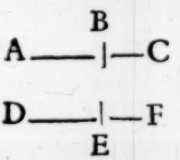


The right line D E, *commenſurable to a line minor* A B, *is alſo a line minor.*

Demonſtration FOr let be done as is before ſhewn, in ſuch ſort as A B and B C, may have the ſame rate to D E and E F, as A C to D F, as in the former Propoſition, D F and E F ſhall be commenſurable to A C and B C, be it in length and power, or in power alone; and ſeeing that as A C is to D F, ſo B C is to E F; and alternately, as A C to B C, a) 22.6. ſo D E to E F, [a] as the ſquare of A C to the ſquare of B C, ſo the ſquare of D F to the ſquare of E F; and in compounding, as the compound of the ſquares of A C and B C, to the ſquare of B C, ſo the compound of the ſquares of D F and E F, to the ſquare of E F: and by permutation, as the compound of the ſquares of A C and B C to the compound of the ſquares of D F and E F, ſo the ſquare of B C to the ſquare of E F: But the ſquare of B C is commenſurable to the ſquare of E F, B C and E F being ſhewn b) 10. 10. commenſurable: Therefore [b] the compound of the ſquares of A C and B C is commenſurable to the compound of the ſquares of D F and E F: But the compound of the ſquares of A C and B C is propoſed rational; c) 9. d. Therefore [c] the compound of the ſquares of D F and E F is rational.

Again, as is ſhewn in the precedent Propoſition, the Rectangle under A C and B C is commenſurable to the Rectangle under D E and E F: but d) C. 24. 10. the Rectangle under A C and B C is propoſed Medial; therefore [d] the Rectangle under D F and E F, ſhall be alſo Medial; and ſeeing that as A C is to B C, ſo D F is to E F: and A C and B C are incommenſurable in e) 10. 10. power, D F and E F [e] ſhall be alſo incommenſurable in power: therefore D F and E F being incommenſurable in power, and making the compound f) 77. 10. of their ſquares rational, and the Rectangle under them Medial: [f] D E ſhall be a Minor: Therefore, &c. Which was to be demonſtrated.

PROP. 107. THEOR. 83.



The right line D E, *commenſurable to a line* A B, *the which with a Space rational, makes the whole medial, the ſame is alſo a line making with a Space rational a whole medial.*

Demonſtration FOr having made the Conſtruction as before, it ſhall be ſhewn as in the precedent Propoſition, that the compound of the ſquares of A C and B C is commenſurable to the compound of the a) C. 24. 10. ſquares of D F and E F, therefore the one being propoſed medial, [a] the other ſhall be alſo Medial.

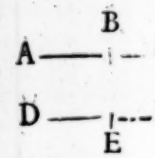
Again,

Again, (as in the 105 Propoſition,) it ſhall be ſhewn that the Rectangle under A C and B C is commenſurable to the Rectangle under D F and E F; therefore the one being propoſed rational, [b] the other ſhall be alſo rational: And laſtly, as is ſhewn in the precedent Propoſition, D F and E F are incommenſurable in power; Wherefore D F and E F being incommenſurable in power, and making the compound of their ſquares medial, and the Rectangle under them rational: [c] D E ſhall make with a Space rational, a whole medial. Which was to be demonſtrated.

b) 9. d.

c) 78. 10.

PROP. 108. THEOR. 84.



The right line D E, *commenſurable to the line* A B, *the which with a Space medial, makes a whole medial, the ſame is alſo a line making with a Space medial a whole medial.*

Demonſtration. FOr having made the Conſtruction as before, we ſhall ſhew as in the 106 Propoſition, that the compound of the ſquares of A C and B C, is commenſurable to the compound of the ſquares of D F and E F; therefore the compound of the ſquares of A C and B C being propoſed Medial; [a] that of the ſquares of D F and E F ſhall be alſo medial.

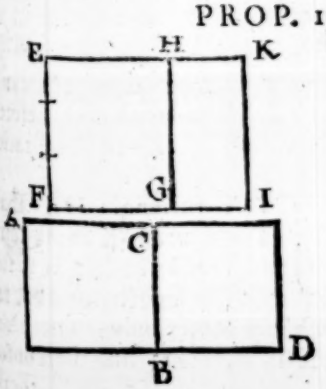
a) C. 24. 10.

Again, (as in the 105 Propoſition,) it ſhall be ſhewn that the Rectangle under A C and B C is commenſurable to the Rectangle under D F and E F; therefore the one being Medial, the other ſhall be alſo Medial. And as in the 106 Propoſition, D F and E F ſhall be incommenſurable in power.

Laſtly, foraſmuch as the compound of the ſquares of D F and E F is commenſurable to the compound of the ſquares of A C and B C, as is ſaid, and the Rectangle under D F and E F is commenſurable to the Rectangle under A C and B C. But the compound of the ſquares of A C and B C, and the Rectangle under A C and B C are propoſed incommenſurable: Therefore the compound of the ſquares of D F and E F, and the Rectangle under D F and E F ſhall be incommenſurable: Wherefore D F and E F being incommenſurable in power, and the compound of their ſquares Medial, and incommenſurable to the compound of their ſquares; [b] D E makes with a Space Medial a whole Medial: Therefore, &c. Which was to be demonſtrated.

b) 79. 10.

PROP. 109. THEOR. 85.



If a Space medial C D, *be cut from a rational* A D, *the right line which is equal in power to the reſt of the Space* A B, *is one of theſe two Irrationals; to wit, a Reſidual, or Minor.*

Demon-

Demonſtration FOr let the rational E F be expoſed, [a] to which let there be applyed the Rectangle E G equal to A B, and to G H the Rectangle H I, equal to A B, and to G H the Rectangle H I, equal to C D, in ſuch ſort as the whole E I may be equal to the whole A D; foraſmuch then as E I equal to A D rational is rational, [b] E K ſhall be rational, commenſurable in length to E F rational.

a) 45. 1.

b) 21. 10.

Again, foraſmuch as H I equal to C D medial is medial: H K ſhall be rational, incommenſurable in length to E F. Wherefore E K being commenſurable in length to E F: But H K incommenſurable in length to the ſame E F, [c] E K and H K ſhall be incommenſurable in length, and being rational, they ſhall be rational, commenſurable in power only; and therefore [d] the remainder E H is a Reſidual, and H K an agreeing line thereto: Therefore E K is more in power then H K, by the ſquare of a line which is thereto commenſurable in length, or incommenſurable.

c) 23. 10.

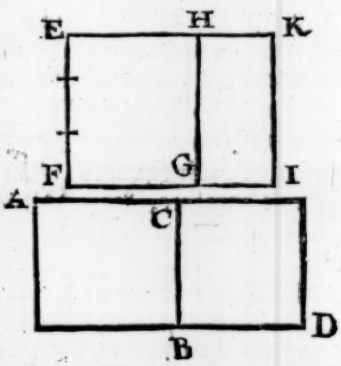
d) 74. 10.

If E K be more in power then H K, by the ſquare of a line which is commenſurable in length thereto, the whole E K being alſo ſhewn commenſurable in length to E F rational: by the Definition, E H ſhall be a firſt Reſidual: Wherefore the line equal in power to the Space E G, conteined under E F rational, and the firſt Reſidual E H; that is to ſay, the Space A B, equal to the ſame, [e] is a Reſidual: But if E K be more in power then H K, by the ſquare of a line which ſhall be incommenſurable thereto in length. The whole E K being ſhewn commenſurable in length to E F rational, according to the Definition, E H ſhall be a fourth Reſidual; Wherefore the line equal in power to the Space E G, conteined uuder E F rational, and E H a fourth Reſidual; that is to ſay, the Space A B equal thereto, [f] is a Minor: Therefore, &c. Which was to be demonſtrated.

e) 92. 10.

f) 95. 10.

PROP. 110. THEOR. 86.



If a Superficie rational C D, *be cut from a medial* A D, *they make two other irrationals; to wit, either a Reſidual firſt medial, or a line making with a Space rational a whole medial.*

Demonſtration LEt be done the ſame Conſtruction as before: Foraſmuch as E I equal to the Medial A D, is medial, E K [a] ſhall be rational, incommenſurable in length to E F.

a) 23. 10.

And foraſmuch as H I equal to the rational C D is rational: H K [b] ſhall be rational, commenſurable in length to E F: Therefore H K being commenſurable in length to E F: but E K incommenſurable in length to the ſame E F, H K and E K [c] ſhall be incommenſurable in length, and are rationals, they are then rational, commenſurable in power only: Therefore [d] the remainder E H is a Reſidual, and H K an agreeing line thereunto; there-

b) 21. 10.

c) 13. 10.

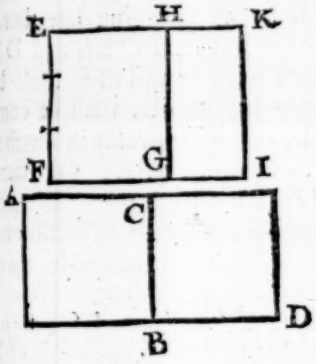
d) 74. 10.

therefore E K is more in power then H K, by the ſquare of a line commenſurable thereto in length, or incommenſurable, if commenſurable, and the agreeing line H K being ſhewn commenſurable in length to E F rational, by the Definition, E H ſhall be a ſecond Reſidual: Therefore the line equal in power to the Space E G, conteined under E F rational, and E H, a ſecond Reſidual, that is to ſay, A B equal thereto, [c] is a Reſidual firſt medial: But if E K be more in power then H K, by the ſquare of a line incommenſurable in length thereto, the agreeing line H K being ſhewn commenſurable in length to E F rational, E H ſhall be a fifth Reſidual, by the Definition: Therefore the line equal in power to the Space E G, conteined under E F rational, and the fifth Reſidual E H, that is to ſay A B, equal thereto, is a line [f] which with a rational makes a whole Medial: Therefore, &c. Which was to be demonſtrated.

c) 93. 10.

f) 96. 10.

PROP. III. THEOR. 87.



If a ſuperficies medial C D, *be cut from a ſuperficies medial* A D, *incommenſurable to the whole, there is made two other irrationals; to wit, a Reſidual ſecond medial, or a line making with a ſuperficies medial, a whole medial.*

Demonſtration. FOr having made the ſame Conſtruction as before: Foraſmuch as E I and H I, equal to the Medials A D and C D are Medials; E K and H K [a] ſhall be rational, incommenſurable in length to E F; and A D and C D, that is to ſay, E I and H I being propoſed incommenſurable, [b] E K and H K having the ſame rate, ſhall be incommenſurable in length, and are rational; therefore they are rational, commenſurable in power only: therefore [c] E H remaining is a Reſidual, and H K an agreeing line thereto; therefore E K is more in power then H K, by the ſquare of a line commenſurable thereto in length, or incommenſurable.

a) 23. 10.

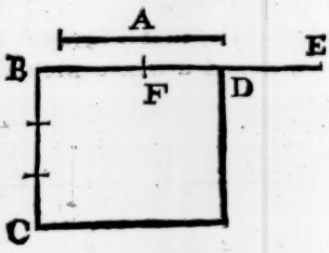
b) 10. 10.

c) 74. 10.

If by the ſquare of a line which may be commenſurable thereto in length; both the one and the other E H and H K being ſhewn not commenſurable in length to E F rational, according to Definition, E H ſhall be a third Reſidual. Wherefore the line equal in power to the Space E G, conteined under the rational E F, and the third Reſidual E H; that is to ſay A B equal thereto, is a Reſidual ſecond Medial: But if E K be more in power then H K, by the ſquare of a line incommenſurable in length thereto, neither the one nor the other E K nor H K, being commenſurable in length to E F rational, as is demonſtrated by the Definition, E H ſhall be a ſixth Reſidual: Therefore the line equal in power to E G, conteined under E F rational,

onal, and E H a ſixth Reſidual; that is to ſay, A B equal thereto, is that which with a Space Medial makes a whole Medial: Therefore, If, &c. Which was to be demonſtrated.

PROP. 112. THEOR. 88.



The line A, *called* Apotome *or* Reſidual, *is not the ſame as the line of two names, or* Binomial.

Demonſtration FOr let A be a Binomial (if poſſible) and having expoſed the rational B C, a let there be applyed to B C the Rectangle C D, equal to the ſquare of A; Foraſmuch then as A is a Reſidual, the breadth B D b ſhall be a firſt Reſidual: Let then D E be an agreeing line thereto; Therefore by the Definition of the firſt Reſidual, B E and D E ſhall be Rational, commenſurable in power only, and B E ſhall be more in power then D E, by the ſquare of a line which ſhall be commenſurable in length thereto; and B E ſhall be commenſurable in length to B C rational: Again A being alſo propoſed to be a Binomial; c the ſame breadth B D ſhall be a firſt Binomial: Let B F be his greateſt name: Therefore by the Definition of the firſt Binomial B F and F D ſhall be rational, commenſurable in power only, and B F ſhall be more in power then F D, by the ſquare of a line commenſurable in length thereto, and B F ſhall be commenſurable in length to the rational B C; therefore as well B E, as B F being commenſurable in length to B C; d B E and B F ſhall be alſo commenſurable in length to one another: Wherefore the whole B E, being commenſurable in length to his part B F, the ſame B E ſhall be alſo commenſurable in length to his part B F, the ſame B E ſhall be alſo commmenſurable in length to the other part F E; and therefore B E being Rational, F E ſhall be alſo Rational.

a) 45. 1.
b) 98. 10.
c) 61. 10.
d) 12. 10.

And foraſmuch as the two B E and F E, are commenſurable in length, B E is incommenſurable in length to D E (B E and D E being Rational, commenſurable in power only,) e the remainder F E ſhall be incommenſurable in length to the ſame D E: But as well F E as D E, is ſhewn Rational: Therefore F E and D E are Rational, commenſurable in power only: Wherefore the remainder F D, f is a Reſidual, and therefore Irrational. But it is alſo ſhewn Rational, which is abſurd: Therefore, &c. Which was to be demonſtrated.

e) 14. 10.
f) 74. 10.

COROL-

COROLLARIE.

From these things demonstrated, may be gathered, that the Residual, and the other Irrationals following the same Residual, are neither Medials, nor the same to one another.

For the square of a Medial *applyed to a* Rational, *makes* [a] *the breadth rational, incommensurable in length to the* Rational *proposed.* a) 23. 10.

But the square of the Residual *applyed to a rational line, makes* [b] *the breadth, a first* Residual. b) 98. 10.

And the square of the Residual *first* Medial, *applyed to a rational line,* [c] *makes the breadth a second* Residual. c) 99. 10.

But the square of the Residual *second* Medial, *applyed to a rational line,* [d] *makes the breadth a third* Residual. d) 100. 10.

Furthermore the square of the Minor applyed to a rational line, [e] *makes the breadth a fourth* Residual. e) 101. 10.

But the square of a line, the which with a space rational, makes a whole Medial, *applyed to a rational line,* [f] *makes the breadth a fifth* Residual. f) 102. 10.

Lastly, the square of a line, which with a Space Medial *makes a whole* Medial, [g] *applyed to a rational line, makes the breadth a sixth* Residual. g) 103. 10.

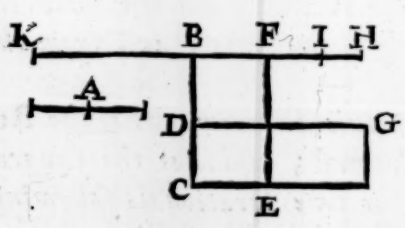
Therefore these breadths being different from the breadth of the Medial, *and to one another; I say, from the breadth* Medial, *being the one is rational, and the others irrational: But to one another; forasmuch as they are not* Residuals *of the same order: it is manifest that the* Residual *and the other Irrationals following do differ among themselves, and from the* Medial.

But forasmuch as it is shewn in this Theorem, that the Residual *is not the same as the* Binomal; *and that the squares of the* Residual, *and of the other five Irrationals following, applyed to a rational line, makes the breadths a First, Second, Third, Fourth, Fifth, and Sixth* Residual; *but the square of the* Binomial, *and the other five Irrationals following applyed to a rational line makes the breadth a First, Second, Third, Fourth, Fifth, and Sixth* Binomial. *It is manifest that the breadths of the* Residual, *and the other five Irrationals following, are not the same as the breadths of the* Binomials, *and the five other Irrationals following, forasmuch as not any*

Residual, *is the same as any* Binomial: *Therefore the* Residual *and the others following, do differ from the* Binomial, *and the others following: Wherefore as well those as the others, do differ from the* Medial, *in such sort as any whatsoever rational line being proposed, there will be* 13 *Irrational lines differing from one another, of which we have discoursed hitherto, and they are these following.*

1 The Medial line.
2 The Binomial line, or line of two names.
3 The first Bimedial.
4 The second Bimedial.
5 The line Major.
6 The line equal in power to a Rational and a Medial.
7 The line equal in power to two Medials.
8 The Residual, or Apotome.
9 The Residual first Medial.
10 The Residual second Medial.
11 The line Minor.
12 The line making with a Superficie Rational, a whole Medial.
13 The line making with a Superficie Medial, a whole Medial.

PROP. 113. THEOR. 89.



The square of a rational line A, *being applyed to a Binomial line* BC, *make the breadth* BF *Residual, of which the names are commensurable, and proportional to the names* BD *and* DC, *of the Binomial* BC, *and over and above the Residual* BF, *is of the same order as the Binomial* BC.

a) 45. 1. *Demonstration* FOr a to DC the lesser name, let there be applyed CG, equal to the square of A; and therefore to BE, making DG the breadth; and let BH be equal to DG and BE, CG being equal, as BC shall be to DC, so DG, or his equal BH, to BF, and in dividing, as

as BD to DC, so HF to FB: But BD is greater then DC; therefore HF shall be greater then FB. Let FI be equal to FB, and let it be as HI to IF, so FB to BK: Therefore in compounding HF shall be to IF, or to his equal FB, as FK to BK: But as HF to BF, so BD is shewn to be to DC: Therefore also as BD shall be to DC, so FK to BK: But BD and DC (names of the Binomial BC) are rational, commensurable in power only: Therefore [b] FK and BK are commensurable in power only. b) 10. 10.

Again, HF being to BF, as FK to BK, [c] the Antecedents HF and FK together, to wit, HK the whole, shall be to FB and BK consequents; that is to say, to FK the whole, as FK to BK: therefore FK is a mean proportional between HK and BK; therefore [d] as HK the first, to BK the third, so the square of HK the first, to the square of FK the second: But forasmuch as CG rational (being equal to the square of A rational, applyed to DC rational,) makes [e] the breadth DG rational, commensurable in length to DC; HB equal to DG, shall be also rational, commensurable in length to DC, and it being shewn that as BD is to DC, so FK is to BK: But as FK is to BK, so HK to FK; also as BD shall be to DC, so HK to FK: therefore [f] as the square of BD to the square of DC, so the square of HK to the square of FK. But the square of BD is commensurable to the square of DC: (BD & DC names of the Binomial BC rational, commensurable in power;) Therefore [g] the square of HK shall be also commensurable to the square of FK; but as the square of HK to the square of FK, so HK to BK: therefore [h] HK is commensurable in length to BK: and therefore [i] to the rest BH: But BH is shewn rational, therefore HK commensurable thereto, is rational; and therefore BK commensurable to HK, is rational: Therefore FK being shewn commensurable in power only to BK, FK shall be also rational; therefore FK and BK being rationals, and shewn commensurable in power only; [k] the remainder BF shall be a Residual; and BK shall be an agreeing line thereto, which is first proposed.

c) 12. 5.
d) C. 20. 6.
e) 21. 10.
f) 22. 6.
g) 10. 10.
h) 10. 10.
i) C. 16. 10.
k) 74. 10.

But forasmuch as the whole HK is shewn commensurable in length to his part BK, BK and BH [l] shall be commensurable in length: Therefore BH being shewn commensurable in length to DC, BK shall be also commensurable in length to DC, as appeares by the 12 Proposition of this Book.

l) 16. 10.

And it being shewn that as BD is to DC, so FK to BK, and by permutation, as BD to FK, so DC to BK, but DC and BK are shewn commensurable in length: Therefore [m] BD and FK shall be also commensurable in length: Therefore FK being commensurable in length to BD, and BK to DC, FK and BK names of the Residual BF, shall be commensurable in length to BD and DC, names of the Binomial BC, which is in the second place proposed.

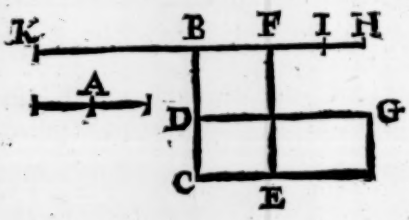
m) 10. 10.

Also it being demonstrated, that as BD is to DC, so FK is to BK: FK and BK, names of the Residual, shall be in the same rate as BD and DC, names of the Binomial BC, which is in the third place proposed.

Lastly, BD is more in power then DC, by the square of a line commensurable in length thereto, or incommensurable; if it be more in power by the square of a line commensurable in length thereto; BD being to DC, as EK to BK, [n] FK shall be also more in power then BK, by the square of a line which shall be commensurable in length thereto, and if BD be more in power then DC, by the square of a line commensurable thereto in length, FK [o] shall be more in power then BK, by the square of a line

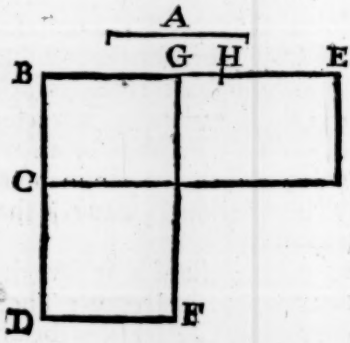
n) 15. 10.
o) 15. 10.

line incommensurable thereto in length; and if B D be commensurable in length to the rational exposed F K, shewn commensurable in length to BD, it shall be also commensurable in length to the same rational, as appears by the 12 Proposition of this Book. But if D C be commensurable in length to the rational B K, by the same reason it shall be commensurable in length to the same rational; and lastly, if neither BD nor DC be commensurable in length to the rational, neither F K P nor B K also shall be commensurable in length to the rational proposed: Therefore BF Residual is of the same order as B C Binomial, as appeares by the second and third Definitions, which is in the fourth place proposed: Therefore, &c. Which was to be demonstrated.



p) 14. 10.

PROP. 114. THEOR. 90.



The square of a rational line A, *being applyed to a Residual* B C, *makes the breadth* B E *a Binomial, whose names are commensurable to the names* B D *and* C D, *of the Residual* B C, *and in the same rate: and over and above, the Binomial* B E *is of the same order as the Residual* B C.

Demonstration FOr let there be also applyed to the whole B D the Rectangle B F, equal to the square of A; that is to say, to C E, making B G the breadth; forasmuch as B F and C E are equal, [a] as BE, shall be to B G, so B D to B C: and by conversion of reason, as B E to G E, so B D shall be to C D.

a) 14, 16. 6.

Let E G also be divided in H, according to the rate of B E to G E, to the end that E A may be to H G as B E to G E: but as B E the whole is to G E the whole, so E H cut off from B E, to H G cut off from G E: B H the remainder of B E, shall be also to H E the remainder of G E, as the whole B E to the whole G E.

But as B E was to G E, so E H to H G; Therefore also as B H shall be to H E, so E H to G H; therefore H E is a mean proportional between B H and G H: Wherefore [b] as B H the first, to G H the third, so the square of B H the first, to the square of H E the second: But as B D to C D, so B E to G E, that is to say, B H to H E, and B D to C D, names of the Residual B C, being rational, commensurable in power only; [c] B H and H E are

b) C. 20. 6.

c) 10. 10.

are also commensurable in power, and therefore the squares of BH and HE commensurable: Therefore BH and GH being in the same rate as the squares of BH and HE, as hath been demonstrated, are commensurable in length, and the whole BH being commensurable in length to its part GH; [d] it shall be also commensurable in length to the other part BG. d) C. 16. 10.

But BD being rational, to wit, the greatest name of the Residual BC, and the Rectangle BF rational, being equal to the square of A rational; [e] its breadth BG, shall be rational, commensurable in length to BD: Therefore [f] BH is commensurable in length to the same rational BD, and therefore is rational, BH and BG having been shewn commensurable in length. e) 21. 10. f) 12. 10.

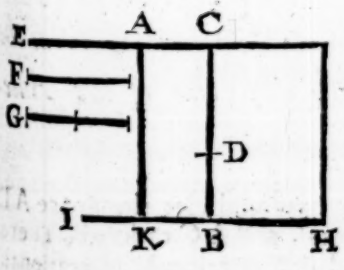
And seeing that BH and HE are shewn commensurable in power only, and BE rational, HE commensurable thereto, is also rational: Therefore BH and HE are rational, commensurable in power only; Therefore [g] BE is a Binomial, which is first proposed. g) 37. 10.

But it being shewn that BH is to HE, as BD to CD, and alternately, BH to BD, as HE to CD; but BH being shewn commensurable in length to BD, [h] HE shall be also commensurable in length to CD: Wherefore BH and HE, names of the Binomial BE, are commensurable in length to BD and CD, names of the Residual BC, which is in the second place proposed. h) 10. 10.

They are also in the same rate. It being demonstrated that BH is to HE, as BD to CD, which is in the third place proposed.

Lastly, either BD is more in power then CD, by the square of a line commensurable in length thereto, or is incommensurable: if commensurable BH shall be also more in power then HE, by the square of a line which shall be commensurable in length thereto; and if incomensurable, it shall be more in power, by the square of a line which shall be incommensurable thereto in length: and if BD be commensurable in length to the rational exposed, BH [i] shall be so also, being commensurable in length to BD, commensurable in length to the Rational. But if CD be commensurable in length to the Rational, AE by the same reason, shall be so also. If neither the one nor the other, BD nor CD, be commensurable in length to the Rational, also [k] neither BH nor HE, shall be commensurable in length thereto: Therefore by the Second and the Third Definitions, BE is a Binomial of the same order as BC, the Residual which is in the fourth place proposed: Therefore, The Square, &c. Which was to be demonstrated. i) 12. 10. k) 14. 10.

PROP. 115. THEOR. 91.

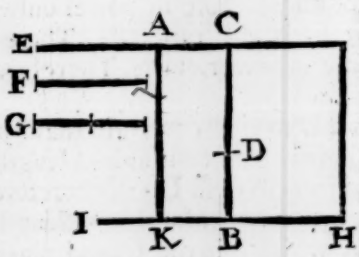


If a Space AB, *be conteined under a Residual* AC, *and under a line called a Binomial* CB, *whose names* CD *and* DB, *are commensurable to the names* CE *and* AE,

AE, *of the Reſidual* AC, *and in the ſame rate* (CD *to* DB, *as* CE *to* AE,) *the right line* F, *equal in power to the ſame Space* AB, *is a Rational.*

a) 45. 1. *Demonſtration* FOr having expoſed the Rational G, let a there be applyed to the Binomial CB, the Rectangle CH, equal to the

b) 113. 10. ſquare of G: Therefore b the breadth BH, ſhall be a Reſidual: Therefore the names HI and BI are commenſurable in length to the names CD and DB, and in the ſame reaſon; to wit, HI to BI, as CD to DB; and therefore as E to AE, and by permutation, as the whole HI to the whole CE: ſo the part cut off BI, to the part cut off AE: Therefore



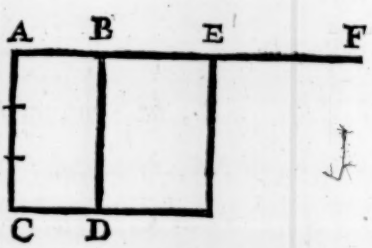
c) 19. 5. c the remainder BH ſhall be to the remainder AC, as the whole HI is to the whole CE: But HI is commenſurable in length to CE, as well HI as CE being commenſurable in length to CD: Therefore

d) 10. 10. d BH is commenſurable in length to AC, and therefore HC is commenſurable to BA: HC being to BA, as HB to AC. But HC being equal to the ſquare of the Rational G, is Rational: Therefore BA commenſurable thereto, ſhall be Rational: Therefore F equal in power to the ſame, ſhall be Rational: Therefore, &c. Which was to be demonſtrated.

COROLLARIE.

From this it is manifeſt, that a Space rational, may be conteined under two Irrational lines: For AB *conteined under* AC *and* CB *a Reſidual and a Binomial, which are Irrationals, is ſhewn to be rational.*

PROP. 116. THEOR. 92.



Of the Medial line AB, *there are made an infinite number of Irrationals, and not one of them the ſame with any one of the Antecedents.*

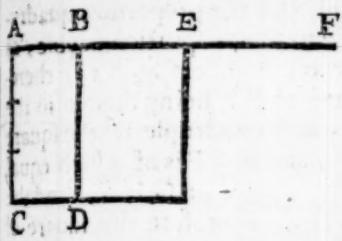
Demonſtration FOr the Rational AC being propoſed, let the Space AD be conteined under AB Medial, and AC rational; Therefore AD conteined under AD rational, and AB irrational, is irrational:

Let

Let B E be equal in power to the Space A D, [a] B E ſhall be irrational; I ſay that B E is not any of the 13 [b] before ſpoken of. a) 11.d. b) c.112.10.

For ſeeing that the ſquare of the Medial applyed to A C rational, makes the breadth [c] rational, incommenſurable in length to A C, and the ſquares of the other 12 irrationals applyed to A C rational, makes the breadths either Binomials, or Reſiduals, as appears by the 61, 62, 63, 64, 65, 66, 98, 99, 101, 102, and 103 Propoſitions of this Book. c) 23.10.

But the ſquare of this irrational B E, applyed to the ſame rational A C, makes the breadth A B Medial: It is manifeſt that B E irrational, doth differ from all the 13; ſeeing that his ſquare applyed to a rational line, makes the breadth different from the breadths made by the ſquares of thoſe 13 lines applyed to the ſame rational: And if the Rectangle D E conteined under B D rational, and B E irrational be accompliſhed, the ſame ſhall be irrational: Let then E F be equal in power to the ſame, which ſhall be irrational.



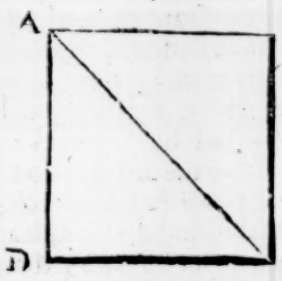
I ſay, [d] again that E F is not any of the 13 before mentioned lines, nor is alſo B E, which is manifeſt, ſeeing that the ſquare of E F applyed to the rational, makes the breadth B E; but the ſquares of the ſame 13, and alſo the ſquare of B E, applyed to the ſame rational, make the breadths differing to B E, as hath been demonſtrated: In like manner, there might be found infinite other irrationals, differing from one another, and from the afore-mentioned 13: Therefore, &c. Which was to be demonſtrated. d) 11. d.

PROP. 117. THEOR. 93.

Let it be propoſed to us to ſhew that in ſquare figures A B, *and* C D, *the Diameter* A C, *is incommenſurable in length to the ſide* A B.

Demonſtration FOr otherwiſe it ſhould be commenſurable in length to A B, and therefore [a] A C and A B ſhould be to one another as a number to a number: Let A C be to A B, as the number E F is to the number G, and let E F and G be the leaſt in their rate: Therefore A C being to A B as E F to G, the ſquare of A C ſhall be alſo to the ſquare of A B as the ſquare number of E F to the ſquare number of G: (for [b] the ſquares being in a double rate of their ſides, and the ſides having equal rates, the rates of the ſquares ſhall be alſo equal, being double the equal rates,) but the ſquare of A C is double to the ſquare of A B (A B and B C being equal;) therefore the ſquare of the number E F, ſhall be double to the ſquare of the number G; and therefore the ſquare of E F having a half, and may be divided into two equal parts, ſhall be even, by the Definition: Therefore E F which produceth it, ſhall be alſo even; (for if it were odde, and multiplying its ſelf, doth produce its ſquare, the ſame ſquare ſhould be alſo odde, ſeeing that an odde number multiplying an odde number produceth a) 5. 10. b) 20. 6.

an odde number, which is abſurd, for it is ſhewn to be even:) But E F and G being the leaſt in their rate, are primes to one another; and E F being ſhewn even, G ſhall be odde: (For if it were alſo even, the Binary ſhould meaſure E F and G; and therefore ſhould not be primes to one another, which is abſurd:) Now let the even number E F be divided into two equal parts at H; foraſmuch then as E F is double to E H, and the ſquares are in a double rate to their ſides, the ſquare of E F ſhall be quadruple to the ſquare of E H, (for the proportion quadruple, is double to the proportion double, as appears by theſe numbers 4, 2, 1;) therefore the ſquare of E F being double to the ſquare of G, and quadruple to the ſquare of E H, the ſquare of E F is of 4 ſuch equal parts, as the ſquare of G is 2, and the ſquare of E H 1. Therefore the ſquare of G is double to the ſquare of E H, being as 2 to 1; and therefore as hath been ſaid of the number E F, the ſquare of G having a half, ſhall be even, and the ſame G alſo even: But it is alſo ſhewn to be odde, which is abſurd: Therefore the Diameter A C is not commenſurable in length to the ſide A B: Therefore incommenſurable in length.



E . . . H . . . F
G . . . . . .

Otherwiſe, (if poſſible,) let the Diameter A C be commenſurable in length to the ſide A B, and that A C and A B may be to one another, as the numbers E F and G, which may be the leaſt of their rate, and therefore primes to one another: Therefore G ſhall not be unity, for the ſquare of A C being double to the ſquare of A B, and as the ſquare of A C to the ſquare of A B, ſo the ſquare of the number E F, to the ſquare of the number G, as it is ſaid in the firſt Demonſtration, the ſquare of E F ſhall be alſo double to the ſquare of G; if therefore G be Unity, and therefore his ſquare alſo Unity, the ſquare of E F ſhall be a Binary, which is abſurd.

And foraſmuch as hath been already ſhewn, the ſquare of E F being double to the ſquare of G, the ſquare of G ſhall meaſure the ſquare of E F; and therefore G the ſide ſhall meaſure E F the ſide: Therefore G meaſuring alſo it ſelf, E F and G ſhall be compounds to one another, having G for their common meaſure, but they are alſo primes to one another, which is abſurd: Therefore the Diameter A C is not commenſurable in length to the ſide A B: Therefore, &c. Which was to be demonſtrated.

The End of the Tenth Element of EUCLIDE.

THE

THE ELEVENTH ELEMENT OF EUCLIDE.

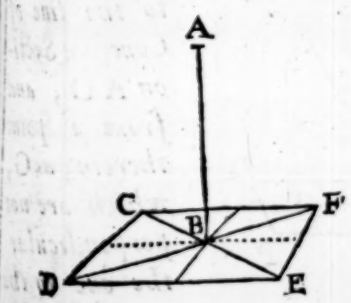
THE ARGUMENT.

IN this Eleventh Book is treated firſt of right lines in relation to Solid Bodies; *viz.* when they are in one Plain, when they are erect, or perpendicular to a Plain, when they are parallel, and how from a point given on high, perpendiculars may be drawn to a given Plain. And alſo of the interſections of Plains, and then of Solid Angles: Laſtly, he ſpeaketh of Solid Parallelepipedons, and ſomething concerning Priſmes.

DEFINITIONS.

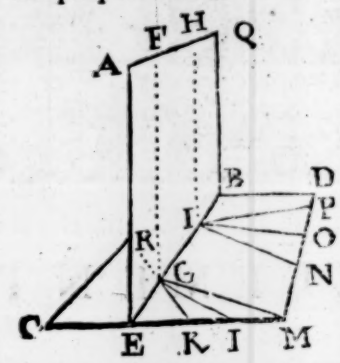
1 *A Solid is that which hath length, breadth, and thickneſſe.*

2 *But the Terms of a Solid are Superficies.*



3 *A right line as* A B, *is raiſed at right angles on a Plain, as* C D E F, *when it maketh right angles as* C B A, D B A, E B A, *and* F B A, *with all the right lines as* B C, B D, B E, and B F, *which do touch it on the Plain propoſed.*

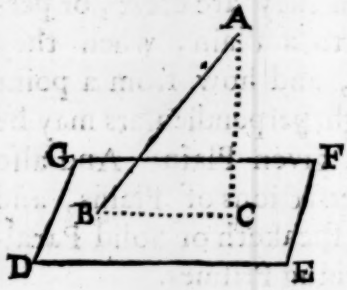
I Say, a line raiſed on thoſe conditions, is ſaid to be perpendicular to the propoſed Plain.



4 *A Plain as* AB, *is raiſed perpendicularly on a Plain* CD, *when all the lines* GF, IH, &c. *drawn on one of the ſaid Plains at right angles to the line of common Section* BE, *are at right angles to the other Plain* CD.

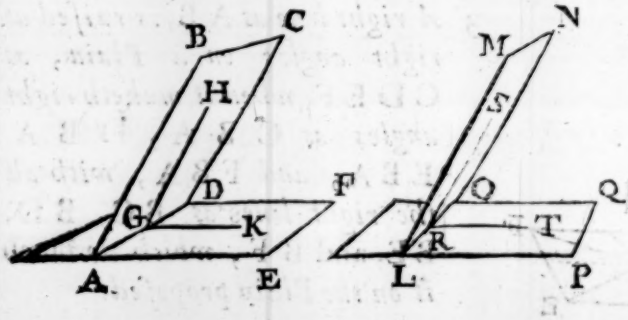
IF FG and HI are perpendicular to the other Plain CD; that is to ſay, if they are at right angles to GK, GL, and GM, IN, IO, and IP, &c. the Plain AB, ſhall be at right angles to the Plain CD, not being inclined toward the one part, or toward the other.

For KG being produced to R, and the angle FGK being put a right angle, FGR ſhall alſo be a right angle, and ſo of the reſt, &c. Wherefore the Plain AB, ſhall be at right angles to the Plain CD. And when a Plain is at right angles to another Plain, the perpendicular lines drawn upon one of them to the Common Section, ſhall be alſo perpendicular to the other Plain.



5 *The inclination of a right line as* AB, *to a Plain as* CDEF, *is the angle* ABG, *conteined under the ſaid right line* AB, *and another right line as* BG, *drawn on the Plain, from the inclining line* AB, *by the point where there falleth a right line as* AG, *perpendicular, drawn from the top* A *of the inclining line* AB, *to the ſaid Plain.*

6 *The inclination of one Plain as* ABCD, *to another Plain, as* EADF, *is the acute angle* HGK, *conteined under the right lines* GH *and* GK, *drawn on both the Plains at right angles to the line of Common Section* AD, *and from a point therein, as* G, *which are not perpendicular the one to the other*



7 A

7 *A Plain as* ABCD, *is ſaid to be alike inclined to a Plain as* EADF, *and another Plain* LMNO, *to another Plain as* PLOQ, *when the angles of inclination* HGK *and* SRT, *are equal the one to the other.*

8 *Parallel Plains are ſuch as do not meet, being prolonged.*

9 *Like Solid figures are ſuch as are conteined under like plains, equal in number.*

10 *Equal and like Solid figures are ſuch are conteined of like plains, equal in number and magnitude.*

11 *A Solid angle, is the meeting of more than two right lines, touching one another in a certain point, and being not in one and the ſame ſuperficie, inclining to all the lines.*

12 *A Pyramyde is a Solid figure conteined of divers plains, which meet or terminate at one and the ſame point, being drawn from another plain, which is for the baſe of the Pyramyde.*

13 *Priſme is a Solid figure, conteined of plains, two of which (thoſe that are oppoſite to one another,) are equal, alike, and parallel, but the others are parallelograms.*

14 *Sphere is a figure conteined when a Semicircle is drawn about the diameter, remaining fixed, until it be again poſited there where it began to move.*

THat is to ſay, Sphere is a Solid figure conteined by the Superficies deſcribed by the revolution of a Semicircle drawn about the Diameter, fixed and unmoveable, until it be returned to the point where it began its motion.

15 *The Axis of a Sphere is a fixed right line, about which the Semicircle moves or turns.*

16 *But the Center of a Sphere is the ſame with that of the Semicircle.*

17 *The Diameter of a Sphere is a right line drawn by the Center and terminating on both parts in the Superficies of the Sphere.*

18 *A Cone is a figure which is conteined when one of the ſides*

which conteins the right angle of a rectangle triangle remaining fixed, the triangle is drawn about, until it return to the place where it began to move, and if the unmoveable right line be equal to the other side, which is moved about the right angle, the Cone shall be rectangle, if it be lesse, the Cone shall be an Ambligon, and if greater, an Oxigon.

THat is to say, a Cone is a Solid, conteined under the Superficies, described by a rectangle triangle, when the same triangle is drawn about one of the sides which conteineth the right angle, the same side remaining fixed until the same triangle be returned where it began its motion.

19 *The Axis of a Cone is the unmoveable right line about which the triangle turneth or moveth.*

20 *But the Base of a Cone is the circle described by the other side, drawn about.*

That is to say, by the other of the two, which conteins the right angle.

21 *A Cylinder is a figure which is conteined, when one of the sides of a rectangle parallelogram, of those that are about a right angle, remaining unmoveable, the parallelogram is drawn about, until it be again constituted, where it began to move.*

THat is to say, a Cylinder is a Solid conteined under the Superficie described by the revolution of a rectangled Parallelogram drawn about one of the sides which conteines one right angle, the same side remaining fixed, until it be returned where it began its motion.

22 *The Axis of a Cylinder is the unmoveable right line about the which the parallelogram is drawn.*

23 *But the Bases of a Cylinder are the circles described by the two opposite sides drawn about.*

24 *Like Cones, and Cylinders, are those whose Axes and Diameters of the Bases have the same proportion the one to the other.*

25 *A Cube is a Solid figure conteined under six equal squares.*

26 *A Tetrahedron is a Solid figure conteined under four triangles, equal and equilateral.*

27 *An Octahedron is a Solid figure conteined under eight triangles, equal and equilateral.*

28 *The*

28 *The Dodecahedron is a Solid figure conteined under twelve equal Pentagons, equilateral and equiangled.*

29 *An Icoſahedron, is a ſolid figure conteine under twenty triangles equal, and equilateral.*

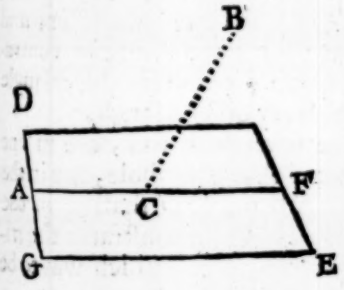
30 *A Parallelepipedon is a ſolid figure conteined under ſix quadrilateral figures, of which the oppoſites are parallels.*

31 *A ſolid figure is ſaid to be inſcribed in a ſolid figure, when all the angles of the figure inſcribed are conſtituted either at the angles, or at the ſides, or laſtly, at the plains of the figure in which it is inſcribed.*

32 *Reciprocally a ſolid figure is ſaid to be circumſcribed about another ſolid figure, when the angles or the ſides, or laſtly, the plains of the circumſcribed figure do touch all the angles of the figure about which it is deſcribed.*

PROPOSITIONS, PROBLEMES, and THEOREMES.

PROPOSITION 1. THEOREM 1.



A part of a right line, as A C, *cannot be in a propoſed plain* D E, *and another part in the aire.*

FOr let A B be the right line, a part whereof, to wit, A C, is on the Plain D E, and let the other part C B be (if poſſible,) in the aire, and ſo draw C F.

Demonſtration FOraſmuch then as A C part of A B a right line, is poſited on the Plain D E, and the other part C B, remaines in the aire, that part of the Plain D E, to wit, from C to F, ſhall be lower or more depreſſed than the other part, to wit, from A to C; and therefore the Superficies D E ſhall not be conteined equally between his lines, which a) is abſurd; ſeeing that D E is propoſed to be a plain Superficies.

a) 7. def. 1.

Other-

Otherwiſe (if it be poſſible, let one part of the right line AB, that is to ſay, AC, be on the Plain DE, and the other part CB in the aire; then in the plain DE there ſhall be ſome right line poſited directly to AC, as CF: Therefore the two right lines AB and AF have one common ſegment
b) 10. c. ſ. 1. AC, which [b] is abſurd: Therefore one part of a right line is not upon a Plain, and the other in the aire. Which was to be demonſtrated.

PROP. 2. THEOR. 2.



If two right lines, as AB *and* CD, *cut one the other, they are in one and the ſame plain, and every triangle as* EFG, *&c. is in one and the ſame plain.*

LEt AB and CD cut one another in E, then in the lines EB and ED, having aſſumed the points F and G, at pleaſure, draw FG, to make the triangle EFG; I ſay the triangle EFG is in one and the ſame Plain, as alſo the right lines AB and CD.

Demonſtration FOr if one part of the triangle EFG, to wit, EHIG, be in the Plain, and another part thereof, to wit, the reſidue FHI, be in the aire; or contrariwiſe, EH and GI, parts of EF and GF, ſhall be alſo in a Plain; but the other parts HF and IF, ſhall be in the
a) 1. 11. aire, which [a] is ſhewn to be impoſſible: And if EFIK, part of the ſame triangle, be ſaid to be in the ſame Plain, but the other part GIK, in the aire; or contrariwiſe, by the ſame reaſon, EK and FI, parts of EGFG, ſhall be in a Plain; but the parts KG and IG ſhall be in the aire, or contrariwiſe.

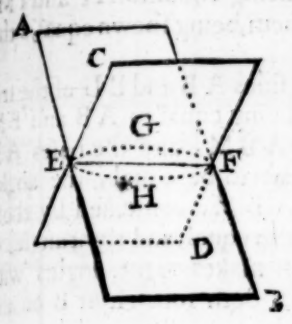
Laſtly, if FHKG, part of the ſame triangle, be acknowledged to be in one and the ſame Plain, and the other part EHK in the aire, or contrariwiſe, FH and GK, parts of FE and GE, ſhall be alſo in one and the ſame Plain, and the other parts HE and KE, in the aire or contra-
b) 1. 11. riwiſe, which is abſurd, as [b] hath been ſhewn; therefore the triangle EFG is in one and the ſame Plain, and ſo of every other triangle.

But ſeeing that EF and EG, ſides of the triangle EFG, are in one
c) 11. 1. and the ſame Plain therewith; but [c] CD and AB the whole, are in the ſame Plain with EF and EG, Therefore AB and CD ſhall be in one and the ſame Plain, to wit, that, in which we have demonſtrated the triangle EFG to be: Therefore, If two right lines, &c. Which was to be demonſtrated.

PROP. 3. THEOR. 3.

If two plains, as AB *and* CD, *cut one another, their common ſection* EF, *is a right line.*

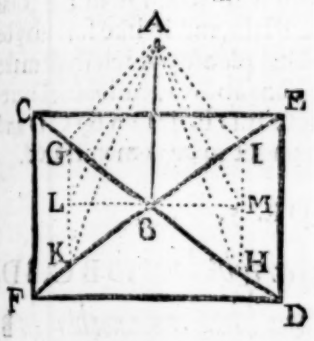
Demon-



Demonſtration FOr if you ſhall ſay it is not ſo, let there be drawn the right line EGF, on the Plain AB, and on the Plain CD, the right line EHF; therefore EGF and EHF being right lines, having the ſame terms E and F, do encloſe a Superficies, which is abſurd; therefore [a] the common ſection EF ſhall be a right line: Therefore, If two Plains, &c. Which ought to be demonſtrated.

a) 12 Ax. or c. f. 1.

PROP. 4. THEOR. 4.



If two right lines, as CD *and* EF, *cut one another, and from the point of their common ſection be drawn another right line, as* AB, *at right angles, that right line* AB *ſhall alſo be at right angles to the Plain where the ſame right lines* CD *and* EF *are drawn.*

Demonſtration FOr ſeeing they cut one another at B, [a] they ſhall be in one and the ſame Plain, which let be CEDF: Let BG and BH be aſſumed, equal to one another, and alſo BI and BK, and draw GK and IH, and on the ſame Plain let there be drawn by the point B, LM, cutting GK and IH at L and M; and from the point A in the aire, let there be drawn to the ſame Plain, the right lines AG, AL, AK, AH, AM, and AI.

a) 2. 11.

Foraſmuch as the ſides BG and GK of the triangle BGK, are equal by Conſtruction to BH and BI of the triangle BHI; [b] and the angles GBK and HBI, conteined of them, are equal at the top B, [c] the baſes GK and HI ſhall be equal to one another, and the angles BKG & BIH alſo equal to one another.

b) 15. 1.
c) 4. 1.

Again, KBL and IBM being equal at the top B, the two angles KBL and BKL of the triangle BKL, ſhall be equal to the two IBM and BIM, of the triangle BIM, and the ſides adjacent BK and BI, are alſo equal by Conſtruction: Therefore the two other ſides KL and LB, are equal to the two others IM and MB, each to his correſpondent ſide.

d) 15. 1.

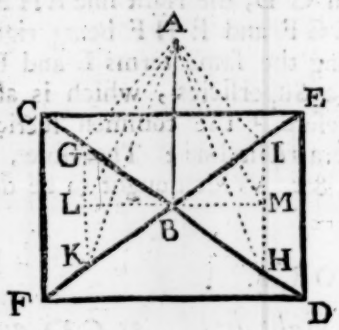
Moreover, the ſides AB and BG of the triangle ABG, being equal to AB and BH, of the triangle ABH, by Conſtruction; and the angles ABG and ABH, conteined of them, equal, (being propoſed right angles) [e] the baſes AG and AH ſhall be equal; and by the ſame AI and AK ſhall be equal.

e) 4. 1.

Furthermore, the ſides AI and IH of the triangle AIH, being equal to AK and KG, of the triangle AKG, and the baſes AH and AG ſhewn to be equal; [f] the angles AIH and AKG, conteined of thoſe ſides, ſhall be

f) 8. 1.

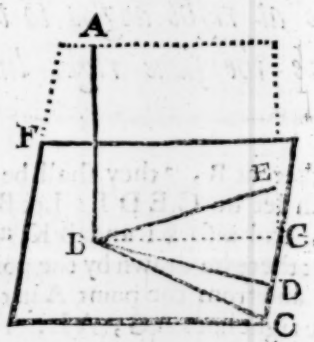
be equal; therefore the ſides A K and K L being equal to A I and I M,
g) 4. 1. and the angles A K L and A I M conteined by them, being ſhewn equal, g the baſes A L and A M ſhall be equal.



Laſtly, the ſides A B and B L of the triangles A B L being equal to A B and B M of the triangle A B M, and the baſes A L and A M ſhewn to be equal, the angles
h) 8. 1. h A B L and A B M, conteined by thoſe
i) 10. def. 1. ſides, ſhall be alſo equal, and right angles i: Therefore A B makes right angles with
k) 3. def. 11. L M, k which doth touch it at B on the Plain propoſed.

By the ſame reaſon it is demonſtrated that A B makes right angles with all the right lines which do touch it in the point B on the ſame Plain, and in like ſort may be taken on the ſaid Plain as many lines as you ſhall pleaſe, which ſhall make right angles therewith, meeting in B, by the reaſons above mentioned: Therefore A B ſhall be at right angles
l) 3. def. 11. l on the Plain C E D F, drawn by C D and E F: Therefore, If two lines, &c. Which ought to be demonſtrated.

PROP. 5. THEOR. 5.



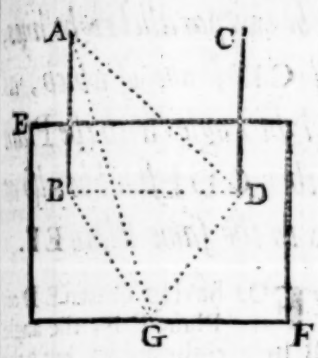
If to three right lines as B C, B D, *and* B E, *touching one another in* B, *another right line* A B *be conſtituted at right angles in the point of common ſection* B, *the ſaid three lines are in one and the ſame Pain* F C.

Demonſtration FOr a ſeeing that any two of them are in one and the
a) 2. 11. ſame Plain, and being drawn forth, will cut one another at B: Let B C and B D be in one and the ſame Plain F C, and (if it be poſſible,) let
b) 2. 11. not B E be in the ſame Plain, but raiſed up in the ayre: But b A B and B E being in one and the ſame Plain, ſeeing they interſect at B, let both the two be in the Plain A E.

And foraſmuch as the Plains F C and A E meet with one another at B,
c) 3. 11. being produced, they muſt of neceſſity cut one another; let c therefore the right line B G be their common ſection; ſeeing that A B is propoſed
d) 4. 11. to be at right angles to B C and B D, d it ſhall alſo be at right angles to
e) 3. def. 11. the Plain F G, drawn by them; and e therefore ſhall be alſo at right angles to B G, which doth touch it at B: Wherefore the right angles A B E and A B G being in the Plain A G, drawn by A B and B E, ſhall be equal the one to the other, the part and the whole, which is abſurd: Therefore the two lines B C and B D being on one and the ſame Plain F C; B E ſhall not be raiſed up in the aire, but ſhall be in the ſame Plain with them: Therefore, If to three right lines, &c. Which ought to be demonſtrated.

PROP.

PROP. 6. THEOR. 9.

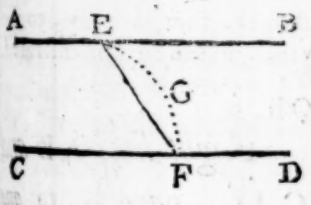


If two right lines as A B *and* C D, *are at right angles on one and the ſame Plain* E F, *the ſame right lines* A B *and* C D *are parallels.*

Demonſtration FOr having drawn B D on the Plain E F, [a] the angles A B D and C D B ſhall be right angles; and on the ſame Plain E F, let D G be drawn perpendicular to B D, and let A B and D G be propoſed equal; and let B G, G A, and A D, be joyned together: Then foraſmuch as the ſides A B and B D of the triangle B A D, are equal to G D and B D of the triangle G D B, by Conſtruction; and the angles A B D and G D B, conteined by them, are alſo equal, being right angles, the baſes A D and G B ſhall be equal. a) 3. def. 11.

Again, [b] the ſides A B and B G of the triangle A B G, being equal to G D and D A of the triangle G D A, and the baſe A G common; [c] the angles A B G and G D A ſhall be equal: But [d] A B G is a right angle, therefore G D A ſhall be a right angle, and the angle G D C being a right angle, G D alſo ſhall be at right angles to the three ſides B D, D A, and D C: Wherefore [e] B D, D A, and D C, ſhall be in one and the ſame Plain; that is to ſay, C D ſhall be on the ſame Plain, drawn by D B and D A: But [f] A B is on the ſame Plain with D B and D A: Therefore C D ſhall be on the ſame Plain with A B; therefore the internal angles A B D and C D B being right angles, A B and C D ſhall be parallels: Therefore, If, &c. Which was to be demonſtrated. b) 4. 1. c) 8. 1. d) 3. def. 11. e) 5. 11. f) 2. 11.

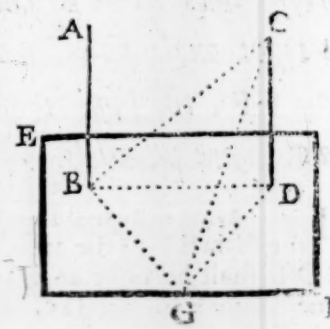
PROP. 7. THEOR. 7.



If there be two parallel right lines, as A B *and* C D, *in both of which be taken two points at pleaſure, (ſuppoſe at* E *and* F,) *the right line* E F, *drawn from thoſe points, is in one and the ſame Plain with the parallels.*

Demonſtration FOr if E F be not in the ſame Plain, let it be in another, and let that other cut the Plain of the parallels by E and F; and let the common ſection of thoſe Plains be E G F, [a] which ſhall be a right line: Therefore the two right lines E F and E G F, having the ſame terms E and F, do encloſe a ſpace, which is abſurd: Therefore the right line E F ſhall not be out of the Plain of A B and C D parallels: Therefore, If there be two, &c. Which was to be demonſtrated. a) 3. 11.

PROP. 8. THEOR. 8.



If there be two parallel right lines, as A B *and* C D, *one of which, as* A B, *is at right angles with the Plain* E F, *the other* C D, *ſhall be alſo at right angles to the ſame Plain* E F.

Demonſtration FOr having drawn B D on the Plain E F, the angle C D B [a] ſhall be a right angle; but [b] the two angles C D B and A B D are equal to two right angles; therefore A B D ſhall be a right angle: Now let B G be drawn on the Plain E F, perpendicular to B D; and let B G and C D be propoſed equal, and let D G, G C, and C D, be joyned.

a) 3. def. 11.
b) 29. 1.

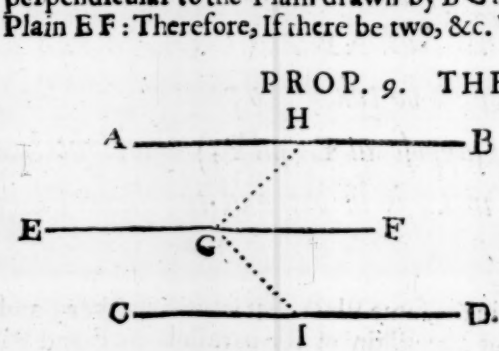
Foraſmuch as the ſides C D and D B, of the triangle C D B, are equal to G B and B D, of the triangle G B D by Conſtruction; and the angles C D B and G D B, conteined of the ſaid ſides are alſo equal, being right angles, [c] the bales C B and G D ſhall be equal.

c) 4. 1.

Again, the ſides C D and D G of the triangle C D G, being equal to G B and B C of the triangle G B C, and the baſe C G common, the angles [d] C D G and G B C, conteined of the ſaid ſides, ſhall be equal: But C D G [e] is a right angle, therefore G B C ſhall be alſo a right angle; wherefore G B ſhall be perpendicular to B D, B C cutting the one and the other at B, being produced, (ſeeing they make an angle with B,) therefore [f] ſhall be perpendicular to the Plain drawn by B D and B C: but A B is alſo in the ſame Plain [g] with B D and B C, being in the ſame Plain with A B and C D parallels; the points B, C, and D being in the parallels A B and C D; therefore G B [h] ſhall be alſo perpendicular to B A: Wherefore A B being perpendicular to B C and B D, A B ſhall be alſo perpendicular to the Plain drawn by B G and B D; that is to ſay, to the Plain E F: Therefore, If there be two, &c. Which was to be demonſtrated.

d) 8. 1.
e) 3. def. 11.
f) 4. 11.
g) 7. 11.
h) 3. def. 11.
4. 11.

PROP. 9. THEOR. 9.



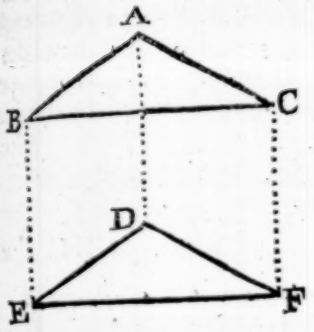
Right lines A B *and* C D, *parallel to one and the ſame right line* E F, *being not in the ſame Plain with them, thoſe right lines are alſo parallel to one another.*

Demonſtration FOr from what point you pleaſe of E F (ſuppoſe from G,) let there be drawn two perpendiculars to E F, to wit, G H on the Plain of the parallels A B and E F, and G I on the Plain of the parallels C D and E F: Therefore E G being perpendicular to G H and G I,

GI, touching one another at G, ſhall be alſo perpendicular to the Plain drawn by GH and GI; wherefore AB and EF being parallels, and EF perpendicular to the Plain drawn by GH and GI; AB ſhall be alſo perpendicular to the ſame Plain. 4. 11. 8. 11.

In like manner, CD and EF being parallels, and EF perpendicular to the Plain drawn by GH and GI, CD ſhall be alſo perpendicular to the ſame Plain: Therefore AB and CD being perpendiculars to the Plain drawn by GH and GI, ſhall be parallels the one to the other: Wherefore, Right lines, &c. Which was to be demonſtrated. 8. 11. 8. 11.

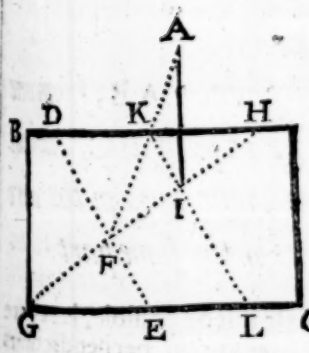
PROP. 10. THEOR. 10.



If two right lines as AB *and* AC, *touching one another, as at* A, *are parallel to two right lines, as* DE *and* DF, *touching one another, as at* D, *and being not in one and the ſame Plain, the ſame right lines ſhall contein equal angles as* BAC *and* EDF.

Demonſtration. FOr let AB and DE be propoſed equal to one another, and alſo AC and DF equal to one another; and let there be drawn BC, EF, BE, AD, and CF; therefore AB and DE being equal and parallel; [a] BE and AD ſhall be alſo be equal and parallel; by the ſame reaſon CF and AD ſhall be parallels and equal: Wherefore BE and CF being equal and parallels to AD, they [b] ſhall be parallels and equal to one another; and therefore [c] ſeeing BC and EF parallel and equal to one another; therefore ſeeing the ſides AB and AC, of the triangle BAC are equal to DE and DF of the triangle DEF, by Conſtruction, each to his correſpondent; and the baſe BC demonſtrated equal to the baſe EF, [d] the angles BAC and EDF ſhall be equal: Therefore, If two right lines, &c. Which was to be demonſtrated. a) 33. 1. b) 9. 11. c) 33. 1. d) 8. 1.

PROP. 11. PROBL. 1.

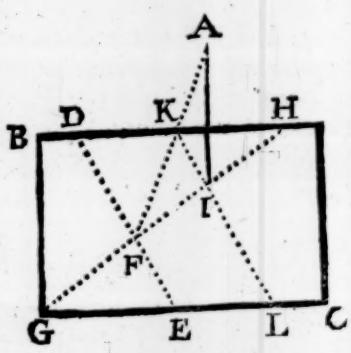


From a point given in the aire, as A, *to draw a right line perpendicular as* AI, *on the plain* BC, *which is under it.*

Conſtruction. LEt the point A be in the aire, from which a perpendicular line ought to be drawn to the Plain BC; let DE be drawn to the ſaid Plain BC at pleaſure, [a] to which let the perpendicular AF be drawn, and on the Plain BC, by F, let GH be drawn perpendicular a) 12. 1.

b) 11. 1. to DE, b to the which from A, let there be drawn the perpendicular AI; I ſay that AI is perpendicular to the ſaid plain BC.

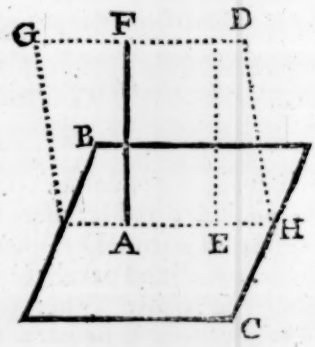
c) 31. 1.



d) 4. 11. e) 8. 11. f) 2. 11. & 3. def. 11. *Demonſtration* FOr c on the plain BC; let there be drawn KL, parallel to DE by I; ſeeing that DF is at right angles to FA, and to FH by Conſtruction; and therefore d perpendicular to the plain drawn by FA and FH; alſo e KI ſhall be perpendicular to the plain drawn by FA and FH: But f AI being in the ſame plain with FA and FH, and touching KI at I, the angle KIA ſhall be a right angle: Wherefore AI ſhall be at right angles to KI and IF; therefore AI ſhall be perpendicular to the plain BC: Therefore g we have drawn a line perpendicular, &c. Which was to be done.

g) 4. 11.

PROP. 12. PROBL. 2.



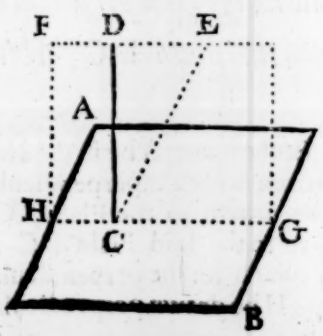
To a given plain as BC, *and from a given point therein as* A, *to draw a right line* AF, *at right angles.*

a) 11. 11. *Conſtruction* FRom a any point, (ſuppoſe D,) given above, let there be drawn DE, perpendicular to the plain BC, which if it fall in the point A, you have your deſire; but if the point E fall elſewhere, having drawn a right line by E and A, let b there be drawn AF parallel to DE, on the plain GH, drawn by DE and EA: I ſay that AF is perpendicular to the given plain BC.

b) 31. 1.

c) 8. 1. *Demonſtration* FOr DE and AF being parallels, and DE perpendicular to the plain BC by Conſtruction; c AF ſhall be alſo perpendicular to the ſame plain BC: Wherefore, To a plain, &c. Which was to be done.

PROP. 13. THEOR. 11.

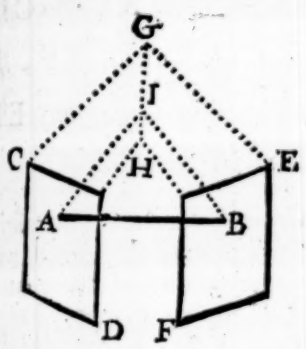


To a given plain, as AB, *from one and the ſame given point as* C, *cannot be drawn two right lines in the aire at right angles on the ſame part.*

a) 2. 11. *Demonſtration* FOr if it be poſſible, let there be drawn two perpendiculars CD and CE, a to the plain AB, and by CD

CD and CE which are in one and the ſame plain; let there be drawn the plain FG, cutting the plain AB by the right line GH; therefore EC and DC being perpendiculars to the plain AB, [b] the angles ECG and DCG ſhall be right angles, and therefore equal, the part to the whole, which is abſurd: Therefore to a given Plain, &c. Which was to be demonſtrated. [b] 3. def. 11.

PROP. 14. THEOR. 12.

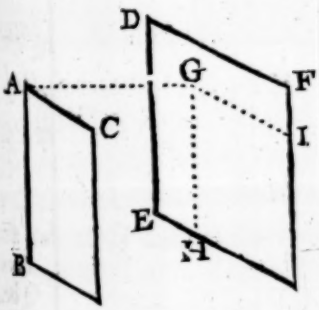


The plaines CD *and* EF, *to which one and the ſame right line* AB *is at right angles, thoſe plains are parallel.*

Demonſtration FOr if it be not ſo, they being produced will meet with one another, which let be on the part of CE, and that GH be the line of their common ſection, in which [a] having taken the point I at pleaſure, let the right lines IA and IB be drawn on the plains GCD and GEF. [a] 3. 11.

Foraſmuch as AB is propoſed to be at right angles to the plains GCD and GEF, [b] the two angles IAB and IBA ſhall be right angles in the triangle ABI: But [c] they are leſſe than two right angles, which is abſurd: Therefore the plains CD and EF being produced, will never meet, they are therefore parallels: Wherefore, The plains, &c. Which was to be demonſtrated. [b] 3. def. [c] 17. 1.

PROP. 15. THEOR. 13.

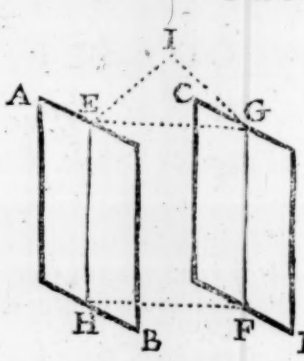


If two right lines AB *and* AC, *touching one another in* A, *are parallels to two other right lines* DE *and* DF; *in like manner touching one the other at* D, *nevertheleſſe being not in one and the ſame plain: the plains* BC *and* EF, *which are drawn by thoſe lines are parallels.*

Demonſtration FOr from the point A, [a] to the plain EF, let there be drawn the perpendicular AG, meeting with the plain EF in the point G; then [b] from the point G let there be drawn GH and GI, on the plain EF, parallel to DE and DF; then AB and GH being parallels to DE, [c] they ſhall be parallel to one another; and therefore [d] the angles BAG and AGH are equal to two right angles: But [e] AGH is a right angle, therefore BAG ſhall alſo be a right angle. In like manner I con- [a] 11. 11. [b] 3. 11. [c] 9. 11. [d] 29. 1. [e] 3. def.

conclude that the angle C A G is a right angle: Wherefore G A being at
f) 4. 11. right angles to the two lines f A B and A C, it ſhall be alſo at right angles
to the plain B C, drawn by A B and A C: But A G is alſo at right angles
g) 14. 11. to the plain E F by Conſtruction: Therefore g the plains B C and E F are
parallels: Therefore, If two right lines, &c. Which was to be demonſtrated.

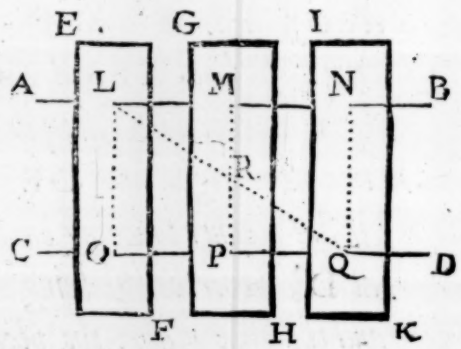
PROP. 16. THEOR. 14.



If two parallel plains A B *and* C D, *are cut by ſome other plain* E F, *the lines of their common ſections* E H *and* G F *ſhall be parallel.*

Demonſtration FOr if they be not parallels, being produced, they will meet with one another, being in the interſecting plain E F; let them meet than in the point I: Foraſmuch as the whole right line H E I, is in one and the ſame plain, to wit, in A B, produced; In like manner, the whole right line F G I is alſo in one and and the ſame plain, to wit, in C D, produced; the plains A B and C D alſo meet with one another in the point I, being produced; which is abſurd, being propoſed parallels; therefore E H and G F are parallels: Wherefore, If two plains, &c. Which was to be demonſtrated.

PROP. 17. THEOR. 15.



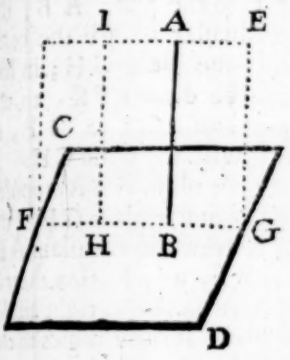
If two right lines A B *and* C D, *are cut by parallel plains* E F, G H, *and* I K, *thoſe right lines ſhall be cut proportionally.*

Demonſtration FOr let the right lines L O and and N Q be drawn on the plains E F and I K; and let L Q be joyned, meeting the plain G H in the point R, from which point R let there be drawn to the points M and P, the lines R M and R P, on the ſame
a) 2. 11. plain G H; a the triangle L N Q ſhall be in one and the ſame plain: In like manner, alſo L O Q ſhall be in one and the ſame plain.

But foraſmuch as the parallel plains G H and I K are cut by the
b) 16. 11. plain of the triangle L N Q; b their common ſections, to wit, the lines M R and N Q ſhall be parallel, by the ſame reaſon R P and L O ſhall be
c) 2. 6. 11. 5. parallels: Wherefore c as L R is to R Q, ſo is L M to M N; in like manner, as L R to R Q, ſo is O P to P Q: Therefore as L M is to M N, ſo is O P to P Q: Therefore, If two right lines, &c. Which was to be demonſtrated.

PROP.

PROP. 18. THEOR. 16.



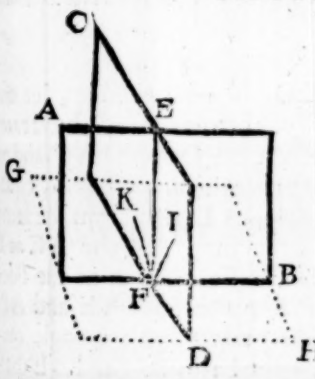
If a right line as A B, *be at right angles to a plain* C D, *all the plains drawn thereby ſhall be alſo at right angles to the ſame plain* C D.

Demonſtration FOr by A B let the plain E F be drawn, cutting the plain C D in the right line F G; and having taken the point H at pleaſure in F G; [a] let there be drawn H I, to the plain E F, parallel to A B; therefore A B and I H being Parallels, and A B propoſed at right angles to the plain C D, [b] I H ſhall be alſo at right angles to the ſame plain C D; [c] therefore perpendicular to the common ſection F G, by the ſame reaſon, all the lines which are drawn parallel to A B, on the plain E F, ſhall be at right angles to the plain C D; and therefore perpendicular to the common ſection F G; therefore [d] the plain E F ſhall be at right angles to the plain C D.

a) 31. 1.
b) 8. 11.
c) 3. def.
d) 4. def.

By the ſame reaſons may be demonſtrated that all the other plains drawn by A B, are at right angles to the plain C D: Therefore, If a right line, &c. Which was to be demonſtrated.

PROP. 19. THEOR. 17.



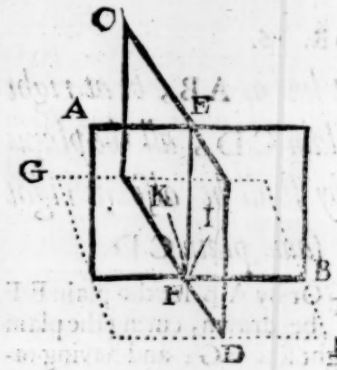
If two plains A B *and* C D, *interſecting one the other, are at right angles to a plain* G H, *their line of common ſection* E F, *ſhall be alſo at right angles to the ſame plain* G H.

Demonſtration FOr E F is either at right angles to B F and D F the common ſections of the plains A B and C D, with the plain G H, or is not, and if E F be at right angles to B F and D F, it [a] ſhall be alſo at right angles to the plain G H, drawn by them: But if E F be ſaid to be at right angles to the one or the other, either B F or D F, let it be only to B F; then foraſmuch as the plain A B is propoſed to be at right angles to the plain G H, [b] the line E F (which line E F is drawn on the plain A B, perpendicular to B F, as a ſection thereof with the plain G H,) ſhall be at right angles to the plain G H, we conclude the ſame, if E F be granted to be at right angles to D F; for then E F ſhall be perpendicular to the plain G H, being propoſed perpendicular to D F, the common ſection of the plain C D, with the plain G H, and drawn to the plain C D, which is at right angles to the plain G H.

a) 4. 11.
b) 4. def.
4. def.

Finally,

11. 1.



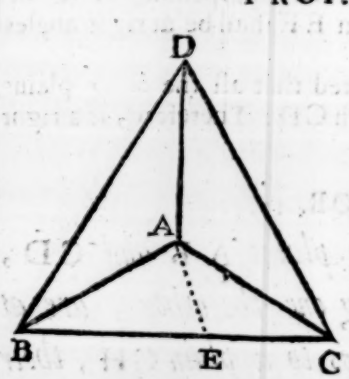
Finally, if E F be not at right angles to B F nor to D F, let there be drawn from the point F to the plain A B, the line F I, perpendicular to B F the common ſection with the plain G H; in like manner let there be drawn F K, to the plain C D, perpendicular to D F, as a ſection thereof, with the plain G H.

Foraſmuch as the plain A B is propoſed to be at right angles to the plain G H, c the line I F (which is drawn in the plain A B, perpendicular to B F, as a ſection thereof with the plain G H,) ſhall be at right angles to the ſame plain G H; by the ſame reaſon K F ſhall be at right angles to the ſame plain G H: Wherefore from the point F to the plain G H, there are drawn two perpendiculars, which f is abſurd: Wherefore E F ſhall be at right angles to the plain G H: Therefore, If two plains, &c. Which was to be demonſtrated.

c) 4. def.

f) 13. 11.

PROP. 20. THEOR. 18.



If a ſolid angle A, *be contained of three plain angles* B A C, C A D, *and* D A B, *two of them taken after what manner ſoever, are greater than the third.*

Demonſtration FOr if it be poſſible, let one of them as B A C, be greater than the two others B A D and D A C, and on the plain drawn by A B and A C; let a there be made the angle B A E, equal to the angle B A D, and the right line A E equal to A D; then from the ſame plain drawn by E, let there be drawn B C, touching A B and A C at B and C; and let there be drawn B D and D C: Foraſmuch as the ſides A D and A B of the triangle B A D, are equal to the ſides A E and A B, of the triangle B A E, each to his correſpondent ſide, and the angle conteined of them alſo equal, by Conſtruction, b the baſes B D and B E ſhall be equal.

a) 23. 1.

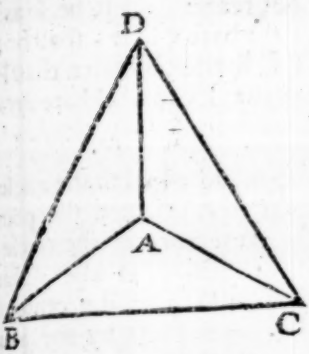
b) 4. 1. 20. 1.

But foraſmuch as the ſides D B and D C are greater than the ſide B C, if the equal right lines B D and B E be cut off, there will remain C D, greater than C E; ſeeing then the ſides A D and A C, of the triangle D A C, are equal to the ſides A E and A C, of the triangle E A C, each to his correſpondent ſide; and the baſe C D greater than the baſe C E; the angle c C A D ſhall be greater than the angle C A E; adding then the equal angles B A D and B A E, the two angles C A D and B A D ſhall be greater than the two C A E and B A E; that is to ſay, the whole angle B A C, which was propoſed to be the greateſt of all: Therefore two of them, which you pleaſe, are much greater than the other. Therefore, &c. Which ought to be demonſtrated.

c) 25. 1.

PROP.

PROP. 21. THEOR. 19.

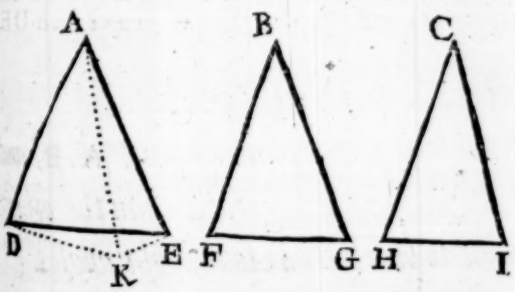


Every ſolid angle A, *is conteined under plain angles* B A C, B A D, *and* D A C, *which are leſſe than four right plain angles.*

Demonſtration FOr having drawn the right lines B C, C D and D B, the three ſolid angles B, C, and D, are made, each of which is conteined under three plain angles, *viz.* B, under C B A, A B D, and D B G, but C under B C A, A C D, and D C B, and D under C D A, A D B, and B D C: But [a] the two angles C B A and A B D, being greater than the angle C B D; and in like manner, the two angles B C A and A C D are greater than B C D; and the two angles C D A and A D B, are greater than C D B, the ſix angles C B A, A B D, B C A, A C D, C D A, and A D B, are greater than the three angles C B D, B C D, and C D B. a) 20. 11.

But [b] theſe three are equal to two right angles; therefore theſe ſix are greater than two right angles; therefore ſeeing that theſe ſix together with the three at the point A, are equal to ſix right angles, becauſe of the three angles B A C, C A D, and D A B; (for [c] the three angles of every triangle are equal to two right angles,) if theſe ſix be taken away, which are greater than two right angles; the three which conſtitute the ſolid angle A, will remain leſſe than four right angles: Therefore, Every, &c. Which ought to be demonſtrated. b) 32. 1. c) 32. 1.

PROP. 22. THEOR. 20.

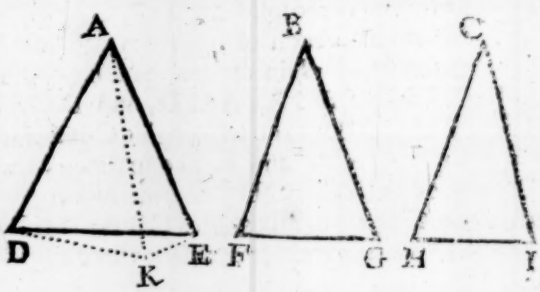


If there be three plain angles A, B, *and* C, *two whereof taken at pleaſure, are greater than the other: But that they be conteined of equal right lines* A D, A E, B F, B G, C H, *and* G I, *it may be ſo ordered, that of the right lines* D E, F G, *and* H I, *joyning together the ſaid equal lines, a triangle may be conſtituted.*

IF the three angles A, B, and C, be equal, [a] the baſes D E, F G, and H I, are equal: Therefore any two of them is greater than the third: But if two angles are only equal, and the third leſſe; [b] two baſes a) 4. 1. b) 4. 1.

c) 24. 1. shall be only equal, and the baſe of the third c leſſe than either of the other; therefore again, the two others (which you pleaſe,) are greater than the other; but if one of the ſaid angles, as A, be greater, let it be ſo as the
d) 24. 1. two others B and C, be equal, or not equal; the baſe d D E, ſhall be alſo the greateſt of all: Wherefore D E and F G ſhall be greater than H I; and D E and H I greater than F G. Now I ſay that F G and H I are greater than D E.

e) 23. 1. *Demonſtration* FOr e let the angle D A K be made equal to the angle B, and let A K be propoſed equal to A D, then the point K will fall under D E; foraſmuch as the circumference of the circle deſcribed at the diſtance A D, on the center A, doth paſſe by the points D K E, becauſe of the equality of the lines A D, A K, and A E; then let D K and K E be joyned together.



Foraſmuch as the two angles B and C are propoſed greater than the angle D A E, and B is equal to the angle D A K by Conſtruction; the angle C ſhall be greater than the other angle K A E; and foraſmuch as the ſides A D and A K of the triangle A D K, are equal to the ſides B F and B G of the triangle B F G, and the angles conteined of them D A K and
f) 4. 1. B are equal; the baſe f D K ſhall be equal to the baſe F G.

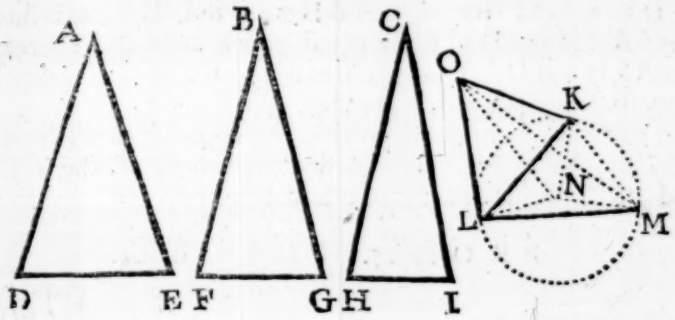
Again, ſeeing the ſides C H and C I of the triangle C H I, are equal to the ſides A K and A E of the triangle A K E; and the angle C demonſtrated
g) 24. 1. greater than the angle K A E, g the baſe H I ſhall be greater than
20. 1. the baſe K E; therefore ſeeing that D K is demonſtrated equal to F G; F G and H I ſhall be greater than D K and K E: But D K and K E are greater than D E: Therefore F G and H I ſhall be yet greater than D E. Which was propoſed.

PROP. 23. PROBL. 3.

To conſtitute a ſolid angle of three plain angles, A, B, and C, two of which taken at pleaſure are greater than the other: But theſe three angles ought to be leſſe than four right angles.

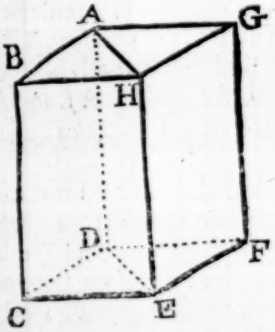
a) 22. 11. *Conſtruction* LEt a the ſix lines A D, A E, B F, B G, C H and C I, be propoſed equal, conteining the ſaid angles A, B, and C, ſubtended of the baſes D E, F G, and H I, and making a triangle of D E, F G, and H I, let the triangle L M K be made, having the three ſides
b) 22. 11. equal to D E, F G, and H I, about which b let there be deſcribed the circle K M L; and from the center N let N M, N L, and N K be drawn;
c) 5. 4. and c having drawn in the aire N O perpendicular, and d cut off, in ſuch
d) 12. 11. ſort as his ſquare, with the ſquare of N L, may be equal to the ſquare of one of the ſides of the ſaid plain angles, and let L O, K O, and M O, be drawn, I ſay that L M O K is a ſolid angle, conteined of three plain angles, equal to A, B, and C.

Demon-



Demonſtration FOr N L, N M, and N K being equal, and N O perpendicular to them; L O, M O, and K O, ſhall be equal; having [e] the like power the one as the other, and ſeeing they have the ſame power as one of the ſides of the given plain angles, they ſhall be equal to the ſides of the ſaid plain angles; the baſes L M, M K, and K L, being equal to the baſes of the given angles: The three angles [f] at the point O ſhall be equal to the three given angles A, B, and C. e) 47. 1. f) 8. 1.

PROP. 24. THEOR. 21.



If a ſolid A B C D, *be conteined under parallel plains*, A C, C F, F H, H A, A F, *and* B E, *the plains oppoſite thereunto are parallelograms alike and equal.*

Demonſtration FOr ſeeing the parallel plains B G and C F are cut by the plain A C; [a] their common ſections A B and C D ſhall be parallel: In like manner, the parallel plains A F and B E, being cut by the plain A C, their common ſections A D and B C ſhall be alſo parallel; therefore the quadrilateral figure A B C D is a parallelogram, and ſo it is demonſtrated that the other figures quadrilateral, are parallelograms: Now I ſay that the oppoſite parallelograms are alike and equal. a) 16. 11.

For ſeeing that A B and B H are parallel to D C and C E, and are not in one and the ſame plain, but in oppoſite plains, [b] the angles A B H and D C E ſhall be equal; by the ſame reaſon the other angles of the parallelogram B G, ſhall be [c] equal to the other angles of the parallelogram C F. b) 10. 11. c) 34. 1.

But foraſmuch as A B is equal to D C, in the parallelogram A C, and B H equal to C E, in the parallelogram B E; as A B is to B H, ſo D C is to C E, &c. and therefore as B H to H G, ſo C E to E F; and for the ſame reaſon, the ſides of the parallelograms B G and C F, about the equal angles, ſhall be proportional: Therefore the parallelograms ſhall be alike.

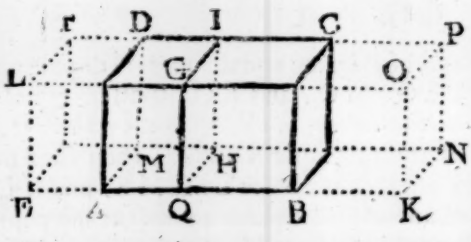
Now having drawn the diameters A H and D E; [d] ſeeing that the ſides d) 34. 1.

AB and BH of the triangle ABH, are equal to DC and CE, of the triangle DCE; and [c] the angle ABH equal to DCE, as is shewn, the triangles [f] ABH and DCE are equal to one another: Therefore [g] the triangles ABH and DCE being the halves of the parallelograms BG and CF, they shall be equal to one another.

c) 16. 11.
f) 4. 1.
g) 34. 1.

In like manner, we might shew that the opposite parallelograms AC and GE, and AF and BE are alike and equal to one another: Therefore, If a solid, &c. Which ought to be demonstrated.

PROP. 25. THEOR. 22.



If a solid Parallelepipedon ABCD, *be cut by a plain* IQ, *parallel to the opposite plains* AD *and* BC, *as the base* AH *shall be to the base* HB, *so the solid* AI *shall be to the solid* IB.

Demonstration. FOr AB being drawn forth towards E and K, let AE be proposed equal to AQ, and BK to BQ, and so finish the parallelograms EM and BN, and the solid AF and PB: Therefore AE and AQ being equal, [a] the parallelograms EM and NQ shall be equal, and [b] LD and DG opposite plains, equal and alike, and so AL and AG alike and equal to MF and MI, which are opposite plains, and so of the rest.

a) 1. 6.
b) 24. 11.

And the solid AF is equal to the solid AI, [c] and the solid BI to the solid BP: wherefore the solid EI is so multiplyed of the solid AI, as the base EH is of the base AH; and the solid QP is so multiplyed of the solid CQ, as the base QN is of the base BH: Therefore after the same manner that EH shall be greater, equal, or lesse than AH, so the solid EI shall be greater, equal, or lesse than the solid AI; also after the same manner that the base QN shall be greater, equal, or lesse than HB, so the solid QP shall be greater, equal, or lesse than the solid QC.

c) 9. def. 11.

Let there be then four magnitudes AH and HB, bases, and HI and QC solids, (whereof from the first and the third, *viz.* from AH base, and AI solid,) there were taken the equimultiplices EH base, and IE solid, and also from the second and fourth, (to wit, from the base BH, and the solid BQ,) the equimultiplices QN and PQ, base and solid, the which are demonstrated greater, equal, or lesse, in what ever multiplication they be taken: Therefore [d] as the base AH is to the base HB, so is the solid AI to the solid CQ: Therefore, &c. Which ought to be demonstrated.

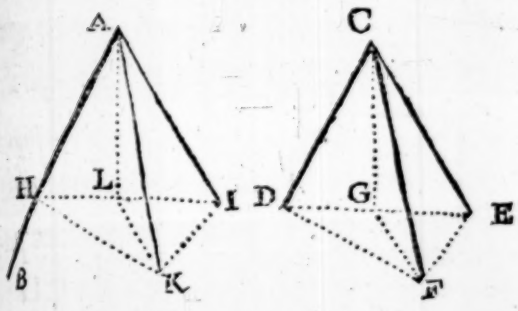
d) 6. def. 5.

COROLLARIE.

From these things it follows that if any Prisme be cut by a plain equidistant to the opposite plains, that the section is a figure equal and alike to the opposite plains; it having been shewn in the first Prisme that the triangle GHI *is equal and alike to the triangle* ABC, *and therefore also to the triangle* DEF; *and there is the same Demonstration in all, and the like is to be said of Parallelepipedons.*

PROP.

PROP. 26. PROBL. 4



To a given right line A B, *and to a point therein* A, *to conſtitute a ſolid angle equal to a ſolid angle given* C.

Conſtruction. LEt [a] there be drawn from F to the plain paſſing by C D and C E, the perpendicular F G, and let D F, D G, E F, F G, and C G be joyned; then let A H be cut off equal to C D; and [b] let the angle H A I be made equal to the angle D C E, and A I equal to C E.

Again, on the plain drawn by A H and A I, let there be conſtituted the angle H A L, equal to D C G, which is on the plain drawn by C D, C E, and A L equal to C G; and [c] from L (to the plain where are A H, A L, and A I,) let there be drawn K L perpendicular, and put equal to F G, and joyn K A: I ſay the ſolid angle A, conteined under the three plain angles H A I, H A K, and K A I, is equal to C, the given ſolid angle.

Demonſtration. FOr having joyned H K, K L, I K, and I L; foraſmuch as the ſides A H and A L of the triangle A H L, are equal to the ſides C D and C G of the triangle C D G, each to his correſpondent ſide, and [d] the angles H A L and D C G equal by Conſtruction, the baſes H L and D G ſhall be eqnal.

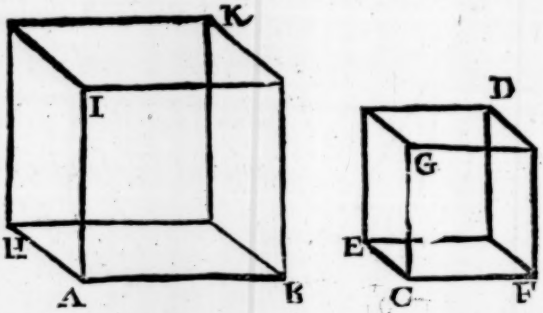
Again, having taken away the equal angles H A L and D C G, from the equal angles H A I and D C E, the remaining angles L A I and G C E are equal; then ſeeing that the ſides A L and A I of the triangle A L I, are equal to C G and C E of the triangle C G E, each to his correſpondent ſide by Conſtruction; [e] L I and G E ſhall be equal: Foraſmuch then as the ſides L H and L K are equal to the ſides G D and G F; and the angles H L K and D G F [f] are right angles, [g] the baſes H K and D F are equal; therefore the ſides A H and A K of the triangle A H K, being equal to C D and C F of the triangle C D F, by Conſtruction: (For ſeeing that A L and L K are equal to C G and G F by Conſtruction, and do contein equal angles, to wit, [h] right angles; the baſes A K and C F [i] are equal,) the angles [k] H A K and D C F ſhall be equal.

Laſtly, L I and I K, being equal to G E and G F, and the angles I L K and E C F right angles, [l] the baſes I K and E F ſhall be equal. Therefore ſeeing the ſides A I and A K of the triangle A I K, are equal to C E and C F of the triangle C E F, by Conſtruction; the angles I A K and E C F ſhall be equal: Therefore the three plain angles H A I, H A K, and K A I, compounding the ſolid angle A, are equal to the three plain angles D C E, D C F, and F C E, compounding the ſolid angle C: Therefore the ſolid angle A is equal to the ſolid angle C: Wherefore, &c. Which ought to be done.

a) 11. 11.
b) 23. 1.
c) 12. 11.
d) 4. 1.
e) 4. 1.
f) 3. def.
g) 4. 1.
h) 3. def.
i) 4. 1.
k) 8. 1.
l) 3. def.

PROP.

PROP. 27. PROBL. 5.

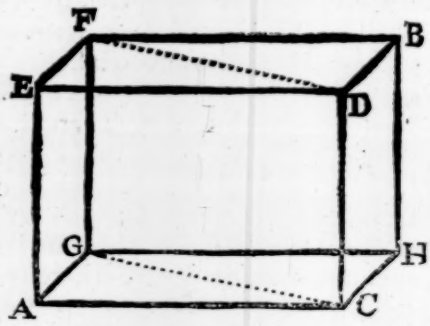


On a given right line A B, *to deſcribe a ſolid parallelepipedon, alike, and alike poſited to a given parallelepipedon* C D.

a) 26. 11. *Conſtruction* ON a the line A B, and at the point therein A, let there be made a ſolid angle, equal to the given ſolid angle C, in ſuch manner as the three plain angles H A I, I A B, and B A H, may b) 12. 6. be equal to the three E C G, G C F, and F C E, and b as C F to C G; ſo let A B be made to A I, and as C G to C E, ſo A I to A H; and in reaſon of equality, as C F to C E, ſo A B to A H; and let the parallelepipedon A K be perfectly finiſhed; to wit, the parallelograms B H, H I, and I B, being accompliſhed; and by the points I, B, and H, having drawn the plains I K, B K, and H K, which are parallels to the parallelograms B H, H I, and I B; I ſay the parallelepipedon A K is alike, and alike poſited to the parallelogram C D.

Demonſtration FOr ſeeing the angles B A H and F C E are equal, and the ſides about them proportional; *viz.* as B A to A H, ſo F C to C E, by Conſtruction; the parallelograms H B and E F, are alike, and alike poſited: In like manner, H I and E G; and I B and G F; therefore the three plains B H, H I, and I B, of the ſolid A K are alike, and alike poſited to the three F E, E G, and G F, of the ſolid C D: But c) 24. 11. c three of them, which you pleaſe, are equal, and alike to the three others oppoſite. Wherefore the ſix plains of the ſolid A K are alike, and d) 9. def. alike poſited to the ſix plains of the ſolid C D: Therefore d the ſolids A K and C D are alike and alike poſited: Wherefore, &c. Which ought to be done.

PROP. 28. THEOR. 23.



If a ſolid parallelepipedon A B, *be cut by a plain drawn by the diagonal lines of the oppoſite plains* A H *and* E B, *the ſolid ſhall be cut by that plain in two equal parts.*

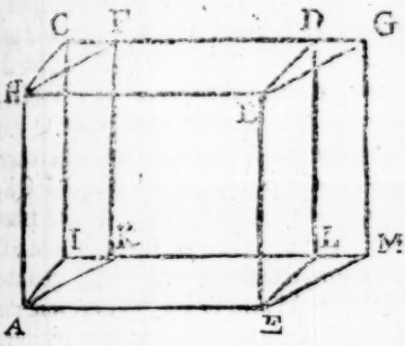
a) 24. 11. *Demonſtration* FOr a ſeeing the Plains A H and E B are parallelograms, equal and alike, their halves, *viz.* the triangles A G C, G C H, E F D, and F D B, ſhall be equal the one to the other.

But

But the ſides about the equal angles G A C, C H G, F E D, and D B F, are proportional; therefore [b] the ſaid triangles ſhall be alſo alike, and the parallelogram A F being equal and alike to the parallelogram C B, and A D to G B, and C F common, the two triangles A G C and E F D, and [c] the parallelograms A F, A D, and C F, of the Priſme A C G F E D, are equal and alike to the two triangles H C G and B D F, and to the parallelogram C B, B G and C F, of the priſme H G C D B F; wherefore [d] the ſaid priſmes ſhall be equal, which compoſing the parallelepipedon A B, the parallelepipedon A B ſhall be cut in two equal parts: Wherefore, &c. Which ought to be demonſtrated.

b) 6. 6.
c) 24. 11.
d) 10. def.

PROP. 29. THEOR. 24.



The ſolids parallelepipedons A C D E *and* A F G E, *conſtituted on one and the ſame baſe* A B, *and of the ſame height, and from which the lines inſiſting, are placed in the ſame right lines, are equal to one another.*

Demonſtration FOr [a] ſeeing the paralellograms A L and A M, made on the ſame baſe A E, and between the ſame parallels, are equal to one another; taking away the common trapezium A L, the triangles A I K and E L M will remain equal.

a) 35. 1.

But [b] foraſmuch as all the ſides of the triangle A I K, are equal to all ſides of the triangle H C F, each to his correſpondent ſide, [c] they ſhall be equiangled and equal, and [d] ſhall have the ſides about the equal angles proportional, and therefore ſhall be alike to one another; and by the ſame reaſon the triangle E L M ſhall be alike and equal to the triangle B D G; Again [e] the parallelogram A C is equal and alike to the parallelogram E D; and in like manner, the parallelogram A F to the parallelogram E G: But [f] I F is equal to L G the baſes I K and L M being equal; (for I L and K M being equal each to A E, are equal to one another; and taking away the common part K L, I K and L M will remain equal;) therefore [g] all the plains of the priſme A I K F C H are equal and alike to all the plains of the priſme E L M G D B; therefore [h] theſe priſmes ſhall be equal: Therefore if the common ſolid A H F K L D B E be added, the parallelepipedons ſhall be made equal, on the ſame baſe, &c. Which was to be demonſtrated.

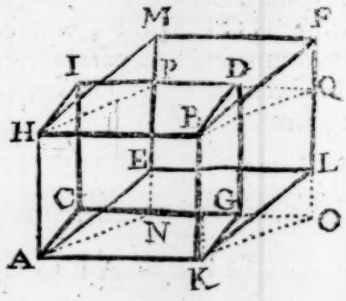
b) 34. 1.
c) Cor. 8. 1.
d) 4. 6.
e) 24. 11.
f) 36. 1.
g) 34. 1.
h) 10. def.

PROP. 30. THEOR. 25.

The ſolids parallelepipedons A I D K *and* A M F K, *made on the ſame baſe* A B, *and of the ſame height, and from which the lines inſiſting are not placed in the ſame right lines, are equal to one another.*

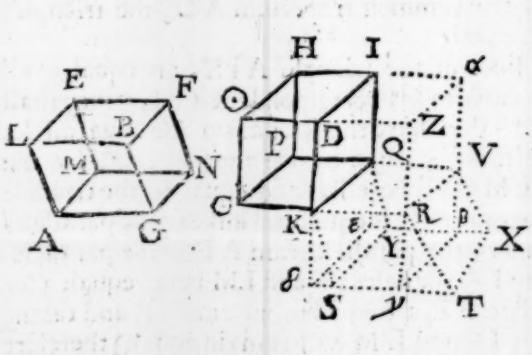
Demon-

Demonſtration FOr ſeeing that the plains C D and E F, oppoſite to the baſe A B, are in one and the ſame plain, becauſe of the ſame height of the parallelepipedons; let the lines C G and I D be prolonged on the ſame plain, of which let C G cut E M and L F, produced in the points N and O, and I D the ſame in the points P and Q; and let the right lines A N, K O, H P, and B Q, be joyned.



a) 34. 1. Foraſmuch [a] as P Q and M F are equal, being oppoſite to the parallelogram F P; and M F is equal to H B, P Q and H B ſhall be equal: But they are parallelograms, H D B being a parallelogram; b) 33. 1. therefore [b] H B and P Q are equal and parallel; and therefore H P Q B is a parallelogram; by the ſame reaſon H P N A, A N O K, and K O Q B, ſhall be paralellograms, and N O Q P is alſo a parallelogram; therefore A P Q R is a paralellepipedon, wherefore c) 29. 1. the [c] paralellepipedon A I D K is equal to the paralellepipedon A P Q K, having the ſame baſe A B, and the lines inſiſting are placed in the ſame right lines C O and I Q; in like manner, the paralellepipedon A M F K ſhall be equal to the ſame A P Q K, having the ſame baſe A B, and the lines inſiſting being in the ſame right lines N M and O F: Wherefore the paralellepipedons A I D K, and A M F K, are equal to one another: Wherefore, &c. Which ought to be demonſtrated.

PROP. 31. THEOR. 26.



The ſolids paralellepipedons A E F G, *and* C H I K, *made on equal baſes* A B *and* C D, *and of the ſame height, are equal to one another.*

Demonſtration FOr firſt of all, let the lines inſiſting A M, G N, L E, and B F, be perpendiculars to the baſe A B, and the inſiſting lines C P, K Q, O H, and D I, perpendiculars to the baſe C D, that being ſo, all the ſaid perpendiculars ſhall be equal the one to the other, the paralellepipedons being of one and the ſame height. Let C K be produced directly, and let K R be put equal to L B, and the angle R K S (on the ſame plain produced, O K,) made equal to the angle B L A, and K S put equal to L A, and finiſh the parallelogram K T, on which conſtitute the paralellepipedons Q S T V, according to the height of the perpendicular R Q: Foraſmuch as the ſides K R and K S are equal to L B and L A, and the angles R K S and B L A equal, the parallelograms K T and L G ſhall be equal and alike.

Again, foraſmuch as the ſides K Q and K S are equal to L E and L A, and

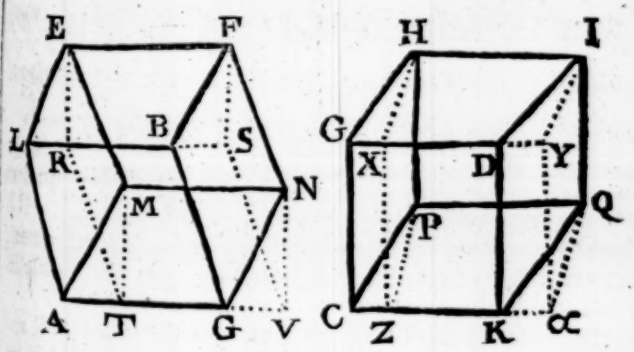
and [a] the angles Q K S and E L A are right angles, K Q and L E being propoſed at right angles to the plains K T and L G, the paralellograms Q S and E A are equal and alike. a) 3. def. 11.

In like manner, the ſides K R and C Q being equal to L B and L E; and the angles Q K R and E L B right angles, the paralellograms K V and L F are alſo equal and alike. Wherefore [b] ſeeing the three plains K T, Q S, and K V, of the paralellepipedon Q S T V are equal and alike to the three plains L G, E A, and L F of the paralellpipedon A E F G, as well thoſe of the one, as thoſe of the other, being equal to the three others oppoſite; the paralellepipedons [c] Q S T V and E A G F, are equal to one another. b) 24. 11. c) 24. 11.

Let D K and T S produced, meet in ϑ, and I Q, X Y in ε; and let the paralellepipedon ♌ϑγV be finiſhed, and H I, βV produced to meet in α, and O D, γ R, in Z, and let the paralellepipedon I K R γ be finiſhed: Foraſmuch as the paralellepipedons ♌ S, T V, and ♌ϑγV have the ſame baſe K V, and the ſame height, to wit, between the ſame parallel plains K V, ad ϑ X, their inſiſting lines K S, K ϑ, R T, R γ, ♌ Y, ♌ ε, V X, and V ζ, are placed in the ſame right lines ΛT and εX, [d] they ſhall be equal to one another: But the paralellepipedon ♌ S T V is equal to the paralellepipedon E A G F; therefore ♌ϑγV ſhall be equal to the ſame E A G F. d) 29. 11.

But [e] foraſmuch as K T and Kγ, paralellograms, are equal to one another, and K T equal to L G and Kγ, it ſhall be alſo equal to L G; that is to ſay, to C D, the baſe L G and C D being put equal; therefore [f] as C D is to D K, ſo is Kγ to D R; but [g] as the baſe C D is to the baſe D R, ſo the ſolid C H I K to the ſolid K I α R; the parallelepipedon C H α R being cut by the plain I K, parallel to the oppoſite plains C H and α R: In like manner, as Kγ, to D R: ſo the ſolid ♌ϑγV, to the ſolid I K R α; ſeeing that the paralellepipedon Iϑγα is cut by the plain K V, parallel to the plains oppoſite Dαϑζ. Therefore the paralellepipedons C H I K and ♌ϑγV, having the ſame proportion to the ſame ſolid I K R α; to wit, as the equal baſes C D and Kγ to the baſe D R; then ſeeing the parallepipedon ♌ϑγV is demonſtrated equal to the paralellepipedon A E F G, the paralellepipedons A E F G, and C H I K ſhall be equal: Which is propoſed. c) 35. 1. f) 7. 5. g) 25. 11.

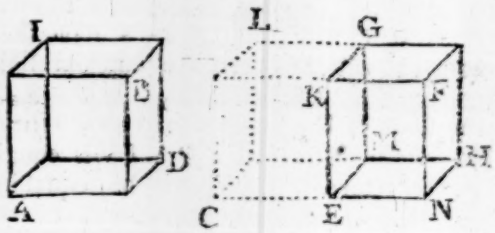
Now ſuppoſe that A M, G N, L E, and B F, and C P, K ♌, O H, and D I, be not perpendiculars to the baſes A B and C D, and [h] from the points E, F, M, and N; let there be drawn the perpendiculars E R, F S, M T, and N V, to the plain of the baſe A B, and from the points H, I, P, and ♌, to the plain where is the baſe C D, let there be drawn the perpendiculars H X I Y, P Z, ♌α, they ſhall be equal, the altitudes of the ſaid paralellepipedons being equal; let there be drawn h) 11. 11.



drawn RS, SV, VT, TR, and XY; Z_α, $_\alpha$I, and ZX, to make the paralellepipedons ETVF, and HZ_αI, the which being of the same height, and the lines insisting perpendicular, they shall be equal to one another, as is demonstrated.

i) 29,30.11. But [i] ETVF is equal to AEFG, having the same base and height, and the paralellepipedon HZ_αI is equal by the same reason to CHIK, therefore AEFG and CHIK are equal to one another, the same demonstration may be made, if the insisting lines of one paralellepipedon are perpendiculars to the base, and of the other not perpendicular: Wherefore, &c. Which ought to be demonstrated.

PROP. 32. THEOR. 27.



The solid paralellepipedons ABCD *and* EFGH, *of the same height are to one another as their bases* AB *and* EF *are.*

a) 45. 1. *Demonstration.* FOr [a] let there be made on EK the parallelogram CK, equal to the paralellogram AB, having the angle CEK equal to the angle ENF; so the paralellograms EE and CK shall make the whole paralellogram, as is shewn in the 45. of the first.

Then if the other plains of the paralellepipedon EFGH are produced towards EG, and that the whole paralellepipedon CFLH be finished;

b) 31. 11. [b] the paralellepipedons ABID and CKLM shall be equal, being made on equal bases AB and CK, by construction, and of the same height by

c) 7. 5. supposition; therefore [c] as the solid CKLM is to the solid EFGH, so

d) 25. 11. is the solid ABID, to the same EFGH: But [d] CKLM is to EFGH as the base CK or the base AB his equal, is to the base EF; therefore the solid ABID shall be also to the solid EFGH, as the base AB is to the base EF: Therefore solid paralellepipedons of the same height, &c. Which ought to be demonstrated.

PROP. 33. THEOR. 28.

Like solid paralellepipedons ABCD *and* EFGH, *are the one to the other in a triple proportion to their homologal sides, or sides of the same proportion, as* AI *and* EK.

Demonstration. FOr, Let AI be produced to L, and let IL be equal to EK, or GR; and DI to M, and let IM be equal to HK or GO. and BI to N; and let IN be equal to KF or GS; then having accomplished the paralellograms LM, NL, and IT, let the paralellepipedon TXIV be finished.

Forasmuch as the sides IL and IM are equal to the sides GR and

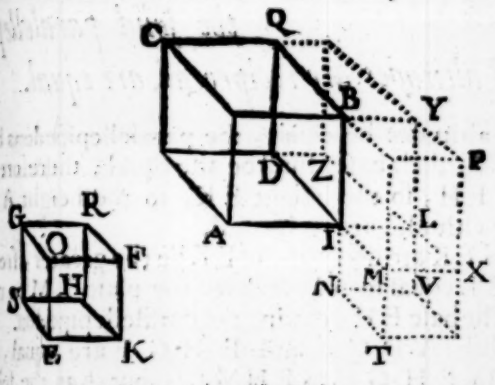
a) 15. 1. [a] GO, and the angles conteined of them also equal, the angle LIM being equal to AID, which because of the like paralellepiped. is equal to

to EKH, or RGO, the parallelograms IX and GF ſhall be alike and equal, and in like manner LN, and RS, and IT and GE.

Therefore the three plains IX, LN, and IT, of the paralellepipedon TXIV are alike and equal to the three plains GF, RS and GE, of the paralellepipedon EFGH: But [b] three plains of each are alike and equal to the three others oppoſite; therefore [c] the paralellepipedons TXIV and EFGH, are alike and equal. b) 24. 11. c) 10. def.

Again, having accompliſhed the parallelograms MB, BL, and LM, finiſh the paralellepipedon MPBL, and the prralellograms IY, DL, and I2, being accompliſhed, let the paralellepipedon IY2Z be alſo finiſhed.

Foraſmuch then as becauſe of the like paralellepipedons ABCD and EFGH, as AI to EK; that is to ſay to IL, ſo DI to HK; that is to ſay to IM, and BI to FK, that is to ſay, to IN. But [d] as AI is to IL, ſo the paralellogram AD to DL; and as DI to IM, ſo the parallelogram AD to LM; and as BI to IN, ſo the paralellogram BL to LN; then as AD to DL, ſo DL to LM; and BL to LN. But [e] as the baſe AD to the baſe DL, ſo the paralellepipedon ADCB to the paralellepipedon DLY2; and as the baſe DL to the baſe LM, ſo the paralellepipedon DLYQ, to the paralellepipedon LMBP, and as the baſe BL to the baſe LN, ſo the paralellepipedon LMBP to the parallelepipedon LNTX; therefore as ADCB to DLYQ, ſo DLYQ to LMBP, and LMBP to LNTX: Wherefore the four magnitudes ADCB, DLYQ, LMBP, and LNTX, are continually proportional; therefore [f] the firſt ADCB, is to the fourth LNTX; that is to ſay, to EFGH in triple proportion to ADCB the firſt, to DLY2 the ſecond. d) 1. 6. e) 32. 11. f) 10. def. 5.

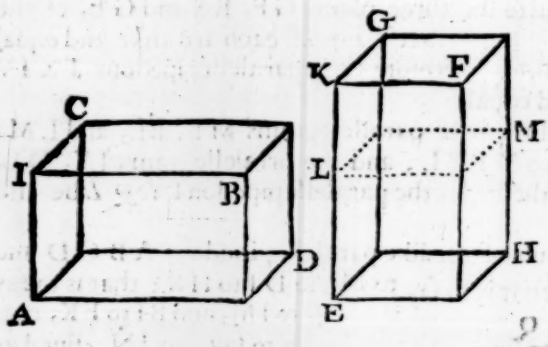


But [g] as ADCB to DLYQ, ſo the baſe AD to the baſe DL; and [h] as AD to DL, ſo the line AI to IL, that is to ſay, to EK; therefore the paralellepipedon ADBC is in triple proportion to the paralellepipedon EFGH, of their homologal ſides; to wit, of AI to AK: Wherefore, &c. Which ought to be demonſtrated. g) 32. 11. h) 1. 6.

COROLLARIE.

From this it is manifeſt that if four right lines are continually proportional; as the firſt to the fourth, ſo the Paralellepipedon deſcribed on the firſt, ſhall be to the like Paralellepipedon, and alike deſcribed on the ſecond: Foraſmuch as it hath been ſhewn, that the Paralellepipedon is to the Paralellepipedon, as the firſt line to the fourth, in triple proportion of the firſt line to the ſecond, to wit, of their homologal ſides.

PROP. 34. THEOR. 29.



The baſes A D *and* E H, *and the altitudes of equal ſolid paralellepipedons* AD CB, *and* EHGF, *are reciprocal; and the ſolid paralellepipedons, whoſe baſes and altitudes are reciprocal, are equal.*

Demonſtration FOr if their altitudes be equal, the paralellepipedons being put equal; their baſes ſhall be alſo equal; therefore as the baſe A D to the baſe E H, ſo the height E K, to the height A I; therefore the baſes and the altitudes are reciprocal.

But if the heights A I and E K are unequal, let E K be the greater, from which let there be cut off E L equal to A I, and let the plain L M be underſtood to be parallel to the baſe E H, cutting the paralellepipedon.

Foraſmuch as the ſolids A D C B and E H G F are equal, as
a) 7. 5. [a] A D C B to E H M L, ſo E H G F to E H M L; but
b) 32. 11. [b] as the ſolid A D C B to the ſolid E H M L; ſo the baſe A D to the baſe E H, the heights A I and E L being put equal; and as the ſolid E H G F to the ſolid E H M L; ſo by the ſame reaſon, the baſe K N to the baſe L N; ſince that, by that reaſon, the ſolids E H G F and E H M L have the ſame height K N and L N, being put for baſes; for they ſhall be between the ſame parallel plains K N and G H; therefore as the baſe A D to the baſe
c) 1. 6. E H, ſo the baſe K N to the baſe L N: But [c] as K N to L N, ſo the line E K to the line E L; that is to ſay, to A I, equal to E L. Therefore as the baſe A D to the baſe E H, ſo the height E K to the height A I: Therefore the heights and the baſes are reciprocal.

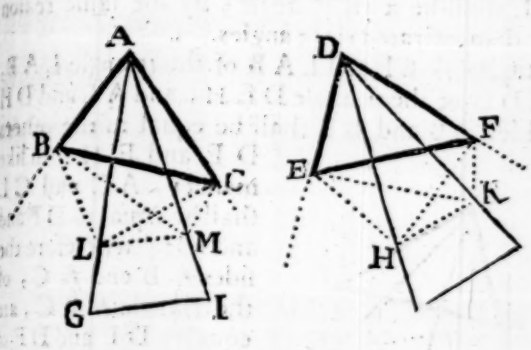
Now let the baſes and heights be reciprocal, I ſay the paralellepipedons are equal; for if the heights E K and A I are equal, the baſe A D being put to the baſe E H, as the height E K to the height A I; the baſes A D and
d) 31. 11. E H ſhall be equal; wherefore [d] the paralellepipedons A D C B and E H G F are equal to one another, having equal baſes, and the ſame altitudes.

But if the height E K be greater, let E L be cut off equal to A I; and let the plain L M be underſtood to be paralel to E H.

Foraſmuch then, as the baſe A D is to the baſe E H, ſo the height E K is to the height A I by ſuppoſition; that is to ſay, to E L, equal to A I:
e) 32. 11. But [e] as the baſe A D to the baſe E H, ſo the ſolid A D C B to the ſolid
f) 1. 6. E H M L, the heights A I and E L being put equal; and [f] as E K to E L,
g) 32. 11. ſo K N to L N; but [g] as the baſe K N to the baſe L N, ſo the ſolid E H G F to the ſolid E H M L, the ſolids E H G F, and E H M L having the ſame height, if K N and L N be put for baſes: For ſo they ſhall be between parallel plains K N and G H, as the ſolid A D C B ſhall be to the ſolid E H M L,

EHML, so the solid EHGF to the same solid EHML: Therefore [h] the solids ADCB and EHGF shall be equal, &c. Which ought to be demonstrated. h) 9. 5.

PROP. 35. THEOR. 30.



If there be two plain angles BAC *and* EDF *equal, at whose tops* A *and* D, *are raised in the aire two right lines* AG *and* DH, *containing equal angles,* BAG *to* EDH, *and* CAG *to* FDH, *with the lines first proposed, each to his correspondent line; and in the lines raised in the aire, be taken any points at pleasure,* G *and* H, *and from them be drawn perpendiculars* GI *and* HK, *to the plains where are the angles first proposed,* BAC *and* EDF: *But at the points which are made on the plains, by the perpendiculars* GI *and* HK, *the right lines* IA *and* KD *are joyned, the same right lines shall contein equal angles with the lines drawn in the aire.*

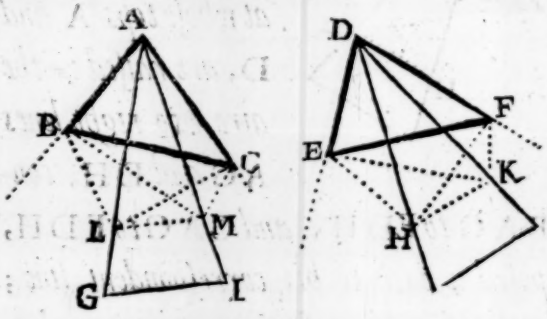
Demonstration. FOr if AG and DH be unequal, let there be taken AL from the greatest AG, equal to DH, and from L let there be drawn LM parallel to GI, to the plain of the triangle AGI; forasmuch as GI and LM are parallels, and GI is at right angles to the plain of the angle BAC, LM [a] shall be also at right angles to the same plain: from the points M and K, let there be drawn the perpendiculars MB, MC, KE, and KF, to AB, AC, DE, and DF; and let BC, BL, LC, EF, EH, and HF, be joyned. a) 8. 11.

And forasmuch as LM is at right angles to the plain BAC, it [b] will make a right angle with AM drawn to the same plain; wherefore [c] the square of AL shall be equal to the squares of AM and ML; but [d] the square of AM is equal to the squares of AC and CM, the angle ACM being a right angle by Construction; therefore the square of AL is equal to the squares of AC, CM, and ML; but [e] the square of CL is equal to the squares of CM and ML, [f] the angle CML being a right angle; therefore the square of AL is equal to the squares of AC and CL; therefore [g] the angle ACL shall be a right angle. b) 3. def. c) 47. 1. d) 47. 1. e) 47. 1. f) 3. def. g) 48. 1.

Again, [h] seeing that the square of AL [i] is equal to the squares of AM and ML: But the square of AM is equal to the squares of AB and BM, the h) 47. 1. i) 47. 1.

the angle ABM being a right angle by Conſtruction; the ſquare of AL is equal to the ſquares of AB, BM, and ML. But the ſquare of BL is
k) 3. def. equal to the ſquares of BM and ML, [k] the angle BML being a right angle; therefore the ſquare of AL is equal to the ſquares of AB and BL;
l) 48. 1. therefore [l] the angle ABL ſhall be a right angle; by the ſame reaſon DFH and DEH ſhall be demonſtrated right angles.

Foraſmuch then as the angles ABL and LAB of the triangle LAB, are equal to DEH and HDE of the triangle DEH, and AL and DH
m) 26. 1. are equal; [m] the other ſides AB and BL ſhall be equal to the others DE and EH: In like manner, AC and CL ſhall be equal to DF and and FH; wherefore the ſides AB and AC, of the triangle ABC, are equal to DE and DF of the triangle DEF, and the angles BAC and EDF, conteined of them, are equal by ſuppoſition: [n] the baſes



n) 4. 1. BC and EF ſhall be equal, and ABC and ACB equal to DEF and DFE, each to his correſpondent angle. But the whole ABM and ACM are equal to the whole DEK and DFK, being all right angles; then the reſidues MBC and MCB ſhall be equal to the reſidues KEF and KFE: Wherefore BC and EF being demonſtrated equal, the ſides
o) 26. 1. [o] BM and CM ſhall be equal to the ſides EK and FK.

Foraſmuch as AC and CM of the triangle ACM are demonſtrated equal to DF and FK of the triangle DFK, and ACM and DFK right
p) 4. 1. angles, [p] the baſes AM and DK ſhall be equal to one another; and BL and EH being demonſtrated equal, their ſquares ſhall be equal: But foraſmuch as the ſquare of BL is equal to the ſquares of BM and ML,
q) 3. def. and the ſquare of EH equal to the ſquares of EK and KH; [q] BML and EKH being right angles, the ſquares of BM and ML ſhall be equal to the ſquares of EK and KH.

Having then taken away the equal ſquares of BM and EK, demonſtrated equal; the remaining ſquares of LM and HK ſhall be equal; and therefore LM and HK equal: Wherefore ſeeing that the ſides AL and AM of the triangle ALM, are equal to DH and DK of the triangle DHK, and the baſe LM equal to the baſe HK, the angles LAM and HDK ſhall be equal: Therefore, If there be two plain angles, &c Which ought to be demonſtrated.

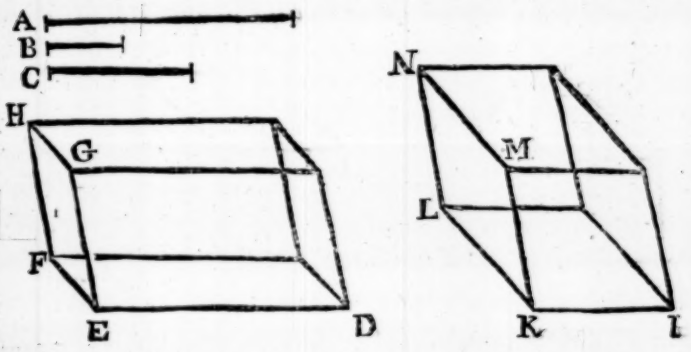
COROLLARIE.

If there be two equal plain angles, from the tops of which are drawn in the aire the equal right lines conteining equal angles with the lines firſt propoſed, each to his correſpondent angle, the perpendiculars drawn from the extremities of thoſe lines raiſed in the aire on the plain of the angles firſt propoſed, ſhall be equal to one another: For the plain angles BAC *and* EDF *being propoſed equal, and* AL *and* DH *raiſed on high, conſtituting the equal angles* LAB *and* HDE, *and* LAC *and* HDF, *it hath been ſhewn that* LM *and* HK *the perpendiculars, are equal to one another.*

PROP.

PROP. 36. THEOR. 31.

If there be three right lines proportional, A, B, *and* C, *the ſolid paralellepipedon* DH *made of them, is equal to the ſolid paralellepipedon* I N, *deſcribed of the mean* B, *provided it be equilateral, but equiangled to the aboveſaid.*



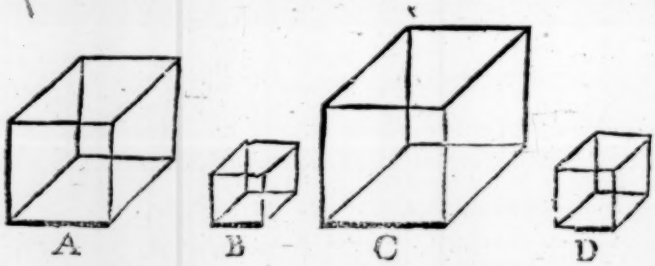
Conſtruction LEt A, B, and C be continually proportional, and let there be made the ſolid angle E of any three plain angles D E F, D E G, and F E G, in ſuch manner as that D E be equal to A, and E F to B, and E G to C, and having accompliſhed the paralellograms D F, F G, and G D, let the paralellepipedon D H be finiſhed, which is ſaid to be conteined of the three lines A, B, and C, or made of them; [a] Then on the right line I K, and to a point therein K, let there be made the ſolid angle K, equal to the ſolid angle E, of the three plain angles I K L, I K M, and L K M, which let be equal to the three angles D E F, D E G, and F E C, in ſuch ſort as that I K, K L, and K M, may be equal each to the mean B, and having accompliſhed the paralellograms I L, L M, and M I; let the paralellepipedon I N be finiſhed, which is ſaid to be conteined under the line B, or deſcribed thereof: I ſay the ſolid D H is equal to the ſolid I N. a) 26. 11.

Demonſtration FOr ſeeing that as D E to I K, ſo is K M to E G, (D E being taken equal to A, and I K and K M to B, and E G to C,) and the angles D E G and I K M equal to B, [b] the paralellograms D G and I M ſhall be equal, having the ſides about the equal angles reciprocal: But foraſmuch as the plain angles D E G and I K M are equal; from whoſe tops are drawn in the aire the lines E F and K L, conteining equal angles with the lines firſt propoſed, by conſtruction, each to his correſpondent angle, the perpendiculars drawn from F and L, to the plain of the baſes D G and I M, *viz.* the altitudes of the paralellepipedons D H and I N, (if the baſes D G and I M [c] be equal to one another:) Therefore [d] the paralellepipedons D H and I N, having the baſes D G and I M equal, and the altitudes alſo equal, they ſhall be equal to one another: Wherefore, If there be three right lines, &c. Which was to be demonſtrated. b) 14. 6. c) C. 35. 11. d) 31. 11.

PROP.

PROP. 37. THEOR. 32.

If four right lines A, B, C, *and* D, *be proportional, the ſolid paralellepipedons* A, B, C, *and* D, *alike, and alike deſcribed of them, ſhall be alſo proportional; and if the ſolid paralellepipedons which are alſo alike and alike deſcribed are proportional, the right lines ſhall be alſo proportional.*



Conſtruction HAving conſtituted on the lines A and B, the two paralellepipedons A and B, alike and alike deſcribed; alſo on C and D deſcribe the two others C and D, alike and alike poſited, whether theſe be alike to the other or not, I ſay that as the ſolid A is to the ſolid B, ſo is the ſolid C to the ſolid D.

a) 33. 11. *Demonſtration* FOr [a] ſeeing the ſolid A is to the ſolid B in triple proportion of the right line A, to the right line B, alſo the ſolid C to the ſolid D, in triple proportion of the right line C, to the right line D, the proportions of the ſolids A to B, and C to D ſhall be equal; foraſmuch as they are triples of the equal proportions, to wit, of the proportion of the right line A to the right line B, and of the right line C to the right line D, which is firſt of all propoſed.

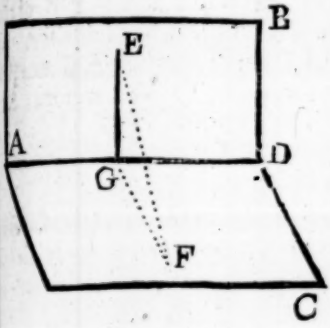
Secondly, let it be as the ſolid A is to the ſolid B, ſo the ſolid C to the ſolid D: I ſay that as the right line A is to the right line B, ſo the right line C is to the right line D.

b) 33. 11. For [b] ſeeing that the ſolid A is to the ſolid B, in triple proportion of the line A to the line B, alſo the ſolid C is to the ſolid D in proportion triple of that of the line C to the line D, the proportions of the lines A to B, and of C to D are equal; foraſmuch as their tripled proportions, to wit, of the ſolid A to the ſolid B, and of the ſolid C to the ſolid D, are put equal. Which was in the ſecond place to be demonſtrated.

PROP. 38. THEOR. 33.

If a plain AB *be perpendicular to a plain* AC, *and from ſome point* E *of thoſe that are in one of the plains* AB, *a line be drawn perpendicular to the other plain* AC, *the perpendicular drawn ſhall fall on the common ſection* AD, *of the plains* AB *and* AC.

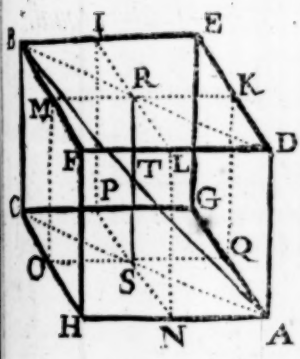
Demon-



Demonſtration FOr otherwiſe, let it fall elſewhere (if it be poſſible, as at F,) and [a] from ſome plain A C, let there be drawn F G perpendicular to A D, and let E G be joyned on the plain A B: Foraſmuch as F G is perpendicular to the common ſection A D, [b] it ſhall be alſo perpendicular to the plain AB; and therefore [c] alſo perpendicular to G E: But E F is alſo perpendicular to F G: Therefore in the triangle E F G, the two angles E F G and E G F are right angles, which is abſurd, being [d] they are leſſe than two right: Therefore the perpendicular drawn from E on the plain A C, ſhall not fall out of the common ſection; therefore ſhall fall on it: Therefore, If a plain, &c. Which was to be demonſtrated.

a) 12. 1.
b) 4. def.
c) 3. def.
d) 17. 1.

PROP. 39. THEOR. 34.



If the ſides of the oppoſite plains A C *and* B D, *of a ſolid parallelepipedon* A B, *are cut in two equal parts, and that by the ſections the plains be drawn, the common ſection of the plains* R S, *and the diameter of the ſolid paralellepipedon* A B *ſhall equally cut one another.*

Demonſtration FOr having joyned R B, R D, S A, and S C, let the two triangles A Q S and C O S be conſidered: Foraſmuch as the ſides A Q and Q S of the triangle A Q S, are equal to the ſides C O and O S of the triangle C O S; (for A Q and C O are the halves of the equal ſides A G and C H and [a] Q S and O S are equal to the two equal ſides A N and H N; A S and H S being paralellograms) and [b] the angle A Q S equal to the alternate angle C O S; the baſes A S and C S ſhall be equal, and the angles A S Q and C S O equal: But [c] the angles A S Q and A S O are equal to two right angles; therefore C S O and A S O are alſo equal to two right angles: Therefore [d] A S and C S ſhall make one only right line: In like manner, B R and D R ſhall be demonſtrated equal, and ſhall make one only right line.

a) 29. 1.
b) 4. 1.
c) 13. 1.
d) 14. 1.

Again, foraſmuch as A D as well as B C, is parallel and equal to F H, becauſe of the paralellograms A F and F C, they [e] ſhall be alſo parallel to one another and equal: Wherefore [f] A C and B D which joyn them at the extreams, are alſo equal and parallel; therefore their halves A S and B R are equal.

e) 9. 11.
f) 33. 11.

But foraſmuch as A C and B D are parallels, [g] A B and R S ſhall be in one and the ſame plain with them, therefore they ſhall cut one another,

g) 7. 11.

h) 15,29.1. to wit, in the point T. But [h] seeing that the two angles A S T and A T S, of the triangle A S T are equal to the two angles B R T and B T R of the

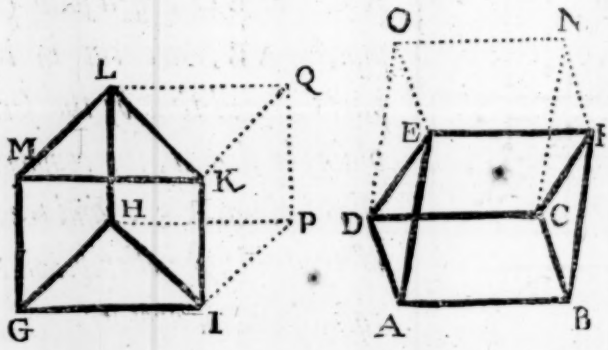
i) 26. 1. triangle B R T, and the side A S to the side B R, [i] the other sides T A and T S shall be equal to the other sides T B add T R: Therefore A B and R S do cut one another in two equal parts in the point T. Wherefore, &c. Which was to be demonstrated.

COROLLARIE.

It follows from this Demonstration that in every paralellepipedon, all the diameters cut one another in two equal parts at one point, to wit T, *(as is here shewn by the line* R S,) *and every plain cutting the paralellepipedon in two equal parts, doth passe by the center thereof, as by the point* T.

PROP. 40. THEOR. 35.

If two Prismes ABCDEF, *and* GHIKLM, *are of the same height, of which the one hath a paralellogram* ABCD, *for the base, and the other a triangle* GHI, *and that the paralellogram be double the triangle, those prismes shall be equal.*



Demonstration FOr let the paralellepipedons A N and Q G be accomplished, prolonging the plains of the triangles, to make the paralellograms B N, A O, G P, and M Q: For having drawn N O and P Q, there shall be made two paralellepipedons A N and G Q, of the same height with the prismes, and to which the opposite plains are parallels, as is easie to be gathered from the 15th. Prop. of this Book

a) 34. 1. For asmuch [a] as the paralellogram G P is double to the triangle G H I, and the paralellogram A C is put double to the same triangle G H I, the

b) 31. 11. paralellograms A C and G P shall be equal; therefore [b] A N and G Q are of the same height, and on equal bases A C and G P, shall be equal to one another; therefore their halves, to wit, the prismes A B C D E F, and

c) 32. 11. G H I K L M, (for [c] the paralellepipedons A N and G Q are cut each in two equal prismes, by the diameters of the opposite plains C F, D E, H I, and L K,) are also equal to one another: Therefore, &c. Which was to be demonstrated.

The End of the Eleventh Element of EUCLIDE.

THE TWELFTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

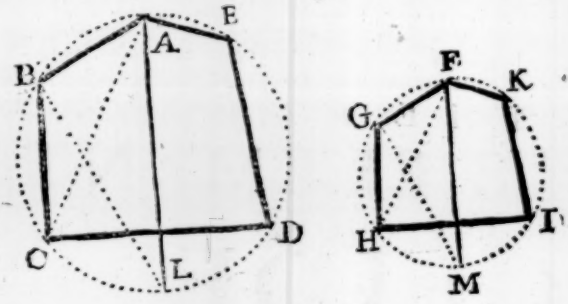
THis Twelfth Book ſetteth forth the Paſſions and Proprieties of Pyramids, Priſmes, Cones, Cylinders and Spheres, and compareth Pyramids to Pyramids and Priſmes, and likewiſe compareth Cones and Cylinders, and laſtly Spheres one to another: But before he treateth of the afore-mentioned bodies, he proveth that like polygonal figures inſcribed in Circles, and alſo the Circles themſelves, are in ſuch proportion the one to the other, as the ſquares of the Diameters of ſuch Circles are, it being abſolutely neceſſary that thoſe things be firſt proved, for the better confirmation of the divers paſſions and proprieties of thoſe bodies.

PROPOSITIONS, PROBLEMS, and THEOREMES.

PROPOSITION 1. THEOREM 1.

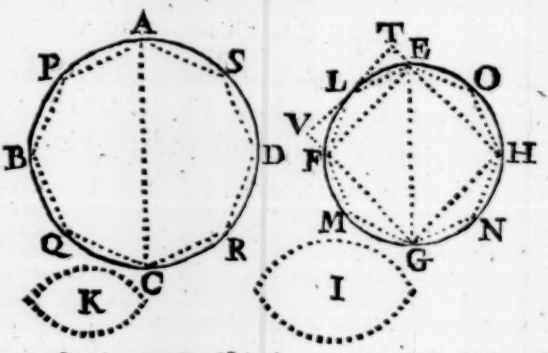
Like polygons ABCDE *and* FGHIK, *inſcribed in circles, are in proportion the one to the other, as the ſquares deſcribed of the diameters* AL *and* FM, *of the circles.*

Demonstration FOr, let AC and FH be drawn, subtending the equal angles ABC and FGH; and let BL and GM be joyned: Then, by reason of the similitude of the polygons, AB being to BC, as
a) 6. 6. FG to GH, [a] the triangles ABC and FGH shall be equiangled, having the sides about the equal angles ABC & FGH, proportional. But [b] the
b) 21. 3. angle ALB is equal to the angle ACB, and the angle FMG equal to FHG; therefore ALB and FMG shall be equal; wherefore ABL and FGM being



c) 31. 3. also equal, [c] *viz.* right angles in the semicir-
d) 32. 1. cles [d] the remaining angles BAL and GFM shall be equal: Therefore
e) 4. 6. [e] as AL to AB, so FM to FG; and alternately, as AL to FM, so AB to FG: Therefore as the square of AL to the square of FM, so the polygon ABCDE described on the right line AB, to the polygon FGHIK described on FG; seeing that as well the squares as the polygons, are like figures, and alike described: Therefore, Like polygons, &c. Which ought to be demonstrated.

PROP. 2. THEOR. 2.



Circles ABCD *and* EFGH, *are in proportion the one to the other, as the squares of their diameters* AC *and* EG.

Demonstration FOr if it be not so, I shall be lesse or greater; Suppose first of all I to be lesse, (if possible,) and that the circle
a) 10. 10. EFGH be greater than I, by the figure K: But [a] cutting off more than the half of the circle EFGH, and more than the half of the rest, so often
b) 6. 4. that at last there may remain a magnitude lesse than K: Let there be [b] inscribed in that circle the square EFGH, which shall be greater than the half the circle, if the four segments of the circle are lesse than K, you have what you require, if not, let the arches EF, FG, GH, and HE, be divided in two equal parts by L, M, N, and O; and let the Octogon be inscribed in the circle, drawing EL, LF, FM, MG, GN, NH, HO, and OE; it appeares that there will be four triangles equal in the four equal segments, and that each one, as FLE, shall exceed the
c) 41. 1. half of his section, the Isosceles triangle ELF [c] being the half of the Rectangle of the same height, described on the base EF, as EV, which is greater than the section FLE,

Now

Now let the eight remaining figures FM, MG, &c. be leſſe than K, for if it were not ſo, it would be required to divide the laſt arches, and alwayes inſcribe the polygons, whereof the laſt ſhall have the ſides double in multitude to thoſe of his precedent, and to take away ſtill more than the half of each ſegment (to wit) his Iſoſcele triangle: It is evident that the laſt ſections ſhall be in the end leſſe than K.

Laſtly, Let then the ſaid eight ſections remaining be leſſe than K: It is manifeſt that the ſaid Octogon ſhall be greater than I, I and K together, being equal to the circle E F G H. Let there be alſo deſcribed an Octogon alike, in the circle A B C D, dividing each of the ſemicircles A B C and A D C in two equal parts, in B and D, and again, each part in two equal parts in P, Q, R, and S; and having joyned the right lines to all the ſaid points; it is manifeſt that the Octogon figure inſcribed, ſhall be alike to the Octogon inſcribed in the circle E F G H; [d] and the ſaid Octogon of the circle A B C D, ſhall be to the Octogon of the circle E F G H, as the ſquare of the diameter A C, to the ſquare of the diameter E G. But as the ſquare of A C, to the ſquare of E G, ſo the circle A B C D is to I: Therefore [e] as the polygon A Q S to the polygon E M O: ſo the circle A B C ſhall be to I. But the polygon A Q S is leſſe than the circle A B C, therefore [f] the polygon E M O ſhall be alſo leſſe than I, which is abſurd; it being demonſtrated to be greater: Therefore the figure I cannot be leſſe than the circle E F G H.

d) 1. 12.

e) 11. 5.

f) 14. 5.

Nor can it alſo be greater; For the circle A B C being to I, as the ſquare of A C to the ſquare of E G; alternately, I ſhall be to the circle A B C, as the ſquare of E G to the ſquare A C; but let it be underſtood as I is to the circle A B C, ſo the circle E F G is to another, as to K; therefore [g] as I is greater than the circle E F G, ſo the circle A B C ſhall be greater than K, and [h] the circle A B C ſhall be to K, as the ſquare of A C to the ſquare of E G, which is contrary to the firſt part of this Propoſition; where it is ſhewn that one of the circles being to a certain figure in the proportion of the ſquares of the diameters, that that figure cannot be leſſe than the other circle: Therefore I cannot be greater than the circle E F G, nor leſſe, as by the firſt part hereof, therefore equal: Therefore, Circles, &c. Which ought to be demonſtrated,

g) 14. 5.

h) 11. 5.

COROLLARIE.

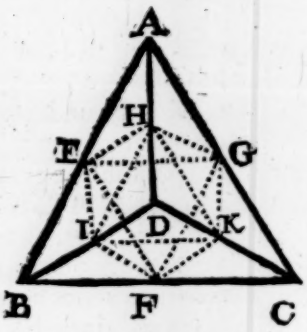
From hence it follows, that as the circle is to the circle, ſo the polygon deſcribed in the one circle, to the like polygon deſcribed in the other circle; ſeeing that as well the circle is to the circle, and the polygon to the polygon, as the ſquare of the diameter to the ſquare of the diameter, as hath been demonſtrated.

PROP. 3. THEOR. 3.

Every pyramid having the baſe A B C *triangular, may be divided into two pyramids* A E G H *and* H I K D, *and alike to one another, having the baſes* A E G *and* H I K, *triangular, and alike to the whole, and in two equal priſmes, which two priſmes are greater than the half of the whole pyramid.*

Demon-

Demonſtration FOr the ſides A D and D B, of the triangle A D B being divided in two equal parts, and therefore proportionally;
a) 2. 6. [a] H I and A B ſhall be parallels, and alſo I K, B C and H K, A C and E G, B C and E F, A C and F G, A B and E H, B D and E I, A D and
b) 9. 11. I F, D C and H G, and D C, are parallels by the ſame reaſon. But [b] F G and H I being parallels to A B, are parallel the one to the other, and ſo G H and F I being parallels to D C, are alſo parallel to one another; therefore A E I H, H E B I, I D H E, E B F G, G H K C, C K I F, and F G H I are paralellograms. But H E and H G being parallel to D B and
c) 10. 11. D C, [c] the angles E H G and B D C are equal; by the ſame reaſon
d) 4. 6. H E G, D B C, and H G E and D C B ſhall be equal; therefore [d] the ſides of the triangle H E G are proportional to the ſides of the triangle D B C, about the equal angles: Wherefore the triangle H E G is alike
e) Cor. 4. 6. to the triangle D B C. But [e] the triangles H A E, H A G, and A E G, are alſo alike to the triangles D A B, D A C, and A B C; therefore
f) 9. def. 11. [f] the pyramid A E G H is alike to the pyramid A B C D.
g) 10. 11. Again, H I and H K being parallels to A B and A C, [g] the angles I H K and B A C ſhall be equal; and in like manner H I K and A B C, and
h) 4. 6. and H K I and A C B equal; [h] wherefore the ſides of the triangle H I K are proportional to the ſides of the triangle A B C, about the equal angles, and the triangle H I K is alike to the triangle A B C: But the



i) Cor. 4. 6. triangles D H I, D I K, and D L H, [i] are
k) 9. def. 11. alike to the triangles D A B, D C B, and D C A; therefore the pyramid [k] H I K D is alike to the pyramid A B C D. But the
l) 21. 6. triangles A H E and H D I being alike to the triangle A D B, as is ſhewn, [l] they ſhall be alſo alike to one another, and being made on equal lines A H and H D, they ſhall be equal; by the ſame reaſon A H G and H D K ſhall be equal and alike, being proved alike to the triangle A D C, and made on equal lines A H and H D, and ſo the triangle A E G and H I K, ſhall be equal and alike, being proved alike to the triangle A B C, and poſited on the equal
m) 34. 1. lines A E and H I, being [m] the oppoſite ſides of the paralellogram A E H I.

Alſo E H G and I D K ſhall be equal and alike, being proved alike to
n) 34. 1. the triangle B D C, and having H E and D I equal [n] to the paralello-
o) 10. de. 11. gram H E I D: Therefore the pyramids A E G H and H I K D [o] are equal and alike, all the triangles of the one being proved equal and alike to all the triangles of the other.

p) 34. 1. Again, [p] E H, H G, and G E, being equal and parallel to B I, I F, and F B, of the paralellograms E H I B, F G H I, and B F G E, the
q) Cor. 8. 1. triangles E H G and B F I [q] ſhall be equiangled and equal to one another, and therefore alike; But they are alſo parallel to E H and H G, by which the plain E H G is drawn, being parallels to B I and I F, by which is drawn the plain B I F; therefore the ſolid B I F G H E, conteined of the two triangles E H G and B I F equal, alike, and parallel, and oppoſite one to the other, and by the three paralellograms E G F B, B E H I, and I F G H is a priſme by the definition. So the ſolid C F G H I K ſhall be

ſhewn

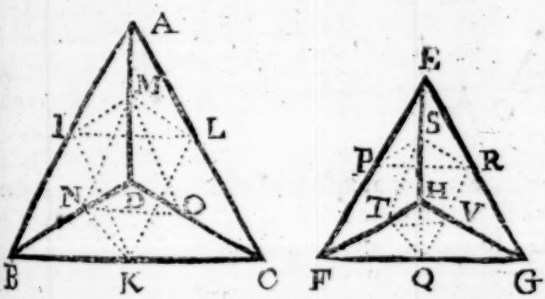
ſhewn to be a priſme: For F C, C G and G F being equal and parallel, to I K, K H, and H I, of the parallelograms C F I K, C G H K, and F G H I, the triangles C F G and H I K ſhall be equal and equiangled to one another, and therefore alike: [r] But they are parallels, C F and C G, by which is drawn the plain C F G, being parallel to K I and K H, by which paſſeth the plain K I H; therefore the ſolid C F G H I K, conteined of the two oppoſite triangles C F G and H I K, ſhall be equal, alike, and parallel and the three parallelograms C F I K, K H G C, and I F G H, is a priſme, and thoſe priſmes E B F G H I, and C F G H I K are of the ſame height, to wit, between the parallel plains B C G E and H I K, and the baſe quadrangular being double to the triangular, [s] they ſhall be equal to one another.

r) 15. 11.

s) 40. 11.

Laſtly, becauſe the priſme E B F G H I is greater than the pyramid E B F I, the whole than its part: But the pyramid E B F I is equal and alike to the pyramid A E G H, and alſo to the pyramid H I K D, as is manifeſt by the equality and ſimilitude of the triangles; the priſmes E B F G H I and C F G H I K ſhall be greater than the pyramyds A E G H, and H I K D; therefore theſe priſmes do exceed the half of the whole pyramid A B C D. For a whole being divided into two unequal parts, the greateſt part exceeds the one half thereof, and the leſſer part wants thereof: Therefore, Every pyramid having the baſe, &c. Which ought to be demonſtrated.

PROP. 4. THEOR. 4.

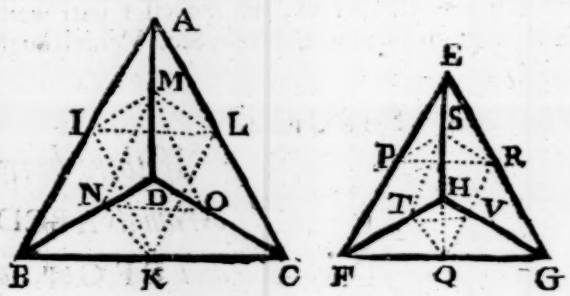


If there be two pyramids A B C D, *and* E F G H, *of the ſame height, having the baſes* A B C *and* E F G *triangular, and that each of them be divided in two pyramids, equal to one another,* A I L M, M N O D, E P R S, *and* S T V H, *and alike to the whole, and in two equal priſmes* I B K L M N, C K L M N O, P F Q R S T, *and* G Q R S T V, *and that in like manner both the one and the other of the pyramids produced by this firſt diviſion, be divided, and that that be alwayes done after one manner; as the baſe* A B C *of one of the pyramids, ſhall be to the baſe* E F G *of the other, ſo alſo all the priſmes which are in the one of the pyramids, ſhall be to all the priſmes of the other equal in number.*

Demon-

Demonſtration FOr ſeeing that as BC to CK, ſo FG to GQ, both the one and the other line being divided in two equal parts, and the triangles ABC and LKC being alike and alike poſited.

a) Cor. 4. 6. Alſo, [a] the triangles EFG and RQG, then b) 22. 5. [b] as the triangle ABC ſhall be to the triangle LKG, ſo the triangle EFG to the triangle RQG; and alternately, as ABC to EFG, ſo LKC to RQG: But as LKC to RQG; ſo the priſme CKLMNO to the priſme GQRSTV, as ſhall be preſently ſhewn; and therefore ſo the priſme IBKLMN to the priſme PFQRSTV; theſe being equal to the others, and as one priſme alone, to wit, IBKLMN, to one priſme alone PFQRST; ſo c) 12. 1. [c] the two priſmes, IBKLMN and CKLMNO together, are to the two priſmes PFQRST and GQRSTV together; therefore alſo as the baſe ABC to the baſe EFG, ſo the two priſmes in the pyramid ABCD, to the two in the pyramid EFGH: In like manner may be ſhewn that the two priſmes that are in the pyramids AILM and MNOD, made in the pyramid ABCD, are to the two priſmes which are in the pyramids EPRS and STVH, made in the pyramid EFGH; as the baſes AIL and MNO of thoſe pyramids, are to the baſes EPR and STV of theſe, and ſo following, making alwayes the ſame diviſion: But as theſe baſes are to the others, ſo the baſe LKC, which is equal and alike to theſe, is to the baſe RQG, which is equal and alike to theſe, as is demonſtrated by the precedent; that is to ſay, ſo the baſe ABC is to the baſe EFG: Therefore alſo as the baſe ABC to the baſe EFG, ſo the priſmes of each pyramid made in the pyramid ABCD, ſhall be to the priſmes of each pyramid made: Therefore alſo as the priſmes of the pyramid ABCD, to the priſmes of the pyramid EFGH, ſo the priſmes as well of the pyramid AILM, are to the priſmes of the pyramid EPRS; as theſe of the pyramid MNOD, to theſe of STVH, and ſo following.



Wherefore ſeeing that as two priſmes of the pyramid ABCD, are to d) 12. 5. two priſmes of the pyramid EFGH, [d] ſo all the priſmes which are in the pyramids ABCD, AILM, and MNOD, &c. together, are to all the priſmes together, which are in the pyramids EFGH, EPRS, and STVH, &c. if thoſe are equal in number to theſe: In like manner, as the baſe ABC to the baſe EFG, ſo all the priſmes of the pyramid ABCD, to all the priſmes of the pyramid EFGH: Therefore, if two pyramids, &c. Which ought to be demonſtrated.

PROP. 5. THEOR. 5.

The pyramids ABCD *and* EFGH, *which are of the ſame height, and having the baſes* ABC *and* EFG *triangular, are the one to the other as their baſes.*

Demon-

Demonſtration FOr if it be not ſo, let it be put as the baſe A B C to the baſe E F G, ſo the pyramid A B C D to the ſolid X, which ſhall be leſſe or greater than the pyramid E F G H.

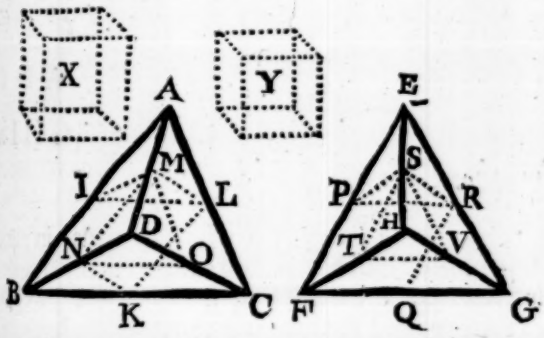
Firſt, Let it be greater by the ſolid Y, and let the pyramid E F G H be divided in two equal pyramids, and in two equal priſmes, according to Prop. 3. of the 12th. Again the two pyramids made in E F G H, let be in like ſort divided in two equal pyramids, and two equal priſmes, and ſo following.

Foraſmuch as, if from the pyramid E F G H be taken more than the half, to wit, the two priſmes P F Q R S T and G Q R S T V, [a] greater than the half of the pyramid E F G H; alſo from the remaining pyramids E P R S and S T V H, more than the halfe, to wit, their priſmes, and ſo following; [b] there will remain in the end a magnitude leſſe than Y, the exceſſe of E F G K, on the ſolid X. Now then let the remaining magnitude be leſſe: But the pyramid E F G H being put equal to the ſolids X and Y, the priſmes remaining in the pyramid E F G H, ſhall be greater than X.

a) 3. 12.

b) 1. 10.

Let the pyramid A B C D be divided into two equal pyramids and two equal priſmes, and in like manner the pyramids A I L M and M N O D, in two equal pyramids, and two equal priſmes; and let that be done ſo many times as in the pyramid E F G H.



Foraſmuch [c] as all the priſmes in the pyramid A B C D, are to all thoſe of the pyramid E F G H, equal in number; as the baſe A B C to the baſe E F G, that is to ſay, as the pyramid A B C D is to X. But all the priſmes in the pyramid A B C D being leſſe than the whole A B C D, alſo [d] all the priſmes in the pyramid E F G H ſhall be leſſe than the ſolid X. But they alſo are proved greater, which is abſurd. Therefore X is not leſſe than E F G H.

c) 4. 12.

d) 14. 5.

Now let the ſolid X be greater than the pyramid E F G H; therefore the pyramid A B C D being put to X, as the baſe A B C to the baſe E F G; by converſion of proportion, X ſhall be to A B C D, as the baſe E F G to the baſe A B C, as X is to the pyramid A B C D, ſo let the pyramid E F G H be put to the ſolid Y, and X being put greater than E F G H; [e] the pyramid A B C D ſhall be greater than Y; therefore as the baſe E F G is to the baſe A B C, ſo the pyramid E F G H is to Y, which is leſſe than the pyramid A B C D, which is abſurd. It being already ſhewn that as the baſe is to the baſe, ſo the pyramid cannot be a ſolid leſſe than the pyramid: Therefore the ſolid X is not greater than the pyramid E F G H, and is alſo ſhewn not to be leſſe, therefore equal. Wherefore ſeeing that as the baſe A B C is to the baſe E F G, ſo the pyramid A B C D is put to X: But [f] the pyramid A B C D is to X, as the pyramid E F G H equal to the ſolid X; alſo as the baſe A B C to the baſe E F H, ſo the pyramid A B C D to the pyramid E F G H: Therefore, The pyramids, &c. Which ought to be demonſtrated.

e) 14. 5.

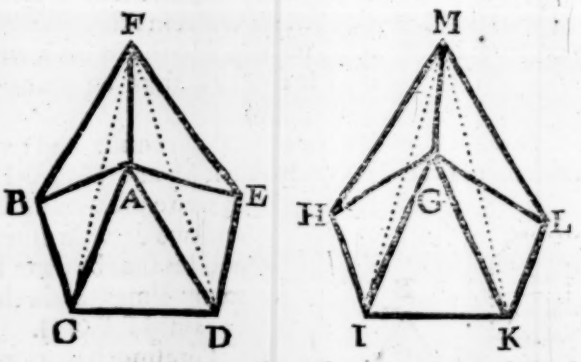
f) 9. 5.

COROLLARIE.

Hence it comes that the pyramids of one height made on the same base, or equal triangular bases, are equal to one another, having the same proportion that their bases have, which are proposed equal, or one and the same.

It follows contrariwise that the equal triangular pyramids made on the same base, or equal bases are of the same height, and that equal triangular pyramids having the same height, have equal bases, or one and the same, the which two things is shewn by the first part of this Corollary; by the same discourse used in the Demonstration of the converse of the 30th. Prop. and 31th. of the Eleventh Book. If another pyramid be made, cut off as well in the height, as in the base.

PROP. 6. THEOR. 6.



Pyramids ABCDEF *and* GHIKLM, *which are of one height, and having the bases polygons,* ABCDE *and* GHIKL, *are the one to the other, as are their bases.*

Demonstration. FOr having reduced the bases into triangles equal in number, each pyramid shall be divided into so many triangular pyramids: But [a] seeing that as the base ABC is to the base ACD, so the pyramid ABCF to the pyramid ACDF; and compoundedly, as the base ABCD to the base ACD, so the pyramid ABCDF to the pyramid ACDF: But [b] again, as the base ACD to the base ADE, so the pyramid ACDF to the pyramid ADEF: Therefore in proportion of equality, as the base ABCD to the base ADE, so the pyramid ABCDF to the pyramid ADEF, Therefore compoundedly, as the base ABCDE to the base ADE, so the pyramid ABCDEF shall be to the pyramid ADEF; by the same reason, as the base GHIKL shall be to the base GKL, so the pyramid GHIKLM to the pyramid GKLM; and by converse proportion, as the base GKL to the base GHIKL; so the pyramid GKLM to the pyramid GHIKLM.

a) 5. 12.

b) 5. 12.

c) 5. 12.

Again, [c] forasmuch as the base ADE is to the base GKL; as the pyramid ADEF, to the pyramid GKLM, there will be four bases ABCDE, ADE, GKL, and GHIKL, in the same proportion as the four pyramids ABCDEF, ADEF, GKLM, and GHIKLM; therefore in proportion of equality, as the base ABCDE to the base GHIKL, so the pyramid ABCDEF to the pyramid GHIKLM; And therefore, Pyramids, &c. Which ought to be demonstrated.

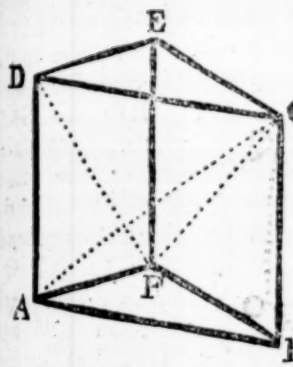
COROLLARIE.

It is manifest that pyramids of the same height made on bases equimultilateral, are

ge in one and the ſame, are equal to one another, having the ſame proportion with their baſes that are put equal to one and the ſame.

Again it followes contrariwiſe that the pyramids equimultilateral, and made on equal baſes, equal to one and the ſame, have one height: And that the pyramids equimultilateral: having the ſame height, have the baſes equal, if they be not on one and the ſame: Theſe two things ſhall be demonſtrated as we have ſaid in the Corollary of the fifth Propoſition of this Book.

PROP. 7. THEOR. 7.



Every priſme as ABCDEF, *having the baſe triangular, may be divided into three pyramids, equal to one another, having the baſes triangular.*

Demonſtration FOr in the three parallelograms let there be drawn three diameters, to wit, AC in the parallelogram ABCD, CF in the parallelogram BCEF, and FD in the parallelogram ADEF: Foraſmuch [a] as the triangles ABC and ADC are equal, and [b] that as the baſe ABC is to the baſe ADC, ſo the pyramid ABCF to the pyramid ADCF, thoſe pyramids having the ſame height, to wit, the perpendicular drawn from the top F, to the plain ABCD, the pyramids ABCF and ADCF, ſhall be equal to one another: In like manner the pyramids ADFG and EFDC, made on the equal baſes ADF and EFD, and the ſame height, to wit, the perpendicular drawn from the top C, on the plain ADEF, ſhall be equal: But the pyramid ADCF is the ſame with the pyramid ADFC; that being conteined of four plains, to wit, of the baſe ADC, and the triangles ADF, ACF, and DCF, and this of the ſame four plains, to wit, of the baſe ADF, and the triangles ADC, ACF, and DCF: Therefore the three pyramids ABCF, ADCF, and EFDC, or CDEF, (which is the ſame with EFCD, compounding the whole priſme, are equal to one another: Therefore, Every Priſme, &c. Which ought to be demonſtrated.

a) 34. 1.
b) 5. 12.

COROLLARIE.

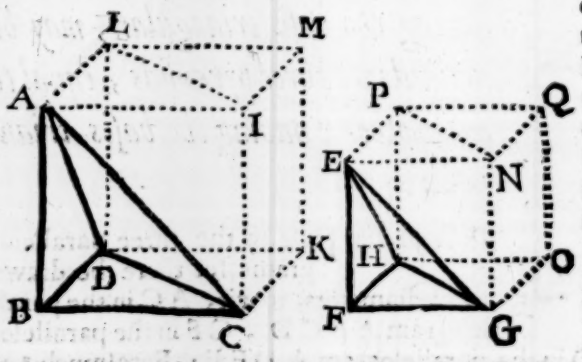
It follows from what hath been ſaid, that every pyramid is the third part of the priſme that hath the ſame height and baſe, or equal thereunto, or elſe, every priſme is the triple of the pyramid which hath the ſame height and ſame baſe, or equal thereunto.

PROP. 8. THEOR. 8.

Like pyramids ABCD *and* EFGH, *which have the baſes* ABC *and* FEG *triangular, are in a triple proportion of their homologal ſides* BC *and* FG.

Demonſtration FOr let the parallelepipedons BM and FQ be finiſhed, the pyramids being alike, the plain angles ABC and EFG ſhall be equal; and as AB to BC, ſo EF to FG, the parallelogrames BI and FN ſhall be alike: In like manner, the angles ABD and EFH being equal, and as AB to BD, ſo EF to FH; alſo the angles DBC and HFG equal; and as DB to BC, ſo HF to FG; the parallelograms AL and BK ſhall be alike to the parallelograms FP and FO.

a) 14. 11. But [a] as well the three ſides BI, BK, and BL of the parallelepipedon BM, are equal and alike to the three oppoſite ſides DM, AM, and CM; as the three ſides FN, FO, and FP, of the parallelepipedon FQ, are equal and alike to the three oppoſite ſides HQ, EQ, and GQ; therefore the ſix plains conteining the ſolid BM, are equal to the ſix plains comprehending the ſolid FQ:



b) 9. def. 11. Wherefore [b] the parallelepipedon BM and FQ are alike: But forasmuch as the lines LIPN being drawn, c) 15. 5. [c] the priſmes DBCILA and HFGNPE have the ſame proportion as the parallepipedons BM and FQ their double, d) 11. 5. and the pyramids ABCD and EFGH the ſame proportion as the ſaid priſmes the triple of them, [d] the pyramids ſhall have alſo the ſame proportion as the parallelepipedons.

e) 33. 11. But [e] the parallelepipedons BM and FQ being in triple proportion of their homologal ſides BC and FG, the pyramids ABCD and EFGH ſhall be alſo in triple proportion of BC to FG: Therefore, &c. Which was to be demonſtrated.

COROLLARIE.

It is alſo manifeſt that the like pyramids whereof the baſes have more than three ſides, are in a triple proportion of their homologal ſides.

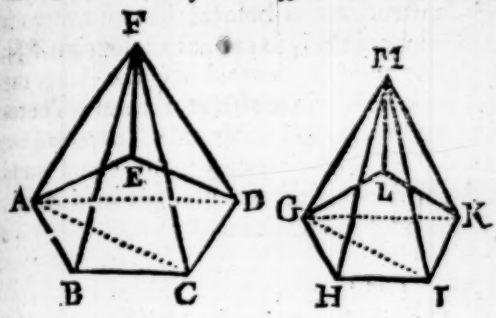
For let ABCDEF *and* GHIKLM, *be like pyramids, having for their baſes rectiline figures of ſeveral ſides, I ſay that thoſe pyramids are in triple reaſon of the homologal ſides* AB *and* GH; *for from* A *and* G *having drawn the lines* AC, AD, GI, *and* GK, *to the oppoſite angles, the like baſes ſhall be divided into an equal number of like triangles, to which* ABC, ACD, *and* ADE, *ſhall be alike to* GHI, GIK, *and* GKL; *therefore the pyramids being alike, the triangles* AFB *and* GMH *are alike, and the angle* FAB *equal to* MGH, *and as* FA *to* AB, *ſo* MG *to* GI; *the triangle* ABC *and* GHI *being alike. and therefore in equal reaſon as* FA *to* AC, *ſo* MG *to* GI.

Again, forasmuch as AC *is to* CB, *ſo is* GI *to* IH, ABC *and* GIH *being alike, and as* GB *to* CF, *ſo* IH *to* IM, *the triangles* BCF *and* HIM *being alike, by reaſon of the ſimilitude of the pyramids, and in equal reaſon as* AC *to* CF, *ſo is* GI *to* IM: *Therefore being as* FA *to* AC, *ſo* MG *to* GI, *and as* AC *to* CF, *ſo* GI *to* IM; *alſo in equal reaſon, as* FA *to* FC, *ſo is* MG

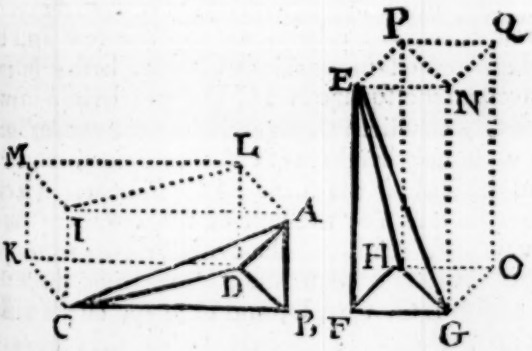
5. 6. *to* MI: *Therefore* AFC *and* GMI *ſhall be equiangled, and therefore alike.*

But AFB, BFC, *and* ABC *are alſo alike to the triangles* GMH, HMI, *and*

and GHI: *Therefore the pyramids* ABCF, *and* GHIM, *are alike, and in like manner alſo* ACDF, GHIKM, *and* ADEF *and* GKLM: *Therefore the pyramids* ABCF, ACDF, *and* ACDEF, *ſhall be to* GHIM, GIKM, *and* GKLM, *each to his correſpondent; in triple reaſon of the homologal ſides* AB, CD, DE, GH, IK, *and* KL, *each to his correſpondent, there being therefore the ſame reaſon of* AB, CD, *and* DE, *to* GH, IK, *and* KL, *the pyramids* ABCF, ACDF, *and* ADEF, *will have one and the ſame reaſon, to wit, triple to* GHIM, *and* GKLM: *Therefore as* ABCF *alone, to* GHIM *alone, ſo all of them together; to wit, the whole* ABCDEF, *to all of them together, to wit, the whole* GHIKLM. *Therefore the pyramid* ABEF *being to the pyramid* GHIM *in triple reaſon of the homologal ſides* AB *and* GH, *the pyramid* ABCDEF *ſhall be alſo to the pyramid* GHIKLM, *in triple reaſon of* AB *to* GH. *Which was propoſed.* 8.def.11. 8.12. 12.5. 8.12.



PROP. 9. THEOR. 9.



The baſes and the altitudes of equal pyramids ABCD, *and* EFGH, *having the baſes* ABC *and* EFG *triangular, are reciprocal; and pyramids having the baſes triangular, whoſe baſes and altitudes are reciprocal, are equal.*

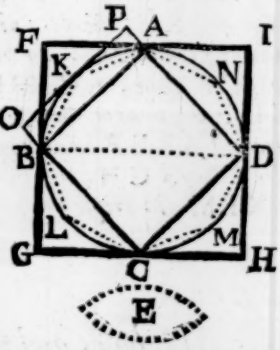
Demonſtration FOr having finiſhed the parallelepipedons BM and FQ, of the ſame height as the pyramids, and having drawn LI and PN, the priſmes DBCILA and HFGNPE, (being triple to the pyramids ſuppoſed equal,) ſhall be equal to oue another; and therefore the paralellepipedons BM and FQ alſo equal, being double to the priſmes; and [a] therefore their baſes and their heights are reciprocal, that is to ſay, as the baſe BI to the baſe FN, ſo the height of the ſolid FQ to the height of the ſold BM [b]. a) 34.11. b) 15.5.

But as the baſe BI to the baſe FN, ſo the triangle ABC to the triangle EFG; therefore alſo as the baſe ABC to the baſe EFG, ſo the height of the pyramid EFGH to the height of the pyramid ABCD; ſeeing the heights of the pyramids are the ſame as thoſe of the paralellepipedons: Therefore the baſes and the heights of equal pyramids are reciprocal.

Now

Now let the baſes and heights be reciprocal; I ſay that the pyramids are equal. For having made the Conſtruction as beforeſaid: Foraſmuch
c) 15. 5. c as A B C is to E F G as the parallelogram B I, to the parallelogram F N, and the heights of the parallelepiped. and the pyram. being the ſame, the baſes of the paralellograms, and their heights are alſo reciprocal; There-
d) 34. 11. fore d the parallelepipedons B M and F Q ſhall be equal: Therefore the priſmes D B C I L A and H F G N P E their halves, are alſo equal; and therefore alſo the pyramids the third parts of the priſmes alſo equal: Therefore, The baſes, &c. Which ought to be demonſtrated.

PROP. 10. THEOR. 10.



Every Cone is the third part of the Cylinder, which hath the ſame baſe as the Cylinder, and the height equal thereto.

Demonſtration FOr otherwiſe the cylinder ſhall be greater or leſſe than the triple of the cone: Let it in the firſt place be greater (if poſſible) as by the magnitude of the ſolid E; that is to ſay, that from the cylinder (whoſe baſe is the circle A B C D,) E being cut off, the reſt ſhall be the triple of the cone, where f the ſame circle A C is the baſe: In the ſame circle let there be inſcribed the ſquare A B C D, and divided in two triangles by the diameter B D, on which triangles let there be underſtood to be made two priſmes of equal height with the cylinder, and the ſquare being greater than the half of the circle, as it appears, it is evident that thoſe two priſmes ſhall exceed the half of the cylinder: But if (the two priſmes taken away,) the reſidue of the cylinder are greater yet than E, on the baſes of the ſaid reſts or ſegments, let there be made the four Iſoſceles triangles A K B, B L C, C M D, and D N A, on the which let there be imagined four priſmes of the ſame height with the cone or cylinder, of which the circle A B C D is the baſe.

It is manifeſt that theſe Iſoſceles triangles are more than the halfe of the baſes of the ſegments reſting of the cylinder, being the halves of the rectangles of the ſame height as B P: Wherefore theſe four priſmes ſhall be more than the half of the four ſegments: But if the eight little ſegments reſting of the cylinder, are not leſſe than E: After the ſame manner let there be ſtill ſubſtracted more than the halfe of what remaineth;
a) 1. 10. and a in the end there will remaine a magnitude leſſe than E; and in brief, let now theſe eight ſegments remaining of the cylinder, being taken together, be leſſe than E.

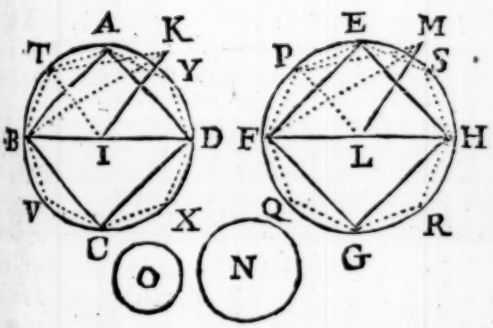
It is therefore evident as the column compounded of theſe ſix priſmes, having for baſe the polygon inſcribed in the circle A C, and of the ſame height as the given cylinder, ſhall be more than the triple of the cone, according to what hath been ſaid before: But each priſme being the triple of his pyramid of the ſame height, and which hath the baſe equal,
b) 7. 12. b may be divided in three ſuch equal pyramids: It followes that all theſe priſmes making the column, which hath for baſe the polygons inſcribed in the circle A C; that is to ſay, that column of the ſame height, as the cylinder

cylinder is the triple of theſe ſix pyramids which do make the pyramid, having the ſame polygon for baſe, and of the ſame height as the ſaid column: Therefore the ſaid pyramid ſhall be greater than the cone of the ſame height, having the circle AC for baſe, which is abſurd, the pyramid being no other then part of the cone: Therefore the cylinder is not greater than the triple of the cone.

Let it be then leſſer (if poſſible,) by the quantity of E, that is to ſay, that taking off E from the cone, that the reſt may be the third of the cylinder: Now from the cone whereof the circle A C is the baſe, let there be taken more than the half, to wit, the pyramid of the ſame height, whereof the baſe is the ſquare A B C D, and from the reſt, to wit, the four ſegments K, L, M, and N, let there be taken more than the halfe, to wit, the pyramid of each ſegment of the ſame height as the ſaid ſegment, and having for baſe the Iſoſceles triangle in the ſame ſegment; and let this cutting off be continued until that the remaining ſegments be leſſer than E, which c will happen, and in five, let the eight ſmall ſegments be leſſe than E. c) 1. 10.

It is therefore manifeſt that the pyramid of the ſame height as the cone, having that octogon for baſe, is greater than the third of the given cylinder, being greater than the cone, having cut off from it E. But the cone thus cut off, is the third of the given cylinder, and the pyramid alſo the third of the column of the ſame height, having for baſe the ſame octogon, as hath been ſaid above: Therefore that column ſhall be greater than the given cylinder, whereof it is a part, which is abſurd: Therefore the given cylinder is neither greater nor leſſe than the triple of the cone; therefore equal: Therefore, &c. Which was to be demonſtrated.

PROP. 11. THEOR. 11.



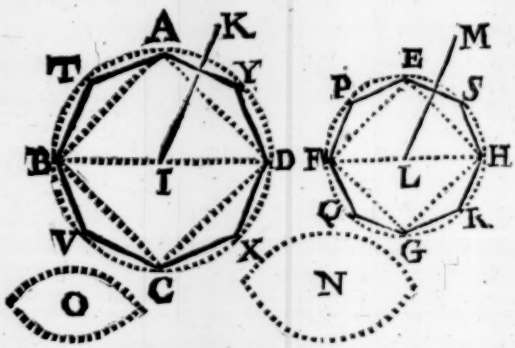
Cones and the cylinders which are of the ſame height, I K *and* L M, *are the one to the other as their baſes* ABC *and* E F G.

Demonſtration FOr if it be not ſo, it ſhall be greater or leſſer, ſuppoſe it in the firſt place to be leſſe, and that by the quantity of O, (if poſſible,) therefore N and O ſhall be equal to the cone F L M, and from the cone F L M, (as in the tenth Propoſition) Let there be taken more than the halfe, to wit, a pyramid of the ſame height as the cone, whereof the baſe is the ſquare E F G H, and from the reſt alſo more than the halfe, to wit, four pyramids of the ſame height,) having for baſes the four Iſoſceles triangles E P F, F Q G, G R H, and H S E, and ſo following, there will remain in the end a magnitude leſſe than O, which let be the eight little ſegments: It follows that the pyramid F M of the ſame height as the cone F M, whereof the baſe is the octogon E P F Q G R H S, ſhall be greater than N.

Within the circle A B C D let there be inſcribed a polygon alike to the po-

polygon inſcribed in the circular baſe E F G H, and on the ſame let there be underſtood to be a pyramid of the ſame height as the cone B I K: But
a) Co. 2. 12. a as the circle A B C is to the circle E F G, ſo the polygon T V X Y is to the polygon P Q R S; but as the circle A B C is to the circle E F G, ſo the
b) 6. 12. cone B K to N, and b as the polygon to the polygon, ſo the pyramid to the
c) 11. 5. pyramid of the ſame height as the cones; therefore c the pyramid A T B V C X D Y K ſhall be to the pyramid E P F Q G K H S M, as the cone B I K to the ſolid N: But A T B V C X D Y G being leſſe than the cone B I K,
d) 14. 5. the part than the whole, ; d alſo the pyramid E P F Q G R H S M ſhall be leſſe than N; but it hath been ſhewn to be greater, which is abſurd: Therefore N is not leſſe than the cone E F G H M.

Secondly, ſuppoſe N to be greater than E F G H M; therefore being put as the circle A B C to the circle E F G, ſo the cone B K to N; by converſion of proportion, as the circle E F G to the circle A B C, ſo N to the cone B K, and as N to B K, ſo the cone E F G H M is put to O, and N be-
e) 14 5.. ing put greater than the cone F M, e the cone D K ſhall be alſo greater than O: Therefore ſeeing that as the circle E F G is to the circle A B C, ſo N to the cone B K; f as the circle E F G H is to the circle A B C, ſo the cone F M to O, leſſe than the cone B K, which is abſurd; being already ſhewn that the cone cannot be to another magnitude leſſe than the other cone, as the baſe of the one to the baſe of the o-



f) 11. 5. other: Therefore N is neither greater nor leſſe than the cone F M; therefore equal: Therefore, being put as the baſe A B C D to the
g) 7. 5. baſe E F G H, ſo the cone B K is to N; but g as the cone B K is to N, ſo the ſame cone B K, to the cone F M; alſo as the baſe A B C D, to the baſe E F G H, ſo the cone A B C D K, to the cone E F G H M.

h) 15. 5. But foraſmuch as the cone A B C D K h is to the cone E F G H M, as the cylinder A B C D K, (which is triple the cone A B C D K,) is to the cylinder E F G H M, (which is triple the cone E F G H M,) alſo as the baſe A B C D ſhall be to the baſe E F G H, ſo the cylinder A B C D K ſhall be to the cylinder E F G H M, which may alſo be demonſtrated; ſo as of the cones, if in lieu of cones and pyramids, be underſtood cylinders and priſmes: Therefore, Cones and cylinders, &c. Which was to be demonſtrated.

CORLLARIE.

Hence it followes that cones and cylinders made on the ſame baſe, or equal baſes and of the ſame height are equal, and contrariwiſe equal cones and cylinders, made on the ſame baſe, or equal baſes, are of the ſame height, and equal cones and cylinders, and of the ſame height, are alſo on equal baſes, or one and the ſame; which ſhall be demonſtrated as in the converſe of the 31th. Prop. of the Eleventh Book.

PROP. 12. THEOR. 12.

Like cones and cylinders are the one to the other in triple pro-

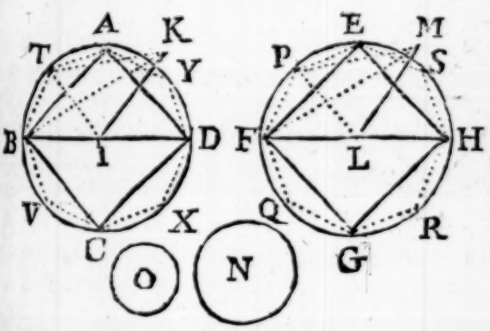
proportion of the Diameters BD *and* FH, *of their baſes* ABC *and* EFG.

Demonſtration IF it be not ſo, let the cone ABCDK be greater or leſſer than the cone EFGHM, by another magnitude N, in triple proportion of the diameter BD to the Diameter FH.

And let N in the firſt place be leſſe than the cone EFGHM, by O, and let there be made the ſame conſtruction as in the precedent Propoſition, ſo as that the pyramid EPFQGRHM be again ſhewn greater than N; and let the right lines KB, KT, MF, and MP, be drawn, to have two triangles BKT and EMP, of the pyramids ATBVCXDYK, and EPFQGRHSM; and let TI and PL be joyned.

Foraſmuch as the cones ABCDK and EFGHM are put alike, [a] the axis IK ſhall be to the axis LM, as the diameter BD to the diameter FH: Therefore as the Semidiameter BI to the Semidiameter FL, ſo the axis IK to the axis LM; and alternately, as BI to IK, ſo FL to LM. a) 24.def.11

Then ſeeing that the angles BIK and FLM [b] are right angles, (the cones being put right,) therefore the axes being perpendicalar to their baſes, [c] the triangles BIK and FLM ſhall be equiangled; therefore as KB to BI, ſo MF to FL. But as BI to BT, ſo FL to FP; becauſe of the ſimilitude of the triangle BIT to FLP, (for ſeeing BIT and FLP, inſiſting on like arches, are equal; and as BI is to IT, ſo FL to LP, as well by the equality of the lines BI and IT, as FL and LP, [d] the triangles BIT and FLP are alike,) and in proportion of equality, as KB to BT, ſo MF to FP. b) 3.def.11. c) 6. 6. d) 6. 6.

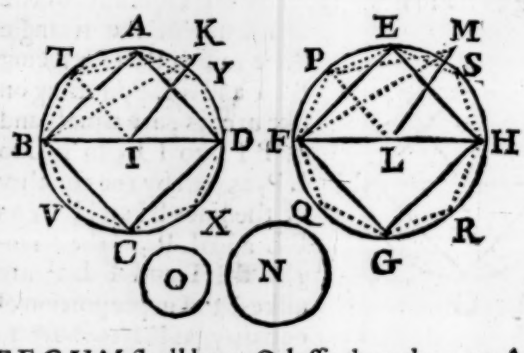


Again, foraſmuch as the ſides KI and IB of the triangle KIB, are equal to KI and IT of the triangle KIT, and the angles conteined of them [e] right angles, the axis IK being put perpendicular to the circle ABC, [f] the baſes KB and KT ſhall be equal; and in like manner MF and MP equal; therefore KB and KT ſhall be proportional to MF and MP, there being proportion of equality on both parts; but ſeeing that as KB to BT, ſo KT to the ſame BT; alſo as MF to FP, ſo MP to the ſame FP; but as KB was to BT, ſo MF to FP: In like manner, as KT ſhall be to BT, ſo MP to FP; and by converſion, as BT to TK, ſo FP to PM; therefore ſeeing that as TK is to KB, ſo PM to MF, and as KB to BT, ſo MF to FP, and as BT to TK, ſo FP to PM, as is ſhewn; the triangles BKT and FMP ſhall have the ſides proportional; therefore [g] are equiangled and alike; and [h] in like manner, the other triangles about the pyramids ATBVCXDY and EPFQGRHSM, ſhall be alike the one to the other, which being equal in number, [i] the ſame pyramids ſhall be alike, and [k] ſhall be in proportion triple of their homologal ſides BT and FP: But as BT to FP, ſo BI to FL; by the ſimilitude of the triangles BIT and FLP; and [l] as BI to FL, ſo BD to FH: Therefore e) 3.def.11. f) 4. 1. g) 6. 6. h) 1. def. 6. i) 9. def. 11. k) Co. 8. 12. l) 11. 5.

the pyramid ſhall be to the pyramid in triple proportion of the diameters B D and F H. But the cone A B C D K was put to N in triple proportion of the ſame diameters: Therefore as the pyramid A T B V C X D Y K to the pyramid E P F Q G R H S M, ſo the cone A B C D K to N: Wherefore ſeeing that the pyramid A T B V C X D Y K is leſſe than the cone
m) 14. 5. A B C D K the part than the whole, [m] the pyramid A P F Q G R H S M ſhall be leſſe than N. But it is ſhewn to be greater, which is abſurd: Therefore N is not greater than the cone E F G H M.

Secondly, ſuppoſe N to be greater than the cone E F G H M: Therefore the cone A B C D K being put to N in proportion triple of the diameter B D to F H, and the pyramid A T B V C X D Y K being to the pyramid E P F Q G R H S M in triple proportion of the ſame diameters, as is ſhewn, as the cone A B C D K ſhall be to N, ſo the pyramid A T B V C X D Y K, to the pyramid E P F Q G R H S M; and alternately, as N to the cone A B C D K, ſo the pyramid E P F Q G R H S M to the pyramid A T B V C X D Y K.

n) Co. 8. 12. Therefore [n] the pyramid E P F Q G R H S M being to the pyramid A T B V C X D Y K in triple proportion of the homologal ſides P F and T B; that is to ſay, of the diameter F H to the diameter B D, N ſhall be alſo to the cone A B C D K in triple proportion of the diameter F H to the diameter B D, as N is to the cone A B C D K, ſo the cone E F G H M is put to O. Therefore the cone E F G H M ſhall be alſo to O in triple proportion of the diameter F H, to the diameter B D, and N being put greater than the cone E F G H K,



o) 14. 5. [o] the cone A B C D K ſhall be alſo greater than O; therefore the cone E F G H M ſhall be to O, leſſe than the cone A B C D K in triple proportion of the diameter F H to the diameter B D, which is abſurd: For it is ſhewn that the cone cannot be in triple proportion of the diameters of the baſes to another magnitude leſſe than the other cone; therefore N is not greater than the cone E F G H M, and is ſhewn not to be leſſe, therefore equal:
p) 7. 5. Wherefore [p] the cone A B C D K hath the ſame proportion to the cone E F G H M, as to N.

Therefore ſeeing that the cone A C B D K is to N in triple proportion of the diameters B D and F H, the cone A B C D K ſhall be alſo to the cone E F G H M, in triple proportion of the ſame diameters.

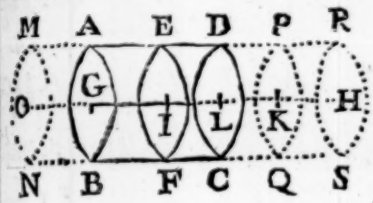
q) 15. 1. But [q] foraſmuch as the cylinders are in the ſame proportion as the cones whereof they are triples, the cylinder ſhall be to the cylinder in the ſame triple proportion of the diameters of the baſes; which ſhall be demonſtrated; as of the cones, if in lieu of cones and pyramids, you take cylinders and priſmes: Therefore, Like cones and cylinders, &c. Which was to be demonſtrated.

PROP. 13. THEOR. 13.

If a cylinder A B C D, *be cut by a plain* E F, *parallel to the*

the oppoſite plains A B *and* C D, *as the cylinder* A F *ſhall be to the cylinder* E C, *ſo the axis* G I *ſhall be to the axis* I L.

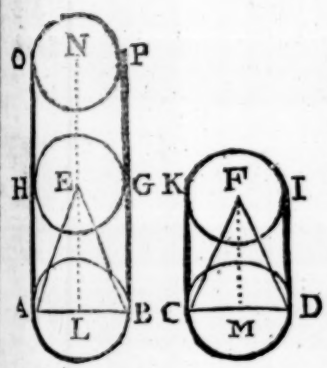
Demonſtration FOr having prolonged the axis G L on both parts, let G O be put equal to G I, and L K and K H each equal to I L, and let it be underſtood that the cylinder is prolonged from both parts to O and H, that being, it is manifeſt that the cylinders A N, A F, E C, C P, and Q R, are all on equal baſes; [a] and therefore A N and A F having the ſame height, that is to ſay, having the axes equal, I G and G O are equal to one another: In like manner, F D, D Q, and Q R, being of the ſame height, are alſo equal to one another; wherefore the axis I O is ſo multiplex of the axis G I, as the cylinder E N, of the cylinder A F, and the axis I H ſo multiplex of the axis I L, as the cylinder F R of the cylinder F D, as is manifeſt. a) C. 11. 12.



Therefore if the axis O I, multiplex of G I firſt magnitude, be equal, greater, or leſſe than I H, multiplex of I L ſecond magnitude; alſo the cylinder E N multiplex of the cylinder E B third magnitude, ſhall be greater, equal, or leſſe than the cylinder F R, multiplex of the cylinder F D fourth magnitude, in any multiplication whatever; wherefore [b] as the axis to the axis, ſo is the cylinder to the cylinder: Therefore, If a cylinder, &c. Which was to be demonſtrated. b) 6. def.

PROP. 14. THEOR. 14.

Cones A B E *and* C D F, *and cylinders* A B G H *and* C D I K, *made on equal baſes* A B *and* C D, *are to one another as the heights* L E *and* M F.



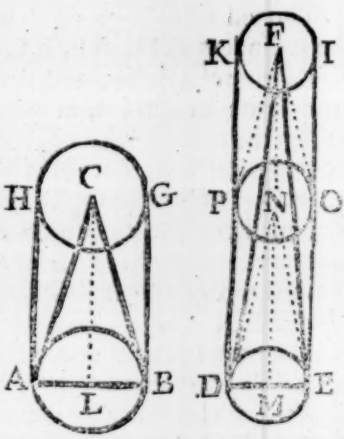
Demonſtration FOr let the cylinder A G be prolonged on the part of G H, with his axis L E, and the rectangle A G; and let the axis E N be taken equal to the axis M F, and about the center N let there be underſtood to be a circle O P, equal and parallel to G H, laſtly, let the cylinder G O be made of the ſame height as the cylinder C I.

Foraſmuch as the cylinders H P and C I, having the baſes and heights equal, [a] are equal; the cylinder A G ſhall have the ſame proportion to them. But [b] the cylinder A G is to the cylinder H P, as the axis or height L E, to the axis or height E N; that is to ſay, to the height M F his equal: Therefore the cylinder A G ſhall be alſo to the cylinder C I, as the height L E to the height M F. a) C. 11. 12. b) 13. 12.

But [c] foraſmuch as the cones A B E and C D F are the third parts of the cylinders A G and C I, [d] they ſhall be in the ſame proportion as the c) 10. 12. d) 15. 5.

cylinders: Therefore the cone ABE ſhall be to the cone CDF, as the height LE, to the height MF: Therefore, Cones, &c. Which was to be demonſtrated.

PROP. 15. THEOR. 15.



The baſes AB *and* DE, *and the heights* LM *and* MF, *of equal cones* ABC *and* DEF, *and cylinders* AG *and* DI, *are reciprocal, and cones and cylinders, whoſe baſes and heights are reciprocal, are equal.*

Demonſtration FOr if the heights LC and MF are equal, the cylinders being put equal, a) Co. 11. 3. a the baſes ſhall be equal; therefore as AB to DE, ſo MF to LC; therefore the baſes and the heights are reciprocal.

And if the one be greater, as MF; let there be cut off MN, equal to LC, and let the plain PO be underſtood to paſſe by N, and parallel to DE, cutting the cylinder DI in two; therefore the two cylinders AG and DI, being put equal; b) 7. 5. b as AG ſhall be to DO, ſo DI to DO; but c) 11. 12. c as the cylinder AG to the cylinder DO, ſo the baſe AB to the baſe DE, the heights CL and MN being equal d) 14. 12. d.

Alſo as DI to DO, ſo the height MF to MN, being the baſes are equal; to wit, one and the ſame DE; therefore alſo as the baſe AB to the baſe DE, ſo the height MF to the height MN; that is to ſay, to his equal LG; therefore the baſes and the heights are reciprocal.

And if the cones ABC and DEF are equal, the cylinders AG and DI ſhall be equal, e) 10. 12. e the cones being the thirds of the cylinders: Therefore from the equality of the cylinders, it followes that the baſes and the heights are reciprocal, as is ſhewn; and therefore from the equality of the cones it will alſo follow that the baſes and the heights are reciprocal; which may alſo be demonſtrated by the diſcourſe uſed about the cylinders, if there be conſtituted two cones, as appears by the figure.

Now let the baſes and heights be reciprocal; I ſay that the cones and the cylinders are equal: If the heights be equal, being to one another as the baſes, their baſes ſhall be alſo equal; therefore f) Co. 11. 12. f the cylinders are equal.

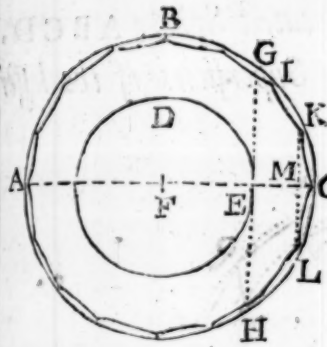
If unequal, let the ſame conſtruction be made as before; therefore the height MF being put to the height LC; that is to ſay, to MN his equal, as the baſe AB to the baſe DE; but g) 11. 12. g as AB to DE, ſo the cylinder AG to the cylinder DO, of the ſame height h) 14. 12. h.

Alſo as the height MF to the height MN, ſo the cylinder DI, to the cylinder DO, the baſes being equal, as the cylinder AG to the cylinder DO, ſo DI to the ſame cylinder DO; therefore i) 9. 5. i the cylinder AG ſhall be equal to the cylinder DI.

And if the baſes and heights of the cones ABC and DEF are reciprocal, the baſes and the heights of the cylinders AG and DI, being the

the ſame baſes and heights of the cones, ſhall be reciprocal: Therefore as hath been ſhewn, the cylinders, and therefore the cones, the third parts of them, are equal: Nevertheleſſe, the ſame demonſtration may be made of the cones that hath been made of the cylinders: Therefore, The baſes, &c. Which was to be demonſtrated.

PROP. 16. PROBL. 1.



Two circles A B C *and* D E, *being about one and the ſame center* F, *to inſcribe in the greateſt of them* A B C, *an equilateral polygon, and of an even number of ſides, and which ſhall not touch the leſſer circle* D E.

Conſtruction. LEt A C be drawn by the center F, cutting the circle D E in E, and by E let G H be drawn perpendicular to A C, [a] which will touch the circle at E: Foraſmuch then as the arch A G C is greater than the arch G C, if from A G C the half be taken A B, and from the reſt B C, the half B I; and of the reſt I C, the half I K, and ſo following; [b] there will remain at laſt an arch leſſe than C G, which let be the ſubtenſe C K; I ſay that C K is one ſide of the polygon which ought to be inſcribed.

a) Co. 16. 3.

b) 1. 10.

Demonſtration. FOr if the arch B I be divided into equal parts in number and magnitude, in the parts of the arch C I, and the quarter of the circle A B; let be divided into ſo many equal as there are equal parts in the arch B C: and alſo the ſemicircle A H G: Then let there be drawn the right lines ſubtending all the arches, which [c] ſhall be equal to the right line C K; foraſmuch as they ſubtend arches equal to the arch C K, and ſo you ſhall have deſcribed an equilateral polygon in the circle A B C, and of an even number of ſides which ſhall not touch the leſſer circle D E.

c) 29. 3.

For from K to A C, let there be drawn the perpendicular K L, cutting A C in M; foraſmuch as the angles G E M and K M E are right angles, [d] H G and K L ſhall be parallels; therefore G H touching the circle D E in the point E, the right line K L ſhall be wholly without the ſaid circle, and ſhall never touch it, never meeting G H: Therefore the right line G K which is farther diſtant from the circle D E, ſhall leſſe touch the circle D E than K L; and therefore alſo neither ſhall the other ſides of the inſcribed polygon, being equal to the ſide C K; and therefore ſhall be ſo farre diſtant from the center F as C K, and ſhall not touch the circle D E: [e] Therefore, Two circles, &c. Which was to be demonſtrated.

d) 29. 1.

e) 14. 3.

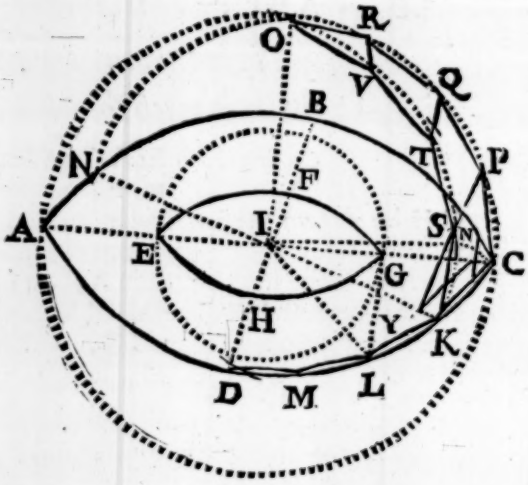
COROLLARIE.

It is manifeſt from this, that if from the extremity of the ſide of the polygon inſcribed, which doth meet with the diameter, be drawn a line perpendicular to the diameter, that line cannot in any wiſe touch the leſſer circle, but ſhall fall wholy with-

without it. For such is the line K L, *which drawn from the extremity* K *of the side* C K, *meeting with the diameter* A C, *perpendicular to the diameter* A C, *is demonstrated not to touch the circle* D E.

PROP. 17. PROBL. 2.

Two Spheres A B C D *and* E F G H, *being about one and the same center* I, *to inscribe in the greatest Sphere* A B C D, *a polyhedron which shall not touch the Superficies of the lesser Sphere* E F G H.



Construction LEt the two Spheres be cut by some plain drawn by the center I, and let the common sections be made in the two Spheres, the plains A B C D and E F G H, (which shall be circles, by the definition of the Sphere,) having the same center I with the Spheres; for the semicircles, by the revolution of which the spheres are described, will agree with the sections A B C D and E F G H: wherefore the said sections shall be circles: Or, seeing that all the right lines drawn from the center I, to the circumferences of the sections are equal; and in them are drawn the diameters A C and B D, cutting one another at right angles in the center I; [a] then in the greatest circle A B C D, let there be inscribed a polygon, that may not touch the lesser circle E F G H; and let C K, K L, L M, and M D, be sides of the quarter C D; and let K N be drawn by the center I; and [b] from I let there be drawn I O, perpendicular to the plain of the circles A B C D and E F G H, meeting in O the superficies of the greatest Sphere, and by A C, I O, and N K, I O, let there be drawn the plains A O C and N O K, they shall be semicircles, the greatest amongst those of the sphere, having the same center, as is said, and I O the line of common section of the plains of the semicircles N O K and A O C, being perpendicular to the plain of the circle A B C D; [c] those semicircles shall be also perpendicular to the plain of the circle A B C D, and the three semicircles A D C, A O C, and N O K, being equal, their diameters being equal, their halves, to wit, the quarters O C,

a) 16. 12.

b) 12. 11.

c) 18. 11.

OC, OK, and DC, ſhall be equal: Wherefore OC and OK ſhall contein each as many ſides of the polygon as DC.

Let thoſe ſides be CP, PQ, QR, RO, KS, ST, TV, and VO, and let there be drawn SP, TQ, and VR, and from P and S, the perpendiculars PX and XY; [d] they ſhall fall on IC and IK, common ſections of the ſquares IOC and IOK, and the plain ICD, raiſed perpendicularly on the ſame plain, [e] they ſhall be alſo parallel, and let XY be alſo drawn. d) 38. 11. e) 6. 11.

Now the arches PC and SK being equal, and [f] the right lines ſubtending (PC and SK) alſo equal; the angles PXC and SYK right angles, and [g] the angles PCX and SKY equal, inſiſting on equal circumferences AOP and NOS, the two angles PCX and PXC of the triangle PXC, are equal to the two angles SKY and SYK, and the ſides PC and SK equal, [h] the two other ſides PX and XC ſhall be equal to the two other ſides SY and YK each to his correſpondent ſide. Therefore the diameters AC and NK being equal, and the ſegments CX and KY alſo equal, AX and NY ſhall be alſo equal, and IX and IY equal. Wherefore as IX to XC, ſo IY to YK; therefore [i] XY is parallel to KC, and PX and SY being ſhewn equal and parallel, SP and YX [k] ſhall be alſo equal and parallel, and [l] SP and KC ſhall be parallel, and [m] SK and PC in one and the ſame plain with them. f) 29. 3. g) 27. 3. h) 26. 5. i) 2. 6. k) 33. 1. l) 9. 11. m) 7. 11.

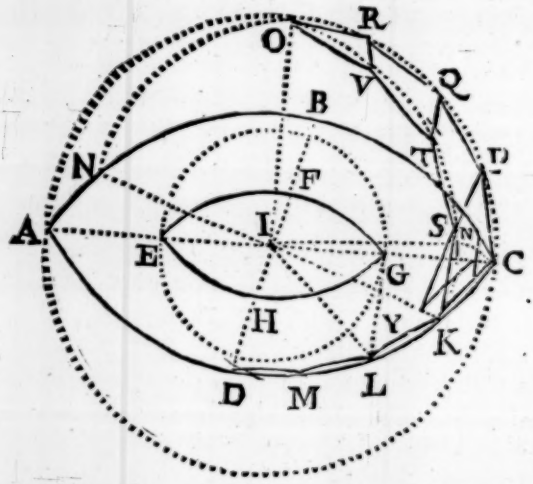
Therefore the whole quadrilateral figure SKCP is in one and the ſame plain, ſo it's ſhewn that each of the quadrilateral figures, as TP and VQ hath all his parts in one and the ſame plain, and the triangle ORV in like manner; and if from the points P, Q, R, S, T, and V, be drawn lines to the center I, there may be imagined a ſolid figure, compoſed of four pyramids, between OC and OK, whoſe tops are in the center I, and their baſes are the quadrilateral figures CS, PT, and QV, and the triangle OVR; and doing the ſame on the other ſides KL, LM, and MD, as on CK, and alſo on the three other quarters BC, BA, and AD, there will be inſcribed a polyhedron in the propoſed Sphere: I ſay it touches not the leſſer Sphere.

For having drawn the line IZ, from the center I, perpendicular to the plain CS, it ſhall be greater than IG, the ſemidiameter of the leſſer Sphere: For to inſcribe the polygon, (having drawn GL perpendicular to AC from the point G, which being greater than CK, the ſide of the inſcribed polygon, as is manifeſt;) let IL be drawn, and alſo CZ and KZ, [n] the angles IZC and IZK being right angles, the ſquare of IC [o] ſhall be equal to the two ſquares of IZ and CZ, and the ſquare of IK equal to the ſquares of IZ and ZK: Wherefore the ſquares of the diameters being equal, the ſquares of IZ and ZK ſhall be equal to the ſquares of IZ and CZ: Therefore if the common ſquare of IZ be taken away, the ſquares of CZ and KZ will remain equal; and therefore KZ and CZ equal; and having drawn the lines from Z to S and P, it will be demonſtrated by the ſame reaſons that they ſhall be equal, as well to one another, as to CZ and ZK; therefore if from Z, taken for the center, there be deſcribed a circle, at the diſtance of one of them, it will paſſe by all the points C, K, S, and P; and therefore ſhall be deſcribed about the ſaid quadrilateral figure, whoſe three ſides CP, CK, and KS, are equal, [p] ſubtending equal arches of equal circles, and the fourth SP leſſer. n) 2. 11. o) 47. 1. p) 29. 3.

For ſeeing that the triangles ICK and IXY are alike, and the ſide

IC

q) 14. 5. I C greater than the ſide I X, q C K ſhall be alſo greater than Y X, that is to ſay, than S P, ſhewn to be equal to Y X, the angle C Z K ſhall be greater than a right angle; (for the four lines drawn from the point Z to the points K, C, P, and S, being demonſtrated equal, the four triangles
r) 4. 1. which are made, whereof the tops are at the point Z, r ſhall have the four angles at Z equal to four right angles, and the baſes of the three triangles S Z K, K Z C, and C Z P, being ſhewn equal, the three angles oppoſed to them ſhall be equal; therefore each one ſhall be greater than a right angle, ſeeing the baſe C K is ſhewn to be greater than S P, and the angle C Z K greater than the angle S Z P, therefore greater than a right angle, and ſo the greateſt of the triangle C Z K; therefore the ſide C K ſhall be greater than C Z, and G L being greater than C K, ſhall be alſo greater than C Z. Wherefore the ſquare of G L ſhall be greater than the ſquare of C Z, and the ſquares of I G and G L being equal to the ſquares of I Z and C Z, (as well thoſe two, as the other two, being equal to



the ſquare of the ſemidiameter, ſeeing that the triangles I G L and I C Z are rectangles,) and the ſquare of G L being greater than the ſquare of C Z, the ſquare of I Z ſhall be alſo greater than the ſquare of I G; and therefore I Z greater than I G; therefore the plain C K S P touches not the leſſer Sphere E F G H; and ſo it may be demonſtrated that all the other plains of the polyhedron in the greateſt Sphere, touches not the ſuperficies of the leſſer Sphere E F G H: Therefore, Two Spheres, &c. Which was to be demonſtrated.

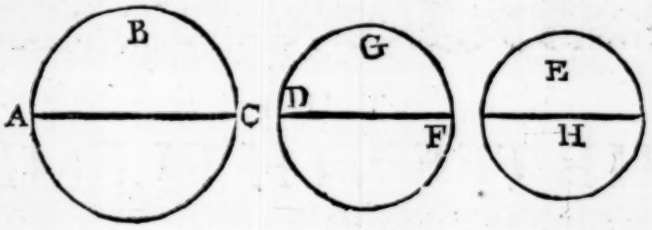
COROLLARIE.

From this Demonſtration follows, that if in another Sphere there be deſcribed a ſolid polyhedron, alike to the aboveſaid, they ſhall be in triple proportion of the diameters of the Spheres in which they are inſcribed; for being alike by the definition of like ſolids, they will contein in their convexitie as many like plains the one as the other. Wherefore they may be divided into as many like pyramids the one as the other, having all of them the ſemidiameter of their Spheres, for one of their ſides: Therefore being taken one and one, ſhall be in triple proportion of their s) Co. 8. 12. *homologal ſides,* t) 12. 5. *which are the ſemidiameters of their Spheres, and t the whole to the whole, alſo in triple proportion of their ſemidiameters;* v) 15. 5. *v and therefore alſo in triple proportion of the diameters intire.*

PROP.

PROP. 18. THEOR. 16.

Spheres A B C *and* D G F, *are the one to the other in triple proportion of their diameters* A C *and* D F.



Demonstration FOr if it be not ſo, it ſhall be greater or leſſer; Firſt then, Suppoſe E be leſſe than D E F, then E may be incloſed in D G F, putting both the two about one and the ſame center: Wherefore [a] in D G F may be inſcribed a ſolid polyhedron, which ſhall not touch the leſſer circle E, and by the ſame reaſon may be deſcribed a ſolid polyhedron in the other, which [b] ſhall be to that which is deſcribed in D G F in triple proportion of the diameter A C to the diameter D F; but the Sphere A B C is put in ſuch proportion to the Sphere E: Therefore [c] as the Sphere A B C to the Sphere E, ſo the polyhedron of the Sphere A B C ſhall be to that of D G F; but the Sphere A B C is greater than his polyhedron, the whole than his part; [d] therefore E ſhall be greater than the polyhedron of the Sphere D G F, the part than the whole, which is abſurd: Wherefore A B C is not to another Sphere leſſe than D G F in triple proportion of the diameter A C to the diameter D F.

a) 17. 12.
c) C. 17. 12.
c) 11. 5.
d) 14. 5.

Now ſuppoſe E to be greater than D G F, if poſſible, and it being put as the Sphere A B C is to the Sphere E, ſo the triple proportion of the diameter A C to the diameter D F; alternately, E ſhall be to A B C in triple proportion of D F to A C; in like manner it may be underſtood that as E is to A B C, ſo D G F is to a fourth proportional, which [e] ſhall be leſſe than A B C, E being put greater than D G F; wherefore this fourth may be inſcribed in A B C, and [f] D G F ſhall be to the ſaid fourth proportional inſcribed in A B C, in triple proportion of D F to A C, which is ſhewn to be abſurd: Therefore E is neither greater nor leſſe than D G F, therefore equal: Therefore, Spheres, &c. Which was to be demonſtrated.

e) 14. 5.
f) 11. 5.

COROLLARIE.

It is manifeſt from this Demonſtration that one Sphere is to another Sphere, as the polyhedron inſcribed in the one, is to a like polyhedron and alike inſcribed in the other, being ſhewn that as well the Spheres as the polyhedrons inſcribed in them, are in triple proportion of their diameters.

The End of the Twelfth Element of EUCLIDE.

THE THIRTEENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

THE chief matter conteined in this Thirteenth Book is concerning the Paſſions of a line divided by extream and mean proportion, a thing of wonderful uſe in Geometry, as by this and the following Books will evidently appear: This Book alſo treateth of the compoſition of the five Regular Bodies, *viz.* the Tetrahedron, Cube, Octohedron, Dodecahedron, and Icoſahedron, and how to inſcribe them in a Sphere given. It alſo ſetteth forth certain ſimilitudes and compariſons of the ſaid Bodies one with another, and alſo with the Sphere within which they are inſcribed, with divers other matters to the ſaid Bodies relating.

PROPOSITIONS, PROBLEMES, and THEOREMES.

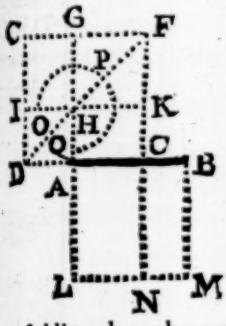
PROPOSITION 1. THEOREM 1.

If a right line AB, *be cut according to extream and mean proportion,* (*as in* C,) *the greateſt ſegment* AC, *compriſed with*

with the half AD, *of the whole* AB, *is in power quintuple of the ſquare which is deſcribed of the half of the whole.*

Conſtruction THat is to ſay, that the ſquare of the greateſt ſegment and of the halfe of the whole, as of one only line, is quintuple of the ſquare of the half of the whole.

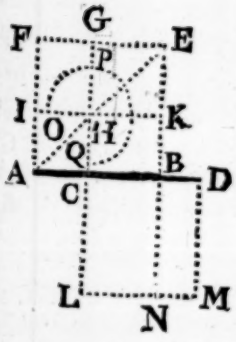
Demonſtration FOr on CD let there be deſcribed the ſquare DF, and having drawn the diameter DF, let there be drawn AG and IK, cutting it in H, parallel to DC and CF, and prolonging GA, let the ſquare AM be made, and prolonging FC to N, [a] AI and KG ſhall be the ſquares of AD and AC; and [b] foraſmuch as AB to AC, as AC to CB, [c] the rectangle CM conteined under AB and CB, ſhall be equal to KG the ſquare of AC, and AB being put the double of AD, and AL equal to AB, and AH to AD, AL ſhall be alſo double to AH. But [d] as AL to AH, ſo the rectangle AN to the rectangle AK: Therefore AN is double to AK; but [e] foraſmuch as AK is equal to IG, AN ſhall be equal to the two AK and IG.



a) Cor. 4. 2.
b) 3. def. 6.
c) 17. 6.
d) 1. 6.
e) 43. 1.

Adding then the equal figures CM and KG, the ſquare AM ſhall be equal to the Gnomon OPQ; therefore the ſquare AM being the quadruple of the ſquare AI (ſeeing that DA is put halfe of AB) the Gnomon OPQ ſhall be quadruple to the ſame AI: Wherefore if to OPQ be added AI, the ſquare FD ſhall be made quintuple to the ſquare AI: Therefore, If a right line, &c. Which was to be demonſtrated.

PROP. 2. THEOR. 2.



If a right line AB *be in power quintuple of his ſegment* AC, *the line* CD *double, the ſaid ſegment being divided according to extream and mean proportion, the other part is the greateſt ſegment of the line firſt cut.*

Conſtruction THat is to ſay, if the ſquare of a right line be quintuple of the ſquare of one part thereof, the double of that part being divided by extream and mean proportion, the greateſt ſegment is the other part of the ſaid line.

Demonſtration FOr [a] having finiſhed the Conſtruction as in the precedent, CI and KG ſhall be the ſquares of AC and CB, and foraſmuch as the ſquare BF is put quintuple to the ſquare CI, if the ſquare CI be taken away, there will remain the Gnomon OPQ, quadruple to the ſame ſquare CI; but the ſquare CM is quadruple to the ſquare CI; ſeeing that CD is double to AC; therefore the Gnomon OP 2 ſhall be equal to CM.

a) Cor. 4. 2.

Again, foraſmuch as C D is put double to A C, and L C is equal to C D, and C H to A C, A C ſhall be double to C H; therefore ſeeing that as L C is to C H, ſo the rectangle L B to the rectangle C K, that ſhall be alſo double to this: But [b] C K is equal to H F; therefore L B is equal to C K and H F; and therefore the ſquare reſting K G, to the rectangle reſting B M.

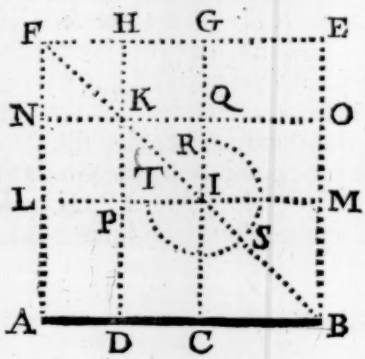


b) 43. 1.

Foraſmuch then as the rectangle B M conteined under C D and B D, is equal to K G the ſquare of C B, and [c] as C D ſhall be to C B, ſo C B to B D: Therefore C D is divided in B by extream and mean proportion, and C B is the greateſt ſegment, &c. Which was to be demonſtrated.

c) 17. 6.

PROP. 3. THEOR. 3.



If a right line A B, *be divided in* C, *according to extream and mean proportion, the line made of the leſſer ſegment, and of the half of the greateſt, is in power the quintuple of the ſquare deſcribed of the half of the greateſt ſegment.*

Conſtruction THat is to ſay, the ſquare of the leſſer ſegment, with the half the greateſt, as of one only line, is the quintuple of the ſquare of the half of the greateſt.

Demonſtration FOr on A B let there be deſcribed the ſquare A E, whoſe diameter is B F, by C and D let there be drawn C G and D H parallel to A F and B E, and cutting B F by I and K, by which let there be drawn L M and N O, parallel to A B and E F, cutting C G and D H, at P and Q, [a] L G, P Q, N H, and D O, are the ſquares of A C, C D, and B D.

a) Cor. 4. 2.

Foraſmuch then as A C is the double of C D, the ſquare L G ſhall be quadruple to the ſquare P Q, but the rectangle C E is equal to the ſquare L G; (for ſeeing that as A B to A C, ſo A C to C B, the [b] rectangle under A B and C B, to wit, C E, is equal to the ſquare of A C, to wit, to L G:) Therefore C E ſhall be alſo quadruple to P Q.

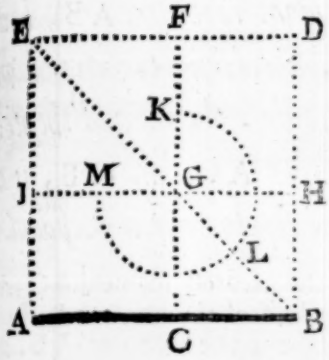
b) 17. 6.

But foraſmuch as the ſquares N H and P Q are equal, A D and C D being equal, their ſides H K and I Q ſhall be equal; and [c] therefore E O and O M oppoſite to them equal; therefore [d] the rectangles I O and Q E ſhall be equal: But [e] I O is equal to I D; therefore Q E ſhall be equal to the ſame I D, adding therefore the common rectangle C O, all the Gnomon R S T ſhall be equal to the rectangle C E, and C E being ſhewn quadruple to the ſquare P Q, the Gnomon R S T ſhall be alſo quadruple to the ſame

c) 34. 1.
d) 36. 1.
e) 43. 1.

ſame ſquare P Q, adding the common ſquare P Q, the ſquare D O deſcribed of B D, ſhall be quintuple to the ſquare P Q, deſcribed of C D: Therefore, If a right line, &c. Which was to be demonſtrated.

PROP. 4. THEOR. 4.



If a right line A B, *be divided in* C, *according to extream and mean proportion, the one and the other ſquare, to wit, that of the whole* A B, *and that of the leſſer ſegment* C B *together are triple to the ſquare deſcribed of the greateſt ſegment* A C.

Demonſtration. FOr on A B let there be deſcribed the ſquare A D, and having drawn B E the diameter, and by G drawn C F, parallel to B D, and H I to A B, cutting the diameter at G, [a] C H and I F ſhall be the ſquares of B C and A C; and foraſmuch then, as A B is to A C, as A C is to C B, [b] the rectangle A H conteined under A B and B C, is equal to I F the ſquare of A C: Therefore A H being equal to C D, the Gnomon K L M with the ſquare C H, ſhall be double to the ſquare I F: Wherefore adding I F and A D the ſquare of A B, with C H ſquare of B C, ſhall be triple to I F ſquare of A C: Therefore, If a right line, &c. Which ought to be demonſtrated.

a) Cor. 4. 2.

b) 17. 6.

PROP. 5. THEOR. 5.



If a right line A B, *be divided in* C, *according to extream and mean proportion, and that to it there be added a line* A D, *equal to the greateſt ſegment* A C, *the whole right line* B D *is cut in extream and mean proportion, and the greateſt ſegment* A B, *is the right line which was at the beginning.*

Demonſtration. FOr upon A B, let there be deſcribed the ſquare B F, and let A E the diameter, be drawn, and C G parallel to B E, cutting A E in the point H, by which let there be drawn I K, parallel to B D, and by D, D L, parallel to A F, meeting I K, prolonged at L, and [a] C K and I G ſhall be the ſquares of A C and B C.

Foraſmuch then, as A B is to A C, as A C is to C B, [b] the rectangle I F, conteined under A B and B C, ſhall be equal to C K, the ſquare of A C: But [c] C K is equal to A L, therefore I F is alſo equal to A L; adding then the common part B K, the rectangle B L, conteined under B D and D A, ſhall be equal to B F, the ſquare of A B: Therefore [d] as B D ſhall

a) Cor. 4. 2.

b) 17. 6.

c) 36. 1.

d) 17. 6.

ſhall be to A B, ſo A B to A D: Wherefore B D is divided at A in extream and mean proportion, and A B is the greateſt ſegment: Which was to be demonſtrated.

PROP. 6. THEOR. 6.



If a rational right line A B, *be divided in* C, *according to extream and mean proportion, the one and the other of the ſegments* A C *and* C B, *is an irrational line, the which is called an Apotome or a Reſidual.*

Demonſtration. FOr let there be added to the greateſt ſegment A C, the right line A D, equal to the half of the whole A B; foraſmuch a) as the ſquare of C D is quintuple to the ſquare of A D, the ſquare of C D ſhall be to the ſquare of A D, as a number to a number: Wherefore b) the ſquares of C D and A D ſhall be commenſurable, and therefore the lines C D and A D commenſurable at leaſt in power. But A D is rational, being the half of A B rational, therefore C D ſhall be alſo rational c).

a) 1. 13.
b) 6. 10.
c) Cor. 24. 8.

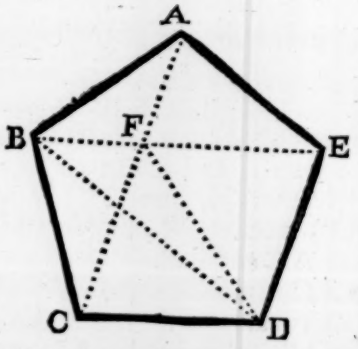
But foraſmuch as the ſquares of C D and A D are the one to the other, as a ſquare number to a ſquare number (being as 5 to 1, ſo 25 to 5,) C D and A D d) ſhall be incommenſurable in length; and therefore only rational, commenſurable in power: Wherefore if from C D rational, be taken A D rational, commenſurable in power only, e) the remain A C ſhall be irrational, which is called a Reſidual.

d) 9. 10.
e) 74. 10.

Again, the rectangle A E being applyed to A B, conteined under A B and C B, ſeeing that as A B to A C, ſo A C to C B, f) the rectangle A E ſhall be equal to the ſquare of A C: Wherefore the ſquare of the Reſidual A C, that is to ſay, the rectangle A E, applyed according to the rational A B, makes g) the other ſide B E, that is to ſay, C B, equal to the ſame, firſt Reſidual: Therefore, If a right line, &c. Which was to be demonſtrated.

f) 17. 6.
g) 98. 10.

PROP. 7. THEOR. 7.



If three angles A, B, *and* C, *of an equilateral Pentagon* A B C D E, (*whether taken in order, or not in order*) *are equal, the ſame Pentagon ſhall be equiangled.*

Demonſtration. FOr to thoſe equal angles let B E, A C, and C D, be ſubtendents, and from F, where B E and A C interſect one another, let F D be drawn: Foraſmuch as the ſides A B and A E, of the triangle A B E, are equal to C B and C D, of the

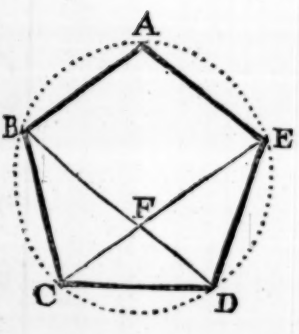
the triangle C D B, and the angles conteined of them put equal, [a] the baſes B E and B D; and the angles A E B and C D B are equal: But [b] the angles B E D and B D E are alſo equal, the ſides B E and B D, being ſhewn equal: Therefore the whole A E D and C D E ſhall be equal.

Again, the ſides A B and A E of the triangle A B E, being equal to B A and B C of the triangle B A C, and the angles conteined of them equal, by ſuppoſition, [c] the baſe B E ſhall be equal to A C, and the angles A B E and A E B equal to B A C and B C A, each to his correſpondent angle: Therefore ſeeing the angles A B E and B A C of the triangle A B F are equal, [d] the ſides B F and A F are alſo equal; therefore if they be taken from the equal lines B E and A C, the reſt F E and F C ſhall be equal: Therefore the ſides F E and E D of the triangle F E D, being equal to F C and C D of the triangle F C D, and F D common, [e] F E D and F C D, conteined of thoſe ſides ſhall be equal: But A E B and B C A are ſhewn to be equal, therefore the whole A E D and B C D are equal; therefore C D E being ſhewn equal to A E D, and the angles A B C, B A E, and B C D, being put equal, the ſame Pentagon ſhall be equiangled.

Secondly, Let the three angles A, C, and D, (not taken in order,) be equal: Foraſmuch then, as the ſides A B and A E of the triangle A B E, are equal to the ſides C B and C D of the triangle C B D, and the angles conteined of them put equal, [f] the baſes B E and B D, and the angles A E B and C D B ſhall be equal: But [g] the angles B E D and B D E are alſo equal, the ſides B E and B D being ſhewn equal; therefore the whole A E D and C D E are equal: Therefore B A E and C D E being put equal, the three angles taken in order A, E, and D, ſhall be equal: Therefore, If three angles, &c. Which was to be demonſtrated.

a) 4. 1.
b) 5. 1.
c) 4. 1.
d) 6. 1.
e) 8. 1.
f) 4. 1.
g) 5. 1.

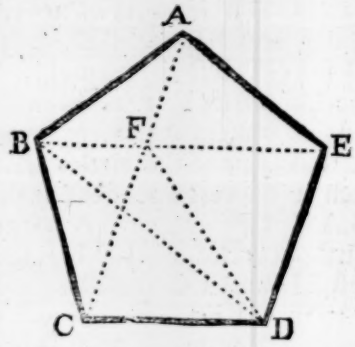
PROP. 8. THEOR. 8.



If two right lines D B *and* C E, *ſubtend two angles* C *and* D, *of a Pentagon equiangled and equilateral* A B C D E, *the which are in order, they ſhall divide one another in extream and mean proportion, and their greateſt ſegments* F B *and* F E, *are equal to the ſide of the Pentagon.*

Demonſtration. FOr [a] having deſcribed a circle about the Pentagon, [b] the five arches A B, B C, C D, D E, and E A, ſhall be equal; and foraſmuch as the ſides C D and C B of the triangle C D B, are equal to the ſides D C and C E of the triangle D C E, and the angles conteined of them put equal; the baſes D B and C E, and the angles C D B and D C E are equal. Wherefore [c] in the triangle C D F, the two angles F C D and F D C being equal, and [d] B F C the external, equal to them, B F C ſhall be the double of D C E; but [e] the angle B C E is alſo double to the ſame D C E: Foraſmuch as the arch B A E is double to the arch D E:

a) 14. 4.
b) 28. 3.
c) 4. 1.
d) 32. 1.
e) 33. 6.



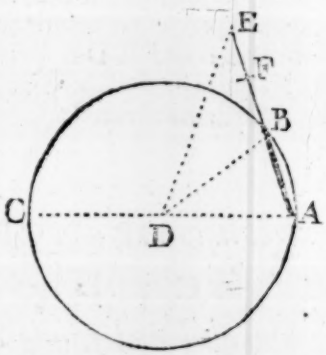
DE: Therefore the angles BFC and BCF are equal; and [f] therefore BF equal to the ſide of the Pentagon BC.

But [g] foraſmuch as the angles BDC and ECD, inſiſting on the equal arches, are equal, the two angles DBC and CDB of the triangle BCD, are equal to the two DCF and FDC of the triangle CFD; therefore [h] the triangles BCD and CFD are equiangled; therefore [i] as BD to DC, that is to ſay, to BF, equal to DC, ſo CD, that is to ſay, BF, his equal, is to FD; therefore BD is cut in F in extream and mean proportion, and the greateſt ſegment BF is ſhewn equal to the ſide of the Pentagon.

In like manner may be ſhewn that CE is cut at F, according to extream and mean proportion, and that his greateſt ſegment EF is equal to the ſide of the Pentagon: Therefore, If two right lines, &c. Which was to be demonſtrated.

f) 6. 1.
g) 17. 5.
h) 32. 1.
i) 4. 6.

PROP. 9. THEOR. 9.



If the ſide EB *of the Hexagon, and the ſide* AB *of the Decagon inſcribed in one and the ſame circle* ABC, *are compounded, the whole right line* EA *is divided in* B, *in extream and mean proportion, and the greateſt ſegment* EB *thereof, is the ſide of the Hexagon.*

Demonſtration FOr from A let there be drawn the diameter AC, and from the center D, let there be joyned DB and DE: Foraſmuch as the arch AB, is the tenth part of the whole circumference of the circle; the arch of the ſemicircle will contein the arch AB five times; therefore the arch BC ſhall be the quadruple of the arch AB: Wherefore [a] the angle BDC ſhall be alſo quadruple of the angle ADB.

Again, foraſmuch as the ſides BD and BE are equal, to wit, ſides of the Hexagon, (for [b] BE ſide of the Hexagon, is equal to the ſemidiameter BD,) [c] the angles BDE and BED ſhall be equal, [d] to which the external angle ABD being equal, ABD ſhall be the double of BED: But [e] BAD is equal to ABD, the ſides DA and DB being equal; therefore BAD ſhall be alſo double of BED; therefore the two angles DAB and DBA together, ſhall be the quadruple of BED.

Therefore [f] ſeeing the external angle BDC is equal to the two DAB and DBA, the external angle BDC ſhall be the quadruple of the ſame BED, but the ſame angle BDC hath been ſhewn the quadruple of ADB,

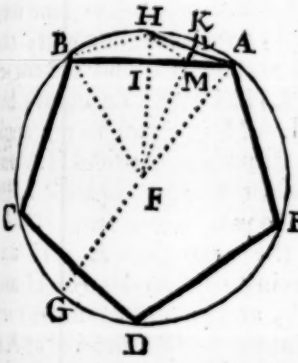
a) 33. 6.
b) Co. 15. 4.
c) 5. 1.
d) 32. 1.
e) 5. 1.
f) 32. 1.

ADB; therefore the angles AED and ADB are equal. Therefore the two angles AED and DAE of the triangle ABE being equal, the angles AED and ADB are equal. Therefore thoſe two equal to the two others ADB and BAD, of the triangle ABD; g the triangles ADE and ABD are equiangled: Therefore as EA ſhall be to AD, or to EB, equal to AD, ſo AD, that is to ſay, EB, his equal, to BA; therefore EA is divided in B in extream and mean proportion, and EB is the ſide of the Hexagon: Therefore, If the ſide, &c. Which was to be demonſtrated. g) 32. 11.

COROLLARIE.

From this Demonſtration it is manifeſt that if the ſide of the Hexagon be divided in extream and mean proportion, his greateſt ſegment ſhall be the ſide of the Decagon deſcribed in the ſame circle. For from the ſide of the Hexagon BE *being taken* BF, *a part equal to* AB, *as the whole* AE *ſhall be to the whole* BF, *a part equal to* AB, *as the whole* AE *ſhall be to the whole* BE, *ſo the part cut off* BE, *to the part cut off* BF; *therefore* h *the reſt* AB, *that is to ſay,* BF, *ſhall be to the reſt* FE, *as the whole* AE *to the whole* BE, *or* BE *the part cut off from* AE *to* BF, *the part cut off from* BF, *Therefore* i BE *ſhall be cut as* AE *in extream and mean proportion in* F, *and* BF *the greateſt ſegment is equal to* AB, *the ſide of the Decagon.* h) 19. 5. i) 3. def. 6.

PROP. 10. THEOR. 10.



If an equilateral Pentagon ABCDE, *be inſcribed in a circle* ABCD, *the ſide of the Pentagon ſhall be equal in power to the ſide of the Hexagon and the ſide of the Decagon, inſcribed in the ſame circle.*

THat is to ſay, That the ſquare of the ſide of the equilateral Pentagon inſcribed in the circle, is equal to the two ſquares together of the ſide of the Hexagon and Decagon inſcribed in the ſame circle.

Demonſtration FOr let the diameter AFG be drawn, and joyn FB, then having divided the arch AB in two equal parts at H, let the right lines AH, BH, and FH, be joyned, dividing the right line AB in I, the right line AH ſhall be the ſide of the Decagon, and BF the ſide of the Hexagon.

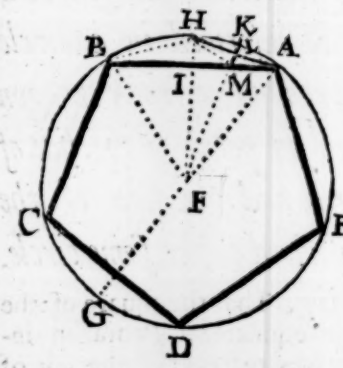
Again, having divided the arch AH in two equal parts in K, let FK be joyned, cutting the right line AH in the point L, and the right line AB in the point M, to which let there be drawn HM: Foraſmuch then, as the arches AH and BH are equal, a the angles AFH and BFH inſiſting on them are equal: Wherefore the ſides AF and FI, of the triangle AFI, being equal to BF and FI of the triangle BFI, and the angles conteined of them alſo equal, b the baſes AI and BI ſhall be equal, and the angles AIF and BIF equal, and therefore right angles; by the ſame reaſon, the right lines AL and HL ſhall be equal, and the angles ALF and HLF right angles. a) 27. 3. b) 4. 1.

Then if from the equal ſemicircles A B C G and A E D G, be taken the equal arches A B C and A E D, there will remain the arches C G and D G equal to one another; therefore the arch C G ſhall be the half of the arch C D. But the arch A H is the half of the arch A B; therefore the arches A B and C D being equal, their halves, which are the arches C G and A H, ſhall be equal; therefore ſeeing that the arch A H is double to the arch H K, the arch C G ſhall be alſo the double of the arch H K.

In like manner, foraſmuch as the arches A B and B C are equal, and the arch A B the double of the arch B H, the arch B C ſhall be alſo the double of the ſame B H: Wherefore the arches C G and B C are equimultiplices, to wit, the doubles of the arches H K and B H; and [c] therefore the whole arch B G ſhall be alſo the double of the whole arch B K, and [d] therefore the angle B F G the double of the angle B F K: But [e] foraſmuch as the ſame angle B F G at the center, is alſo the double of the angle F A B, the angles B F M and F A B are equal; and therefore the triangles A B F and F B M having the angles F A B and M F B equal, and the angle A B F common, ſhall be equiangled; therefore [f] as A B to B F, ſo B F to B M: Therefore [g] the rectangle conteined under A B and B M ſhall be equal to the ſquare of the right line B F.

c) 1. 5.
d) 33. 6.
e) 20. 3.
f) 4. 6.
g) 17. 6.

Again, foraſmuch as the ſides A L and L M of the triangle A L M, are equal to H L and L M of the triangle H L M, and the angles conteined of them equal, to wit, right angles; the baſes [h] A M and H M, and the angles L A M and L H M ſhall be equal; but [i] the angle L A M is equal to the angle H B A: Foraſmuch as the ſides H A and H B are equal, therefore the angle L H M ſhall be alſo equal to the ſame H B A; therefore [k] the triangles A B H and A H M, having the angles A B H and A H M equal, and H A M common, they ſhall be equiangled: Wherefore [l] as A B to A H, ſo A H to A M. Therefore [m] the rectangle under A B and A M ſhall be equal to the ſquare of A H. But the rectangle under A B and B M is ſhewn equal to the ſquare of B F; therefore the rectangles under A B and B M, and under A B and A M together, are equal to the ſquares of B F and A H. But [n] the rectangle under A B and B M, and under A B and A M, are equal to the ſquare of A B; therefore the ſquare of A B, that is to ſay; the ſide of the pentagon is equal to the ſquares of B F, and A H the ſide of the Hexagon, and the ſide of the Decagon: Therefore, If in a circle, &c. Which was to be demonſtrated.



h) 4. 1.
i) 5. 1.
k) 32. 1.
l) 4. 6.
m) 17. 6.
n) 2. 2.

COROLLARIE.

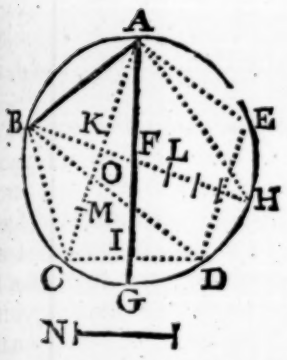
It is manifeſt from this Demonſtration that the right line drawn from the center which doth divide one arch in two equal parts, doth alſo divide the right line ſubtending it, at right angles in two equal parts it being ſhewn that F H *dividing the arch* A B *in two equal, divideth alſo the right line* A B *in two equal parts, and at right angles to* I, *and the ſame Demonſtration ſerveth for all the others.*

It is manifeſt alſo that the Diameter of the circle drawn from the angle of the Pintagon, doth divide alſo the arch ſubtending the ſide oppoſite to the ſame angle in two

equal

equal parts, and alſo the ſaid ſide in two equal parts and at right angles, it being ſhewn that the diameter A G *divideth in two equal parts the arch* C D, *ſubtend of the ſide* C D. *Therefore by what hath been demonſtrated, it divideth alſo the ſaid ſide* C D *in two equal parts and at right angles, the ſame Demonſtration may be made of every equilateral Polygon inſcribed in a circle, the number of the ſides being eade.*

PROP. 11. THEOR. 11.



If in a circle A B C D, *whoſe diameter is rational, there be inſcribed an equilateral Pentagon* A B C D E, *the ſide of the Pentagon* A B, *is an irrational line, called Minor.*

Demonſtration. FOr let the diameters A G and B H be drawn, interſecting at the center F, and let A G cut C D at I, and let A C and A H be joyned; and A C cut B H at K, and let there be taken F L, the quarter part of F H, and C M the quarter part of A C: Foraſmuch as B H is put rational, F L and B F the aliquot parts thereof, ſhall be rational, it being commenſurable; and therefore the whole B L, compounded of them, [a] being commenſurable to each of them, F L and B F ſhall be rational. a) 16. 10.

Again, F B drawn from the center, cutting the arch A B C in two equal parts at B, [b] it will cut the right line A C alſo in two equal parts at K, and at right angles: But [c] A G cuts C D at I, in two equal parts, and at right angles, therefore the triangles A C I and A F K, having the angles C I A and A K F right, and C A I common, [d] ſhall be equiangled; therefore [e] as C I to C A, ſo F K to F A; and alternately, as C I to F K, ſo C A to F A, or to F H, equal to F A. But [f] as C A to F H, ſo C M the quarter part of C A, to F L, quarter part of F H: Therefore as C I to F K, ſo C M to F L, and alternately; as C I to C M, that is to ſay, as C D the double of C I, to C K the double of C M, ſo F K to F L; and in compounding, as C D and C K together, to C K, ſo F K and F L together, to wit, K L, to F L; and [g] therefore as the ſquare of the compounded of C D and C K, to the ſquare of C K, ſo the ſquare of K L to the ſquare of F L. b) C. 10. 12. c) C. 10. 13. d) 32. 1. e) 3. 6. f) 15. 5. g) 22. 6.

But foraſmuch as, if A C be cut in extream and mean proportion (B D being drawn) [h] his greateſt ſegment O A, ſhall be equal to the ſide of the Pentagon, to wit, to C D, [i] the ſquare of the compounded of C D or O A the greateſt ſegment, and of C K the half of the whole, ſhall be the quintuple of the ſquare of C K, the half of the whole; wherefore the ſquare of K L ſhall be alſo the quintuple of the ſquare of F L. Therefore the ſquare of K L ſhall be commenſurable to the ſquare of F L, and therefore K L and F L alſo commenſurable at leaſt in power, &c. But F L is ſhewn rational, therefore K L ſhall be alſo rational. h) 8. 13. i) 1. 13.

Now foraſmuch as B F is 4 ſuch parts as F L is 1, and ſuch 5 parts as is B L; F L is 1 thereof, the ſquare of B L ſhall be 25 ſuch parts as the ſquare of F L 1, (as appeares by theſe numbers 1, 5, 25. For [k] the ſquares are in a double proportion of their ſides. k) 20. 6.

But the ſquare of K L is ſhewn to be 5 ſuch parts as the ſquare of F L is 1; therefore the ſquare of B L ſhall be 25 ſuch parts as the ſquare of K L is 5: Therefore the ſquare of B L ſhall be the quintuple of the ſquare of K L [l].

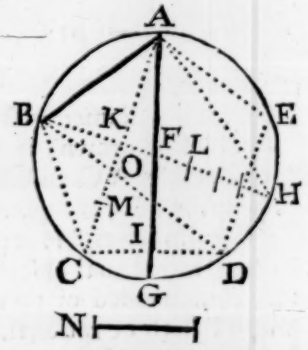
l) Co. 24. 8.

Foraſmuch then, as the ſquares of B L and K L are not the one to the other as a ſquare number to a ſquare number, (being as 25 to 5, or 5 to 1,) [m] B L and K L ſhall be incommenſurable in length, and being ſhewn rational, they ſhall be rational, commenſurable in power only; therefore if from the rational B L, be taken the rational K L, commenſurable in power only, [n] the remaining B K ſhall be irrational, named Reſidual, and his agreeing line ſhall be K L.

m) 9. 10.

n) 74. 10.

But now let B L be greater in power than K L, by the ſquare of N; foraſmuch as the ſquare of B L is equal to the ſquares of K L and N.



But the ſquare of B L was 5 ſuch parts as the ſquare of K L 1, the remaining ſquare of N ſhall be four ſuch parts as the ſquare of B L is 5: Therefore the ſquares of B L and N ſhall be to one another as a number to a number: Therefore [o] commenſurable, and B L and N commenſurable at leaſt in power. But B L is ſhewn rational; therefore N ſhall be ſo alſo, and [p] the ſquares of B L and N being not the one to the other as a ſquare number to a ſquare number (being as 5 to 4,) B L and N ſhall be incommenſurable in length, therefore rational, commenſurable in power only.

o) 6. 10.

p) Co. 24. 8.

Wherefore the whole B L being rational, commenſurable in length to the rational B H, (for B L is 5 ſuch parts as B H is 8,) and being more in power than his agreeing line K L, by the ſquare of N ſhewn incommenſurable in length thereto, B K ſhall be the fourth Reſidual by the Definition.

Laſtly, Foraſmuch [q] as A B is a mean proportional between B H and B K, (the triangle A B H having the angle B A H right, from which is drawn A K, perpendicular to the baſe,) [r] the ſquare of A B ſhall be equal to the rectangle under B H and B K; therefore A B being in power as the ſuperficies contained under the rational B H, and the fourth Reſidual B K; [s] A B ſhall be a Line Minor: Therefore, If in a Circle, &c. Which was to be demonſtrated.

q) Co. 8. 6.

r) 31. 3.

s) 17. 6.

PROP. 12. THEOR. 12.

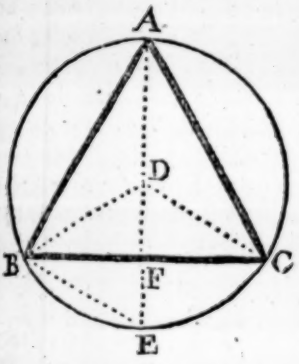
If a equilateral triangle A B C *be inſcribed in a circle* A B C, *the ſide* A B *of the triangle is triple in power to the line drawn from the center of the circle to the circumference.*

Conſtruction THat is to ſay, that the ſquare of the ſide of the equilateral triangle, inſcribed in the circle, is triple the ſquare of his ſemidiameter.

Demonſtration FOr having drawn the diameter A E, cutting the arch B C in two equal parts at E, [a] and the right line B C alſo in two

a) C. 16. 13.

two equal parts and at right angles at F, the arch B C being the third part of the circumference, the arch B E ſhall be the ſixth part, and the right line B E being drawn, ſhall be the ſide of an Hexagon, and [b] equal to the ſemidiameter: Foraſmuch then as the ſquare of A E is equal to the ſquares of A B and B E, [c] the angle A B E being a right angle in the ſemicircle: But [d] the ſquare of A E is the quadruple of the ſquare of B E, ſeeing that A E is the double of E B, the ſquares of A B and B E together, ſhall be in like manner the quadruples of the ſame ſquare of B E; and therefore ſuch four parts as are the ſquares A B and B E, the ſquare of B E ſhall be one, and ſo the ſquare of A B ſhall be three ſuch parts: Therefore the ſquare of A B is the triple of the ſquare of B E, the which is equal to the ſemidiameter: Therefore, If a triangle, &c. Which was to be demonſtrated.



b) Co. 15. 4.
c) 47. 1.
d) 31. 3.

COROLLARIE.

From theſe things it followes, that the diameter of the circle is ſeſquitertia in power to the ſide of the equilateral triangle inſcribed in the ſame circle; that is to ſay, the ſquare of the diameter is in proportion ſeſquitertia, or as 4 to 3, to the ſquare of the ſide of the equilateral triangle inſcribed in the ſame circle; for the ſquare of A B *being ſhewn triple the ſquare of the ſemidiameter* A D, *the ſquare of* A B *being put* 3, *the ſquare of* A D *ſhall be* 1, *and therefore the ſquare of* A E *quadruple the ſquare of* A D, *ſhall be* 4. *Wherefore the ſquare of* A E *is ſeſquitertia of the ſquare of* A B, *that is to ſay, as* 4 *to* 3; *and* [c] *foraſmuch then as* A F *is to* A B *as* A B *to* A F *perpendicular, drawn from one angle on the ſide. The ſquare of* A B *ſhall be alſo to the ſquare of* A F *as* 4 *to* 3.

c) Cor. 8. 6.

Secondly, it may be gathered that D E *the ſemidiameter is cut in two equal parts at* F, *by the ſide* B C *of the triangle* A B C, *for the ſquare of* A B *being triple the ſquare of* B E, *if the ſaid ſquare of* A B *be put* 12, *the ſquare of* B E *ſhall be* 4. *But* [f] *the ſquare of* B E *is equal to the ſquares of* B F *and* F E, *therefore the ſquares of* B F *and* F E *ſhall be* 4. *But the ſquare of* B F *is* 3, *(the ſquare of* A B *being quadruple the ſquare of* B F) *for* A B *is the cube of* B F; *therefore the ſquare of* F E *ſhall be* 1; *therefore the ſquare of* B E *quadruple the ſquare of* F E; *therefore* D E *ſhall be double to* F E. *Wherefore* D E *is cut in two equal parts at* F.

f) 47. 1.

PROP. 13. PROBL. 1.

To conſtitute a pyramid, and encompaſſe it with a given ſphere, and demonſtrate that the diameter A B *of the ſphere is ſeſquialtera in power to the ſide of the pyramid.*

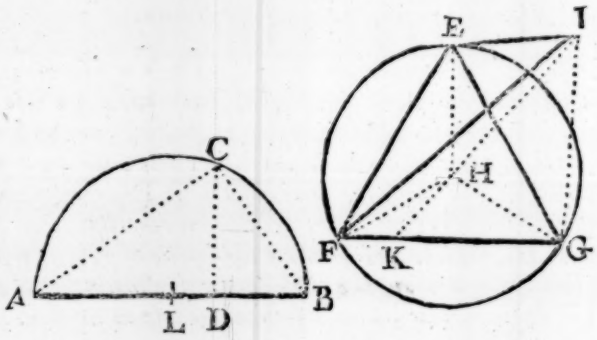
Conſtruction LEt A B be the diameter of the given ſphere, about which let there be deſcribed the ſemicircle A C B, and let there be taken B D, the third part of A B, in ſuch manner as that A B be double to D B, and A B triple to the ſame D B, and ſeſquialtera of A D; and having drawn C D perpendicular to A B, let A C and C D be joyned, and let the circle F E G be deſcribed at the ſpace H E, put equal to D C, on

on the center H, in which let the equilateral triangle E F G be inſcribed, and H F and H G joyned, each of which ſhall be equal to D C, being equal to H E, and from H let there be raiſed H I, at right angles to the plain of the circle, and put equal to A D; and from I let there be drawn I E, I F, and I G. I ſay that the ſolid contained of the four triangles F E G, I E F, I F G, and I E G, is a Pyramid or Tetrahedron.

Demonſtration FOraſmuch as the ſides D A and D C of the triangle A D C, are equal to H I and H E of the triangle I H E, and [a] the angles contained of them put right angles, [b] the baſe A C ſhall be equal to the baſe I E: In like manner, A C ſhall be ſhewn equal to I F and I G.

a) 3. def. 11.
b) 4. 1.

Again, Foraſmuch as the three lines A D, D C, and D B, are proportional, as A D to D B, ſo the ſquare of A D to the ſquare of D C; and in compoſing, as A B to D B, ſo the ſquares of A D and D C together, to the ſquare of D C. But the ſquares of A D and D C are equal to the ſquare of A C; therefore alſo as A B to D B, ſo the ſquare of A C to the ſquare of D C; therefore A B being the triple of D B, alſo the ſquare of A C ſhall be the triple of the ſquare H E, H E being equal to D C; then the ſquare of E F is alſo the triple of the ſame ſquare of H E; therefore the ſquare of A C ſhall be equal to the ſquare of E F; and therefore A C equal to E F: Wherefore A C being ſhewn alſo equal to I E, I F, and I G, the four triangles E F G, E F I, F G I, and G E I, ſhall be equilateral and equiangled to one another, therefore there is made a Pyramid or Tetrahedron, whoſe baſe is E F G, and his top I: I ſay that the ſaid pyramid is comprehended in the given ſphere, whoſe diameter is A B, and the diameter of the ſphere A B is ſeſquialtra in power to the ſide F E or A C.



Let I H be prolonged, perpendicular to K, in ſuch ſort as that I K be equal to D B, and the whole I K to the whole A B; foraſmuch then as the three lines A D, D C, and D B, are proportional, to which I H, H E, and H K, are equal; I H, H E, and H K, ſhall be alſo proportional: Wherefore H E being perpendicular to I K, and mean proportional between I H and H K, the ſemicircle deſcribed about I K, in the plain of I K and H F, will paſſe by E, as ſhall be by and by ſhewn.

In like manner, as well the ſemicircle deſcribed about I K, in the plain of I K and H F, will paſſe by F, as the ſemicircle deſcribed about I K, in the plain of I K and H G by G; therefore each of theſe ſemicircles drawn about the diameter remaining fixed, ſhall deſcribe a ſphere, which ſhall comprehend the Tetrahedron conſtituted, ſeeing that it paſſeth by all the angles thereof E, F, G, and I; and that ſphere being equal to the ſphere given, the diameter I K being put equal to the diameter A B, the given ſphere ſhall comprehend the ſame Tetrahedron.

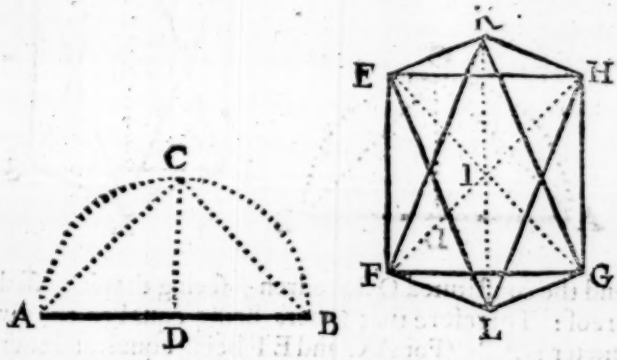
But

But forasmuch [c] as A B, A C, and A D, are proportional, as A B to A D, [d] so the square of A B shall be to the square of A C. But A B is put sesquialtra to the square of A D. Therefore the square of A B shall be sesquialtra to the square of A C; and therefore seeing that A B is the diameter of the sphere, and A C equal to E F the side of the pyramid. It is manifest that the diameter of the sphere is sesquialtra in power to the side of the Tetrahedron or pyramid: Therefore we have constituted, &c. Which was to be done.

c) Co. 8. 6.
d) Co. 20. 6.

PROP. 14. PROBL. 2.

To constitute an Octohedron, and to encompasse it with one and the same Sphere as the Pyramid, and shew that the diameter A B *of the Sphere, is double in power to the side of the said Octohedron.*



Construction LEt A B be the diameter of the sphere comprehending the constituted pyramid, about which let there be described the semicircle A C B, and from the center D let there be drawn D C, perpendicular to A B, and let A C and B C be joyned, which [a] shall be equal to one another, and taking E F equal to A C or B C, let there be described thereon the square E F G H, to which let there be drawn the diameters E G and F H, cutting one another in I, and I E, I F, I G, and I H, shall be equal, [b] being semidiameters of the circle described about the square.

a) 4. 1.
b) 9. 4.

Then from I let there be drawn K L, on both parts, perpendicular to the plain of the square, and I K and I L put equal to E, and from K to L let there be drawn K E, K F, K G, K H, L F, L E, L H, and L G: I say that the solid conteined of the eight triangles K E F, K F G, K G H, K F H, L E F, L E H, L H G, and L F G, is the Octohedron required.

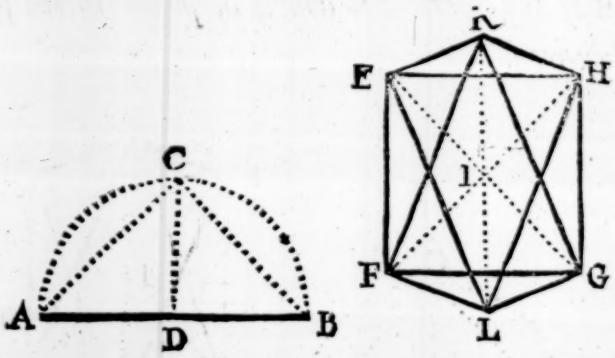
Demonstration FOr seeing the sides E I and I K of the triangle F I K, are equal to E I and I H of the triangle E I H, and the angles conteined of them right angles, (to wit, E I K [c] right, and E I G [d] being equal to G I H by consequence, by reason of the equality of the sides E I and I H, and G I and I H, and of the bases E H and G H, [e] the base E K shall be equal to the base E H.

c) 3. def. 11.
d) 8. 1.
e) 4. 1.

In like manner, K H shall be equal to E H; therefore K E H shall be an equi-

equilateral triangle, and ſo we conclude that the other triangles ſhall be equilateral, and for that cauſe equal to the triangle K E H, being their ſides are equal to the ſides of the ſquare E F G H, each to each: Therefore the Octohedron is conſtituted of eight triangles equilateral and equal, the which I ſay is conteined of the ſphere, whoſe diameter is A B, and of the which the pyramid conſtituted is conteined; and the diameter of the ſphere A B, is double in power to the ſide E F or A C.

f) 3. def. 11. Now foraſmuch[f] as I E is perpendicular to K L, and the mean proportional between the ſegments K I and I L, (K I, I E, and I L, being equal, and therefore in the ſame proportion;) the ſemicircle deſcribed about K L on the plain of K L and I F, ſhall paſſe by E; In like manner, the ſemicircle deſcribed about K L, on the plain of K L and I F, ſhall paſſe by F, and the ſame of the points G and H: Therefore each of theſe ſemicircles drawn about the fixed diameter K L, will deſcribe a ſphere, the which ſhall



comprehend the conſtituted Octohedron; ſeeing that it paſſeth by all the angles thereof: Therefore that ſphere being equal to the given ſphere, whoſe diameter is A B; (For A C and E F being equal, and therefore their ſquares equal; and as well the ſquare of A C donble of the ſquare of A D, as the ſquare of F E of the ſquare of E I, A D and E I, that is to ſay, K I, ſemidiameters are equal; therefore the whole A B and K L diameters, ſhall be equal, and therefore their ſpheres equal;) the ſame Octohedron ſhall be conteined of the given ſphere.

But foraſmuch as the ſquare of A B is equal to the equal ſquares of A C and B C, the ſquare of A B ſhall be double the ſquare of A C. Wherefore A B being the diameter of the ſphere, and A C equal to the ſide of the Octohedron E F; it is manifeſt that the diameter of the ſphere is double in power to the ſide of the Octohedron: Therefore we have conſtituted, &c. Which ought to be done.

COROLLARIE.

Hence it is manifeſt that in the Octohedron the three diameters cut one another at right angles in the center of the ſphere, as K L, E G, *and* F G, *at* I, *all the angles at* I *being ſhewn right angles; therefore* K I *being perpendicular to* E G, F H *ſhall be ſo alſo to the plain made by* E G *and* F H; *therefore the plains* K F L H *and* E L G K, *drawn by* K I, *ſhall be at right angles to* E F G H; *and by the ſame reaſon will cut one the other at right angles, the which are alſo ſquares, having for their ſides thoſe of the Octohedron, which are equal; and the angles right angles: Foraſmuch as (the ſquare of the diameter of the ſphere being double the ſquare of the ſide of the Octohedron, as is ſhewn,) the ſquare of the diameter* H F *is equal to the*

ſquares

ſquares of the ſides of the Octohedron FK *and* KH, *& the angle* FKH *ſhall be a right angle, and ſo of the others.* g) 48. 1.

So then it is evident, that the Octohedron may be divided into two Pyramids alike and equal, having for baſe the common ſquare EFGH, *and four equal triangles and alike, as well on one part as on the other.*

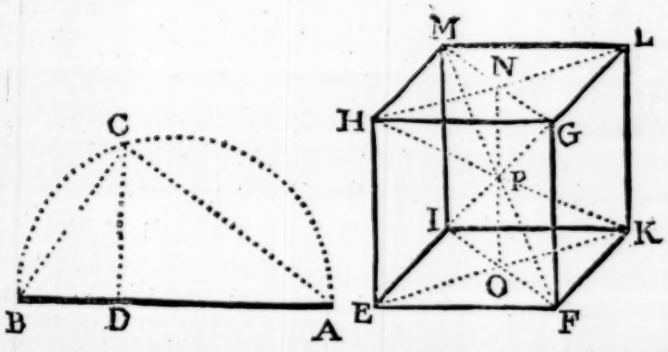
It followes alſo that if the Tetrahedron and the Octohedron, are deſcribed in one and the ſame ſphere, the ſide of the Tetrahedron ſhall be ſeſquialtera in power to the ſide of the Octohedron, for ſuppoſing that ſquare of the diameter of the ſphere be divided into ſix parts; h *then the diameter of the ſphere being ſeſquialtera in power to the ſide of the Tetrahedron, the ſquare of the ſide of the Tetrahedron is four ſuch parts as the ſquare of the diameter of the ſphere is ſix.* h) 13. 13.

Again, i *the diameter of the ſphere being double in power to the ſide of the Octohedron, the ſquare of the ſide of the Octohedron ſhall be three ſuch parts as the ſquare of the diameter of the ſphere is ſix; therefore the ſquare of the ſide of the Tetrahedron being four ſuch parts as the ſquare of the ſide of the Octohedron is three; it is manifeſt that the ſquare of the ſide of the Tetrahedron is ſeſquialtera to the ſquare of the ſide of the Octohedron.* i) 14. 13.

Laſtly, ſeeing that in the ſquare EFGH, EH *is parallel to* FG, *and in the ſquare* ELGK, EK *is parallel to* GL; k *the plain* EHK *drawn by* EH *and* EK, *ſhall be parallel to the plain* FGL, *drawn by* FG *and* GL, *there being the ſame reaſon given of the other baſes of the Octohedron, it followes that the oppoſite baſes of the Octohedron are parallel to one another.* k) 15. 11.

PROP. 15. PROBL. 3.

To conſtitute a Cube, and encloſe it in one and the ſame ſphere, as the foregoing figures, and to demonſtrate that the diameter AB *of the ſphere, is triple in power to the ſide of the ſaid Cube.*



Conſtruction. LEt AB be the diameter of the ſphere, which doth contein the precedent figures, and about the which there be deſcribed the ſemicircle ABC, and from AB let there be taken DB the third part, then having drawn DC perpendicular to AB, let AC and BC be joyned, and let EF and GH the ſquare of EF, be deſcribed, which let be taken equal to BC, on which let there be drawn at right angles EI, FK, GL, and HM, alſo equal to BC, at whoſe extreams let there be joyned IK, KL, LM, and MI: Foraſmuch then, as EI and FK are at right angles to the plain EFGH, a EI and FK ſhall be parallel; but they are equal, each being put equal to BC, that is to ſay, to EF; there- a) 6. 11.

b) 33. 1. fore [b] E F and I K are parallel and equal; therefore the paralellogram E F K I, in which E I, I K, and K F, being each equal to E F, all the four

c) 34. 1. are equal: But [c] all the four angles are right angles, E I K and F K I being equal to the two oppoſite angles K F E and I E F; therefore E F K I is a ſquare; by the ſame reaſon F K L G, G L M H, and H M I E, are ſquares,

d) 24. 11. and ſo I K L M ſhall be a ſquare, [d] being alike and equal to the oppoſite

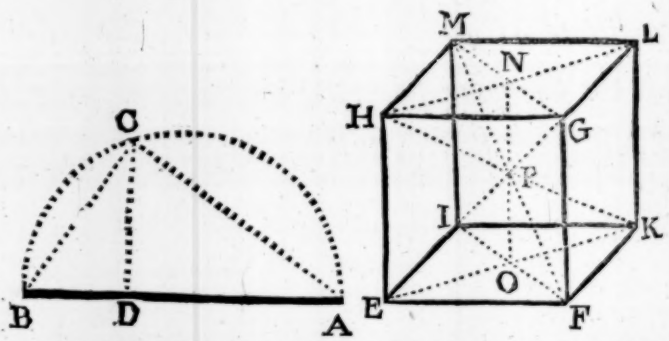
e) 15. 11. ſquare E F G H; for E L [e] is a ſolid paralellepipedon, having the oppoſite plains parallel, to wit, drawn by parallel lines touching one another: Wherefore E L ſhall be a cube, the which I ſay is comprehended of the ſphere, whoſe diameter is A B; for let E K, F I, G M, and H L, be the diameters of the oppoſite plains E F K I and G H M L, by which let there be drawn the plains E K L H and F I M G.

f) 28. 11. *Demonſtration* FOraſmuch as, [f] as well the plain E K L H as F I M G, cuts the cube in two equal parts, both the one and the other will paſſe by the center of the cube, to wit, by P, in which point alſo all

g) C. 39. 11. the diameters of the cube [g] are divided into two equal parts, therefore the common ſection of the plains, to wit, N O, ſhall paſſe by the ſame point P. But foraſmuch as the plains E K L H and F I M G are rectangles, (for H E being perpendicular to E I and E F, by reaſon of the ſquares E M

h) 4. 11. and E K, [h] it ſhall be alſo perpendicular to the plain E K; and therefore

i) 3. def. 11. [i] to the line E K, therefore the angle H E K is a right angle, by the ſame reaſon the other angles on the plain E K L H, ſhall be right angles, and



alſo on the plain F I M G. Therefore the plains E K L H and F I M G are rectangles,) and equal, the ſides of the one being equal to the ſides of the other, and their diameters E L, H K, F M, and G I, are equal, as is manifeſt from the 34th. of the firſt: Therefore P E, P L, P H, P K, P F, P M, P G, and P I, ſhall be equal: Wherefore the ſemicircle about E L, deſcribed from the center P, aud drawn about the diameter E L remaining fixed, will deſcribe a ſphere paſſing by all the angles of the cube, which ſhall be ſhewn equal to the ſphere, whoſe diameter is A B.

k) 47. 1. For ſeeing that the ſquare of E K is equal [k] to the equal ſquares of E F and F K; and therefore double the ſquare of E F, that is to ſay, the ſquare of K L; the ſquares of E K and K L ſhall be triple ſquare of K L; but

l) 47. 1. [l] the ſquare of E L is equal to the ſquares of E K and K L, the angle E K L being a right angle to the rectangle E K L H: Therefore the ſquare of E L ſhall be triple the ſquare of K L, that is to ſay, of the ſquare of B C. But the

m) Co. 8. 6. ſquare of A B is triple the ſame ſquare of B C, [m] (for A B, B C, and B D,

n) Co. 20. 6. are proportional,) therefore [n] as A B to B D, ſo the ſquare of A B to the ſquare of B C.

Then ſeeing that A B is triple to B D, the ſquare of A B ſhall be alſo triple

triple to the ſquare of BC; therefore the ſquares of EL and AB ſhall be equal: Wherefore EL and AB the diameters of the ſpheres, ſhall be equal, and therefore the ſaid ſpheres alſo equal.

But foraſmuch as the ſquare of the diameter EL is ſhewn triple to the ſquare of the ſide of the cube KL, it is manifeſt that the diameter of the ſphere is triple in power to the ſide of the cube: Therefore we have conſtituted, &c. Which was to be done.

COROLLARIE. I.

From this it is manifeſt, that all the diameters of the cube are equal to one another and doe cut one another in two equal parts in the center of the ſphere, and by the ſame reaſon, the right lines joyning the centers of the oppoſite ſquares, divide themſelves in two equal parts in the ſame center.

For it is demonſtrated that the diameters of the cube EL, HK, FM, *and* GI, *are equal, and do cut one another in two equal parts in the center* P.

Let NO *joyning the centers* N *and* O *of the oppoſite ſquares be alſo cut equally in two parts in the ſame center, we ſhall ſhew that the two angles* OFP *and* OPF *of the triangle* OPF, *are equal to* NMP *and* NPM *of the triangle* NMP, *(for* OFP o *is equal to* NMP *the alternate angle between* FI *and* GM p *parallels,* o) 29. 1.
being common ſections of the parallel plains EK, GM, *and* q OPF *equal to* p) 16. 11.
NPM, *oppoſite to the top.) But the ſides adjacent* PF *and* PM *are equal, being* q) 15. 1.
ſemidiameters, therefore PO *and* PN *ſhall be alſo equal; therefore* NO *ſhall be cut in two equal parts at* P, *and ſo of the other lines conjoyning the centers of the oppoſite ſquares.*

COROLLARIE II.

Again, the power of the diameter of the ſphere or of the cube, is equal to the powers of the ſides of the Tetrahedron and cube together; For r *the ſquare of the diameter* r) 13. 13.
is nine ſuch parts as the ſquare of the ſide of the Tetrahedron is ſix. But s *the ſaid* s) 15. 13.
ſquare of the diameter of is of nine ſuch parts as the ſquare of the ſide of the cube is three. Therefore the ſquare of the ſide of the Tetrahedron and the ſquare of the ſide of the cube together are of nine parts even as the ſquare of the diameter is: Therefore the ſquare of the diameter is equal to the ſquares of the ſaid ſides; that is alſo manifeſt of the ſemicircle deſcribed about the diameter of the ſphere where the ſide of the Tetrahedron is AC, *and of the cube* BC; *But it appeares that the diameter of the ſphere* AB *hath power as the line* AC *and* BC; *ſeeing that* ACB *is a right angle.*

PROP. 27. PROBL. 4.

To conſtitute an Icoſahedron and encloſe it with one and the ſame ſphere, as the figures before ſaid, and ſhew that the ſide of the Icoſahedron is a line irrational, the which is called Minor.

Conſtruction. LEt AB be the diameter of the ſphere, conteining the precedent figures, about which let there be deſcribed the ſemicircle ADB, and let BC be taken, the fifth part of AB, to the end that AB may be quintuple to BC, and to AC ſeſquiquarta, but AC quadruple of BC. Then having drawn DC perpendicular to AB, let AD and bD be drawn, and at the ſpace of EF, which let be equal to BD, let there be deſcribed a circle from the center E, in which let there be deſcribed an equilateral pentagon FGHIK.

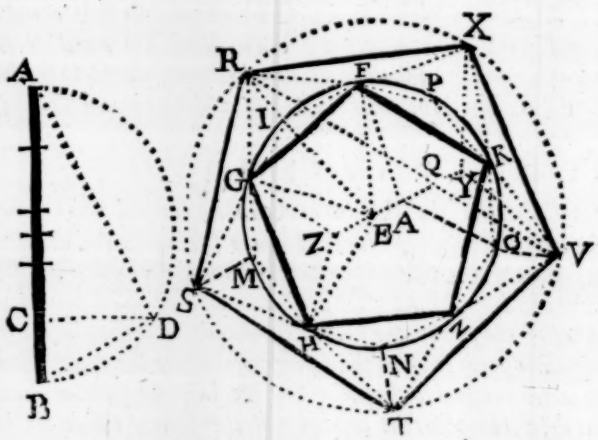
Afterwards having divided the arches F G, G H, H I, I K, and K F, in two equal parts, by L, M, N, O, and P, let the right lines (F L, L G, G M, and M H, &c. be joyned,) to wit, the ſides of the decagon: Then from the center E, and from the points L, M, N, O, and P, let there be erected the perpendiculars to the plain of the circle F G H I K, to wit, E Q, L R, M S, N T, O V, and P X, the which let be put equal to the ſemidiameter E F, or to C D, and they ſhall be all equal to one another:

a) 6. 11. But [a] they are parallel, therefore the right lines which joyn them, to wit, E L, Q R, E M, Q S, E N, Q T, E O, Q V, E P, and Q X, (the which notwithſtanding we have not drawn all to avoid confuſion,)

b) 33. 1. [b] ſhall be equal to one another, two to two; and therefore E L and E M being equal to the ſemidiameter E F; Q R, Q S, Q T, Q V, and Q X, ſhall be alſo equal, both to one another, and to the ſemidiameter E F, or

c) 15. 1. to C D [c].

But foraſmuch as the plain drawn by Q R and Q S, is parallel to the plain F G H I K, drawn by E L and E M; by the ſame reaſon the plain drawn by Q S and Q T, is parallel to the ſame plain F G H I K, drawn by E M and E N. But the plain drawn by Q R and Q S, agrees with the plain by Q S and Q T, to the right line Q S; therefore theſe two plains will make together one only plain, as ſhall be preſently demonſtrated. In like manner, the plain by Q T and and Q V, and this plain will make one only plain, and alſo the plain by Q V and Q X, and alſo the plain by Q X & Q R. There are then five equal lines in one and the ſame plain Q R, Q S, Q T, Q V, and Q X; therefore if from Q, at the ſpace Q R, there be deſcribed a circle in the ſame plain, it will paſſe by the other points R, S, T, V, and X, and ſhall be equal to the circle F G H I K; let the points R, S, T, V, and X, be joyned by the



d) 6. 11. lines R S, S T, T V, V X, and X R; foraſmuch [d] as L R and P X are equal and parallel; if there be underſtood a right line drawn from L to P,

e) 33. 1. [e] L P and R X ſhall be alſo equal and parallel: Therefore [f] in equal cir-

f) 28. 3. cles, they ſhall take away equal arches: But L P takes the fifth part of the circle F G H I K, to wit, $\frac{2}{10}$, F L and L P: Therefore R X ſhall alſo take the fifth part of the circle R S T V X.

Even ſo we ſhall conclude that the other right lines R S, S T, T V, and V X, taking the fifth parts: Wherefore R S T V X is an equilateral pentagon, having all the ſides equal to the pentagon F G H I K. For from the angles of the pentagon R S T V X, let there be drawn to the angles of the pentagon F G H I K, the right lines R F, R G, S G, S H, T H, T I, V I, V K, X K, and and X F: Foraſmuch then as L K perpendicular, is put equal to the ſemidiameter E F, that is ſo ſay, to the ſide of the Hexagon of

g) 10. 13. the circle F G H I K, and L F being the ſide of the decagon, [g] the ſquare of

of the side of the pentagon of the same circle, shall be equal to the squares of L R and L F. But [h] the square of F R is equal to the same squares, [i] the angle F L R being a right angle; therefore the square of F R is equal to the square of the side of the pentagon: Therefore F R shall be equal to the side of the pentagon L P or F G, that is to say, to K X.

And by the same reason, the other lines R G, S G, and S H, &c. shall be equal to the other sides of the one and the other pentagon; and therefore the ten triangles R F X, R F G, R G S, S G H, S H T, T H I, T I V, V I K, V K X, and X K F, shall be equilateral and equal to one another, their sides being equal to these of the equilateral pentagon; then after, let the perpendicular E Q be prolonged on both parts, in such manner as Q Y and E Z may be equal each to the side of the decagon of the circle F G H I K, or R S T V X; and let V Q, V Y, X Q, X Y, G E, G Z, H E, and H Z, be joyned: Forasmuch as Q X is the semidiameter of the circle R S T V X, that is to say, the side of the hexagon, and Q Y is the side of the decagon of the same circle, [k] the square of the side of the pentagon X V is equal to the squares of Q X and Q Y; but [l] the square of X Y, is equal to the same squares; the angle X Q Y being [m] a right angle, (for the plains of the circles F G H I K and R S T V X, being shewn parallel, and [n] E Q being perpendicular to the plain of that circle, by construction, E Q shall be also perpendicular to the plain of the circle, as it appeares, and therefore perpendicular to Q X in the same plain:) Therefore the square of X Y is equal to the square of X V, therefore X Y shall be equal to X V: so X V shall be shewn equal to V Y; therefore the triangle V Y X is equilateral.

Even so it will be demonstrated, if the right lines R Y, S Y, and T Y, be drawn (the which notwithstanding have not been drawn for other reason than to avoid confusion) that the four triangles R Y X, R Y S, S Y T, and T Y V, are equilateral and equal to the triangle V Y X, that is to say, to the ten first, all their sides being equal to the side of the pentagon: In like manner, G Z H shall be equilateral, (seeing that E G is the side of the hexagon, to wit, the semidiameter, and E Z the side of the decagon, and the angle G E Z a right angle,) and [o] also the four triangles H Z I, I Z K, K Z F, and F Z G, (from which we have drawn the lines to avoid confusion) all of them shall be equal to the fifteen first, each to his correspondent, for the same cause: Therefore those twenty triangles being equilateral and equal, and each joyned to other, by the right lines, to wit, their sides; there shall be constituted of them an Icosahedron, the which I say is comprehended of the sphere, whereof A B is the diameter.

For let E Q be divided in two equal parts by A, and let A F, A Y, and A V, be drawn. Forasmuch as the sides A Q and Q V, of the triangle A Q V, are equal to A Q and Q X, of the triangle A Q X; seeing that Q V and Q X are semidiameters of the circle A S T V X; but the angles A Q V and A Q X are [p] equal, to wit, right angles, [q] the bases A V and A X shall be equal.

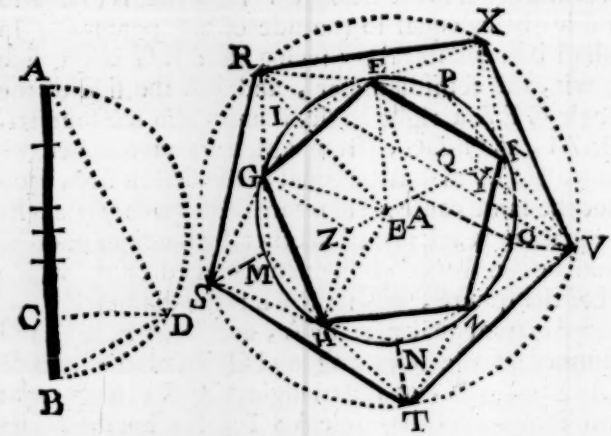
In like manner, if the right lines be drawn from A and Q, to R, S, and T, the right lines A R, A S, and A T, shall be shewn equal both to one another, and to A V and A X. Again, forasmuch as the sides A Q and Q X, of the triangle A Q X, are equal to A E and E F of the triangle A E F: Seeing that Q X and E F are semidiameters of equal circles, and that E Q is divided in two equal parts at A. But the angles A Q X and A E F are equal, to wit, [r] right angles, the bases A X and A F shall be [s] equal:

In

b) 47. 1.
i) 3. def. 11.
k) 10. 13.
l) 47. 1.
m) 3. def. 11.
n) 14. 11.
o) 3. def. 11.
p) 3. def. 11.
q) 4. 1.
r) 3. def. 11.
s) 4. 1.

In like manner, if from A E there be drawn the right lines to G, H, I, and K, it will be ſhewn that A G, A H, A I, and A K, will be equal to A X; and therefore the ten lines drawn to the ten angles F, G, H, I, K, R, S, T, V, and X, are equal: But foraſmuch as Q E is the ſemidiameter, that is to ſay, the ſide of the hexagon of the circle F G H I K, and E Z
t) 9. 13. the ſide of the decagon of the ſame circle, t Q Z ſhall be divided in E by extream and mean proportion, and the greateſt ſegment ſhall be Q E: Wherefore the leſſer ſegment Z E; taking E A the half of the greateſt ſegment, that is to ſay, A Z, is in power as the quintuple of the ſquare deſcribed of E A. But A F is in power alſo as the quintuple of the ſame ſquare of E A, (for the ſquare of E F being quadruple of the ſquare of A E, E F being the double of E A, the ſquare of E F and E A together, ſhall be the
v) 47. 1. quintuples of the ſquare of E A. And v the ſquare of A F being equal to the ſquares of E F and E A, the ſquare of A F ſhall be alſo the quintuple of the ſquare of E A;) therefore the ſquares of A Z and A F ſhall be equal; and therefore A Z and A F equal; then ſeeing that Z Y is cut in two equal parts at A, (foraſmuch as, if to the equal lines A E and A Q, there be added the equal lines E Z and Q Y, the whole Z A and Y A are made equal;) all the right lines drawn from A, to all the angles of the Icoſahedron ſhall be equal: Wherefore the ſphere deſcribed about the diameter Z Y, from the center A, ſhall paſſe by all the angles of the conſtituted Icoſahedron: I ſay it is equal to the ſphere of the diameter A B.

w) 15. 5. x) 22. 6. *Demonſtration* FOr ſeeing w that as A Z to A E, ſo Y Z the double of A Z, to Q E the double of A E, and x ſo as the ſquare of A Z to the ſquare of A E, ſo the ſquare of Y Z to the ſquare of Q E, and the the ſquare of A Z being ſhewn quintuple of the ſquare of A E; alſo the ſquare of Y Z ſhall be quintuple of the ſquare of Q E, that is to ſay, of the ſquare of B C; Q E being put equal to the ſemidiameter E F, that is to ſay, to B C: But y the ſquare of A B is alſo quintuple of the ſame ſquare of B C: (For A B being to B C, as B C to B D, as z A B ſhall be to B D, ſo the ſquare of A B to the ſquare of B C; and therefore A B being quintuple of the ſquare of B D, the ſquare of A B ſhall be alſo quintuple of the ſquare of B C:) therefore the ſquares of Y Z and A B are equal: Wherefore X Z and A B alſo equal, and therefore the ſpheres deſcribed about the ſame ſhall be equal.



y) Co. 20. 6. z) Cor. 8. 6.

Laſtly, foraſmuch as the diameter of the ſphere A B is put rational, (for in comparing of it, it is ſhewn that the ſide of the Icoſahedron is a line irrational, called Minor,) and is in power as the quintuple of the ſquare of B C, or of E F his equal, E F the ſemidiameter of the circle F G H I K ſhall be alſo rational; (for a the ſquares of A B and E F being

a) 6. 10.

to

to one another as a number to a number, to wit, as 5 to 1, or 10 to 2, shall be commensurable: Therefore AB and EF commensurable at least in power: Therefore seeing that AB is rational, EF shall be so also;) therefore the whole diameter of the circle FGHIK shall be rational: Wherefore FG the side of the pentagon, that is to say, of the Icosahedron, shall be a line irrational called Minor: Therefore we have constituted, &c. Which was to be done. 11. 13.

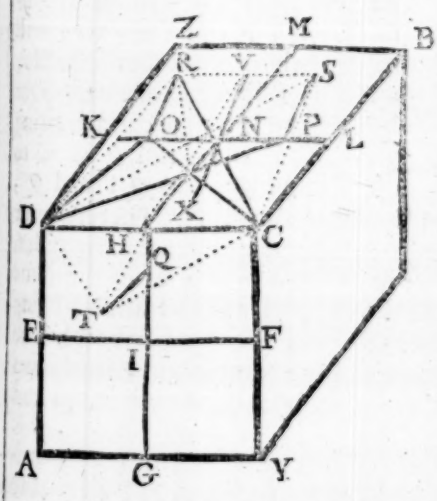
COROLLARIE.

From this it may be concluded that the diameter of the sphere is quintuple in power to the semidiameter of the circle which doth encompasse the five sides of the Icosahedron, to wit, of that whereof the Icosahedron is constituted, and which passeth by the five angles of the Icosahedron. For it is shewn that the square of the diameter AB *is quintuple to the square of* BC, *that is to say, to the semidiameter* EF.

Secondly, In like manner it is manifest that the diameter of the sphere is compounded of the side of the hexagon, that is to say, of the semidiameter, and the two sides of the decagon of one and the same circle; for YZ *diameter of the sphere is compounded of* E 2 *side of the Hexagon, and of* 2Y *and* EZ, *sides of the decagon* FGHIK.

Thirdly, It appeares lastly, that the opposite sides of the Icosahedron, as RX *and* HI, *are parallel, for* RX *is shewn parallel to the right line which shall be drawn from* L *to* P: *Therefore* HI *being also parallel to the same* LP, *(the alternate angles* HIL *and* ILP *made of the right line drawn from* L *to* P, [b] *being equal; subtending equal arches, to wit, each* $\frac{1}{10}$ *of the circumference,) it is manifest that* RX *and* HI *are parallels, and so of the other opposite sides.* b) 27. 3.

PROP. 17. PROBL. 5.

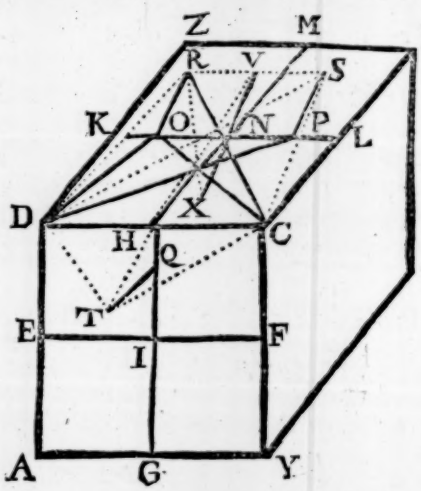


To constitute a Dodecahedron, and enclose it with one and the same sphere, as the figures aforesaid, and demonstrate that the side of the Dodecahedron is an irrational line, the which is called Residual.

Construction LEt there be proposed two plains of the cube aforesaid at right angles to one another, AC and BD, whose sides may be cut each in two equal parts, by E, G, F, H, K, L, and M; and let the lines EF, GH, KL, and HM, be drawn, to cut one another in the points I and N; then IH, KN, and NL, [a] are cut by extream and mean proportion, by O, P, and 2, whose greatest segments are I2, PN, and NO, and [b] at the exteriour parts of the plains BD and CA, let there be drawn OR, PS, and 2T, perpendiculars, equal to PN, NO, and I2, and let DT, DR, SR, CT, and a) 30. 6. b) 12. 11.

and C S, be joyned: I ſay that D R S C T is an equilateral and equiangled pentagon, and is on one and the ſame plain.

Demonſtration SUppoſe it otherwiſe, Let D P, D O, and D S be joyned in the plain D B and D S; ſeeing that N K is cut by extream and mean proportion at O, and N O is the greateſt ſegment, the
c) 4. 13. ſquares [c] of N K and of K O, ſhall be triple of the ſquare of N O, N K, and K D, being equal, and O N and O R alſo equal by conſtruction, the ſquares of D K and K O ſhall be alſo the triple of the ſquare of N O, that is to ſay, of the ſquare of O R; but the ſquare of D O is equal to the
d) 47. 1. two ſquares of D K and K O, ([d] the angle O K D being a right angle;) therefore the ſquare of D O is alſo the triple of the ſquare of O R: Wherefore adding the ſaid ſquare of O R, the two ſquares of D O and O R, ſhall be the quadruples of the ſquare of O R: But the ſquare of D R
e) 47. 1. is equal to the two ſquares of D O and O R: ([e] the angle D O R being a right angle,) it ſhall be then alſo quadruple of the ſquare of O R; and R S being the double of R O, his ſquare ſhall be alſo the quadruple of the ſquare of R O: Wherefore D R and R S ſhall be equal. By the ſame reaſon, the three other ſides D T, T C, and C S, ſhall be ſhewn equal to one another, and alſo to D R and R S: Therefore the pentagon is equilateral.



It is alſo in one and the ſame plain: For let there be drawn N V parallel to O R, and let there be alſo drawn V H and T H, they will meet with one another directly, and V H T ſhall be one only right line: For H I being cut according
f) 3. def. 6. to extream and mean proportion at Q, [f] I H ſhall be to I Q as I Q to Q H, and I H and H N are equal, and N V and I Q, and Q T and Q I, alſo equal: Wherefore as H N to N V, ſo Q T to Q H. Therefore the triangles V N H and H Q T, having two ſides proportional to two ſides, and being conjoyned at the angle H, in ſuch ſort as the ſides of the ſame
g) 6. 11. proportion, N H, and Q T, and N V and Q H [g] are parallels, being
h) 32. 6. perpendicular to the plains A C and D B, [h] the line V H T ſhall be one only right line, the which being in one and the ſame plain, it followes that the whole pentagon D T C S V R drawn by the ſame, is in one and the ſame plain.

It is alſo equiangled, For K N being cut in extream and mean proportion, and O N is the greateſt ſegment, if K N be added to N P, equal to the greateſt ſegment N O, the whole K P ſhall be divided by extream and mean proportion at N, and K N ſhall be the greateſt ſegment; there-
i) 4. 13. fore [i] the ſquares of K P and N P, or of P S equal to N P, are the triple of the ſquare of K N, or of the ſquare of K D, equal to K N.

And adding the ſquare of K D, with the two ſquares of K P and P S, theſe three ſquares ſhall be quadruple of the ſquare of K D; but the
k) 47. 1. ſquare of P D is [k] equal to the two ſquares of P K and K D; therefore the

the two ſquares of D P and P S ſhall be quadruples of the ſquare of K D; but [l] the ſquare of D S is equal to the two ſquares of D P and P S: Therefore the ſquare of D S ſhall be alſo quadruple of the ſquare of K D, and the ſquare of D C is alſo quadruple of the ſquare of D K, (D C being the double of D K:) Wherefore D C and D S ſhall be equal. l) 47. 1.

And all the ſides of the Pentagon being equal, the two triangles D R S and D T C ſhall have the ſides D R and R S equal to D T and T C, each to his correſpondent ſide, and the baſes D S and D C equal: Therefore [m] the angle D R S ſhall be equal to D T C, and ſo it ſhall be ſhewn that the angle R S C is equal to the angle D T C. Wherefore three angles of the Pentagon being equal, [n] it ſhall be equiangled, and being equilateral on D C, one of the twelve equal ſides of the cube. m) 8. 1. n) 7. 13.

And if on each ſide of the eleven remaining ſides, there be made a Pentagon, there will be a ſolid figure conteined of twelve Pentagons, equilateral and equiangled, which ſhall be a Dodecahedron by the Definition.

It may alſo be inſcribed in the propoſed ſphere, for V N being perpendicular to the plain B D, drawn from N, the center of that plain being prolonged, [o] ſhall cut the diagonal of the cube in two equal parts at X, which [p] ſhall be the center of the ſphere incloſing the cube, and X N is equal to the half of the ſide of the cube. o) 39. 11. p) Co. 15. 3.

Let X R be drawn, the ſquares of K P and N P being [q] ſhewn triples, the ſquare of K N and V X being equal to K P, and R V to N P, (N X being equal to K N, and V N to N P:) Wherefore the ſquares of R V and V X are triple the ſquare of K N: But the ſquare of R X [r] is equal to the two ſquares of R V and V X: Wherefore the ſquare of R X ſhall be alſo triple the ſquare of K N. But [s] the ſquare of the diameter of the ſphere deſcribed about the cube, is triple the ſquare of the ſide of the cube, and the parts being in the ſame proportion as their equimultiplices, [t] the ſquare of the ſemidiameter ſhall be triple the ſquare of the half ſide. But K N is the half of the ſide of the cube, therefore R X ſhall be the ſemidiameter of the ſphere deſcribed about the cube A B, whoſe center is X. q) 4. 13. r) 47. 1. s) 15. 13. t) 15. 5.

In like manner is ſhewn, that all the right lines drawn from X, to the other angles of the Dodecahedron, are equal to the ſemidiameters of the ſphere deſcribed about the cube: Therefore one and the ſame ſphere may be circumſcribed about a cube and a Dodecahedron.

Laſtly, the ſide of the Dodecahedron is an irrational line, called a Reſidual, for the two half ſides of the cube K N and N L, being divided by extream and mean proportion, [v] the whole K L ſhall be to the two greateſt ſegments together, to wit, to O P, as O P, which is the two ſame and the two greateſt ſegments, to the two leſſer, to wit, to the line compounded of P L and K O: Therefore if K L the ſide of the cube be divided by extream and mean proportion, O P ſhall be the greateſt ſegment: But K L being cut after this manner, is rational, (his ſquare being commenſurable to the ſquare of the diameter of the ſphere propoſed rational, [w] being the third part thereof:) Wherefore O P the greateſt ſegment, or R S the ſide of the Dodecahedron his equal, ſhall be an irrational line, called Reſidual: Therefore, &c. Which was to be done. v) 15. 5. w) 15. 18.

COROLLARIE.

It followes from this Demonſtration that the ſide of the cube being divided in

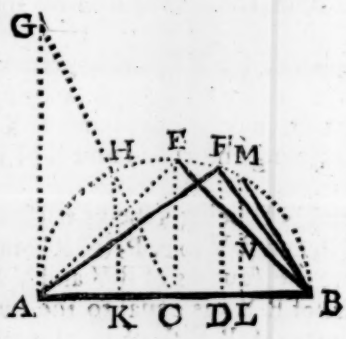
extream and mean proportion, the greateſt ſegment is the ſide of the dodecahedron inſcribed in the ſame ſphere.

And the ſide of the cube is equal to the right line ſubtending the angle of the pentagon of the dodecahedron. But foraſmuch as the ſame is divided in extream and mean proportion, x *the greateſt ſegment is the ſide of the pentagon,* y *the right line compounded of the ſaid ſubtence of the angle of the pentagon, that is to ſay, of the ſide of the cube, and of the greateſt ſegment, to wit, of the ſide of the dodecahedron, is divided in the like ſort, and the leſſer ſegment is the ſide of the dodecahedron, and the greateſt ſegment is the ſide of the cube: It followes that a right line being divided by extream and mean proportion, the leſſer ſegment ſhall be the ſide of the dodecahedron, and the greateſt ſegment ſhall be the ſide of the cube inſcribed in the ſame ſphere.*

x) 8. 13.
y) 5. 13.

PROP. 18. PROBL. 6.

To expound the ſides of the five aforegoing figures, and to compare them to one another.



THat is to ſay; To find the ſides of the five figures inſcribed in one and the ſame ſphere, and their proportion, the diameter of the ſphere being given.

Conſtruction. LEt the diameter of the ſphere be A B, and let it be divided in two equal parts at C; and let it be divided at D, ſo as that A D may be the double of D B, and let the ſemicircle A E B be deſcribed, and let C E and D F be drawn at right angles to A B, from C and D, and let A F, F B, and E B, be joyned; then foraſmuch as A D is the double of D B, A B ſhall be the triple of B D; and by converſion of proportion, B A ſeſquialtra of A D; but as B A to A D, ſo the ſquare of B A to the ſquare of A F: For a the triangle A F B is equiangled to the triangle A F D, therefore the ſquare of B A is ſeſquialtra to the ſquare of A F. But the diameter of the ſphere is alſo ſeſquialtra in power to the ſide of the pyramid, and A B is the diameter of the ſphere; therefore A F is equal to the ſide of the pyramid.

a) 8. 6.

Again, Foraſmuch as A D is the double of D B, A B ſhall be the triple of B D: But as A B to B D, ſo the ſquare of A B to the ſquare of F B: Therefore the ſquare of A B is the triple of the ſquare of B F: But b the diameter of the ſphere is triple in power to the ſide of the cube, and A B is the diameter of the ſphere: Therefore B F is the ſide of the cube, and A C being equal to C B, A B ſhall be the double of B C; but as A B to B C, ſo the ſquare of A B to the ſquare of B E: Therefore the ſquare of A B is the double of the ſquare of B E, and c the diameter of the ſphere is double in power to the ſide of the Octohedron, and A B is the diameter of the given ſphere: Wherefore B E is the ſide of the Octohedron.

b) 15. 13.

c) 14. 13.

From A let there be drawn A G at right angles to A B, and put equal to the ſame A B, and having joyned G C, cutting the circumference at H, from H let there be drawn H K, perpendicular to A B.

De-

Demonstration FOrasmuch as A G is the double of A C (for G A is equal to A B) but [d] as G A to A C, so H K to K C, H K shall be double to K C; therefore the square of H K is the quadruple of the square of K C; therefore the squares of H K and K C, that is to say, the square of H C is quintuple of the square of K C; but H C is equal to C B: Therefore the square of B C is the quintuple of the square of C K. d) 4. 6.

And forasmuch as A B is the double of B C, of which A D is the double of D B, the remaining B D shall be the double of C D: Wherefore B C is the triple of C D; therefore the square of B C is the noncuple of the square of C D; but the square of B C is the quintuple of the square of C K: Therefore the square of C K is greater than the square of C D, and K C greater than C D. Let C L be put equal to K C, and from L let there be drawn L M, at right angles to A B, and let M B be joyned.

And forasmuch as the square of B C is the quintuple of the square of K C, and B A the double of C B, and K L the double of C K, [e] the square of A B shall be quintuple of the square of K L; but [f] the diameter of the sphere is quintuple in power of the semidiameter of the circle of which the Icosahedron is described, and A B is the diameter of the sphere: Therefore K L is the side of the hexagon of the same circle. e) 15. 5. f) C. 16. 13.

Moreover, Forasmuch as the diameter of the sphere is composed of the side of the Hexagon, and of two sides of the decagon described in the same circle, and A B is the diameter of the sphere, K L the side of the Hexagon, and A K equal to L B, both the one and the other, A K and L B, shall be the side of the Decagon, described in the same circle of which the Icosahedron is described; and forasmuch as the side of the Decagon is L B, and that of the Hexagon M L, for it is equal to K L, it being also equal to H K, for that they are equally distant from the center, and both the one and the other H K and K L is the double of K C; M B shall be the side of the pentagon; for that which is of the pentagon, is the same as that of the Icosahedron: Therefore M B is the side of the Icosahedron.

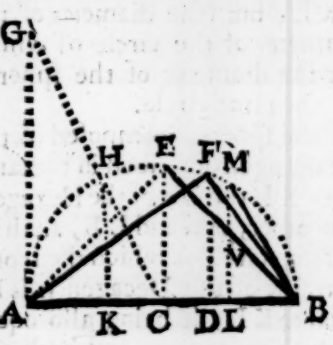
And forasmuch as F B is the side of the cube, let it be divided in extream and mean proportion at N, and let B N be the greatest segment, N B shall be the side of the Dodecahedron, and the diameter of the sphere being shewn sesquialtra in power to the side of the pyramid A F, and double in power to B E, the side of the Octohedron, and triple in power to F B, the side of the cube, of which parts the diameter of the sphere is 6 in power of the same parts, the side of the pyramid shall be 4, of the Octohedron 3, and of the cube 2: Therefore the side of the pyramid is sesquitertia in power to the side of the Octohedron, and to the side of the cube double in power, and the side of the Octohedron is sesquialtra in power to the side of the cube; therefore the aboveſaid sides of the three Figures, to wit, of the Pyramid, of the Octohedron, and of the Cube, are the one to the other in rational proportion: But the two others, to wit, of the Icosahedron, and of the Dodecahedron, are not in rational proportion, neither to one another, nor to the aboveſaid, to wit, the Minor and Residual.

But we shall demonstrate it so as that the side B M of the Icosahedron, is more than the side B N, of the Dodecahedron.

For seeing the triangle F D B is equiangled to F A B, as D B shall be to B F, so F B to B A, and having three lines proportional, as the first shall be to the third, so the square of the first, to the square of the second.

Therefore as D B to B A, ſo the ſquare of D B to the ſquare of B F; and by converſion of proportion, as A B to B D, ſo the ſquare of F B to the ſquare of B D. But A B is the triple of B D; therefore the ſquare of F B is the triple of the ſquare of B D: But the ſquare of A D is the quadruple of the ſquare of D B, for A D is the double of D B: Therefore the ſquare of A D is greater than F B; and therefore A L much greater than F B; and A L being divided by extream and mean proportion, the greateſt portion is L K: Foraſmuch as L K is the ſide of the Hexagon, and K A of the Decagon, and F B being divided by extream and mean proportion, the greateſt portion is B N; therefore K L is greater than B N: but K L is equal to L M, therfore L M is greater than B N; but B M is greater than M L: therefore M B the ſide of the Icoſahedron, ſhall be greater than B N the ſide of the Dodecahedron.

Otherwiſe, Foraſmuch as A D is the double of D B, A B ſhall be the triple of B D; but as A B to B D, ſo the ſquare of A B to the ſquare of B F: Foraſmuch as the triangle F A B is equiangled to the triangle F B D; therefore the ſquare of A B is triple to the ſquare of B F. But the ſquare of A B is ſhewn quintuple to the ſquare of K L; therefore five ſquares of K L are equal to three of B F; but three of B F are greater than ſix of thoſe which are made of B N; therefore alſo five of K L are greater than ſix of B N; therefore one of K L is greater than one of B N; therefore one of K L is greater than one of B N; but K L is equal to L M; therefore L M is greater than B N. Wherefore M B is much greater than B N. Which was to be demonſtrated.



But we ſhall demonſtrate it ſo as that three of F B, are greater than ſix of thoſe of B N.

Foraſmuch as B N is greater than N F, the rectangle under F B and B N ſhall be greater than the rectangle under B F and F N; therefore that which is conteined of F B and B N, with that of B F and F N, is greater than the double of that which is conteined under B F and F N; but that which is conteined of F B and B N, with that conteined under B F and F N, is the ſquare of F B; but that which is conteined under B F and F N, is equal to the ſquare of B N; for F B is divided by extream and mean proportion at N, and the rectangle conteined under the extreams is equal to the ſquare of the mean: Therefore the ſquare of F B is greater than the double of the ſquare of B N: Wherefore one ſquare of F B is greater than two of B N, and therefore three of F B are greater than ſix of thoſe which are made of B N. Which was to be demonſtrated.

The End of the Thirteenth Element of EUCLIDE.

THE

THE FOURTEENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

IN this Book which is a continuation of the former ſubject, is intreated of the Dodecahedron and Icoſahedron deſcribed in the ſame ſphere: And although this be commonly accounted the Fourteenth Book in order of *EUCLIDES* Elements, yet by others it is ſuppoſed to be the Firſt Book of *Hypſicles* of *Alexandria*, concerning the five regular Bodies, as it may appear by the following Preface of the ſaid *Hypſicles*.

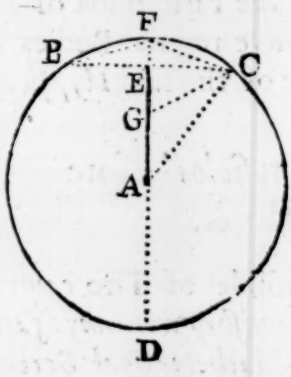
The Preface of *Hypſicles* before the Fourteenth Book.

FRiend Protarchus, *when that* Baſilides of Tire *came into* Alexandria, *having familiar friendſhip with my father, by reaſon of his knowledge in the Mathematical Sciences, he remained with him a long time, yea even all the time of the Peſtilence. And ſometime reaſoning between themſelves of that which* Apollonius *had written touching the compariſon of a Dodecahedron and of an Icoſahedron inſcribed in one and the*

the self same sphere, what proportion such bodies have the one to the other, they judged that Apollonius *had somewhat erred therein. Wherefore they (as my father declared unto me) diligently weighing it, wrote it perfectly. Howbeit afterward I happened to find an other Book written of* Apollonius, *which conteined in it the right demonstration of that which they sought for: which when they saw, they much rejoyced. As for that which* Apollonius *wrote, may be seen of all men, for it is in every mans hand. And that which was of us more diligently afterward written again. I thought good to send and dedicate unto you, as to one whom I thought worthy commendation, both for that deep knowledge which I know you have in all kinds of learning, and chiefly in Geometry, so that you are able readily to judge of those things which are spoken, and also for the great love and good will which yon bear towards my father and me. Wherefore vouchsafe gently to accept this, which I send unto you. But now I think it is time to end our Preface, and to begin the matter.*

PROPOSITIONS, and THEOREMES.

PROPOSITION 1. THEOREM 1.



The perpendicular right line AE, *drawn from the center* A, *to the side* BC, *of the Pentagon inscribed in the circle* BDC, *is the half of the one and the other side, to wit, of that of the Hexagon and of the Decagon together, inscribed in the same circle.*

Demonstration. FOr having continued AE on both parts, to finish the diameter DF, and having taken EG equal to EP, let AC, CG, CF, and BF, be joyned: Foraſmuch a) as AE cutteth BC in two equal parts, the two sides BE and EF, of the triangle BEF, shall be equal to CE and EF of the triangle CEF, and the angles conteined of them being

a) 3. 5.

being equal, to wit, right angles, [b] the bases C F and B F shall be equal, and [c] the arches C F and B F shall be equal; therefore B F C being the fifth part of the whole circumference C F, shall be the tenth part; therefore the right line C F is the side of the Decagon. b) 4. 1. c) 28. 3.

Again, the sides C E and E F of the triangle C E F, being equal to C E and E G, and the angles conteined of them right angles, [d] the bases C F and C G shall be equal, and the angles C F G and C G F equal, and the arch C F being the fifth part of the half circumference D C F, being the tenth part of the whole, the arch D C shall be the quadruple of C F: Wherefore [e] the angle D A C shall be also the quadruple of C A F [f]. d) 4. 1. e) 33. 6. f) 20. 5.

But the angle C F D at the circumference, is the half of D A C at the center, therefore C F D; and therefore C G F shewn equal thereto, shall be the double of C A F; but [g] C G F the external angle, is equal to the two internal angles G A C and G C A, of the triangle G A C, and G A C being shewn the half of the same C G F; G C A shall be the half of the same C G F; and therefore G A C and G C A equal, and G A and G C equal. g) 32. 1.

But C G is shewn equal to C F, therefore A G shall be also equal to C F; therefore adding the equal lines G E and E F, A E shall be equal to G F and F E together; therefore the three lines A E, F E, and F C, together, that is to say, the two together A F and F C, shall be the double of A E; and contrariwise, A E perpendicular to the side of the Pentagon, shall be the half of the two together A F and F C, to wit, of the side of the Hexagon and the Decagon: Therefore, The perpendicular right line, &c. Which was to be demonstrated.

COROLLARIE.

From whence it follows that the perpendicular drawn from the center to the right line fitted in the circle, doth cut the arch subtending the said line in two equal parts, being demonstrated that A E *drawn from the center* A, *perpendicular to* C B, *divides the arch* B C *in two equal parts at* F.

Secondly, That the perpendicular from the center to the side of the said Pentagon, is equal to the perpendicular from the same center drawn on the side of the equilateral triangle, and to the half of the side of the Decagon inscribed in the same circle.

For as it is here demonstrated, the perpendicular drawn from the center to the side of the Pentagon, is equal to the half of the side of the Hexagon and of the Decagon together: But the perpendicular drawn from the center on the side of the equilateral triangle, is [h] *equal to the half of the semidiameter, or side of the Hexagon. Therefore it is manifest that the same perpendicular drawn from the center to the side of the Pentagon, is equal to the perpendicular drawn to the side of the equilateral triangle, and to the half of the side of the Decagon together.* h) C. 12. 13.

PROP. 2. THEOR. 2.

If two right lines A B *and* C D, *be cut in* E *and* F, *in extream and mean proportion, they shall be alike cut, to wit, in the same proportion.*

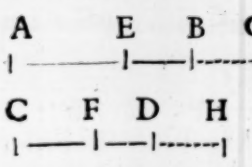
Demonstration. FOr seeing that as A B is to A E, so A E to E B, and as C D to C F, so C F to F D, [a] the rectangle conteined under A B and E B, shall be equal to the square of A E, and the rectangle under C D and F D equal to the square of C F. a) 17. 6.

There-

Therefore as the rectangle under A B and E B ſhall be to the ſquare of A E, ſo the rectangle under C D and F D to the ſquare of C F (there being proportion of equality on the one part and the other:) Wherefore as the quadruple of the rectangle under A B and E B, to the ſquare of A E, ſo the quadruple of the rectangle under C D and F D, to the ſquare of C F, and by compounding, as the quadruple of the rectangle under A B and E B, and the ſquare of A E to the ſame ſquare of A E, ſo the quadruple of the rectangle under C D and F D, and the ſquare of C F to the ſquare of C F: But [b] four times the rectangle under A B and E B, with the ſquare of A E, is equal to the ſquare of A G, compounded of A B and E B.

b) 8. 2.

And four times the rectangle under C D and F D, with the ſquare of C F, is equal to the ſquare of C H, compounded of C D and F D; therefore as the ſquare of A G compounded of A B and E B, to the ſquare of A E, ſo the ſquare of C H compounded of C D and F D, to the ſquare of C F: Therefore [c] as A G to A E, ſo C H to C F, and in compounding, as A G and A E, that is to ſay, the double of A B, is to A E, ſo C H and C F, that is to ſay, the double of C D, is to C F; and by permutation, as the double of A B to the double of C D, ſo A E to C F: But [d] as the double of A B to the double of C D, ſo A B to C D; therefore as A B ſhall be to C D, ſo A E to C F; therefore ſeeing that as the whole A B is to the whole C D, ſo the part cut off A E, to the part cut off C F; alſo [e] the remainder E B ſhall be to the remainder F D, as the whole to the whole; therefore [f] as A E to C F, ſo E B to F D; and alternately, as A E to E B, ſo C F to F D, and ſo according to all or any other manner of arguing in proportion, it will be ſhewn, that the whole lines and their parts, are proportional to one another: Therefore, If two right lines, &c. Which was to be demonſtrated.



c) 22. 6.

d) 15. 5.

e) 19. 5.

f) 11. 5.

PROP. 3. THEOR. 3.

One and the ſame circle conteines the Pentagon of the Dodecahedron, and the triangle of the Icoſahedron, inſcribed in one and the ſame ſphere.

BEfore I ſhall ſhew this Propoſition, I ſhall ſhew that the ſquare of the ſide of the Pentagon, with the ſquare of the line ſubtending one angle thereof, is the quintuple of the ſquare of the ſemidiameter of the circle where the ſaid Pentagon is inſcribed.

Let C H be the ſide of the Pentagon inſcribed in the circle, and let C A be the line ſubtending one of the angles of the Pentagon, and F G the ſemidiameter, [a] cutting C H, and the arch C G H, in two equal parts: I ſay that the ſquares of C A and C H together, are the quintuple of the ſquare of F G.

a) C.10.13.

Demonſtration FOr the right line C G being drawn, ſhall be the ſide of the Decagon, and the ſquare of A G ſhall be the quadruple of the ſquare of F G the ſemidiameter: Wherefore the two ſquares of

of AC and CG, [b] equal to the ſaid ſquare of AG, ſhall be alſo the quadruple of the ſquare of FG: Therefore the three ſquares of AC, CG, and GF, together, ſhall be the quintuple of the ſquare of GF; but the ſquares of CG and of GF together, ſides of the Hexagon and of the Decagon, are [c] equal to the ſquare of CH, the ſide of the Pentagon; therefore the ſquares of AC and of CH, are quintuples of the ſquare of GF. Which was propoſed. Now

b) 47. 1.

c) 10. 13.

Let IK be the diameter of the ſphere conteining the Dodecahedron and the Icoſahedron; and let ABCHE be one Pentagon of the ſame Dodecahedron, and LMN a triangle of the Icoſahedron: I ſay that one and the ſame circle conteineth the Pentagon ABCHE, and the triangle LMN; that is to ſay, that the circles which circumſcribe ABCHE and LMN, are equal, and that it is no other than one and the ſame; for having drawn AC ſubtending the angle B of the Pentagon, [d] AC ſhall be the ſide of the cube inſcribed in the ſame ſphere.

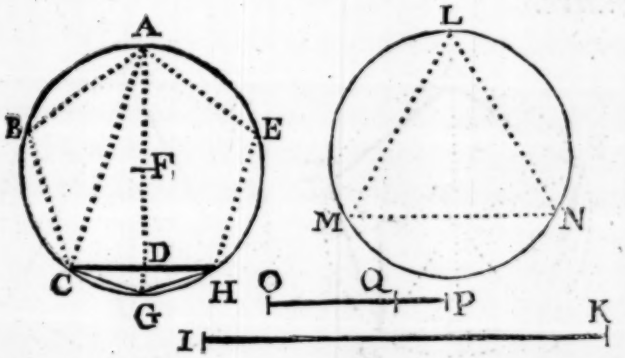
d) C. 17. 13

Let there be taken OP, ſo as that the ſquare of IK the diameter of the ſphere, may be quintuple the ſquare of OP, and OP equal to the ſemidiameter of the circle where the Icoſahedron is inſcribed: And let OP [e] be cut in extream and mean proportion at Q; O Q the greateſt ſegment ſhall be the ſide of the Decagon inſcribed in the ſame circle, whereof OP [f] is the ſemidiameter or ſide of the Hexagon. But [g] CA being divided by extream and mean proportion, his greateſt ſegment is

c) C. 16. 13.

f) C. 9. 13.

g) 7. 13.



AB, the ſide of the Pentagon: Therefore [h] as AC the whole, to OP the whole, ſo the greateſt ſegment AB, to the greateſt ſegment OQ: Therefore [i] as the ſquare of AC to the ſquare of OP, ſo the ſquare of AB to the ſquare of OQ, and [k] at the triple of the ſquare of AC to the quintuple of the ſquare of OP, ſo the triple of the ſquare of AB, to the quintuple of the ſquare of OQ. But the triple of the ſquare of AC is equal to the quintuple of the ſquare of OP ([l] the ſquare of IK the diameter of the ſphere being put equal, as well to the triple of the ſquare of AC the ſide of the cube, as to the quintuple of the ſquare of OP:) Wherefore the triple of the ſquare of AB ſhall be alſo equal to the quintuple of the ſquare of OQ: But [m] ML the ſide of the triangle of the Icoſahedron is equal to the ſide of the Pentagon inſcribed in the circle, whereof OP is the ſemidiameter: Therefore the ſquare of the ſaid ſide being [n] equal to the ſquares of OP and OQ, the ſides of the Decagon and of the Hexagon, five times the ſquare of ML ſhall be equal to five times the ſquare of OP, and to five times that of OQ, or to three times the ſquare of BA, and to three times the ſquare of AC; therefore the ſquares of BA and AC

h) 2. 14.

i) 22. 6.

k) 4. 5.

l) 15. 13.

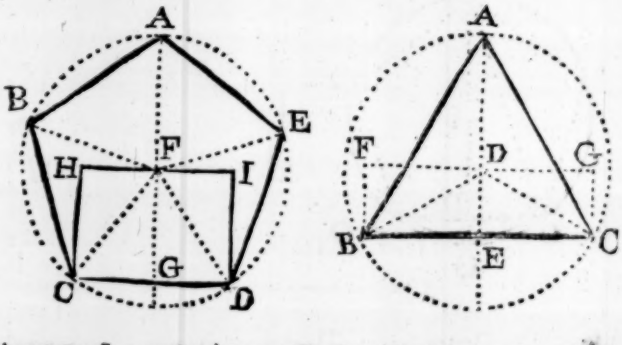
m) C. 16. 13

n) 10. 13.

being quintuples of the ſquare of the ſemidiameter F A (as is demonſtrated) three times the ſquare of AB and three times the ſquare of AC ſhall be equal to fifteen times the ſquare of F A the ſemidiameter: But five times the ſquare of ML the ſide of the equilateral triangle, is equal to fifteen times the ſquare
o) 12. 13. of the ſemidiameter of the circle L M N (each ſquare o of M L being triple the ſquare of the ſemidiameter.) Wherefore the three ſquares of B A and A C being equal to five of M L, fifteen of F A the ſemidiameter, ſhall be equal to fifteen of the ſemidiameter of the circle L M N: Wherefore each ſquare of them ſhall be equal to each ſquare, and the ſemidiameter to the ſemidiameter: Therefore the circles A B C and L M N ſhall be equal: Which was to be demonſtrated.

PROP. 4. THEOR. 4.

If from the center F *of the circle, circumſcribing the Pentagon of the Dodecahedron* A B C D E, *be drawn a right line* F G, *perpendicular to one ſide* C D, *of the ſame pentagon, thirty times the rectangle conteined under the ſaid ſide and the the perpendicular, ſhall be equal to the Superficies of the Dodecahedron.*



Demonſtration FOr from the center F, having drawn the right lines F A, F B, F C, F D, and F E, to all the angles of the Pentagon, the Pentagon ſhall be divided into five equal triangles, as [a] it appeares,
a) Cor. 8. 1. all the ſides being equal, and all the baſes; let there be made the rectangle C I, conteined under C D and F G: Foraſmuch [b] as the rectangle C I
b) 4. 1 1. is the double of the triangle F C D, and by conſequents the double of each of the four remaining triangles, equal to F C D; the rectangle C I taken five times, ſhall be equal to ten ſuch triangles; that is to ſay, to two Pentagons of the Dodecahedron (five triangles being equal to one Pentagon;) therefore the ſextuple of the rectangle C I (that is to ſay, ſix rectangles, ſuch as is C I,) taken five times, to wit, thirty times the rectangle C I ſhall be alſo equal to the ſextuple of the two Pentagons (that is to ſay, to twelve Pentagons) to wit, to the whole ſuperficies of the Dodecahedron: Therefore, If from the center, &c. Which was to be demonſtrated.

In like manner may be ſhewn, that if from the center of the circle circumſcribing the triangle of the Icoſahedron A B C, be drawn a perpendicular

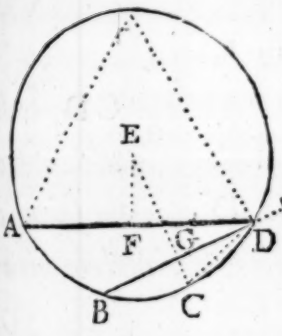
dicular D E on one side, to wit, on B C; the rectangle B G conteined under B C and D E taken thirty times, shall be equal to the superficies of the Icosahedron.

For the rectangle B G being the double of the triangle D B C, that is to say, equal to the said triangle taken two times, the rectangle B G taken thirty times shall be equal to the triangle D B C taken sixty times; but sixty such triangles as D B C make twenty triangles of the Icosahedron, seeing that three of them make one, as it appeares in the triangle A B C, where the sides D A, D B, and D C, are equal, and the three bases A B, B C, and C A, equal by supposition: Therefore B G taken thirty times, shall be equal to the whole superficies of the Icosahedron.

COROLLARIE.

Whence it followes, that as the rectangle C I *is to the rectangle* B G, *so the whole superficies of the Dodecahedron, to the superficies of the Icosahedron.*

PROP. 5. THEOR. 5.



As the Superficies of the Dodecahedron is to the Superficies of the Icosahedron, so the side of the cube is to the side of the Icosahedron inscribed in one and the same sphere.

Construction IN the circle A B C D, [a] enclosing the Pentagon of the Dodecahedron, and the triangle of the Icosahedron; let B D be the side of the Pentagon, and A D the side of the triangle; and from the center E let there be drawn E F and E G, perpendicular to A D and B D, and let F G be prolonged to the circumference in the point C, and let C D be drawn, E C shall cut the arch B C D in two equal parts, and C D shall be the side of the Decagon, as [b] we have shewn. a) 13. 14. b) 1. 14.

And let B D be prolonged to H, in such manner as that B H be equal to the side of the cube inscribed in the same sphere.

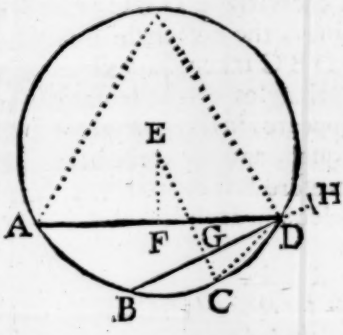
I say now that the superficies of the Dodecahedron is to the superfices of the Icosahedron, as B H the side of the cube, is to A D the side of the Icosahedron.

Demonstration FOr E C being the side of the Hexagon, and D C the side of the Decagon, inscribed in one and the same circle, E C and C D, taken as one only line, [c] shall be divided by extream and mean proportion, and E C the greatest segment, and [d] E G is the half of the whole compounded of E C and C D, and E F is the half of the greatest segment E C ([e] the semidiameter being cut in two equal parts by A D) and E C being divided by extream and mean proportion, the half E F shall be the greatest segment, as is manifest; for the half of the greatest segment and the half of the lesser, makes the half of the whole; [f] their whole being to one another as their parts: But B H the side of the cube, being divided by extream and mean proportion, [g] B D the side of the c) 9. 13. d) 1. 14. e) C. 12. 13. f) 15. 5. g) C. 17. 13.

b) 2. 14. Dodecahedron ſhall be his greateſt ſegment; therefore [h] as the whole B H to B D his greateſt ſegment, ſo the whole E G ſhall be to E F his greateſt ſegment; therefore [i] the rectangle under B H and E F extreams, ſhall be equal to the rectangle under B D and E G means, and the rectangles under B H and E F [k] being to the rectangle under A D and E F, as B H is to A D; and as B H ſhall be to A D, ſo the rectangle under B D and E G (ſhewn equal to the rectangle under B H and E F) ſhall be to the rectangle under A D and E F; therefore [l] the rectangle under B D and E G being to the rectangle under A D and E F, as the ſuperficies of the Dodecahedron to that of the Icoſahedron, alſo as B H the ſide of the cube, ſhall be to A D the ſide of the Icoſahedron, ſo the Superficies of the Dodecahedron ſhall be to the Superficies of the Icoſahedron. Which was to be demonſtrated.

i) 16. 6.

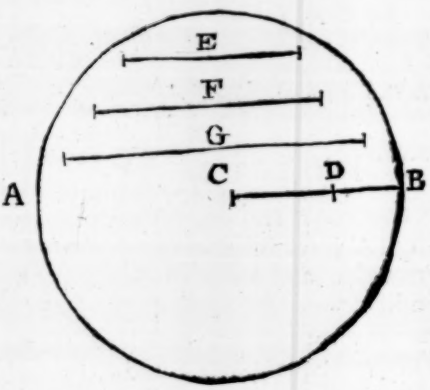
k) 1. 6.



l) Cor. 4. 14.

PROP. 6. THEOR. 6.

If a right line C D, *be divided by extream and mean proportion at* D, *as the ſquare of the whole* C B, *and the ſquare of the greateſt ſegment* C D, *are to the ſquares of the whole* C B, *and the leaſt ſegment* D B, *ſo the ſquare of the ſide of the cube* G, *is to the ſquare of the ſide of the Icoſahedron* F, *inſcribed in one and the ſame ſphere with the cube.*



Demonſtration FOr E being the ſide of the Pentagon, and F the ſide of the triangle inſcribed in the circle propoſed: If G the ſide of the cube be divided by extream and mean proportion, [a] E ſhall be his greateſt ſegment: Wherefore F being the ſide of the triangle inſcribed in the given circle, [b] his ſquare ſhall be triple the ſquare of the ſemidiameter C B, and [c] the ſquares of C B and D B are triple the ſquare of C D: Therefore the ſquare of F is to the ſquare of C B, as the two ſquares of C B and D B are to the ſquare of C D; and by permutation, the ſquare of F ſhall be to the two ſquares of C B and D B, as the ſquare of C B to the ſquare of C D: But as the ſquare of C B is to the ſquare of C D, ſo the ſquare of G to the ſquare of E (ſeeing that as [d] C B the whole is to his greateſt ſegment C D, ſo G the whole, is to E his greateſt ſegment: There-

a) C. 17. 13.

b) 12. 13.

c) 4. 13.

d) 2. 14.

Therefore [e] as the ſquare of C B ſhall be to the ſquare of CD, ſo the ſquare G to the ſquare of E;) therefore [f] as the ſquare of F is to the ſquares of C B and D B, ſo the ſquare of G to the ſquare of E; and alternately, as the ſquare of F to the ſquare of G, ſo the ſquares of C B and D B are to the ſquare of E, which is the ſide of the Pentagon, [g] equal to the ſquares of C B and C D, the ſides of the Hexagon and of the Decagon which are inſcribed in one and the ſame circle: Therefore the ſquare of G the ſide of the cube, is to the ſquare of F the ſide of the Icoſahedron, as the ſquares of C B and C D are to the ſquares of C B and D B: Wherefore, If a right line, &c. Which was to be demonſtrated.

e) 22. 6.
f) 11. 5.
g) 10. 13.

PROP. 7. THEOR. 7.

As the ſide of the Cube is to the ſide of the Icoſahedron, ſo the ſolid of the Dodecahedron is to the ſolid of the Icoſahedron inſcribed in one and the ſame ſphere.

Demonſtration FOraſmuch as the Pentagon of the Dodecahedron, and the triangle of the Icoſahedron inſcribed in one and the ſame ſphere, are [a] inſcribed in one and the ſame circle or equal circles, and that in the ſphere the equal circles are equally diſtant from the center; for the perpendiculars drawn from the center of the ſphere, to the plain of circles, are equal, and fall in the centers of the ſame circles: Therefore the perpendiculars drawn from the center of the ſphere to the center of the circle conteining the Pentagon of the Dodecahedron, and the triangle of the Icoſahedron, are equal: Wherefore the pyramids having for baſes the Pentagons of the Dodecahedron, and the triangles of the Icoſahedron, are of the ſame height: But [b] the pyramids of the ſame height are to one another as their baſes: Therefore as the Pentagon to the triangle, ſo the pyramid which hath for baſe the Pentagon, to the Dodecahedron, and the center of the ſphere for its top; is to the pyramid, whereof the baſe is the triangle of the Icoſahedron, and the center of the ſphere the top.

a) 3. 14.
b) 5. 6. 12.

Therefore alſo as 12 Pentagons are to 20 triangles, ſo 12 Pyramids having the baſes Pentagonal, are to 20 Pyramids having the baſes triangular: But 12 Pentagons are the ſuperficies of the Dodecahedron, and 20 triangles are the ſuperficies of the Icoſahedron: Therefore as the ſuperficies of the Dodecahedron to the ſuperficies of the Icoſahedron, ſo 12 Pyramids having the baſes pentagonal, to 20 Pyramids, having the baſes triangular, and 12 Pyramids having the baſes Pentagonal, are the Solid of the Dodecahedron, and 20 pyramids having the baſes triangular, are the ſolid of the Icoſahedron.

Therefore as the ſuperficies of the Dodecahedron to the ſuperficies of the Icoſahedron, ſo the Solid of the Dodecahedron to the Solid of the Icoſahedron: But as the ſuperficies of the Dodecahedron to the ſuperficies of the Icoſahedron, ſo is ſhewn the ſide of the cube to the ſide of the Icoſahedron; ſo the Solid of the Dodecahedron to the Solid of the Icoſahedron. Which was to be demonſtrated.

COROLLARIE.

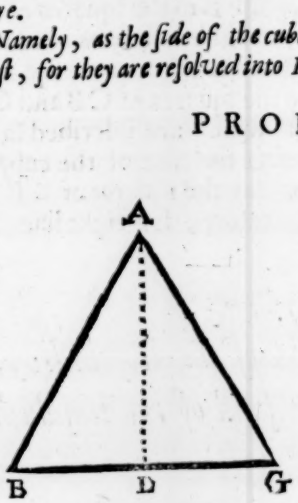
The Solid of a Dodecahedron is to the Solid of an Icoſahedron, as the ſuperficies of

of the one are to the ſuperficies of the other, being deſcribed in one and the ſelf ſame ſphere.

Namely, as the ſide of the cube is to the ſide of the Icoſahedron, as was before manifeſt, for they are reſolved into Pyramids of one and the ſelf ſame altitude.

PROP. 8. THEOR. 8.

If the ſide A B *of an equilater triangle* A B G, *be rational, the ſuperficies* A B G *ſhall be irrational, called Medial.*



Demonſtration FOraſmuch as the line A B is in power ſeſquitertia to the line A D, [a] of what parts the line A B conteineth in power 12 of the ſame parts, the line A D conteineth in power 9: Wherefore the reſidue B D conteineth in power of the ſame parts 3. (For the line A B conteineth in power the lines A D and B D [b]: Wherefore the lines A D and D B are rational and commenſurable to the rational line A B [c]: But foraſmuch as the power of the line A D is to the power of the line D B in proportion as 9 a ſquare number, is to 3 a number not ſquare: Therefore they are not in the proportion of ſquare numbers [d], and therefore they are not commenſurable in length [e]. Wherefore that which is conteined under the lines A D and D B, which are rational lines commenſurable in power only, is Medial [f]: But that which is conteined under the lines A D and D B is double to the triangle A B D [g]. Wherefore that which is conteined under the lines A D and D B, is equal to the whole triangle A B G (which is double to the triangle A B D [h].) Wherefore the the triangle A B G is Medial. If therefore the ſide, &c. Which was required to be proved.

a) C. 12. 13.
b) 47. 1.
c) 6. 10.
d) C. 25. 8.
e) 9. 10.
f) 22. 10.
g) 41. 1.
h) 1. 6.

COROLLARIE.

If an Octohedron and a Tetrahedron be inſcribed in a Sphere, whoſe Diameter is Rational, their Superficies ſhall be Medial.

For thoſe Superficies conſiſting of equilater triangles, whoſe ſides are commenſurable to the diameter which is rational [i]; and therefore they are rational. But they are commenſurable in power onely to the perpendicular line, and therefore they contein a Medial triangle as was before manifeſted.

i) 13, 14. 13.

PROP. 9. THEOR. 9.

If a Tetrahedron and an Octohedron be inſcribed in one and the ſelf ſame ſphere, the baſe of the Tetrahedron ſhall be ſeſquitertia to the baſe of the Octohedron, and the Superficies of the Octohedron ſhall be ſeſquialtera to the Superficies of the Tetrahedron.

Demon-

Demonſtration FOraſmuch as the diameter of the ſphere is in power ſeſquialtera to the ſide of the Tetrahedron [a], and the ſame diameter is in power double to the ſide of the Octohedron [b]: Therefore of what parts the diameter conteineth in power 6, of the ſame the ſide of the Tetrahedron conteineth in power 4, and of the ſame the ſide of the Octohedron conteineth in power 3. Wherefore the power of the ſide of the Tetrahedron is to the power of the ſide of the Octohedron in the ſame proportion that 4 is to 3, which is ſeſquitertia. And like triangles (which are the baſes of the Solids) deſcribed of thoſe ſides, ſhall have the one to the other the ſame proportion that the ſquares made of thoſe ſides ſhall have. For both the triangles are the one to the other, and alſo the ſquares are the one to the other in double proportion of that in which the ſides are [c]: Wherefore of what parts one baſe of the Tetrahedron was 4, of the ſame are four baſes of the Tetrahedron 16. Likewiſe of what parts of the ſame one baſe of the Octohedron was 3, of the ſame are eight baſes of the Octohedron 24. Wherefore the baſes of the Octohedron are to the baſes of the Tetrahedron, in that proportion that 24 is to 16, which is ſeſquialtera. If therefore a Tetrahedron and an Octohedron be inſcribed in one and the ſelf ſame ſphere, the baſe of the Tetrahedron ſhall be ſeſquitertia to the baſe of the Octohedron, and the ſuperficies of the Octohedron ſhall be ſeſquialtera to the ſuperficies of the Tetrahedron: Which was required to be proved.

a) 13. 13.
b) 14. 13.
c) 20. 6.

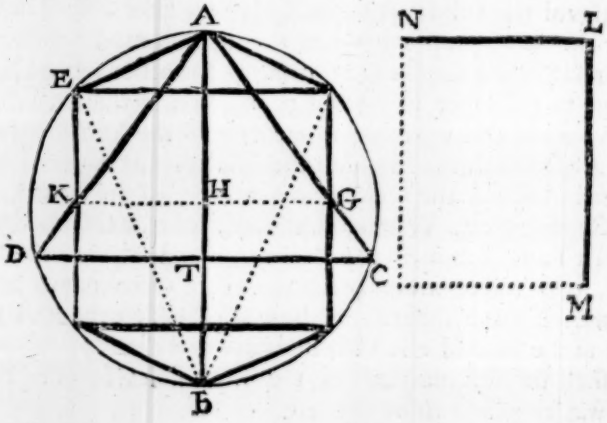
PROP. 10. THEOR. 10.

A Tetrahedron A D C, *is to an Octohedron* A E K B G, *inſcribed in one and the ſelf ſame ſphere* A D B C, *in proportion as the rectangle Parallelogram conteined under the line* N L, *which conteineth in power* $\frac{27}{64}$ *parts of the ſide of the Tetrahedron* A C, *and under the line* M L, *which is ſubſeſquioctava to the ſame ſide of the Tetrahedron, is to the ſquare of the diameter of the ſphere* A B.

Demonſtration 1. FOraſmuch as the line drawn from the angle A, by the center H, perpendicular upon the baſe of the Tetrahedron, falleth upon the center I of the circle which conteineth that baſe, and maketh the right line H T the ſixth part of the diameter A B [a]; therefore the line H A (which is drawn from the center to the circumference) is triple to the line H T; and therefore the whole line A T is to the line A H as 4 is to 3. Let the Tetrahedron A D C be cut by a plain G H K, paſſing by the center H, and being parallel to the baſe D T C [b]. Now then the triangle A D C of the Tetrahedron ſhall be cut by the right line K G, which is parallel to the line D C [c]: Wherefore as the line A T is to the line A H, ſo is the line A C to the line A G [d]: Wherefore the line A C is to the line A G ſeſquitertia, that is, as 4 to 3. And foraſmuch as the triangles A D C, A K G, and the reſt which are cut by the plain K H G, are like the one to the other [e]: The pyramids A D and A K G ſhall be like

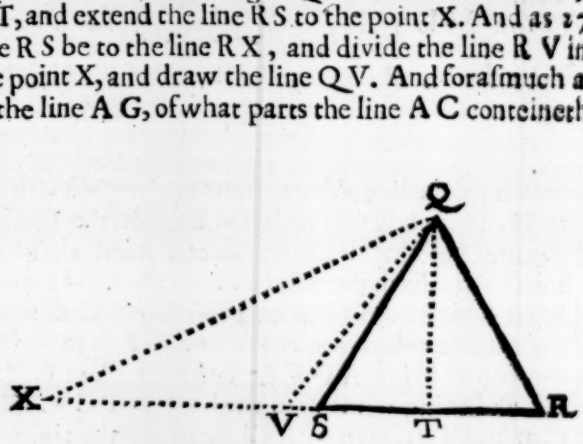
a) 13. 13.
b) C. 15. 11.
c) 16. 11.
d) 2. 6.
e) 5. 6.

f) 7. def. 11. like the one to the other [f]: Wherefore they are in triple proportion of that
g) 8. 12. in which the ſides of A C and A G are [g]. But the proportion of the ſides
b) 2. 8. A C to A G is as the proportion of 4 to 3. Now then if [h] you find out four
i) 15. def. 1. of the leaſt numbers in continual proportion, and in that proportion that 4 is to 3, which ſhall be 64, 48, 36, and 27, it is manifeſt [i] that the extreams 64 to 27 are in triple proportion of that in which the proportion given 4 to 3 is: Or the quantity of the proportion of 4 to 3, which is 1 and $\frac{1}{3}$) being twice multiplyed into it ſelf, there ſhall be produced the proportion



of 64 to 27: Wherefore the Pyramis or Tetrahedron A D C is to the Pyramis A K G as 64 is to 27, which is triple to the proportion of 4 to 3. And foraſmuch as the line A C is to the line A G in length ſeſquitertia of what parts the line A C conteineth in power 64, of the ſame parts doth the line
k) 2. 6. A G contein in power 36. For [k] the proportion of the powers or ſquares, is duple to the proportion of the ſides which are as 64 to 48.

Demonſtration 2. NOw then upon the line R S, which let be equal to the line A G, let there be deſcribed an equilateral triangle
l) 1. 1. Q R S [l], and from the angle Q drawn to the baſe R S, a perpendicular line
m) C. 6. 10. Q T, and extend the line R S to the point X. And as 27 is to 64, ſo [m] let the line R S be to the line R X, and divide the line R V into two equal parts in the point X, and draw the line Q V. And foraſmuch as the line R S is equal to the line A G, of what parts the line A C conteineth in power 64, of the ſame parts the line R S conteineth in power 36, for it is proved that the line A G conteineth in power 36 of thoſe parts: And of what parts the line R S conteineth in power 36, of the ſame parts the line Q T conteineth in power



n) C. 12. 13. 27 [n]. Wherefore of what parts the line A C conteineth in power 64, of the ſame parts the line Q T conteineth in power 27. Wherefore the right line Q T ſhall be equal to the right line L N by ſuppoſition. Again, foraſmuch as the line R S is put equal to the line A G, and of

and of what parts the line R S conteineth in length 27, of the ſame parts is the line R X put to contein in length 64, and of what parts the line R X conteineth in length 64, of the ſame the line A C (which is in length ſeſquitertia to the line A G or R S) conteineth 36. Wherefore the line R V (which is the half of the line R X) conteineth in length of the ſame parts 32, of which the line A C conteined in length 36. Wherefore the line R V is to the line A C ſubſeſquioctava, and therefore the line R V is equal to the line L M, which is alſo ſubſeſquioctava to the ſame line A C. And foraſmuch as the line N L is equal to the line Q T, and the line L M to the line R V (as before hath been proved) the rectangle Parallelogram conteined under the lines Q T and K V, ſhall be equal to the rectangle conteined under the line N L, which is in power $\frac{27}{64}$ to the ſide A C, and under the line L M, which is in length ſubſeſquioctava to the ſame ſide A C. But that which is conteined under the lines Q T and R V is double to the triangle Q V R [o], and to the ſame triangle Q V R is the triangle Q X R double [p]. Wherefore the whole triangle Q X R is equal to that which is conteined under the lines Q T and R V, and therefore is equal to the Parallelogram M N. And foraſmuch as the line R X by ſuppoſition conteineth in length 64 of thoſe parts of which the line R S conteineth 27, and the triangles Q R X and Q R S are [q] in the proportion of their baſes; that is as 64 to 27: But as 64 is to 27, ſo is the Pyramis or Tetrahedron A D C to the Pyramis A K G: Wherefore as the Parallelogram N M or the triangle Q R X is to the triangle Q R S, ſo is the Pyramis A D C to the Pyramis A K G: And foraſmuch as the ſemidiameter A H is the altitude of the Pyramis A K G, and alſo of the two equal and like pyramids of the Octohedron which have their common baſe in the ſquare of the Octohedron [r]: Therefore as the baſe of the Pyramis A K G (which is the triangle Q R S) is to two ſquares of the Octohedron, that is, to the ſquare of the diameter A B, which is equal to thoſe ſquares [ſ]; ſo is the Pyramis A K G to the Octohedron A E B [t]. And foraſmuch as the Parallelogram M N is to the baſe Q R S, as the Pyramis A D C is to the Pyramis A K G, and the baſe Q R S is to the ſquare of the line B E, as the Pyramis A K G is to the Octohedron A E B: Therefore by proportion of equality, taking away the meanes [v], as the Parallelogram N M is to the ſquare of the line B E, ſo is the Pyramis A D C to the Octohedron A E B, inſcribed in one and the ſame ſphere. But the parallelogram N M is conteined under the line N L, which by ſuppoſition is in power $\frac{27}{64}$ to L M, which is alſo by ſuppoſition in length ſubſeſquioctava to the ſame line A C: Wherefore, &c. Which was required to be proved.

o) 41. 1.
p) 1. 6.
q) 1. 6.
r) C. 14. 13.
ſ) 47. 1.
t) 6. 12.
v) 22. 5.

PROP. 11. THEOR. 11.

If a Cube be conteined in a ſphere, the ſquare of the diameter doubled, is equal to all the Superficies of the Cube taken together. And a perpendicular line drawn from the center of the Sphere to any baſe of the Cube, is equal to half the ſide of the Cube.

Demonſtration FOraſmuch [a] as the diameter of the ſphere is in power triple to the ſide of the Cube: Therefore the ſquare of the diameter doubled is ſextuple to the baſe of the ſame Cube. But the ſextuple of

a) 15. 13.

the power of one of the ſides conteineth the whole Superficies of the Cube,
b) 21. de. 11. for the Cube is compoſed of ſix ſquare Superficies,[b] whoſe ſides therefore are equal: Wherefore the ſquare of the diameter doubled is equal to the whole Superficies of the cube. And foraſmuch as the diameter of the Cube, and the line which falleth perpendicularly upon the oppoſite baſes of the Cube do cut the one the other into two equal parts, in the center of the
c) 2 C. 15. 13 Sphere which conteineth the Cube[c], and the whole right line which coupleth the centers of the oppoſite baſes, is equal to the ſide of the Cube
d) 33. 1. [d]; for it coupleth the equal and parallel ſemidiameters of the baſes: Therefore the
e) 15. 5. halfe thereof ſhall be equal to the half of the ſide of the Cube[e]: If therefore a Cube, &c. Which was required to be proved.

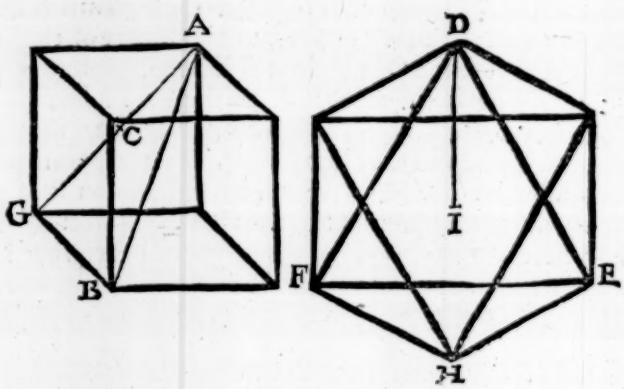
COROLLARIE.

If two thirds of the power of the diameter of the ſphere be multiplyed into the perpendicular line equal to half the ſide of the Cube, there ſhall be produced a Solid equal to the Solid of the Cube.

For it is before manifeſt that two third parts of the power of the diameter of the ſphere are equal to two baſes of the Cube: If therefore unto each of thoſe two thiads be applyed half the altitude of the Cube,
f) 31. 11. they ſhall make each of thoſe Solids equal to half of the Cube[f]; for they have equal baſes: Wherefore two of thoſe Solids are equal to the whole Cube.

PROP. 12. THEOR. 12.

One and the ſelf ſame circle conteineth both the ſquare of a Cube A B G, *and the triangle of an Octobedron deſcribed in one and the ſelf ſame Sphere.*



Conſtruction LEt the diameter of the Sphere be A B or D H, and let the lines drawn from the Centers (that is, the ſemidiameters of the circles which contein the baſes of thoſe Solids) be C A and I D. Then I ſay that the lines C A and I D are equal.

Demonſtration FOraſmuch as A B the diameter of the Sphere which conteineth the Cube, is in power triple to B G the ſide of the
a) 15. 13. Cube[a], unto which ſide A G the diameter of the baſe of the Cube is in power

power double [b], conteineth the baſe [c]. Therefore AB the diameter of the ſphere is in power ſeſquialter to the line AG: namely, of what parts the line AB conteineth in power 12 of the ſame line AG, ſhall contein in power 8. And therefore the right line AC which is drawn from the center of the Circle to the circumference, conteineth in power of the ſame parts 2. Wherefore the diameter of the Sphere is in power ſextuple to the line which is drawn from the center to the circumference of the circle which conteineth the ſquare of the Cube. But the diameter of the ſame Sphere which conteineth the Octohedron is one and the ſame with the diameter of the Cube, namely, DH is equal to AB, and the ſame diameter is alſo the diameter of the ſquare, which is made of the ſides of the Octohedron: Wherefore the ſaid diameter is in power double to the ſide of the ſame Octohedron [d]. But the ſide DF is in power triple to the line drawn from the center to the circumference of the circle which conteineth the triangle of the Octohedron (namely, to the line ID) [e]. Wherefore the ſame diameter AB or DH, which was in power ſextuple to the line drawn from the center to the circumference of the circle which conteineth the ſquare of the Cube, is alſo ſextuple to the line ID, drawn from the center to the circumference of the circle which conteineth the triangle of the Octohedron. Wherefore the lines drawn from the centers of the Circles to the circumferences which contein the baſes of the Cube and of the Octohedron are equal, and therefore the Circles are equal [f]. Wherefore one and the ſelf ſame Circle, &c. Which was required to be proved.

b) 47. 1.
c) 9. 4.
d) 14. 13.
e) 12. 13.
f) 1. def. 3.

COROLLARIE.

Hereby it is manifeſt, that Perpendiculars coupling together in a Spere, the Centers of the Circles which contein the oppoſite baſes of the Cube and of the Octohedron, are equal.

For the Circles are equal, and the lines which paſſing by the center of the Sphere, do couple together the centers of the baſes, are alſo equal. Wherefore the Perpendicular which coupleth together the oppoſite baſes of the Octohedron, is equal to the ſide of the Cube, for either of them is the altitude erected.

PROP. 13. THEOR. 13.

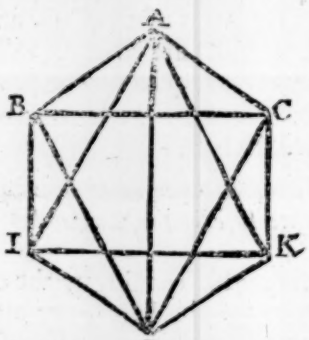
An Octohedron ABCD, *is to the triple of a Tetrahedron* EFGH, *conteined in one and the ſame Sphere, in that proportion as their ſides are.*

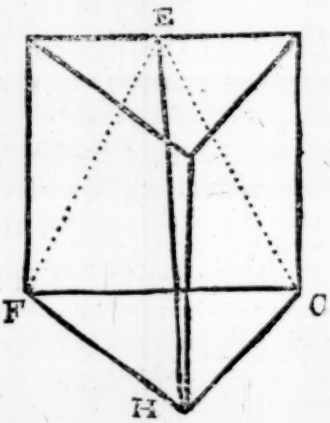
Conſtruction UPon the baſe FGH erect a Priſme, which is done by erecting from the angles of the baſe perpendicular lines, equal to the altitude of the Tetrahedron, which Priſme ſhall be triple to the Tetrahedron EFGH [a]. Then I ſay that the Octohedron is to the Priſme which is triple to the Tetrahedron, as the ſide BC is to the ſide FG.

a) 1 C. 7. 12.

Demonſtration FOr, foraſmuch as the ſides of the oppoſite baſes of the Octohedron are right lines touching one another, and are parallels to other right lines which touch one another, for the ſides of the ſquares which are compoſed of the ſides of the Octohedron, are oppoſite:

Wherefore the oppoſite plain triangles, namely, A B C and K I D, ſhall
b) 15. 11. be parallels, and ſo the reſt [b]. Let the diameter of the Octohedron be the line A D. Now then the whole Octohedron is cut into four equal and like Pyramids, ſet upon the baſes of the Octohedron, and having the ſame altitude with it, and being about the diameter A D, namely the Pyramis ſet upon the baſe B I D, and having his top the point A, and alſo the Pyramis ſet upon the baſe B C D, having his top the ſame point A. Likewiſe the Pyramis ſet upon the baſe I K D, and having his top the ſame point A: and moreover the Pyramis ſet upon the baſe C K D, and
c) 8. def. 11. having his top the former point A, which Pyramids ſhall be equal [c] (for they each conſiſt of two baſes of the Octohedron, and of two triangles conteined under the diameter A D, and two ſides of the Octohedron.) Wherefore the Priſme which is ſet upon the baſe of the Octohedron, and having the ſame altitude with it, namely, the altitude of the parallel baſes, as it is manifeſt by the former, is equal to three of thoſe Pyramids
d) C. 7. 12. of the Octohedron [d]. Wherefore that priſme ſhall have to the other Priſme under the ſame altitude compoſed of the four Pyramids of the
e) Co. 7. 12. whole Octohedron, the proportion of the triangular baſes [e]: And foraſmuch as 4 Pyramids are unto 3 Pyramids in ſeſquitertia proportion to the





baſe of the Priſme which conteineth three Pyramids of the ſame Octohedron and are ſet upon the baſe of the Octohedron; and under the altitude thereof, that is in ſubſeſquitertia proportion to the baſe of the Octohedron. But the baſe of the ſame Octohedron is in ſeſquitertia proportion
f) 10. 14. to the baſe of the pyramis [f]: Wherefore the triangular baſes, namely, of the Priſme which conteineth four pyramids of the Octohedron, and is under the altitude thereof, are equal to the triangular baſes of the Priſme, which conteineth three Pyramids under the altitude of the Pyramis E F G H. But the Priſme of the Octohedron is equal to the Octohedron, and the Priſme of the Pyramis E F G H is proved triple to the ſame Pyramis E F G H. Now then the Priſmes ſet upon equal baſes, are the one to
g) C. 25. 11. the other as their altitudes are [g]: namely, as are the Parallelepipedons
h) 31. 11. their doubles [h]. But the altitude of the Octohedron is equal to the ſide of
i) C. 13. 14. the Cube conteined in the ſame ſphere [i], and the ſide of the Cube is in power to the altitude of the Tetrahedron in that proportion that 12 is to
k) 18. 13. 16 [k]. And the ſide of the Octohedron is to the ſide of the Pyramis in that
l) 18. 13. proportion that 18 is to 24 [l], which proportion is one and the ſame with the proportion of 12 to 16. Wherefore that Priſme which is equal to the Octo-

Octohedron, is to the Prisme which is triple to the Tetrahedron in that proportion that the altitudes, or that the sides are. Wherefore an Octohedron is to the triple of a Tetrahedron, &c. Which was required to be demonstrated.

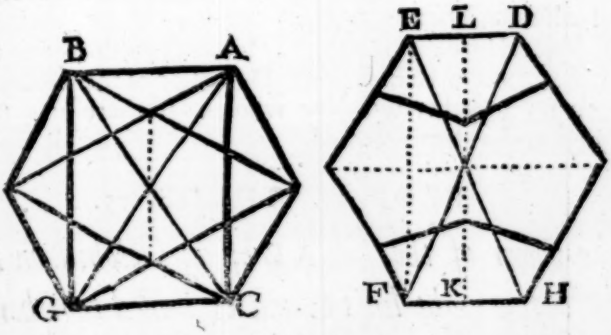
COROLLARIE.

The sides of a Tetrahedron and of an Octohedron are proportional with their altitudes.

For the sides and altitudes were in power sesquitertia. Moreover the diameter of the Sphere is to the side of the Tetrahedron as the side of the Octohedron is to the side of the Cube, namely, the powers of each is in sesquialter proportion [m]. m) 18. 13.

PROP. 14. THEOR. 14.

If a rational line AG *or* DF, *conteining in power two lines* AB *and* BG, *make the whole and the greater segment, and again conteining in power two lines* EF *and* ED, *make the whole and the lesser segment, the greater segment* AB *shall be the side of the Icosahedron* ABGC, *and the lesser segment* ED *shall be the side of the Dodecahedron* DEFH, *conteined in one and the same Sphere.*

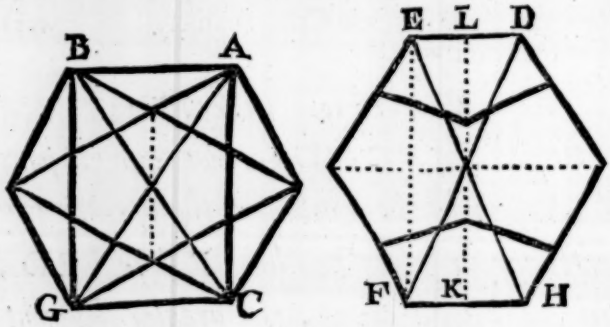


Construction Suppose that AG be the diameter of the sphere which conteineth the Icosahedron ABGC; and let BG subtend the sides of the Pentagon described of the sides of the Icosahedron [a]. Moreover, upon the same diameter AG, or DF equal unto it, let there be described a Dodecahedron [b], whose opposite sides ED and FH, let be cut into two equal parts in the points I and K, and draw a line from I to K. And let the line EF couple two of the opposite angles of the bases which are joyned together. Then I say that AB the side of the Icosahedron, is the greater segment which the diameter AG conteineth in power together with the whole line, and the line ED is the lesser segment which the same diameter AG or DF conteineth in power together with the whole. a) 16. 13. b) 17. 13.

Demonstration For seeing that the opposite sides AB and GC of the Icosahedron being coupled by the diameters AG and BC, are equal and parallels, [c] the right lines BG and AC which couple them c) 2C. 16. 13

d) 33. 1. e) 8. 1. f) 29. 1. g) 47. 1. h) 8. 13.

them together are equal and parallel [d]: Moreover, the angles B A C and A B G being ſubtended of equal diameters, ſhall [e] be equal, and [f] they ſhall be right angles. Wherefore the right line A G conteineth in power the two lines A B and B G [g]. And foraſmuch as the line B G ſubtendeth the angle of the Pentagon compoſed of the ſides of the Icoſahedron, the greater ſegment of the right line B G ſhall be the right line A B [h], which line A B together with the whole line B G, the line A G conteineth in power. And foraſmuch as the line I K coupling the oppoſite and parallel



ſides E D and F H of the Dodecahedron, maketh at thoſe points right angles [i], the right line E F which coupleth together equal and parallel lines E I and F K, ſhall be equal to the ſame line I K [k]. Wherefore the angle D E F ſhall be a right angle [l]: Wherefore the diameter D F conteineth in power the two lines E D and E F. But the leſſer ſegment of the line I K is E D the ſide of the Dodecahedron [m]. Wherefore the ſame line E D is alſo the leſſer ſegment of the line E F (which is equal to the line I K:) Wherefore the diameter D F conteining in power the two lines E D and E F [n], conteineth in power E D the ſide of the Dodecahedron the leſſer ſegment, together with the whole: If therefore, &c. Which was required to be demonſtrated.

i) C. 17. 13. k) 33. 1. l) 29. 1. m) C. 17. 13 n) 47. 1.

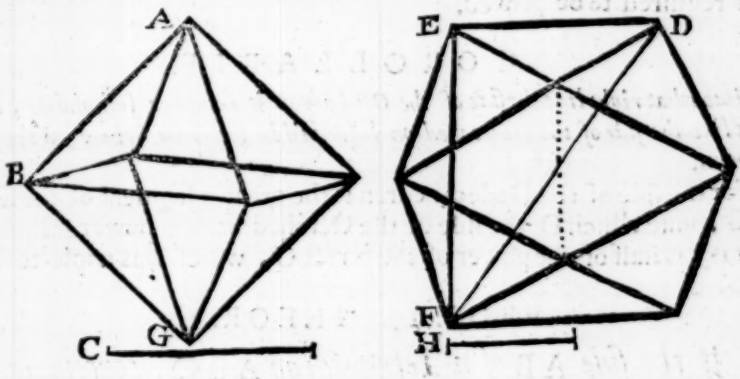
PROP. 15. THEOR. 15.

If the power of the ſide A B *of an Octohedron* A B G, *be expreſſed by two right lines* C *and* H *joyned together by an extream and mean proportion, the ſide* D E *of the Icoſahedron* D E F, *conteined in the ſame Sphere, ſhall be duple to the leſſer ſegment* H.

Demonſtration FOraſmuch as [a] it was manifeſt that E D the ſide of the Icoſahedron is the greater ſegment of the line E F, and that the diameter D F conteineth in power the two lines E D and E F, namely, the whole and the greater ſegment; but by ſuppoſition, the ſide A B conteineth in power the two lines C and H joyned together in the ſame proportion. Wherefore the line E F is to the line E D, as the line C is to the line H [b]. And alternately, [c] the line E F is to the line C, as the line E D is to the line H; and foraſmuch as the line D F conteineth in power the two lines E D and E F, and the line A B conteineth in power the two lines

a) 15. 14. b) 2. 14. c) 16. 5.

lines C and H. Therefore the ſquares of the lines E F and E D are to the ſquare of the line D F, as the ſquares of the lines C and H to the ſquare

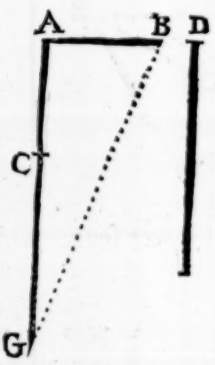


of the line A B. And alternately, the ſquares of the lines E F and E D are to the ſquares of the lines C and H, as the ſquare of the line D F is to the ſquare of the line A B. But D F the diameter is [d] in power double to A B the ſide of the Octohedron inſcribed (by ſuppoſition) in the ſame Sphere. Wherefore the ſquares of the lines E F and E D are double to the ſquares of the lines C and H. And therefore one ſquare of the line E D is double to one ſquare of the line H [e]. Wherefore E D the ſide of the Icoſahedron, is in power dupla to the line H, which is the leſſer ſegment. Therefore, &c. Which was required to be demonſtrated.

d) 14. 13.

e) 12. 5.

PROP. 16. THEOR. 16.



If the ſide A B *of a Dodecahedron, and the right line of which the ſaid ſide is the leſſer ſegment* A G *be ſo ſet, that they make a right angle (as in the point* A*) the right line* D *which conteineth in power half the line* B G, *ſubtending the angle, is the ſide of an Octohedron conteined in the ſame Sphere.*

Demonſtration FOraſmuch as the line A G maketh the greater ſegment G C the ſide of the Cube conteined in the ſame ſphere [a], and the ſquares of the whole line A G, and of the leſſer ſegment A B, are triple to the ſquare of the greater ſegment G C [b]: Moreover, the diameter of the Sphere is in power triple to the ſame line G C, the ſide of the Cube. [c]. Wherefore the line B G is equal to the diameter, for it conteineth in power the two lines A B and A G [d]; and therefore it conteineth in power the triple of the line G C. But the ſide of the Octohedron conteine in the ſame Sphere, is in power triple to half the diameter of the Sphere [e]. And by ſuppoſition, the line D conteineth in power the half of the

a) C. 17. 13.

b) 4. 13.

c) 15. 13.

d) 47. 1.

e) 14. 13.

the line B G. Wherefore the line D (conteining in power the half of the ſame diameter) is the ſide of an Octohedron. If therefore, &c. Which was required to be proved.

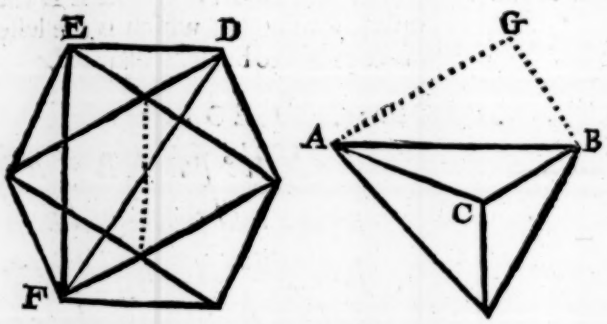
COROLLARIE.

Unto what right line the ſide of the Octohedron is in power ſeſquialter, unto the ſame line the ſide of the Dodecahedron inſcribed in the ſame Sphere, is the greater ſegment.

For the ſide of the Dodecahedron is the greater ſegment of the ſegment C G, unto which D the ſide of the Octohedron is in power ſeſquialtera, that is, is half of the power of the line E G, which was triple to the line C G.

PROP. 17. THEOR. 17.

If the ſide A B *of a Tetrahedron* A B C, *contein in power two right lines* A G *and* G B, *joyned together by an extream and mean proportion, the ſide* E D *of a Icoſahedron* E D F, *deſcribed in the ſame Sphere, is in power ſeſquialter to the leſſer right line* G B.



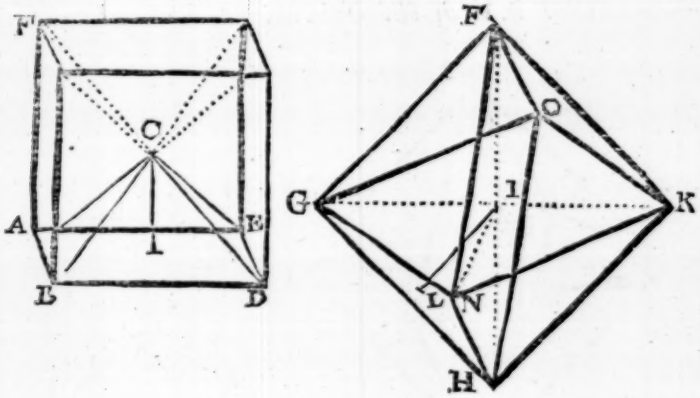
Demonſtration FOraſmuch as [a] the ſide E D is the greater ſegment of the line E F, which ſubtendeth the angle of the Pentagon. But as the whole line E F is to the greater ſegment E D, ſo is the ſame greater ſegment to the leſſer [b]; and by ſuppoſition A G was the whole line, and G B the greater ſegment: Wherefore as E F is to E D, ſo is A G to G B [c]. And alternately, the line E F is to the line A G, as the line E D is to the line G B. And foraſmuch as (by ſuppoſition) the line A B conteineth in power the two lines A G and G B, therefore [d] the angle A G B is a right angle: But the angle D E F is a right angle [e]. Wherefore the triangles A G B and F E D are equiangled [f]: Wherefore their ſides are proportional, namely, as the line E D is to the line G B, ſo is the line F D to the line A B [g]. But by what hath been demonſtrated, F D is the diameter of the Sphere which conteineth the Icoſahedron, which diameter is in power ſeſquialter to A B the ſide of the Tetrahedron inſcribed in the ſame Sphere [h]. Wherefore the line E D the ſide of the Icoſahedron, is in power ſeſquialter to G B the greater ſegment, or leſſer line. If therefore, &c. Which was required to be done.

a) 15. 14.
b) 30. 6.
c) 2. 14.
d) 48. 1.
e) 15. 14.
f) 6. 6.
g) 4. 6.
h) 13. 13.

PROP.

PROP. 18. THEOR. 18.

The Superficies of a Cube A B C D, *is to the Superficies of an Octohedron* F G H K, *inſcribed in one and the ſame Sphere, in that proportion that the Solids are.*



Conſtruction. THe four diameters of the Cube are A C, B C, D C, and E C, produced on each ſide, and the three diameters of the Octohedron are F H, G K, and O N. Draw from the center of the Cube to the baſe A B E D, a perpendicular line C R, and from the center of the Octohedron draw to the baſe G N H, a perpendicular line I L.

Demonſtration. AND foraſmuch as the three diameters of the Cube do paſſe by the center C. Therefore [a] there ſhall be made of the Cube ſix Pyramids, as the Pyramis A B D E C, equal to the whole Cube; for there are in the Cube ſix baſes, upon which fall equal perpendiculars from the center, for the baſes are conteined in equal circles of the Sphere. But in the Octohedron the three diameters do make upon the ten baſes eight Pyramids, having their tops in the center [b]. Now the baſes of the Cube and of the Octohedron are conteined in equal circles of the Sphere [c]. Wherefore they ſhall be equally diſtant from the center, and the perpendicular lines C R and I L ſhall be equal. Wherefore the Pyramids of the Cube ſhall be under one and the ſame altitude with the Pyramids of the Octohedron, namely, under the perpendicular line, drawn from the center to the baſes. Wherefore ſix Pyramids of the Cube are to eight Pyramids of the Octohedron, being under one and the ſame altitude, in that proportion that their baſes are [d]; that is, one Pyramis ſet upon ſix baſes of the cube, and having to his altitude the perpendicular line, which Pyramis is equal to the ſix Pyramids, [e] is to one Pyramis ſet upon the eight baſes of the Octohedron, being equal to the Octohedron, and alſo under one and the ſame altitude, in that proportion that ſix baſes of the Cube, which contein the whole Superficies of the Cube are to eight baſes of the Octohedron, which contein the whole Superficies of the Octohedron, For the Solids of thoſe Pyramids are in proportion to one another as their baſes are [f]. Wherefore, &c. Which was required to be proved.

a) 2 Cor. of 15. 13.

b) 3 Cor. of 14. 13.

c) 13. 14.

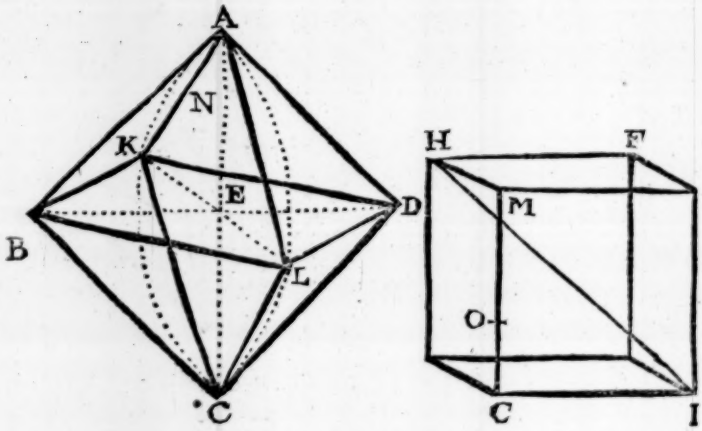
d) 6. 12.

e) 6. 12.

f) 6. 12.

PROP. 19. THEOR. 19.

If a Cube FGHIM, *and an Octohedron* AECDB, *be conteined in one and the ſame Sphere* ABCD, *they ſhall be in proportion to one another as the ſide* MG *of the Cube, is to the ſemidiameter* AE *of the Sphere.*



Demonſtration FOraſmuch as the diameter AC is in power double to BK the ſide of the Octohedron [a], and is in power triple to MG the ſide of the Cube [b]; therefore the ſquare BKDL ſhall be ſeſquialter to FM the ſquare of the Cube. From the line AE cut off a third part AN, and from the line MG cut off likewiſe a third part GO [c]. Now then the line EN ſhall be two third parts of the line AE, and ſo alſo ſhall the line MO be of the line MG. Wherefore the Parallelepipedon ſet upon the ſame baſe BKDL, and having his altitude the line EA, is triple to the Parallelepipedon ſet upon the ſame baſe, and having his altitude the line AN [d]. But it is alſo triple to the Pyramis ABKDL, which is ſet upon the ſame baſe, and is under the ſame altitude [e]. Wherefore the Pyramis ABKDL is equal to the Parallelepipedon which is ſet upon the baſe BKDL, and hath to his altitude the line AN. But unto that Parallelepipedon is double the Parallelepidedon which is ſet upon the ſame baſe BKDL, and hath to his altitude a line double to the line AN [f]; and unto the Pyramis is double the Octohedron ABKLDC [g]. Wherefore the Octohedron ABKDLC is equal to the Parallelepipedon ſet upon the ſame baſe BKLD, and having his altitude the line EN [h]. But the Parallelepipedon ſet upon the baſe BKDL, which is ſeſquialter to the baſe FM, and having to his altitude the line MO, which is two third parts of the ſide of the Cube MG, is equal to the Cube FGI [i]. (For it was before proved that the baſe BKDL is ſeſquialter to the baſe FM.) Now then theſe two Parallelepipedons, namely, the Parallelepipedon which is ſet upon the baſe BKDL (which is ſeſquialter to the baſe of the Cube) and hath to his altitude the line MO (which is two third parts of MG the ſide of the Cube) which Parallelepipedon is proved equal to the Cube and the Parallelepipedon ſet upon the ſame baſe BKDL, and having hi, altis

a) 14. 13.
b) 15. 13.
c) 9. 6.
d) C. 31. 11.
e) 2 C. 7. 12.
f) C. 31. 1.
g) 2 Cor. of 14. 13.
h) 15. 5.
i) 2 part of 34. 11.

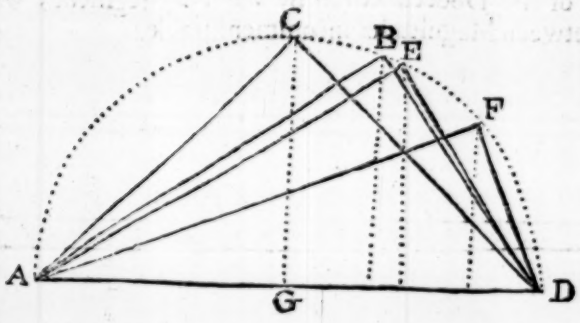
altitude the line E N, which Parallelepipedon is proved equal to the Octohedron) theſe two parallelepipedons (I ſay) are to one another as the altitude M O is to the altitude E N [k]. Wherefore alſo as the altitude M O is to the altitude E N, ſo is the Cube F G H I M to the Octohedron A B K D L C [l]. But as the line M O is to the line E N, ſo is the whole line M G to the whole line E A [m]. Wherefore as M G the ſide of the Cube is to E A the ſemidiameter, ſo is the Cube F G H I M to the Octohedron A B K D L C inſcribed in one and the ſame Sphere. If therefore, &c. Which was required to be demonſtrated.

k) C. 31. 11.
l) 7. 5.
m) 18. 5.

COROLLARIE.

Diſtinctly to notifie the powers of the ſides of the five Solids by the power of the Diameter of the Sphere.

The ſides of the Tetrahedron and of the Cube do cut the power of the Diameter of the Sphere into two ſquares, which are in proportion double to one another. The Octohedron cutteth the power of the Diameter into two equal ſquares. The Icoſahedron into two ſquares, whoſe proportion is double to the proportion of a line divided by an extream and mean proportion, whoſe leſſer ſegment is the ſide of the Icoſahedron. And the Dodecahedron into two ſquares, whoſe proportion is quadruple to the proportion of a line divided by an extream and mean proportion, whoſe leſſer ſegment is the ſide of the Dodecahedron. For A D the Diameter of the Sphere conteineth in power A B the ſide of the Tetrahedron, and B D the ſide of the Cube, which B D is in power half of the ſide A B. The Diameter alſo of the Sphere conteineth alſo in power A C and C D, two equal ſides of the Octohedron. But the Diameter conteineth in power the whole line A E, and the greater ſegment thereof E D, which is the ſide of the Icoſahedron [n]. Wherefore their powers being in dupla proportion of that in which the ſides are [o], have their proportion dupla to the proportion of an extream and mean proportion. Again the Diameter conteineth in power the whole line A F, and his leſſer ſegment F D, which is the ſide of the Dodecahedron [p]. Wherefore the whole having to the leſſer a double proportion of that which the extream hath to the mean, namely, of the whole to the greater ſegment [q], it followes that the proportion of the power is double to the doubled proportion of the ſides [r]; that is, is quadruple to the proportion of the extream and of the mean [s].



n) 15. 14.
o) 1 C. 20. 6.
p) 15. 14.
q) 10. de. 5.
r) 1 C. 20. 6.
s) def. 6.

By this meanes therefore the Diameter of a Sphere being given, there ſhall be given the ſide of every one of the bodies inſcribed. And foraſmuch as three of thoſe bodies have their ſides commenſurable in power only, and not in length unto the Diameter given (for their powers are in the proportion of a ſquare number to a number not ſquare. Wherefore they have not the proportion of a ſquare number to a ſquare number [t]. Where-

t) C. 25. 8.

v) 9. 10. Wherefore alſo their ſides are incommenſurable in length [v]: Therefore it is ſufficient to compare the powers and not the lengths of thoſe ſides to one another; which powers are conteined in the power of the Diameter, namely, from the power of the Diameter let there be taken away the power of the Cube, and there ſhall remain the power of the Tetrahedron, and taking away the power of the Tetrahedron, there remaineth the power of the Cube; and taking away from the power of the Diameter half the power thereof, there ſhall be left the power of the ſide of the Octohedron. But foraſmuch as the ſides of the Dodedecahedron and of the Icoſahedron are proved to be irrational, for the ſide of the Icoſahedron is a leſſer line [w], and the ſide of the Dodecahedron is a Reſidual line [x]; therefore thoſe ſides are unto the Diameter which is a Rational line, incommenſurable both in length and power. Wherefore their compariſon cannot be defined or deſcribed by any proportion expreſſed by numbers [y], neither can they be compared to one another; for irrational lines of divers kinds are incommenſurable to one another; for if they ſhould be commenſurable, they ſhould be of one and the ſame kind [z], which is impoſſible. Wherefore we ſeeking to compare them to the power of the diameter, though they could not be more aptly expreſſed then by ſuch proportions, which cut that rational power of the Diameter according to their ſides, dividing the power of the Diameter by lines in that proportion, as the greater to the leſſer ſegment, to put the leſſer ſegment to be the ſide of the Icoſahedron, and dividing the ſaid power of the Diameter by lines in proportion as the whole to the leſſer ſegment, to expreſſe the ſide of the Dodecahedron by the leſſer ſegment, which may well be done between Magnitudes incommenſurable.

w) 16. 13.

x) 17. 13.

y) 8. 10.

z) 103, 105. 10.

The End of the Fourteenth Element of EUCLIDE.

THE

THE FIFTEENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

THis Fifteenth and laſt Book of *EUCLIDE*, or rather the Second Book of *Appollonius* or *Hypſicles*, teacheth the inſcription and circumſcription of the five regular bodies one within & about another, a thing undoubtedly pleaſant and delectable in mind to contemplate, and alſo profitable and neceſſary to practice. For without practice in act it is very hard to ſee and conceive the conſtructions and demonſtrations of the Propoſitions of this Book, unleſſe a man have a very deep, ſharp, and fine imagination. Wherefore I would wiſh the diligent ſtudent in this Book (to make the ſtudy thereof more pleaſant unto him) to have preſently before his eyes, the bodies formed and framed of paſted paper: And then to draw and deſcribe the Lines and Diviſions, and Superficies, according to the conſtructions of the Propoſitions. In which Deſcriptions if he be careful and diligent, he ſhall find all things in theſe Solid matters, as cleer and manifeſt to the eye, as were things before taught only in plain or ſuperficial figures.

DE-

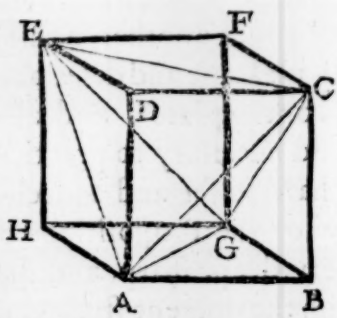
DEFINITIONS.

1 *A ſolid figure is then ſaid to be inſcribed in a ſolid figure, when the angles of the figure inſcribed touch together at one time, either the angles of the figure circumſcribed, or the ſuperficies, or the ſides.*

2 *A ſolid figure is then ſaid to be circumſcribed about a ſolid figure, when together at one time either the angles, or the ſuperficies, or the ſides of the figure circumſcribed, touch the angles of the figure inſcribed.*

PROPOSITIONS, and PROBLEMES.

PROPOSITION 1. PROBLEM 1.



In a given Cube ABCDE FGH, *to inſcribe a Pyramid.*

Conſtruction LEt the given Cube be A B C D E F G H, to inſcribe therein a Pyramid or Tetrahedron; from one angle thereof E, let there be drawn to the baſes conſtituting it three diameters E A, E G, and E C, from the extremities of which A, G, and C, let there be alſo drawn the diameters A G, G C, and C A, to the other three baſes which joyne the extremities of the three firſt Diameters.

Demonſtration FOraſmuch as the diameters of the equal ſquares are equal to one another, being double in power [a] to the ſides of the equal ſquares: It is manifeſt that the four triangles A C E, G A C, G A E, and G C E compounded of the ſaid diameters, are equilateral and equal to one another; therefore do conſtitute a Pyramid; and all the angles of the Pyramid being compared to the angles of the Cube, [b] it is manifeſt that it is inſcribed in the Cube. Therefore in a given Cube we have inſcribed a Pyramid. Which was to be done.

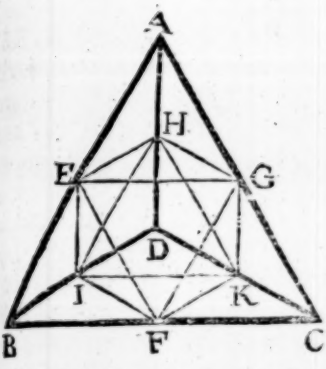
a) 47. 1.

b) 3. def. 11.

PROP.

PROP. 2. PROBL. 2.

In a given Pyramid to inſcribe an Octohedron.



Conſtruction LEt the given Pyramid be ABCD, in which an Octohedron ought to be inſcribed. Having divided all the ſides in two equal parts at E, F, G, H, I, and K, and drawn EF, FG, GE, HI, IK, KH, EI, IF, FK, KG, GH, and HE, there ſhall be conſtituted eight triangles, four of which EHI, IHK, KHG, and HGE, are on the plain EIKG: But the four others IFE, EFG, GFK, and KFI, below the ſame plain.

Demonſtration BUt foraſmuch as the ſides AE and AH of the triangle AEH, are equal to AG and AH of the triangle AGH, being the halves of the equal lines AB, AD, and AC, and the angles conteined of them EAH and GAH equal, being angles of the equilateral triangles ABD and ACD, [a] the baſes EH and GH ſhall be equal, and by the ſame reaſon EH and IH and the others, in like manner: Wherefore theſe eight triangles are equilateral and equal to one another; and therefore do compound the Octohedron EIGHF, [b] the which is inſcribed in the Pyramid, ſeeing that all the angles do touch all the ſides of the Pyramids, by the Conſtruction. Therefore in a Pyramid we have inſcribed an Octohedron. Which was to be done.

a) 4. 1.

b) 31. de. 11

COROLLARIE.

Then foraſmuch as the three plains EIKG, GHIF, *and* FKHE, [c] *are equal ſquares, cuting one another at right angles each of which cutteth the Pyramid* ABCD *in two equal parts (for the ſquare* EK *divideth it in the Pyramid* HIKD, *and in the Priſme* HIKGAE, *and alſo in the Pyramid* EBFI, *and in the Priſme* EFCGKI: *But it appeares that the Pyramid is equal to the Pyramid, and the Priſme to the Priſme: In like manner the other ſquares* [d] GI *and* FH *do divide the ſame Pyramid* ABCD *in two equal parts:) It is manifeſt that if in the Tetrahedron there be inſcribed an Octohedron, that the Tetrahedron ſhall be cut in two equal parts, by three equal ſquares, the which do cut one another at right angles, and the Tetrahedron in two equal parts.*

c) Cor. 1. of 14. 13.

d) 8. 12.

PROP. 3. PROBL. 3.

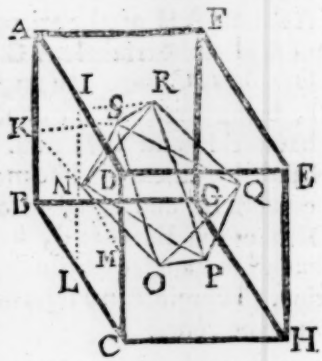
In a given Cube to inſcribe an Octohedron.

Conſtruction LEt the given Cube be AH, in which an Octohedron ought to be inſcribed. Let the ſides of the baſe AD and CD, be divided in two equal parts by I, K, L, and M, which let be joyned by IL and KM, cutting one another in N, which is the center of the ſquare BD, as appeares by the demonſtration of the 8th. of the 4th. and ſo the centers of the other baſes, O, P, Q, R, and S, ſhall be found. Therefore all the right lines drawn from the ſaid centers to the middle of the baſes,

baſes, as are N I, R I, N K, S K, &c. are equal to the halves of the ſides of the Cube, or the ſquares, as appeares by the ſaid Demonſtration of the eighth of the fourth.

Laſtly, if the ſaid centers be joyned by N O, O P, P Q, Q R, R S, S N, N P, P R, R N, S O, O Q, and Q S, there will be conſtituted eight triangles, four of which N S R, R S Q, Q S O, and O S N, are on the plain N O Q R, and the other four O P Q, Q P R, R P N, and N P O, below the ſame Plain.

Demonſtration BUt foraſmuch as the ſides I N and N R, of the triangle I N R, are equal to K N and R S of the triangle K N S, being all halves of the equal ſides of the Cube as is ſaid, alſo the angles conteined of them are equal; for N I and I R being parallel to B A and A F, a) 10. 11. [a] the angle N I R is equal to the right angle K A F of the ſquare A G; by the ſame reaſon N R S being equal to the right angle I A F of the ſquare b) 4. 1. A E, [b] the baſes N R and N S ſhall be equal; ſo we ſhall demonſtrate all the other lines equal both to one another, and to theſe two: If from the centers be drawn right lines to the middle of the ſides, in ſuch manner as that each two be drawn to the middle of the ſide common to two ſquares of the Cube, from which theſe two points, from whence come the right lines, are the centers; ſo as N I and R I which are drawn to I, halfe of the ſide A D, common to the ſquares A C and A E, whoſe centers are N and R: In like manner N K and S K are drawn to K, the middle of A B, common to the ſquares of A C and A G, whoſe centers are N and S, &c. Therefore we have conſtituted eight equilateral triangles, and equiangled to one another, which do conſtitute the Octohedron N O Q R S P, which c) d. 31. 11. [c] is inſcribed in the Cube, ſeeing that all his ſix angles touch all the ſix baſes of the Cube at their centers. Therefore in the given Cube we have inſcribed an Octohedron, &c. Which was to be done.



d) 1. 15. e) 2. 15. Otherwiſe, [d] in the Cube let there be inſcribed a Pyramid, and [e] in that Pyramid let there be inſcribed an Octohedron, and what was propoſed is done. For ſeeing that according to the Demonſtration of the ſecond Propoſition of this Book, the angles of the Octohedron do touch the ſides of the Pyramid; but the ſides of the Pyramid (by the Demonſtration of the firſt Propoſition of this Book) are in the plains of the baſes of the Cube, being the diameters of the ſame baſes: It is manifeſt that the angles of the Octohedron do touch in like manner the baſes of the Cube; and therefore that the Octohedron is inſcribed in the Cube.

COROLLARIE.

Then foraſmuch as the diameters of the Octohedron inſcribed in the Cube, N Q *and* R O *(if they were drawn)* f) 1 C. 14. 13 [f] *cut one another at right angles, and joyn the centers of the oppoſite baſes of the Cube. It is manifeſt that the right lines which joyn together the centers of the oppoſite baſes of the Cube divide themſelves not only in two equal parts,* g) 1 C. 15. 13 [g] *as is ſhewn; but alſo at right angles, as we have here ſhewn.*

PROP.

PROP. 4. PROBL. 4.

In a given Octohedron ABCDEF, *to inscribe a Cube.*



Demonstration FFor asmuch as six Pyramids quadrangular are conteined in the Octohedron whose tops are the six angles of the Octohedron, and each two constitute an Octohedron: Let one of them, to wit, the Pyramid ABCDE, whose base isthe square ABCD, and the equilateral triangles ABE, EBC, CED, and DEA, whose centers are G, H, I, and K, by the which let there be drawn LM, MN, NO, and OL parallel to AB, BC, CD, and DA, the triangles LME, EMN, NEO, and OEL, shall be equilateral,[a] being alike to the triangles equilateral aboveſaid, and therefore equal to one another, having the sides common, and the bases LM, MN, and NO, by consequence equal, and if from E by the center K, be drawn EP, it will divide the angle at E in two equal parts; therefore LK shall be equal to KO; that is to say, that L is divided into two equal parts by K the center; and in like manner LM, MN, and NO, shall be cut each in two equal parts by the centers G, H, and L; therefore the four triangles GLK, KOI, GMH, and HNI, are Isosceles triangles ([b] the angles GLK, KOI, INH, and HMG, being equal to the right angles of the square GKIH) therefore the bases GK, KI, IH, and HG, shall be equal, and shall contein right angles. Wherefore GHIK is a square, and if in the same manner, in the other five Pyramids of the Octohedron, the centers of the triangles are joyned by right lines, there shall be described in like manner five squares, which shall be equal to one another, having the sides common. Wherefore six such squares shall compound a Cube, [c] which shall be described in the Octohedron, seeing that the eight angles thereof do touch the eight bases of the Octohedron at their centers. Therefore in the Octohedron given we have inscribed a Cube. Which was to be done.

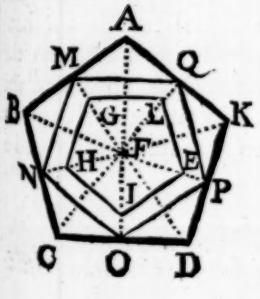
a) Cor. 4.6.

b) 10. 11.

c) 31. de. 11.

PROP. 5. PROBL. 5.

In a given Icosahedron to inscribe a Dodecahedron.

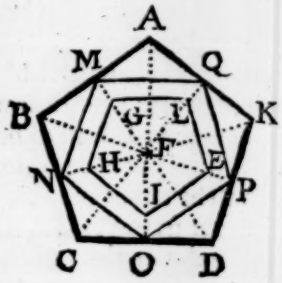


Construction LEt one of the twelve Pyramids of the Icosahedron, whose bases are Pentagons, ABCDEF, and the base the Pentagon ABCDE, and the equilateral triangles ABF, BCF, CDF, DEF, and EAF, and their centers G, H, I, K, and L, joyned, by GH, HI, IK, KL, and LG. Again, from the top F, by the centers of the triangles let there be drawn FM, FN, FO, FP, and FQ, cutting [a] AB, BC, CD, DE, and EA, at M, N, O, P, and Q, in such sort as MA, MB, NB, NC, OC, OD, PD, PE, QE, and QA, may be equal and comprehend equal angles, to wit, of

a) C. 10. 13.

b) 4. 1. c) 8. 1. the Pentagon [b] MN, NO, OP, P ♀, and Q M, joyning the points M, N, O, P, and ♀, are also equal; therefore [c] the angles MF N, NF O, OF P, PF ♀, and ♀FM, are equal, the sides FM, FN, FO, FP, and FQ, being equal, to wit, perpendiculars of the angles of equilateral triangles drawn to the opposite bases.

Demonstration FOr FB and BM of the triangle FBM, being equal to FB and BN of the triangle FBN, and conteining equal d) 4. 1. angles, to wit, of the equilateral triangles, the bases [d] FN and FM shall be equal, and so of the others. But forasmuch as FG, FH, FI, FK, and FL, semidiameters of the equal circles circumscribing the equilateral e) 4. 1. triangles are equal, aud contein equal angles as is demonstrated; [e] GH, HI, IK, KL, and LG, shall be equal; and seeing that as well AMQ, ♀MN, MNB, as BNM, NMO, and ONC, are equal to two right angles. But AMQ and NMB are equal to BNM and ONC, the remainders ♀MN and MNO shall be equal. By the same reason the other angles NOP, OPQ, and PQM, shall be equal both to one another and to these: Wherefore the Pentagon MNOPQ shall be equilateral and equiangled to the plain of the Pentagon ABCDE. Lastly, FM, FN, FO, FP, and F ♀, being cut proportionally at G, H, I, K, and L: Forasmuch also as FG, FH, FI, FK, and FL, are equal lines; drawn from the center;



f) 2. 6. g) 10. 11. [f] GH, HI, IK, KL, and LG, shall be parallel to MN, NO, OP, PQ, and QM; and [g] therefore the angles IGH, GHI, HIK, IKL, and LKG, are equal to the equal angles ♀MN, MNO, NOP, OP♀, h) 15. 11. PQM, therefore also equal to one another, and [h] the plains drawn by GH, HI, and HI, IK, being parallels to the plain of the Pentagon MNOP♀, drawn by MN, NO, and OP, and meeting one another at HI, shall make one only plain. In like manner may be shewn that the plains IK, KL, and KL, LG, and LG, GE, will make one and the same plain with the plain drawn by GH, HI, and IK. Wherefore GHIKL is an equilateral Pentagon and equiangled, having the angles and the sides equal; and if in the same manner to the other eleven Pyramids of the Icosahedron, there be joyned right lines to the centers of the triangles, there will in like manner be described the equilateral triangles equiangled, the which having the sides common, shall be equal to one another: Wherefore twelve such Pentagons shall constitute a Dodecahedron, the i) 31. d. 11. which [i] shall be inscribed in the Icosahedron, the twenty angles of the Dodecahedron being constituted at the centers of the twenty bases of the Icosahedron. Therefore in the Icosahedron given we have inscribed a Dodecahedron. Which was to be done.

PROP. 6. PROBL. 6.

In an Octohedron given ABGDEI, *to inscribe a trilateral equilateral Pyramis.*

Construction TAke four bases of the Octohedron, that is, three which close in the lowest triangle EGD, namely, AEG, BED, IGD,

IGD, and let the fourth be AIB, which is oppoſite to the loweſt triangle before put, namely, to EGD. And take the centers of thoſe four baſes, which let be the points H, C, N, and L, and upon the triangle HCN erect a Pyramis HCNL. Now foraſmuch as theſe two baſes of the Octohedron, to wit, AGE and ABI, are ſet upon the right lines EG and BI, which are oppoſite to one another in the ſquare GEBI of the Octohedron; from the point A draw by the centers of the baſes, to wit, by the centers H and L, perpendicular lines AHF and ALK, cutting the lines EG and BI into two equal parts in the points F and K [a]: Wherefore a right line drawn from the point F to the point K, ſhall be a parallel, and equal to the ſides of the Octohedron, to wit, to EB and GI [b]. And the right line HL, which cutteth the equal ſides AF and AK proportionally (for AH and AL are drawn from the centers of equal circles, to the circumferences) is a parallel to the right line FK [c], and alſo to the ſides of the Octohedron, to wit, to EB and IG [d]. Wherefore as the line AF is to the line AH, ſo is the line FK to the line HL [e].

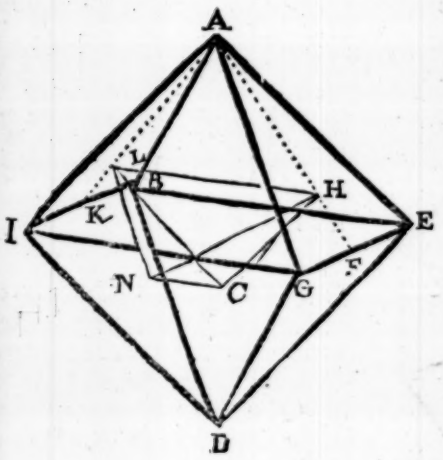
a) C. 12. 13.

b) 33. 1.

c) 2. 6.

d) 9. 11.

e) 4. 6.



Demonſtration FOr the triangles AFK and AHL are like [f]: But the line AF is in ſeſquialter proportion to the line AH: (for the ſide EG maketh HF the half of the right line AH [g]:) Wherefore FK or GI the ſide of the Octohedron is ſeſquialter to the right line HL. And by the ſame reaſon may we prove that the ſides of the Octohedron are ſeſquialter to the reſt of the right lines which make the pyramis HNCL, to wit, to the right lines HN, NC, CL, LN, and CH: Wherefore thoſe right lines are equal, and therefore the triangles which are deſcribed of them, to wit, the triangles HCN, HNL, NCL, and CHL, which make the Pyramis HNCL, are equal and equilateral. And for aſmuch as the angles of the ſame Pyramis, to wit, the angles H, C, N, and L, do end in the centers of the baſes of the Octohedron: Therefore it is inſcribed in the ſame Octohedron [h]. Wherefore, &c. Which was required to be done.

f) Cor. 2. 6.

g) C. 12. 13.

h) 1 def. 15.

COROLLARIE I.

The baſes of a Pyramis inſcribed in an Octohedron, are parallels to the baſes of the Octohedron.

For ſeeing the ſides of the baſes of the Pyramis touching one another, are parallels to the ſides of the Octohedron, which alſo touch one another: As for example, HL was proved to be a parallel to GI, and LC to DI. Therefore [i] the plain Superficies which is drawn by the lines HL and LC, is a parallel to the plain Superficies drawn by the lines GI and DI, and ſo alſo of the reſt.

i) 15. 11.

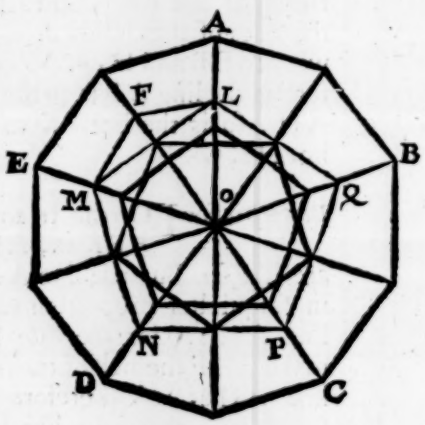
COROLLARIE II.

A right line joyning together the centers of the oppoſite baſes of the Octohedron, is ſeſquialter to the perpendicular line drawn from the angle of the inſcribed Pyramis, to the baſe thereof.

For ſeeing the Pyramis and the Cube which conteineth it, do in the ſame points end their angles [k]: Therefore they ſhall both be incloſed in one and the ſame Octohedron [l]. But the diameter of the Cube joyneth together the centers of the oppoſite baſes of the Octohedron, and therefore is the diameter of the ſphere which conteineth the Cube and the Pyramis inſcribed in the Cube [m]; which diameter is ſeſquialter to the perpendicular which is doawn from the angle of the Pyramis to the baſe thereof; for the line which is drawn from the center of the ſphere to the baſe of the Pyramis is the ſixth part of the diameter [n].

k) 1. 15.
l) 4. 15.
m) 13, 14. 13
n) 2C. 13. 13

PROP. 7. PROBL. 7.



In a Dodecahedron given AB CD E, *to inſcribe an Icoſahedron.*

Conſtruction LEt the centers of the Circles, which contein ſix baſes of the Dodecahedron, be the points L, M, N, P, Q, and O. And draw thoſe right lines O L, O M, O P, and OQ, as alſo the right lines LM, MN, N P, P Q, and QL.

Demonſtration AND now foraſmuch as equal and equilateral Pentagons are conteined in equal Circles: Therefore perpendicular lines drawn from their centers to the ſides, ſhall be equal [a], and ſhall divide the ſides of the Dodecahedron into two equal parts [b]: Wherefore the foreſaid perpendicular ſhall concurre in the point of the Section, wherein the ſides are divided into two equal parts, as LF and MF do. And they alſo contein equal angles, to wit, the inclination of the baſes of the Dodecahedron [c]: Wherefore the right lines LM, MN, NP, PQ, and QL, and the reſt of the right lines which joyn together two centers of the baſes, and which ſubtend the equal angles conteined under the ſaid equal perpendicular lines, are equal to one another [d]: Wherefore the triangles OLM, OMN, ONP, OP Q, OQL, and the reſt of the triangles which are ſet at the centers of the Pentagons are equilateral and equal. Now foraſmuch as the 12 Pentagons of a Dodecahedron contein 60 plain ſuperficial angles, of which 60 every 3 make one ſolid angle of the Dodecahedron: It followes that a Dodecahedron hath 20 ſolid angles, but each of theſe ſolid angles is ſubtended of each of the triangles of the Icoſahedron, to wit, of each of thoſe triangles which joyn together the centers of the Pentagons which make the ſolid angle, as hath been proved. Wherefore

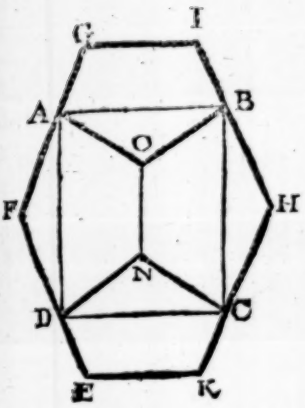
a) 14. 3.
b) 3. 3.
c) 3C. 18. 11
d) 4. 1.

20 equal and equilateral triangles, which ſubtend the 20 ſolid angles of the Dodecahedron, and have their ſides which are drawn from the centers of the Pentagons common, do make an Icoſahedron [c]; and it is inſcribed in the Dodecahedron given [f], for that the angles thereof do all at one time touch the baſes of the Dodecahedron. Wherefore, &c. Which was required to be done.

c) 25. de. 11.
f) 1. def. 15.

PROP. 8. PROBL. 8.

In a Dodecahedron given, to include a Cube.



Conſtruction DEſcribe [a] a Dodecahedron: And take the 12 ſides of the Cube, each of which ſubtend one angle of each of the 12 baſes of the Dodecahedron; for the ſide of the Cube ſubtendeth the angle of the Pentagon of the Dodecaheron [b]: If therefore in the Dodecahedron deſcribed we draw the twelve right lines ſubtended under the foreſaid 12 angles, and ending in 8 angles of the Dodecahedron, and concurring together in ſuch ſort that they be in like ſort ſituate (as was plainly proved [c];) then ſhall it be manifeſt that the right lines drawn in this Dodecahedron from the foreſaid 8 angles thereof, do make the foreſaid Cube, which therefore is included in the Dodecahedron, for that the ſides of the Cube are drawn in the ſides of the Dodecahedron.

a) 17. 13.
b) 2 Cor. of 17. 13.
c) 2 Cor. of 17. 13.

Demonſtration AS for Example, take four Pentagons of a Dodecahedron, to wit, AGIBO, BHCNO, CKEDN, and DFAON; and draw theſe right lines AB, BC, CD, and DA, which four right lines make a ſquare, for that each of thoſe right lines do ſubtend equal angles of equal Pentagons, and the angles which thoſe four right lines contein are right angles: Wherefore the ſix baſes being ſquares, do make a Cube [d]; and for that the eight angles of the ſaid Cube are ſet in eight angles of the Dodecahedron: Therefore is the ſaid Cube inſcribed in the Dodecahedron [e]: Wherefore, &c. Which was required to be done.

d) 21. d. 11.
e) 1. def. 15.

PROP. 9. PROBL. 9.

In a Dodecahedron given ABGD, *to include an Octohedron* AGBDCI.

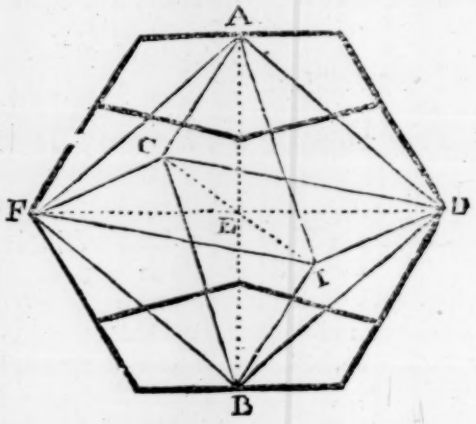
Conſtruction TAke [a] the 6 ſides which are oppoſite to one another, thoſe 6 ſides I ſay whoſe ſections wherein they are divided into two equal parts, are coupled by three right lines within the center of the ſphere wherein the Dodecahedron is conteined, do cut one another perpendicularly. And let the points wherein the foreſaid ſides are cut into two equal parts be A, B, G, D, C, and I. And let the foreſaid three right lines joyning togther the ſaid ſections, be AB, GD, and CI, and let the center of the ſphere be E.

a) 3 C. 17. 13

Demon-

c) Cor. 3. of 17. 13.
c) 4. 1.

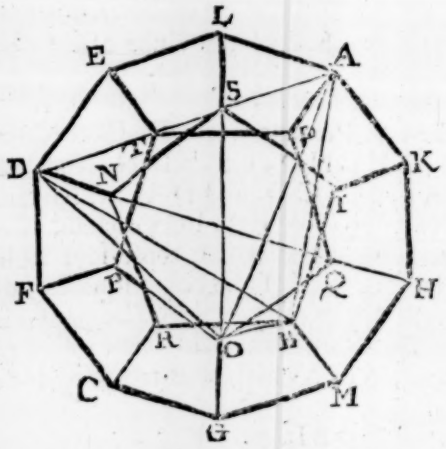
Demonſtration NOw foraſmuch [b] as thoſe three right lines are equal, it followeth [c] that the right lines ſubtending the right angles which they make at the center of the ſphere, which right angles are conteined under the halves of the ſaid three right lines, are equal to one another, that is, the right lines A G, G B, B D, D A, C A, C G, C B, C D, and I A, I G, I B, and I D, are equal to one another. Wherefore alſo the eight triangles C A G, C G B, C B D, C D A, I A G, I G B I B D, and I D A, are equal and equilateral. And therefore A G B D C I is an Octohedron [d]. And the ſaid Octohedron is included in the Dodecahedron [e]; for that all the angles thereof do at one time touch the ſides of the Dodecahedron. Wherefore, In the, &c. Which was required to be done.



d) 23. de. 11
e) 1. def. 15.

PROP. 10. PROBL. 10.

In a Dodecahedron given A B C D, *to inſcribe an equilateral trilateral Pyramis.*



Conſtruction TAke three baſes of the Dodecahedron, meeting at the point S, to wit, the baſes A L S I K, D N S L E, and S I B R N, and of thoſe three baſes take the three angles at the points A, B, and D, and draw theſe right lines A B, B D, and D A; and let the diameter of the ſphere conteining the Dodecahedron be S O, and there draw theſe right lines A O, B O, and D O.

a) 17. 13.

Demonſtration NOw foraſmuch as [a] the angles of the Dodecahedron are ſet in the Superficies of the Sphere deſcribed about the Dodecahedron: Therefore if upon the diameter S O, and by the point A, be deſcribed a ſemicircle, it ſhall make the angles S B O and S D O right angles. Wherefore the diameter S O conteineth in power both the lines S A and A O; or the lines S B and B O, or elſe S D and D O; but the lines S A, S D, and S B, are equal to one another; for they each ſubtend one of the angles of equal Pentagons: Wherefore the other lines re-

maining, to wit, A O, B O, and D O, are equal to one another, and by the ſame reaſon may be proved that the diameter H D, which ſubtendeth the two right lines H A aud A D, conteineth in power both the ſaid two right lines, and alſo conteineth in power both the right lines H B and B D, which two right lines it alſo ſubtendeth. And moreover, by the ſame reaſon, the diameter A C, which ſubtendeth the right lines C B and B A, conteineth in power both the ſaid right lines C B and B A. But the right lines H A, H B, and C B, are equal to one another, for that each of them alſo ſubtendeth one of the angles of equal Pentagons: Wherefore the right lines remaining, to wit, A D, B D, and B A, are equal to one another. And by the ſame reaſon may be proved that each of theſe right liues A D, B D, and B A, is equal to each of the right lines A O, B O, and D O: Wherefore the ſix right lines A B, B D, D A, A O, B O, and D O, are equal to one another: And therefore the triangles which are made of them, to wit, the triangles A B D, A O B, A O D, and B O D, are equal and equilateral; which triangles therefore do make a Pyramis A B D O, whoſe baſe is A B D, and tops the point O, each of the angles of which Pyramis, to wit, the angles at the points A, B, D, and O, do in the ſame points touch the angles of the Dodecahedron. Wherefore the ſaid Pyramis is inſcribed in the Dodecahedron [b]. Wherefore, In a Dodecahedron, &c. Which was required to be done.

b) 1. def. 15.

PROP. 11. PROBL. 11.

In an Icoſahedron given, to inſcribe a Cube.

IT was manifeſt [a] that the angles of a Dodecahedron are ſet in the centers of the baſes of the Icoſahedron. And [b] it was proved that the angles of a Cube are ſet in the angles of a Dodecahedron. Wherefore the ſame angles of the Cube ſhall of neceſſity be ſet in the centers of the baſes of the Icoſahedron. Wherefore the Cube ſhall be inſcribed in the Icoſahedron [c]. Wherfore, &c. Which was required to be done.

a) 7. 15.
b) 8. 15.
c) 1. def. 15.

PROP. 12. PROBL. 12.

An Icoſahedron given, to inſcribe a trilateral equilateral Pyramis.

IT was manifeſted [a] that the angles of a Cube are ſet in the centers of the baſes of the Icoſahedron: And [b] it was plain that the four angles of a Pyramis are ſet in four angles of a Cube. Wherefore it is evident [c] that a Pyramis deſcribed of right lines joyning together theſe four centers of the baſes of the Icoſahedron, ſhall be inſcribed iu the ſame Icoſahedron. Wherefore, In an Icoſahedron, &c. Which was required to be done.

a) 11. 15.
b) 1. 15.
c) 1. def. 15.

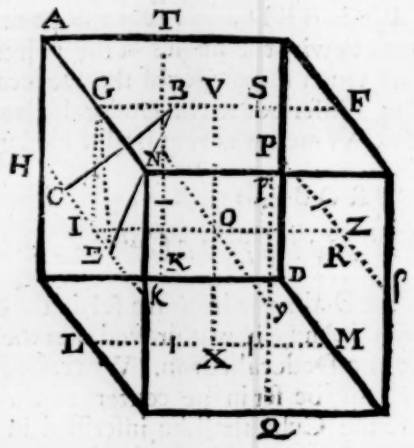
PROP. 13. PROBL. 13.

In a Cube A D F L, *to inſcribe a Dodecahedron.*

Conſtruction 1. DIvide each of the ſides of the Cube into two equal parts in the points T, H, K, P, G, L, M, F, and *p k* Q ſ. And draw theſe right lines T K, G F, *p* Q, H *k*, P ſ, and L M, which lines again

again divide into two equal parts in the points N, V, Y, I, Z, and X, and draw the lines N Y, V X, and I Z. Now the three lines N Y, V X, and I Z, together with the diameter of the Cube, ſhall cut one another into
a) 39. 11. two equal parts in the center of the Cube [a]: Let that center be the point O. And not to ſtand long about the Demonſtration, underſtand all theſe right lines to be equal and parallels to the ſides of the Cube, and to cut
b) 29. 1. one another at right angles [b]. Let their halves, to wit, F V, G V, H I and K I, and the reſt ſuch like, be divided by an extream and mean pro-
c) 30. 6. portion [c], whoſe greater ſegments let be the lines F S, G B, H C, and *k* E, &c. and draw the lines G I, G E, B C, and B E.

Demonſtration NOw foraſmuch as the line G I is equal to the whole line G V, which is the half of the ſide of the Cube; and the line I E is equal to the line B V, that is, to the leſſer ſegment: Therefore the ſquares of the lines C I and I E, are triple to the ſquare of the line
d) 4. 13. G B [d]. But unto the ſquares of the lines G I and I E, the ſquare
e) 47. 1. of the line G E is equal [e]; for the angle G I E is a right angle: Wherefore the ſquare of the line G E is triple to the ſquare of the line G B; and foraſmuch as the line F G is erected perpendicularly to the plain A G K L [f], for it is erected perpendicularly to the two lines A G and G I; therefore the angle B G E is a right angle, for the line G E is drawn in the plain A G *k* L. Wherefore the line B E conteining in power



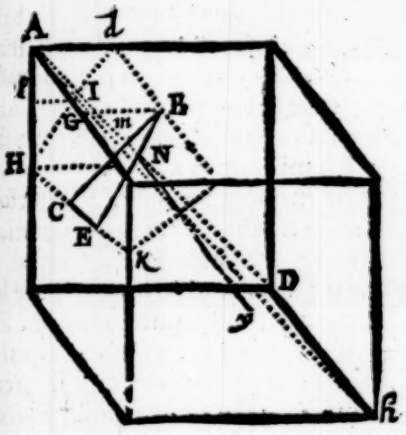
f) 4. 11.
g) 47. 1. the two lines B G and G E [g], is in power quadruple to the line G B (for the line G E was proved to be in power triple to the ſame line G B:)
h) 20. 6. Wherefore the line B F is in length double to the B G [h]. But (by Conſtruction) the line C E is double to the line I E: Wherefore the halves G B and I E are in proportion the one to the other as their doubles B E and C E:
i) 15. 5. Wherefore [i] the line C E is the greater ſegment of the line B E, divided by an extream and mean proportion. And foraſmuch as the ſame thing may be proved touching the line B C: Therefore the lines B E and B C are equal, making an Iſoſceles triangle.

Conſtruction 2. NOw let us prove that three angles of the Pentagon of the Dodecahedron are ſet at the points B, C, and E, and the other two angles are ſet between the lines B C and B E.

Foraſmuch as the circle which conteineth the triangle B C E circum-
k) 11. 4. ſcribeth the Pentagon, whoſe ſide is the line C E [k]: Extend the plain of the triangle B C E, by the parallel lines *d* B and H E, cutting the line A D, to wit, the diameter A D, the baſe of the Cube in the point I; and let it cut the liue A H the diameter of the Cube in the point M. And by the point I, draw in the baſe A D a parallel to the line A *d*, which let be I *l*.

Demon-

Demonstration FOraſmuch as from the triangle A H N, there is by the parallel line *l* I, taken away the triangle A *l* I, like unto the whole triangle A H N [l], the lines A I and *l* I ſhall be equal. But as the line H A is to the line A *d*, ſo [m] is the line H *l* to the line *l* I, or to the line *l* A, which is equal to the line *l* I; and the greater ſegment of the line H A, (which is half the ſide of the Cube) is as before hath been proved, the line A *d*, that is, the line G B, which is equal to the line A *d* [n]: Wherefore the greater ſegment of the line H *l* is the line *l* A, and as the whole line H *l* is to the greater ſegment, ſo ſhall the ſame greater ſegment H *l*, be to the leſſer ſegment *l* A [o]. Wherefore the line H A is divided by an extream and mean proportion in the point *l*. But in the triangle A H N, the line N A which is drawn from the center of the baſe A D, is in the point I, cut like unto the line A H, by the parallel line *l* I [p]; for the lines H N and I *l* are parallels by conſtruction: Wherefore the line N A is in the point I divided by an extream and mean proportion by the Superficies *d* B E H. And foraſmuch as the line Y O N, which coupleth the centers of the oppoſite baſes, is a parallel to the line H E; a plain Superficies extended by the line Y O N, parallel to the plain *d* B E H, the two plains ſhall cut the lines A O and A N (the ſemidiameter of the Cube, and the ſemidiameter of the baſe A D) into the ſame proportions in the points *m* and I [q]. But the line A N is in the point I divided by an extream and mean proportion: Wherefore the ſemidiameter of the Cube is in the point *m* divided by an extream and mean proportion by the plain of the triangle B C E. And foraſmuch as the reſt of the triangles deſcribed in the Cube after the like manner, may by the ſame reaſons be proved to be in a plain, which cutteth the ſemidiameter of the Cube by an extream and mean proportion, it is manifeſt that three plains of the Dodecahedron ſhall under every angle of the Cube concurre in one and the ſame point of the ſemidiameter, being cut by an extream and mean proportion.

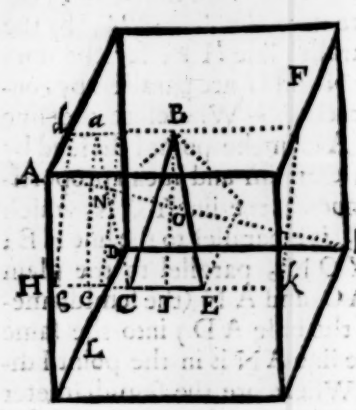


l) Cor. 2. 6.
m) 2. 6.
n) 33. 1.
o) 5. 13.
p) 2. 6.
q) 17. 11.

Conſtruction 3. NOw reſteth to prove that the right lines which couple that point of the ſemidiameter, with the angles of the triangle B E C are equal; whereby may be proved that the Pentagons are equilateral and equiangled.

Take the two baſes of the Cube, whereon are ſet the triangle B C E, to wit, the baſes A F and A *k*, take alſo the ſame diameter of the Cube, that was before, to wit, A *h*, and let the ſide ſet at the point *n* of the ſection of the diameter by an extream and mean proportion, be the line C *n* or B *n*, and let the center of the Cube be as before the point O, and extend the line C *n* to the line B *d*, and let it concurre with it in the point *a*.

Demonſtration FOraſmuch as the plain which paſſeth by the line H C E, and the center O (cutting the Cube into two equal parts, is parallel to A F the baſe of the Cube by Conſtruction: Imagine that by the point *n* be extended a plain Superficies parallel to the former parallel plains which ſhall cut the ſemidiameter O A, and the line C *a* proportionally in the point *n* [r]: For thoſe lines do touch the extream parallel plains extended by the lines H E and E O, and by the lines A *d* and *d* B; but it is proved that the line O A is divided by an extream and mean proportion in the point *n*: Wherefore the line C *a* is divided alſo by an extream and mean proportion in the point *n*. Again, foraſmuch as B C E is an Iſoſceles triangle, and it is proved that the line B I cutteth the baſe C E into two equal parts in the point I, the angles B I C and B I E ſhall be right angles. Imagine by the line B I and the center O, a plain to paſſe (cutting the Cube into two equal parts) perallel to the baſe A D: And unto thoſe plains let there be imagined another parallel plain paſſing by the point *n*, which let be *n e*, which ſhall cut the ſemidiameter A O, and the half ſide of the Cube, to wit, the line I H, like, in the points *n* and *e*. Wherefore the line I H is in the point *e* divided by an extream and mean proportion:



Wherefore the line H *e* is equal to the line C I or I E; to wit, each are leſſer ſegments. And foraſmuch as the line I *e* is to the line I C (which is equal to the line *e* H) as the whole is to the greater ſegment, take away from the whole line I *e* the greater ſegment I C; there ſhall remain the leſſer ſegment C *e* [s]: Wherefore the line I *e* is divided by an extream and mean proportion in the point C. Again, unto the ſame plaines imagine another plain to paſſe by the point *a*, parallel to them, and let the ſame be *a g*. Now then [t] the lines C *a* and C *g* are in like ſort cut in the points *n* and *e*. But the line C *a* was in the point *n* cut by an extream and mean proportion. But the line I C is to the line C *e* as the greater ſegment is to the leſſer: Wherefore the line C *e* is to the line *e g*, as the greater ſegment is to the leſſer; and therefore their proportion is as the whole line I C to the greater ſegment C *e*; and as the greater ſegment C *e* is to the leſſer ſegment *e g*: Wherefore the whole line C *e g* which maketh the greater ſegment and the leſſer, is equal to the whole line I C or I E. And foraſmuch as two parallel plain ſuperficies (to wit, that which is extended by I O B, and that which is extended by the line *a g*) are cut by the plain of the triangle B C E, which paſſeth by the lines *a g* and I B, their common ſections *a g* and I B ſhall be parallels [v]. But the angle B I E or B I C is a right angle: Wherefore the angle *a g* C is alſo a right angle [w], and thoſe right angles are conteined under equal ſides, to wit, the line *g* C is equal to the line C I, and the line *a g* to the line B I [x]: Wherefore the baſes C *a* and C B are equal [y]: But of the line C B the line C E was proved to be the greater ſegment; wherefore the ſame line C E is alſo the greater ſegment of the line C *a*: But *e n* was alſo the greater ſegment of the ſame line C *a*: Wherefore unto the line

C E

r) 17. 11.
17. 11.
s) 5. 13.
t) 17. 11.
v) 16. 11.
w) 29. 1.
x) 33. 1.
y) 4. 1.

CE the line *c n*, which is the ſide of the Dodecahedron, and is ſet at the Diameter, may be proved equal to lines equal to the line C E. Wherefore the Pentagon inſcribed in the circle, wherein is conteined the triangle B C E, [z] is equiangled and equilateral. And foraſmuch as two Pentagons ſet upon every one of the baſes of the Cube, do make a Dodecahedron, and ſix baſes of the Cube do receive twelve angles of the Dodecahedron, and the eight ſemidiameters do in the points where they are cut by an extream and mean proportion, receive the reſt: Therefore the twelve Pentagon baſes conteining 20 ſolid angles do inſcribe the Dodecahedron in the Cube [a]. Wherefore, In a Cube, &c. Which was required to be done.

z) 11. 4.

a) 1. def. 15.

COROLLARIE I.

The Diameter of the Sphere which conteineth the Dodecahedron, conteineth in power theſe two ſides, to wit, the ſide of the Dodecahedron, and the ſide of the Cube wherein the Dodecahedron is inſcribed.

For in the firſt figure, a line drawn from the center O to the point B the angle of the Dodecahedron, to wit, the line O B, conteineth in power the two lines O V the half ſide of the Cube, and V B the half ſide of the Dodecahedron [b]. Wherefore [c] the double of the line O B, which is the Diameter of the Sphere conteining the Dodecahedron, conteineth in power the double of the other lines O V and V B, which are the ſides of the Cube and of the Dodecahedron.

b) 47. 1.
c) 15. 5.

COROLLARIE II.

The ſide of a Cube divided by an extream and mean proportion, maketh the leſſer ſegment the ſide of the Dodecahedron inſcribed in it; and the greater ſegment the ſide of the Cube inſcribed in the ſame Dodecahedron.

For it was before proved that the ſide of the Dodecahedron is the greater ſegment of B E the ſide of the triangle B E C; but the ſide B E (which is equal to the lines G B and S F) is the greater ſegment of G F, the ſide of the Cube; which line B E (ſubtending the angle of the Pentagon) was [d] the ſide of the Cube inſcribed in the Dodecahedron.

d) 8. 15.

COROLLARIE III.

The ſide of a Cube is equal to the ſides of a Dodecahedron inſcribed in it, and circumſcribed about it.

For it was manifeſt by this Propoſition that the ſide of a Cube maketh the leſſer ſegment the ſide of the Dodecahedron inſcribed in it, to wit, as in the firſt figure, the line B S the ſide of the Dodecahedron, is the leſſer ſegment of the line G F the ſide of the Cube, and it was proved [e] that the ſame ſide of the Cube ſubtendeth the angle of the Pentagon of the Dodecahedron circumſcribed; and therefore it maketh the greater ſegment the ſide of the Dodecahedron, or of the Pentagon [f]. Wherefore it is equal to both thoſe ſegments.

e) 17. 13.

f) 1 Cor. of 17. 13.

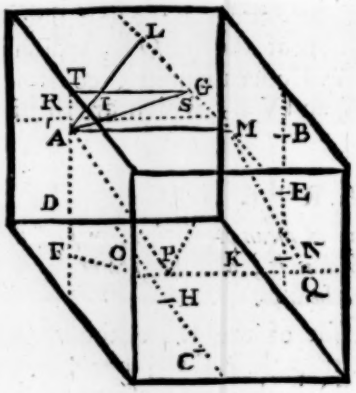
PROP. 14. PROBL. 14.

In a Cube given A B C, *to inſcribe an Icoſahedron.*

Conſtruction. LEt the centers of the baſes of the given Cube be the points D, E, G, H, I, and K, by which points, draw in the baſes

unto the other ſides parallels not touching one another: And divide the lines drawn from the centers, as the line D T, &c. by an extream and mean proportion in the points A, F, L, M, N, B, P, Q, R, S, C, and O,
a) 30. 6. [a] and let the greater ſegments be about the centers: And draw the lines A L, A G, A M and T G.

Demonſtration FOraſmuch as the lines cut are parallels to the ſides of the Cube, they ſhall make right angles to one another
b) 29. 1. [b]: And foraſmuch as they are equal, their ſections ſhall be equal, for that
c) 2. 14. the ſections are like [c]: Wherefore the line T G is equal to the line D T, for they are each half ſides of the Cube. Wherefore the ſquare of the whole line T G, and of the leſſer ſegment T A, is triple to the ſquare
d) 4. 13. of the line A D the greater ſegment [d]: But the line A G conteineth in power the lines A T and T G, for the angle A T G is a right angle. Wherefore the ſquare of the line A G is triple to the ſquare of the line A D: And foraſmuch as the line M G L is erected perpendicularly to the plain paſſing by the lines A T and T G, which is parallel to the baſes of the Cube
e) C. 14. 11. [e]; therefore the angle A G L is a right angle. But the line L G is equal to the line A D, for they are the greater ſegments of equal lines: Wherefore the line A G (which is in power triple to the line A D) is in power triple to the line L G: Wherefore adding to the ſame ſquare of the line A G the ſquare of the line L G, the ſquare of the line A L, which
f) 47. 1. [f] conteineth in power the two lines A G and G L, ſhall be quadruple to the line A D or L G. Wherefore the line A L is
g) 20. 6. double to the line A D [g]; and therefore is equal to the line A F, or to the line L M. And by the ſame reaſon may we prove that every one of the other lines which couple the next ſections of the lines cut, as the lines A M, P F, P M, M Q, and the reſt, are equal: Wherefore the triangles A L M, A P F, A M P, P M Q, and the reſt ſuch like are equal,
h) 4, 8. 1. equiangled, and equilateral [h]. And foraſmuch as upon every one of the lines cut of the Cube are ſet two triangles, as the triangles A L M and B L M, there ſhall be made 12 triangles. And foraſmuch as under every one of the eight angles of the Cube are ſubtended the other eight triangles, as the triangle A M P, &c. of twelve and eight triangles ſhall be produced twenty triangles, equal and equilateral, con-
i) 25. de. 11. teining the ſolid of an Icoſahedron [i], which ſhall be inſcribed in the given
k) 1. def. 15. Cube A B C [k], the invention of the demonſtration of this dependeth of the ground of the former. Wherefore, In a given Cube, &c. Which was required to be done.



COROLLARIE I.

The Diameter of a Sphere which conteineth an Icoſahedron, conteineth two ſides,

ſides, to wit, the ſide of the Icoſahedron, and the ſide of the Cube which conteineth the Icoſahedron.

For if we draw the line A B, it ſhall make the angles at the point A right angles; for that it is a parallel to the ſides of the Cube: Wherefore the line which coupleth the oppoſite angles of the Icoſahedron at the points F and B, conteineth in power the line A B (the ſide of the Cube) and the line A F (the ſide of the Icoſahedron) [l] which line F B is equal to the diameter of the ſphere, which conteineth the Icoſahedron [m].

l) 47. 1.
m) 16. 13.

COROLLARIE II.

The ſix oppoſite ſides of the Icoſahedron divided into two equal parts, their ſections are coupled by three equal right lines, cutting one another into equal parts, and perpendicularly in the center of the Sphere which conteineth the Icoſahedron.

For theſe three lines are the three which couple the centers of the baſes of the Cube, which do in ſuch ſort in the center of the Cube cut one another [n], and therefore are equal to the ſides of the Cube. But right lines drawn from the center of the Cube to the angles of the Icoſahedron, every one of them ſhall ſubtend the half ſide of the Cube, and the half ſide of the Icoſahedron (which half ſides contein a right angle:) Wherefore thoſe lines are equal. Whereby it is manifeſt that the foreſaid center is the center of the ſphere which conteineth the Icoſahedron.

n) Co. 3. 15.

COROLLARIE III.

The ſide of a Cube divided by an extream and mean proportion, maketh the greater ſegment the ſide of an Icoſahedron deſcribed in it.

For the half ſide of the Cube maketh the half of the ſide of the Icoſahedron the greater ſegment: Wherefore alſo the whole ſide of the Cube maketh the whole ſide of the Icoſahedron the greater ſegment [o], for the ſections are like [p].

o) 15. 5.
p) 2. 14.

COROLLARIE IV.

The ſides and baſes of the Icoſahedron, which are oppoſite to one another, are parallels.

Foraſmuch as every one of the oppoſite ſides of the Icoſahedron, may be in the parallel lines of the Cube, to wit, in thoſe parallels which are oppoſite in the Cube, and the triangles which are made of parallel lines, are parallels [q]. Therefore the oppoſite triangles of the Icoſahedron, as alſo the ſides, are parallels to one another.

q) 15. 11.

PROP. 15. PROBL. 15.

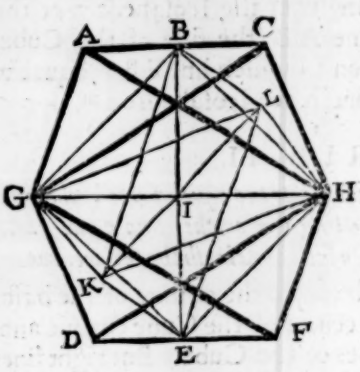
In an Icoſahedron given A C D F, *to inſcribe an Octohedron* B G E H K L.

Conſtruction LEt [a] there be taken the three right lines which cut one another into two equal parts perpendicularly, and which couple the ſections into two equal parts of the ſides of the Icoſahedron; which let be B E, G H, and K L, cutting one another in the point I, and draw the lines B G, G E, E H, and H B.

a) 2 C. 14. 15

Demon-

Demonſtration FOraſmuch as the angles at the point I are (by Conſtruction) right angles, and are conteined under equal lines; the
b) 4. 1. baſes G B and H E ſhall make a ſquare [b]. Likewiſe unto thoſe baſes ſhall be equal the lines drawn from the points K and L, to every one of the points B, G, E, and H; and therefore the triangles which make the Pyramis B G E H K, ſhall be equal and equilateral. And by the ſame reaſon ſhall the reſt of the triangles which make the other Pyramis B G E H L upon the ſame baſe B G E H, be equal and equilateral. Wherefore B G E H K L ſhall be an Octohedron [c], and ſhall be inſcribed in the Icoſahedron [d]: Wherefore, In an Icoſahedron, &c.

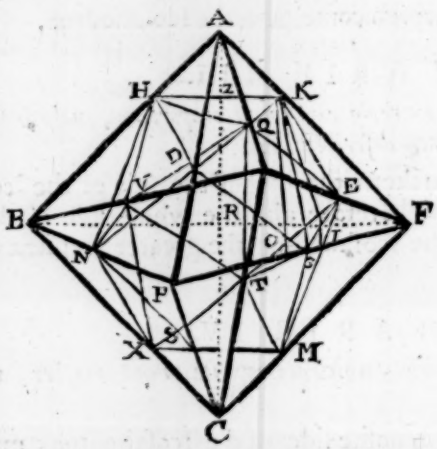


c) 23. d. 11.
d) 1. dc. 15.

Which was required to be done.

PROP. 16. PROBL. 16.

In an Octohedron given ABCFPL, *to inſcribe an Icoſahedron.*



Conſtruction LEt the ſix angles of the Octohedron be A, B, C, F, P, and L, and draw the lines AC, BF, and PL, cutting one another perpendicularly in the point R [a]; and let every one of the twelve ſides of the Octohedron be divided by an extream and mean proportion in the points H, X, M, K, D, S, N, G, V, E, Q, and T; and let the greater ſegments be the lines BH, BX, FM, FK, AD, AQ, CS, CI, PN, PG, LV, and LE, and draw the lines HK, XM, GE, NV, DS, and QT.

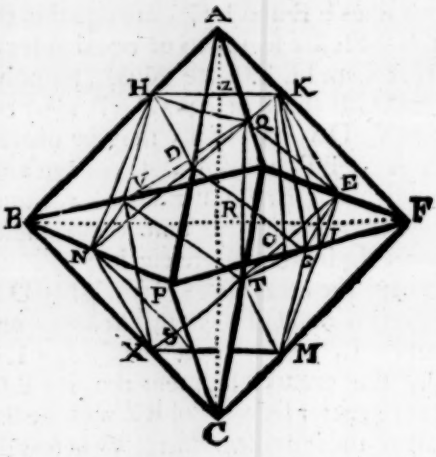
a) 2. Cor. of 14. 13.

Demonſtration FOraſmuch as in the triangle ABF, the ſides are cut proportionally, to wit, as the line BH is to the line HA, ſo
b) 2. 14. is the line FK to the line KA [b]; therefore the line HK ſhall be a parallel
c) 2. 6. to the line BF [c]. And foraſmuch as the line AC cutteth the line HK in the point Z, and the line ZK is a parallel to the line RF, the line RA
d) 2. 6. ſhall be cut by an extream and mean proportion in the point Z [d]; to wit, ſhall be cut like unto the line FA; and the greater ſegment thereof
e) 3. 1. ſhall be the line ZR. Unto the line ZR put the line RO equal [e], and
f) 33. 1. draw the line KO. Now then the line KO ſhall be equal to the line ZR [f]. Draw the lines KG, KE, and KI. And foraſmuch as the triangles ARF
g) 6. 6. and AZK, are equiangled [g], the ſides AZ and ZK ſhall be equal to
h) 4. 6. one another [h]; for the ſides AR and RF are equal: Wherefore the line ZK

ZK ſhall be the leſſer ſegment of the line RA. But if the greater ſegment RZ be divided by an extream and mean proportion, the greater ſegment thereof ſhall be the line ZK, which was the leſſer ſegment of the whole line RA [i]. And foraſmuch as the two lines FE and FG, are equal to the two lines AH and AK, to wit, each are leſſer ſegments of equal ſides of the Octohedron, and the angles HAK and EFG are equal, namely, are right angles [k], the baſes HK and GF ſhall be equal [l]. And by the ſame reaſon the other lines XM, NV, DS, and QT, may be proved equal. And foraſmuch as the lines AC, BF, and PL, do cut one another in two equal parts, and perpendicularly by Conſtruction; the lines HK and GE (which ſubtend angles of triangles like unto the triangles whoſe angles the lines AC, BF, and PL, ſubtend,) are cut in two equal parts in the points Z and I [m]; ſo alſo are the other lines NV, XM, DS, and QT, (which are equal to the lines HK and GE) cut in like ſort, and they ſhall cut the lines AC, BF, and PL, like. Wherefore the line KO (which is equal to RZ) ſhall make the greater ſegment the line RO, which is equal to the line ZK, (for the greater ſegment of RZ was the line ZK) and therefore the line OI ſhall be the leſſer ſegment: When as the whole line RI is equal to the whole line RZ. Wherefore the ſquares of the whole line KO and of the leſſer ſegment OI, are triple to the ſquare of the greater ſegment RO [n]: Wherefore the line KI, which conteineth in power the two lines KO and OI, is in power triple to the line RO [o]; for the angle KOI is a right angle. And foraſmuch as the lines FE and FG (which are the leſſer ſegments of the ſides of the Octohedron) are equal; and the line FK is common to them both, and the angles KFG and KFE (of the triangles of the Octohedron) are equal; the baſes KG and KE ſhall [p] be equal: And therefore the angles KIE and KIG, which they ſubtend, are equal [q]: Wherefore they are right angles [r]. Wherefore the right line KE (which conteineth in power the two lines KI and IE [s], is in power quadruple to the line RO (or IE); for the line KI is proved to be in power triple to the ſame line RO: But the line GE is double to the line IE. Wherefore the line GE is alſo in power quadruple to the line IE [t]. Wherefore the two lines KE and GE are equal. And by the ſame reaſon may the reſt of the lines, to wit, HK, HN, NV, VX, and XS, and the other lines which couple the ſections of the ſides of the Octohedron, be proved equal to the ſame lines KE and GE. Wherefore the triangles deſcribed of them, to wit, GEK, GKD, GDS, GSM, and GME, ſhall be equal and equilateral [v], making a ſolid angle at the point G, which is therefore the angle of an Icoſahedron [w], and is ſet in the ſection G of the ſide PF. And by the ſame reaſon may be proved that the reſt of the eleven ſolid angles of the Icoſahedron, are ſet in the ſections of every one of the ſides of the Octohedron, to wit, in the points E, N, V, H, K, M, X, D, S, Q, and T. Wherefore there are twelve angles of the Icoſahedron. Moreover, foraſmuch as every one of the baſes of the Octohedron do each contein triangles of the Icoſahedron, as in the Pramis ABCFP (which is the halt of the Octohedron) the triangle FCP receiveth in the ſections of his ſides the triangle GMS, and the triangle CPB conteineth the triangle NXS, and the triangle BAP conteineth the triangle HND: And moreover, the triangle APF conteineth the triangle KDG, and the ſame may be proved in the oppoſite Pyramis ABCFL: Wherefore there ſhall be eight triangles. And foraſmuch

i) 5. 13.
k) 14. 13.
l) 4. 1.
m) 4. 6.
n) 4. 13.
o) 47. 1.
p) 4. 1.
q) 8. 1.
r) 13. 1.
s) 47. 1.
t) 20. 6.
v) 8. 1.
w) 16. 13.

aſmuch as beſides theſe triangles, to every one of the ſolid angles of the Octohedron, are ſubtended two triangles, as the triangles KEG and MEG, to the angle F, and the triangles HNV and XNV to the angle B, alſo the triangles NDS and GDS to the angle P; likewiſe the triangles DHK and ϱHK to the angle A: Moreover the triangles EϱT and VϱT to the angle L, and finally, the triangles SXM and TXM to the angle C, theſe twelve triangles being added to the eight former triangles, ſhall produce twenty triangles equal and equilateral coupled together; which ſhall make an Icoſahedron x. And it ſhall be inſcribed in the Octohedron given ABCFPL y; for the twelve angles thereof are ſet in twelve like ſections of the ſides of the Octohedron. Wherefore, In an, &c. Which was required to be done.



x) 25. d. 11.

y) 1. dc. 15.

COROLLARIE I.

The ſide of an equilateral triangle being divided by an extream and mean proportion, a right line ſubtending within the triangle, the angle which is conteined under the greateſt ſegment and and the leſſer, is in power duple to the leſſer ſegment of the ſame ſide.

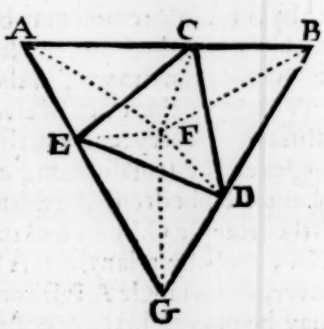
For the line KE which ſubtendeth the angle KFE of the triangle AFL, which angle KFE is conteined under the two ſegments KF and FE, was proved equal to the line HK, which conteineth in power the two leſſer ſegments HA and AK z; for the angle HAK is a right angle: Wherefore the line KE or HK is in proportion duple to the line AK.

z) 47. 1.

COROLLARIE II.

The baſes of the Icoſahedron are concentrical (that is, have one and the ſame center) with the baſes of the Octohedron which conteineth it.

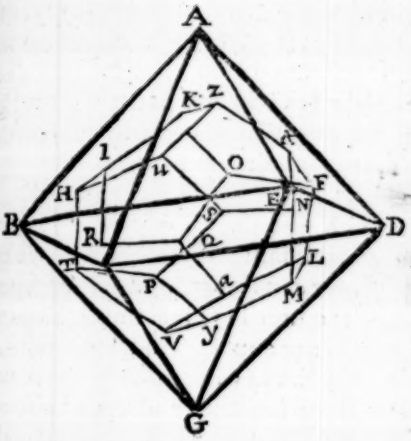
For ſuppoſe that ABG be the baſe of an Octohedron, conteining ECD the baſe of an Icoſahedron; and let the center of the baſe ABG be the point F, and draw the lines FA, FB, FC, and FE. Now then the two lines FA and AE ſhall be equal to the two lines FB and BC; for they are lines drawn from the center, and are alſo ſeſſer ſegments, and they contein the halves of equal angles. Wherefore a the baſes FC and FE are equal, and by the ſame reaſon unto them ſhall be equal the other line FD. Wherefore making the center the point F, with the diſtance FE, deſcribe a circle, and it ſhall be circumſcribed about the



a) 4. 1.

the triangle C E D, and ſo ſhall the point F the center of the baſe of the Octohedron, be the center of C E D, the baſe of the Icoſahedron.

PROP. 17. PROBL. 17.



In an Octohedron given ABGDEC, *to inſcribe a Dodecahedron.*

Conſtruction LEt the twelve ſides of the Octohedron be cut by an extream and mean proportion [a]. It was manifeſt that of the right lines which couple theſe ſections are made twenty triangles, of which eight are concentrical with the baſes of the Octohedron [b].

a) 16. 15.

b) 3 Cor. of 16. 15.

Demonſtration IF therefore in every one of the centers of the twenty triangles be inſcribed [c] every one of the twelve angles of the Dodecahedron, we ſhall find that eight angles of the Dodecahedron are ſet in the eight centers of the baſes of the Octohedron, to wit, thoſe angles I, u, ſt, O, M, a, P, and X, and of the other twelve ſolid angles, there are two in the centers of the two triangles which have one ſide common under every one of the ſolid angles of the Octohedron, to wit, under the ſolid angle A, the two ſolid angles K and Z; under the ſolid angle B, the two ſolid angles H and T; under the ſolid angle G, the two ſolid angles Y and V; under the ſolid angle D, the two ſolid angles F and L; under the ſolid angle E, the two ſolid angles S and N; and under the ſolid angles C, the two ſolid angles Q and R. And foraſmuch as in the Octohedron are ſix ſolid angles, under them ſhall be ſubtended twelve ſolid angles of the Dodecahedron; and ſo are made twenty ſolid angles, compoſed of twelve equal and equilateral ſuperficial Pentagons [d]; which therefore contein a Dodecahedron [e]. And it is inſcribed in the Octohedron [f], for that every one of the baſes of the Octohedron do receive angles thereof. Wherefore, &c. Which was required to be done.

c) 5. 15.

d) 5. 15.

e) 24. d. 11.

f) 1. def. 15.

PROP. 18. PROBL. 18.

In a trilateral and equilateral Pyramis, to inſcribe a Cube.

Conſtruction LEt the baſe of the given Pyramis be A B C, and his top the point D; and let it be comprehended in a Sphere [a]; and let the center of that ſphere be the point E, and from the ſolid angles A, B, C, and D, draw right lines paſſing by the center E, unto the oppoſite baſes of the Pyramis, and they ſhall fall perpendicularly upon the baſes, and ſhall alſo fall upon the centers of the circles which contein the baſes [b]. Let the center of the triangle A B C be the point G; and let the center of the triangle A D C be the point H, and of the triangle A D B, let the point N be the center; and laſtly, let the point F be the center of the other tri-

a) 13. 13.

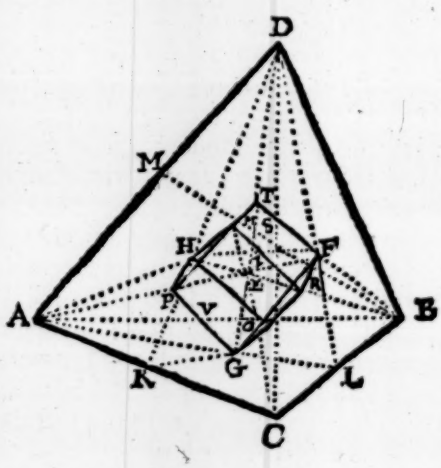
b) C. 13. 13.

triangle D B C. And let the right lines falling upon thoſe centers be D E G, B E H, C E N, and A E F. And by thoſe centers G, H, N, and F, let there be drawn from the angles to the oppoſite ſides, theſe right lines A G L, D H K, B N M, and D F L, which ſhall fall perpendicularly upon the ſides B C, C A, A D, and C B [c]; and therefore they ſhall cut them into two equal parts in the points K, L, and M [d]. Again let the lines which were drawn from the ſolid angles to the oppoſite baſes be divided into two equal parts, to wit, the line D G in the point T, the line C N in the point O, the line A F in the point P, and the line B H in the point R, and draw the lines H T, F T, H O, and F O.

c) C. 12. 13.
d) 3. 3.

Demonſtration FOraſmuch as the lines G K and G L, which are drawn from the center of one and the ſame triangle A B C, to the ſides, are equal, and the lines D K and D L are equal, for they are the perpendiculars of equal and like triangles; and the line D G is common to them. Wherefore [e] the angles K D G and L D G are equal. And foraſmuch as the lines H D and D F are drawn from the center of equal circles, which contein the equal triangles A D C and D B C, therefore they are equal; and the line D T is common to them both, and they contein equal angles, as before hath been proved. Wherefore the baſes H T and F T are equal [f]. And by the ſame reaſon if we draw the lines C F and C H, may we prove that the other lines H O and F O are equal to the ſame lines H T and F T, and alſo to one another. Wherefore alſo after the ſame manner may be proved that the reſt of the lines which couple the centers of the triangles, and the ſections of the perpendicular into two equal parts, as the lines N P, G R, G P, R N, N T, P H, G O, and R F, are equal. And foraſmuch as from every one of the centers of the baſes are drawn three right lines to the ſections into two equal parts of the perpendiculars, and there are four centers, it followeth that thoſe equal right lines ſo drawn are twelve in number, of which every three and three make a ſolid angle in the four centers of the baſes, and in the four ſections, into two equal parts of the perpendiculars: Wherefore that ſolid hath eight angles, conteined under twelve equal ſides, which make ſix quadrangled figures, to wit, H O F T, P G R N, P H O G, G O F R, F R N T, T N P H. Now we will prove that thoſe quadrangled figures are rectangled.

e) 8. 1.
f) 4. 1.



Foraſmuch as upon D C the common baſe of the triangles A D C and B D C, falleth the perpendiculars A S and B S, which are drawn by the centers H and F, either of theſe lines H S and S F ſhall be the third part of either of theſe lines A S and S B; for the line A H is double to the line H S, and divideth the baſe D C into two equal parts [g]: Wherefore in the triangle A B S, the ſides A S and B S are cut proportionally in the points

g) C. 12. 13.

H

H and F; and therefore the line H F is a parallel to the ſide A B [h]. Wherefore the triangles A S B and H S F are equiangled [i]: Wherefore the baſe H F ſhall be the third part of the baſe A B [k]. We may alſo prove that the line T O is the third part of the line D C; for the lines E C and E D, which are drawn from the center of the Sphere which conteineth the Pyramis are equal; and the line E N (which is drawn from the center to the baſe) is the third part of the line E C, ſo alſo is the line G E the third part of the line E D [l]; for it is the ſixth part of the diameter of the Sphere which conteineth the Pyramis. And the line O N is the half of the whole line N C. Wherefore the reſidue E O is the third part of the line E C, and ſo alſo is the line E T the third part of the line E D. Wherefore the line T O in the triangle D E C is a parallel to the line D C, and is a third part of the ſame [m]; as the line H F was proved the third part of the line A B. But A B and D C being ſides of the Pyramis, are equal: Wherefore the lines H F and T O being the third parts of equal lines, are equal [n]. Wherefore [o] the angles H T F and T F O are equal; and by the ſame reaſon, the angles oppoſite to them, to wit, the angles F O H and O H T are equal to one another; and alſo are equal to the ſaid angles H T F and T F O; but theſe four angles are equal to four right angles [p]: Wherefore the angles of the quadrangle H O F Γ are right angles. And by the ſame reaſon may the angles of the other five quadrangled figures be proved right angles. Now reſteth to prove that the foreſaid quadrangles are each of them in one and the ſame plain.

Take the quadrangle H O F Γ, and foraſmuch as in the triangle A S B, the line H F is proved a parallel to the line A B; therefore it cutteth the lines S V and S B proportionally, in the points I and F [q]. Now then foraſmuch as S F was proved the third part of the line S B, the line S I ſhall alſo be the third part of the line S V. Moreover, foraſmuch as the line V S which coupleth the Sections into two equal parts of the oppoſite ſides of the Pyramis, to wit, of the ſides A B and D C, is by the center E divided into two equal parts [r] (for it is the diameter of the Octohedron inſcribed in the Pyramis) therefore the line S T is two third parts of the half line S E. And by the ſame reaſon, Foraſmuch as in the triangle D E C, the line T O is proved to be a parallel to the ſide D C, it ſhall in the ſame triangle cut the lines C E and S E proportionally, in the points O and I [ſ]. But the line E O is proved to be a third part of the line E C: Wherefore the line E I is alſo a third part of the line E S. Wherefore the reſidue I S ſhall be two third parts of the whole line E S. Wherefore the point I cutteth either of the lines T O and H F. Wherefore the two lines H I F and T I O, cutting one another, are in one and the ſame plain [t]. And therefore the points H, T, F, and O, are in one and the ſame plain. Wherefore the rectangled figure H O F T being quadrilateral and equilateral, and in one and the ſame plain, is a ſquare, by the definition of a ſquare. And by the ſame reaſon may the reſt of the baſes of the ſolid be proved to be ſquares equal and plain, or ſuperficial: Now then the ſolid is comprehended of ſix equal ſquares (which are conteined of twelve equal ſides) which ſquares make eight ſolid angles, of which four are in the centers of the baſes of the Pyramis, and the other four are in the middle ſections of the four perpendiculars. Wherefore the ſolid H O F T P G R N is a Cube [v], and is inſcribed in the Pyramis [w]. Wherefore, In a trilateral, &c. Which was required to be done.

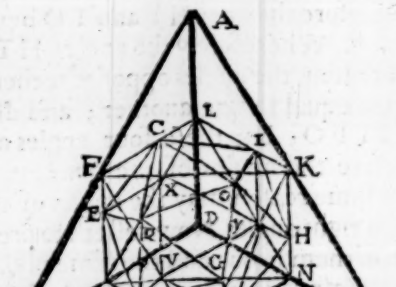
h) 2. 6.
i) 6. 6.
k) 4. 6.
l) C. 13. 13.
m) 2, 4. 6.
n) 15. 5.
o) 8. 1.
p) C. 32. 1.
q) 2. 6.
r) Co. 2. 15.
ſ) 2. 6.
t) 2. 11.
v) 21. d. 11.
w) 1. d. 15.

COROLLARIE

The line which cutteth into two equal parts the oppoſite ſides of the Pyramis, is triple to the ſide of the Cube inſcribed in the Pyramis, and paſſeth by the center of the Cube.

For the line S E V, whoſe third part the line S I is, cutteth the oppoſite ſides C D and A B into two equal parts: But the line E I (which is drawn from the center of the Cube to the baſe) is proved to be a third part of the line E S: Wherefore the ſide of the Cube which is double to the line E I, ſhall be a third part of the whole line V S, which is (as hath been proved) double to the line E S.

PROP. 19. PROBL. 19.



In a trilateral equilateral Pyramis given A B G D, *to inſcribe an Icoſahedron.*

Conſtruction LEt each of the ſides of the given Pyramis be divided into two equal parts in the points F, M, K, L, P, and N, and in every one of the baſes of that Pyramis, deſcribe the triangles L F P, P M N, N K L, and

a) 4. 1. F M K, which triangles ſhall be equilateral [a]; for the ſides ſubtend equal angles of the Pyramis, conteined under the halves of the ſides of the ſame Pyramis: Wherefore the ſides of the ſaid triangles are equal: Let

b) 30. 6. thoſe ſides be divided by an extream and mean proportion [b], in the points C, E, Q, R, S, T, H, I, O, V, Y, and X. Now then, thoſe ſides are cut into the ſame proportion

c) 2. 4. [c]; and therefore they make the like Sections

d) 9. 5. equal [d]. Now I ſay that the foreſaid points do receive the angles of the Icoſahedron inſcribed in the Pyramis A B G D.

In the foreſaid triangles, Let there again be made other triangles, by coupling the ſections, and let thoſe triangles be T R S, I O H, G E Q, and V X Y, which ſhall be equilateral; for every one of their ſides do ſubtend equal angles of equilateral triangles, and thoſe ſaid equal angles are conteined under equal ſides (to wit, under the greater ſegment and the

e) 4. 1. leſſer.) And therefore the ſides which ſubtend thoſe angles are equal [e].

Demonſtration NOw let us prove that at each of the foreſaid points, as for example at T, is ſet the ſolid angle of an Icoſahedron. Foraſmuch as the triangles T R S and T Q O are equilateral and equal, the four right lines T R, T S, T Q, and T O, ſhall be equal. And foraſmuch as F P N K is a ſquare, cutting the Pyramis A B G D into two

f) Co. 2. 15. equal parts [f], the line T H ſhall be in power double to the line T N or

g) 47. 1. N H [g]. For the lines T N or N H are equal, for that by Conſtruction they are each of them leſſer ſegments, and the line R T or T S is in power

h) C. 16. 15. double to the ſame line T N or N H [h]; for it ſubtendeth the angle of the triangle conteined under the two ſegments. Wherefore the lines T H, T S,

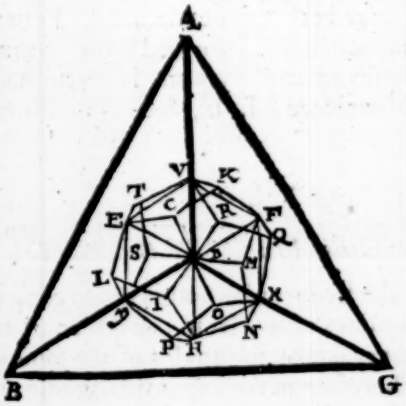
TS, TR, TQ, and TO, are equal, and ſo alſo are the lines HS, SR, RQ, QO, and OH, which ſubtend the angles at the point T equal. For the line QR conteineth in power the two lines P2 and PR, the leſſer ſegments, which two lines the line TH alſo conteined in power. And the reſt of the lines do ſubtend angles (of equilateral triangles) conteined under the greater and the leſſer ſegments. Wherefore the five triangles TRS, TSH, THO, TOQ, and TQR, are equilateral and equal, making the ſolid angle of the Icoſahedron at the point T, [i] in the ſide PN of the triangle PNM. And by the ſame reaſon, in the other ſides of the four triangles PNM, NKL, FMK, and LFP, (which are inſcribed in the baſes of the Pyramis) (which ſides are twelve in number, ſhallbe ſet twelve angles of the Icoſahedron, conteined under twenty equal and equilateral triangles, of which four are ſet in the four baſes of the Pyramis, to wit, thoſe four triangles TRS, HOI, CEQ, and VXY, four triangles are under four angles of the Pyramis; that is, the four triangles CIX, YSH, ERV, and T2O, and under every one of the ſix ſides of the Pyramis are ſet two triangles, to wit, under the ſide DG, the triangle THS and THO; under the ſide DB, the triangles R2E and R2T; under the ſide DA the triangles CO2 and COI, under the ſide AB the triangles EXC and EXV; under the ſide BG the triangles SVR and SVY; and under the ſide AG, the triangles IYH and IYX. Wherefore the ſolid being conteined under twenty equilateral and equal triangles, ſhall be an Icoſahedron [k], and ſhall be inſcribed in the Pyramis ABGD [l]; for all his angles do at one time touch the baſes of the Pyramid. Wherefore, In a trilateral, &c. Which was required to be done.

i) 16. 13.

k) 23.d.11.
l) 1.def. 15.

PROP. 20. PROBL. 20.

In a trilateral equilateral Pyramis given ABGD, *to inſcribe a Dodecahedron.*



Conſtruction LEt each of the ſides of the given Pyramis be cut into two equal parts, and draw the lines which couple the ſections, which being divided by an extream and mean proportion, and right lines being drawn by the ſections, ſhall receive twenty triangles, making an Icoſahedron [a].

a) 19. 15.

Now then if we take the centers of thoſe triangles, we ſhall there find the twenty angles of the Dodecahedron inſcribed in it [b]. And foraſmuch as four baſes of the foreſaid Icoſahedron are concentrical with the baſes of the Pyramis [c], there ſhall be placed four angles of the Dodecahedron, to wit, the four angles E, F, H, and D, in the four centers of the baſes; and of the other ſixteen angles, under every one of the ſix ſides of the Pyramis are ſubtended two, to wit, under

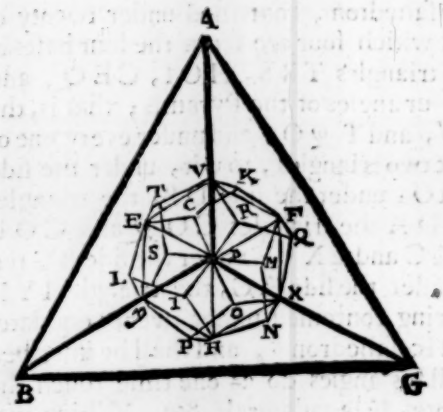
b) 5. 15.
c) 2 Cor. of 6. 15.

under the side A D, the angles C and K; under the side B D, the angles L and I; under the side G D, the angles M and N; under the side A B, the angles T and S; under the side B G, the angles P and O; and under the side A G, the angles R and Q; so there rests four angles, whose true places we will now appoint.

Demonstration FOrasmuch as a Cube conteined in one and the same Sphere with the Dodecahedron, is inscribed in the same Dodecahedron[d]; it followes that a Cube and a Dodecahedron circumscribed about it, are conteined in one and the same bodies, for that their angles meet in the same points. And as was proved[c] that four angles of the Cube inscribed in the Pyramis are set in the middle sections of the perpendiculars which are drawn from the solid angles of the Pyramis to the opposite bases. Wherefore the other four angles of the Dodecahedron are also as the angles of the Cube set in those middle sections of the perpendiculars, to wit, the angle V, is set in the middest of the perpendicular A H; the angle Y in the middest of the perpendicular B F; the angle X in the middest of the perpendicular G E; and lastly, the angle D in the middest of the perpendicular D, which is drawn from the top of the Pyramis to the opposite base. Wherefore those four angles of the Dodecahedron may be said to be directly under the solid angles of the Pyramis, or they may be said to be set at the perpendiculars. Wherefore the Dodecahedron after this manner set, is inscribed in the Pyramis given[f]; for that upon each of the bases of the Pyramis is set an angle of the Dodecahedron inscribed. Wherefore, In a, &c. Which was required to be done.

d) 17. 13. and 8. 15.

c) 18. 15.



f) 1. def. 15.

PROP. 21. PROBL. 21.

In every one of the regular Solids to inscribe a Sphere.

Demonstration IT was declared[a] that the five regular Solids are so conteined in a sphere that right lines drawn from the center of the sphere, or of the solid inscribed, to every one of the angles of the solid inscribed, are equal, which right lines therefore make Pyramids, whose tops are in the center of the Sphere, or of the Solid, and the bases are every one of the bases of those Solids. And forasmuch as those bases are in every Solid equal and like to one another, and described in equal circles, those circles shall cut the Sphere; for the angles which touch the circumference of the circle, touch also the Superficies of the Sphere: Wherefore perpendiculars drawn from the center of the Sphere to the bases, or to the plain Superficies of the equal circles, are equal: Wherefore making the center the center of the Sphere which conteineth the Solid, and the distance

a) 13, 14, 15, 16, and 17. of the 13.

some

ſome one of the equal perpendiculars, deſcribe a Sphere, and it ſhall touch each of the baſes of the Solid, neither ſhall the Superficies of the Sphere paſſe beyond thoſe baſes, when as thoſe perpendiculars are the leaſt lines which are drawn from the center to the baſes. Wherefore, We have [b], &c. Which was required to be done. b) C.18.13.

COROLLARIE.

The regular figures inſcribed in Spheres, and alſo the Spheres circumſcribed about them, or conteining them, have one and the ſame center.

Namely, their Pyramids, the angles of whoſe baſes touch the Superficies of the Sphere, do from thoſe angles cauſe equal right lines to be drawn to one and the ſame point, making the tops of the Pyramids in the ſame point; and therefore they make the centers of the Spheres in the ſame tops, when as the right lines drawn from thoſe angles to the crooked Superficies, wherein are ſet the angles of the baſes of the Pyramids are equal.

An Advertiſement.

Of theſe Solids, only the Octohedron receiveth the other Solids inſcribed one within another. For the Octohedron conteineth the Icoſahedron inſcribed in it, and the ſame Icoſahedron conteineth the Dodecahedron inſcribed in the ſame Icoſahedron; and the ſame Dodecahedron conteineth the Cube inſcribed in the ſame Octohedron. And laſtly, the ſame Cube circumſcribeth the Pyramis inſcribed in the ſaid Octohedron. But this happeneth not in the other Solids.

The End of the Fifteenth Element of EUCLIDE, *after* CAMPANE *and* FLUSSAS.

THE

THE SIXTEENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.



HAving in the Fifteenth Book ſhewed how to inſcribe the five regular Solids one within another, we ſhall here in this Sixteenth Book compare thoſe Solids ſo inſcribed one with another, and declare their Paſſions and Properties; which in this Book (aacording to *Fluſſas*) is excellently well performed, for the which he deſerveth perpetual praiſe and commendations: In the peruſal and practice whereof the Studious Reader ſhall take much delight, and have occaſion offered him to invent and contrive greater variery of Properties and Paſſions incident to the ſaid Regular Bodies.

PROPOSITIONS, and THEOREMES.

PROPOSITION 1. THEOREM 1.

A Dodecahedron and a Cube inſcribed in it, and a Pyramis inſcribed in the ſame Cube, are conteined in one and the ſame Sphere.

Demon-

Demonſtration FOr the angles of the Pyramis are ſet in the angles of the Cube wherein it is inſcribed [a]; and all the angles of the Cube are ſet in the angles of the Dodecahedron circumſcribed about it [b]. And all the angles of the Dodecahedron are ſet in the Superficies of the Sphere [c]. Wherefore thoſe three Solids inſcribed one within another, are conteined in one and the ſame Sphere [d]: Therefore, A Dodecahedron, &c. Which was to be demonſtrated.

a) 1. 15.
b) 8. 15.
c) 17. 13.
d) 1. def. 15.

COROLLARIE.

Theſe three Solids likewiſe are ſet in one and the ſelf-ſame Icoſahedron, or Octohedron, or Pyramis.

For they are inſcribed in one and the ſame Icoſahedron [e], and they are inſcribed in one and the ſame Octohedron [f]. Laſtly, they are inſcribed in one and the ſame Pyramis [g]; for the angles of all thoſe Solids are ſet in the centers of the baſes of the circumſcribed Icoſahedron, or Octohedron, or Pyramis.

e) 5, 11, & 12. of the 15.
f) 4, 6, & 16. of the 15.
g) 18, & 19. of the 15.

PROP. 2. THEOR. 2.

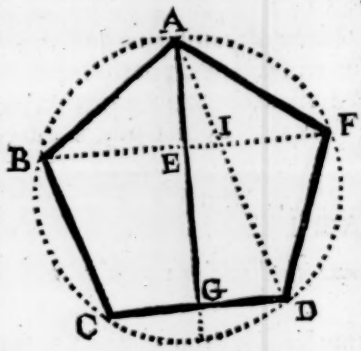
The proportion of a Dodecahedron circumſcribed about a Cube, to a Dodecahedron inſcribed in the ſame Cube, is triple to an extream and mean proportion.

Demonſtration FOraſmuch as [a] it was proved that the ſide of a Dodecahedron inſcribed in a Cube, is the leſſer ſegment of the ſide of that Cube divided by an extream and mean proportion; and the ſide of the Dodecahedron circumſcribed about the ſame Cube is the greater ſegment of the ſide of the ſame Cube [b], the ſide of the Dodecahedron circumſcribed ſhall be to the ſide of the Dodecahedron inſcribed; as the greater ſegment of a right line divided by an extream and mean proportion, is to the leſſer ſegment of the ſame, which proportion is called an extream and mean proportion [c]. But the proportion of like Solid Polihedrons is triple to the proportion of the ſides of like proportion [d]: Wherefore the proportion of the Dodecahedron circumſcribed about the Cube, is to the Dodecahedron inſcribed in the ſame Cube in triple proportion of the ſides joyned together by an extream and mean proportion: Therefore, The proportion, &c. Which was to be demonſtrated.

a) 2 Cor. of 13. 15.
b) 13. 15.
c) 30. 6.
d) C. 17. 12.

PROP. 3. THEOR. 3.

In every equiangled and equilateral Pentagon ABCDF, *a perpendicular* AG *drawn from one of the angles* A, *to the baſe* CD, *is divided in the point* I *by an extream and mean proportion, by a right line* BF, *ſubtending the ſame angle* BAF.



Demonſtration FOraſmuch as the angles GAF and GAB are equal [a], and the angles ABF and AFB are equal [b]; therefore the angles remaining at the point E, of the triangles AEB and AEF, are equal; for that they are the reſidues of two right angles [c]. But the angle EGC is by conſtruction a right angle. Wherefore the lines BF and CD are parallels [d]. Wherefore as the line DI is to the line IA, ſo is the line GE to the line EA [e]. But the line DA is in the point I divided by an extream and mean Proportion [f]. Wherefore the line GA is in the point E divided by an extream and mean proportion [g]. Wherefore, In every equiangled and equilateral Pentagon, &c. Which was to be demonſtrated.

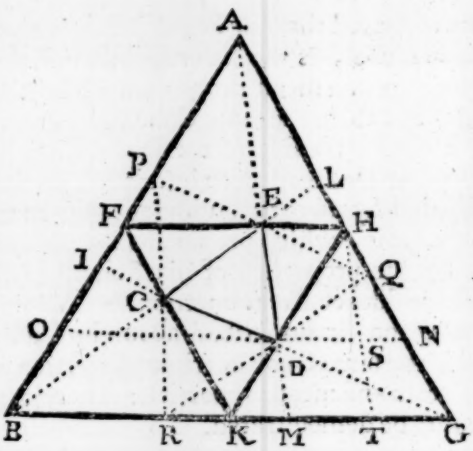
a) 27. 3.
b) 5. 1.
c) C. 32. 1.
d) 28. 1.
e) 2. 6.
f) 8. 13.
g) 2. 14.

COROLLARIE.

The line which ſubtendeth the angle of a Pentagon, is a parallel to the ſide oppoſite to the angle.

As was manifeſt in the lines BF and CD.

PROP. 4. THEOR. 4.



If from the angles A, B, *and* G, *of the baſe of a Pyramis* ABG, *be drawn to the oppoſite ſides* AB, BG, *and* GA, *the right lines* GI, AM, *and* BL, *cutting the ſaid ſides by an extream and mean proportion, they ſhall contein the baſe of the Icoſahedron* CDE; *inſcribed in the Pyramis, which baſe ſhall be inſcribed in an equilateral triangle* FKH, *whoſe angles* C, D, *and* E, *cut the ſides of the baſe of the Pyramis by an extream and mean proportion.*

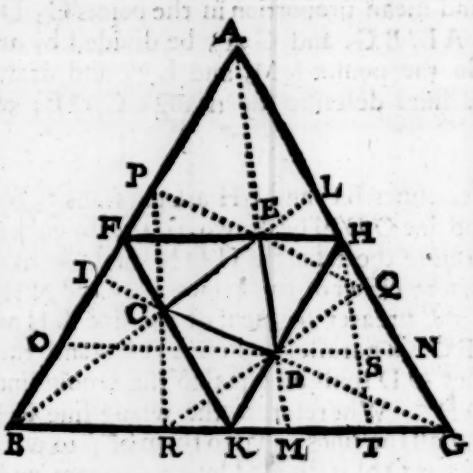
Conſtruction THe equilateral triangle is deſcribed by dividing the ſides of the baſe of the Pyramis into two equal parts: And the baſe of the Icoſahedron is inſcribed in the Pyramis, by dividing the ſides FK, KH,

KH, and HF, by an extream and mean proportion in the points C, D, and E [a]. Again, Let the ſides AB, BG, and GA, be divided by an extream and mean proportion in the points I, M, and L [b]; and draw AM, BL, and GI: I ſay thoſe lines deſcribe the triangle CDE, of the Icoſahedron.

a) 19. 5.
b) 30. 6.

Demonſtration FOr foraſmuch as the lines BG and FH are parallels [c], by the point D, let the line ODN be drawn parallel to either of the lines BG and FH. Wherefore the triangle HDN ſhall be like to the triangle HKG [d]: Wherefore either of theſe lines DN and NH, ſhall be equal to the line DH, the greater ſegment of the line KH or FH. And foraſmuch as the line FO is a parallel to the line HK, and the line OD to the line FH; the line OD ſhall be equal to the whole line FH, in the parallelogram FODH [e]. Wherefore as the whole line FH is to the greater ſegment FE, ſo ſhall the lines equal to them be, to wit, OD and DN [f]. Wherefore the line ON is divided by an extream and mean proportion in the point D [g]. But the triangles AOD, AFE, and ABM, are like to one another; and ſo alſo are the triangles ADN, AEH, and AMG [h]. Wherefore as FE is to EH, ſo is OD to DN, and BM to MG. Wherefore the line AM cutting the lines FH and ON, like to the line BG, in the points E, D, and M, deſcribeth ED the ſide of the triangle of the Icoſahedron ECD, which is deſcribed in the ſections E, C, and D, by ſuppoſition. And by the ſame reaſon, the lines BL and GI, ſhall deſcribe the other ſides EC and CD, of the ſame triangle. By the point E, let there be drawn to GI, a parallel line PEQ. Now foraſmuch as the lines BM and FE are parallels, the line AM is in the point E, cut like to the line AB in the point F [i]: Wherefore the line AE is equal to the line EM, and unto the line EM are alſo equal either of the lines GD and DI, which are cut like unto the foreſaid lines. Again, foraſmuch as in the triangle ADI, the lines DI and EP are parallels, as the line DI is to the line EP, ſo is the line AD to the line AE: But as the line AD is to the line AE, ſo is the line DG to the line EQ [k]: Wherefore as the line DI is to the line EP, ſo is the line DG to the line EQ; and alternately, as the line DI is to the line DG, ſo is the line EP to the line EQ: But the lines DI and IG are equal; wherefore alſo the lines EP and EQ are equal. And foraſmuch as the line AH is equal to the line FH, whoſe greater ſegment is the line HN; therefore the whole line AN is divided by an extream and mean proportion in the point H [l]. But as the line AN is to the line AH, ſo is the line AD to the line AE [m] (for the lines FH and ON are parallels) and again, as the line AD is to the line AE, ſo [n] is the line AG to the line AQ, and the line AI to the line AP; for the lines PQ and GI are parallels: Wherefore the lines AG and AI are divided by an extream and mean proportion in the points Q and P, and the line AQ ſhall be the greater ſegment of the line AG or AB. And foraſmuch as the whole line AG is to the greater ſegment AQ, as the greater ſegment AI is to the reſidue AP, the line AP ſhall be the leſſer ſegment of the whole line AB or AG: Wherefore the line PEQ (which by the point E paſſeth parallel to the line GI) cutteth the lines AG and BA by an extream and mean proportion in the points Q and P. And by the ſame reaſon, the line PR (which by the point C paſſeth paralelly to the line AM) ſhall fall upon the ſections P and R; ſo alſo ſhall

c) 2. 6.
d) Cor. 2. 6.
e) 34. 1.
f) 7. 5.
g) 2. 14.
h) Cor. 2. 6.
i) 2. 6.
k) 2. 6.
l) 5. 13.
m) 2. 6.
n) 2. 6.



o) 34. 1.

the line R Q (which by the point D, paſſeth parallelly to the line B L) fall upon the ſections R and Q: Wherefore either of the lines P E and E Q ſhall be equal to the line C D, in the Paralellograms P D and Q C o. And foraſmuch as the lines P E and E Q are equal, the lines P C, C R, R D, and D Q, ſhall be likewiſe equal. Wherefore the triangle P R Q is equilateral, and cutteth the ſides of the baſe of the Pyramis in the points P, Q, and R, by an extream and mean proportion. And in it is inſcribed the baſe E C D of the Icoſahedron conteined in the foreſaid Pyramis: If therefore from the angles, &c. Which was to be demonſtrated.

COROLLARIE

The ſide of an Icoſahedron inſcribed in an Octohedron, is the greater ſegment of the line, which being drawn from the angle of the baſe of the Octohedron, cutteth the oppoſite ſide by an extream and mean proportion.

p) 16. 15. For P F K H is the baſe of the Octohedron, which conteineth the baſe of the Icoſahedron C D E, unto which triangle F K H, the trian- H K G is equal, as hath been proved. By the point H draw to the line M E a parallel H T, cutting the line D N in the point S. Wherefore E S, D T, and E T, are Paralellograms; and therefore the lines E H and M T, are equal; and the lines E M and H T are like cut in the points D and S q. Wherefore the greater ſegment of the line H T, is the line H S, which is equal to E D, the ſide of an Icoſahedron. But r the line T K is cut like to the line H K, by the parallel D M. And therefore ſ it is divided by an extream and mean proportion. But the line T M is equal to the line E H. Wherefore alſo the line T K is equal to the line E F or D H: Wherefore the reſidues E H and T G are equal. For the whole lines F H and K C are equal. Wherefore K G the ſide of the triangle H K G, is in the point T divided by an extream and mean proportion, by the right line H T; and the greater ſegment thereof is the line E D, the ſide of the Icoſahedron inſcribed in the Octohedron, whoſe baſe is the triangle H K G (or the triangle F K H) which is equal to the triangle H K G t.

q) 34. 1.
r) 2. 6.
ſ) 2. 14.
t) 16. 15.

PROP. 5. THEOR. 5.

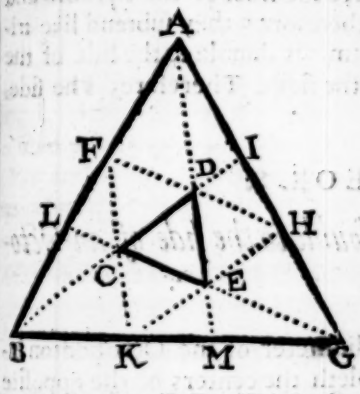
The ſide A G, *of a Pyramis divided by an extream and mean proportion, maketh the leſſer ſegment* A I *in power double to the ſide* C E, *of the Icoſahedron inſcribed in it.*

Conſtruction LEt A B G be the baſe of a Pyramis, and let the baſe of the Icoſahedron inſcribed in it be C D E, deſcribed of three right

right lines, which being drawn from the angles of the baſe A B G, cut the oppoſite ſides by an extream and mean proportion [a]; to wit, of the three lines A M, B I, and G L.

a) 4. 16.

Demonſtration FOraſmuch as [b] it was proved that the triangle C D E is inſcribed in an equilateral triangle, whoſe angles cut the ſides of A B G, the baſe of the Pyramis, by an extream and mean proportion; let that triangle be F H K, cutting the line A B in the point F. Wherefore the leſſer ſegment F A is equal to the ſegment A I [c] (for the lines A B and A G are cut like:) Moreover the ſide F H of the triangle F H K, is in the point D cut into two equal parts [d]; and F C E D is a Paralellogram: Wherefore the lines C E and F D are equal [e]. And foraſmuch as the line F H ſubtendeth the angle B A G, of an equilateral triangle, which angle is conteined under the greater ſegment A H, and the leſſer ſegment A F; therefore the line F H is in power double to the line A F, or to the line A I, the leſſer ſegment [f]. But the ſame line F H is in power quadruple to the line C E [g] (for the line F H is double to the line C E:) Wherefore the ſquare of the line A I being the half of the ſquare of the line F H, is in power double to the line C E, to which the line F H was in power quadruple. Wherefore, The ſide of a Pyramis divided, &c. Which was to be demonſtrated.



b) 4. 16.

c) 2. 14.

d) 4. 16.

e) 33. 1.

f) C. 16. 15.

g) 4. 2.

COROLLARIE.

The ſide of an Icoſahedron inſcribed in a Pyramis, is a Reſidual line.

For the diameter of the Sphere which conteineth the five regular bodies, being rational, is in power ſeſquialtera to the ſide of the Pyramis [h]. And therefore the ſide of the Pyramis is rational, by the definition, which ſide being divided by an extream and mean proportion, maketh the leſſer ſegment a Reſidual line [i]; Wherefore the ſide of the Icoſahedron being commenſurable to the ſame leſſer ſegment (for the ſquare of the Icoſahedron is the half of the ſquare of the ſaid leſſer ſegment) is a Reſidual line.

h) 13. 13.

i) 6. 13.

PROP. 6. THEOR. 6.

The ſide of a cube conteineth in power half the ſide of an equilateral triangular Pyramis inſcribed in the ſaid cube.

Demonſtration FOraſmuch as the ſide of the Pyramis inſcribed in the Cube, ſubtendeth two ſides of the Cube which contein a right angle [a], it is manifeſt [b] that the ſide of the Pyramis ſubtending the ſaid ſides, is in power double to the ſide of the Cube. Wherefore alſo the ſquare of the ſide of the Cube is the halfe of the ſquare of the ſide of the Pyramis: Therefore, &c. Which was to be demonſtrated.

a) 1. 15.

b) 47. 1.

PROP.

PROP. 7. THEOR. 7.

The ſide of a Pyramis is double to the ſide of an Octohedron inſcribed in it.

Demonſtration FOraſmuch as [a] it was proved that the ſide of the Octohedron inſcribed in a Pyramis, coupleth the middle ſections of the ſides of the Pyramis: Wherefore the ſides of the Pyramis and of the Octohedron, are parallels [b]; and therefore [c] they ſubtend like triangles: Wherefore [d] the ſide of the Pyramis is double to the ſide of the Octohedron, to wit, in the proportion of the ſides. Therefore, The ſide, &c. Which was to be demonſtrated.

a) 2. 15.
b) Co. 39. 1.
c) Cor. 2. 6.
d) 4. 6.

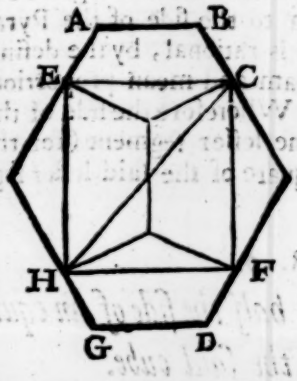
PROP. 8. THEOR. 8.

The ſide of a cube is in power double to the ſide of an Octohedron inſcribed in it.

Demonſtration IT was proved [a] that the diameter of the Octohedron inſcribed in the Cube, coupleth the centers of the oppoſite baſes of the Cube: Wherefore the ſaid diameter is equal to the ſide of the Cube. But the ſame is alſo the diameter of the ſquare made of the ſides of the Octohedron, to wit, is the diameter of the ſphere which conteineth it [b]: Wherefore that diameter being equal to the ſide of the Cube, is in power double to the ſide of that ſquare, or to the ſide of the Octohedron inſcribed in it [c]. The ſide therefore, &c. Which was to be demonſtrated.

a) 3. 15.
b) 14. 13.
c) 47. 1.

PROP. 9. THEOR. 9.



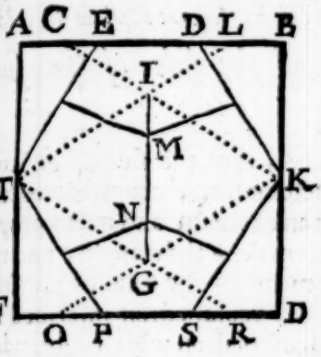
The ſide AB, *of a Dodecahedron* ABGD, *is the greater ſegment of the line which conteineth in power half the ſide* CH, *of the Pyramis inſcribed in the ſaid Dodecahedron.*

Demonſtration FOr ſeeing that EC, the ſide of the Cube, being divided by an extream and mean proportion, maketh the greater ſegment the line AB, the ſide of the Dodecahedron [a] (for they are conteined in one and the ſame ſphere [b]) and the line EC the ſide of the Cube, conteineth in power the half of the ſide CH [c]. Wherefore AB the ſide of the Dodecahedron, is the greater ſegment of the line EC, which conteineth in power the half of the line CH, which is the ſide of the Dodecahedron inſcribed in the Pyramis. The ſide therefore, &c. Which was required to be proved.

b) 1 Cor. of 17. 13.
b) 1. 16.
c) 6. 16.

PROP.

PROP. 10. THEOR. 10.



The ſide C L, *of an Icoſahedron, is the mean proportional between the ſide* A B, *of the Cube circumſcribed about the Icoſahedron* C L I G O R, *and the ſide* E D, *of the Dodecahedron* E D M N P S, *inſcribed in the ſame Cube.*

Demonſtration. FOraſmuch as C L is the greater ſegment of A B [a], and the ſide E D is the leſſer ſegment of the ſame ſide A B [b]: It followeth that A B the ſide of the Cube, being divided by an extream and mean proportion, maketh the greater ſegment C L, the ſide of the Icoſahedron inſcribed in it, and the leſſer ſegment E D the ſide of the Dodecahedron likewiſe inſcribed in it. Wherefore as the whole line A B, the ſide of the Cube, is to the greater ſegment C L, the ſide of the Icoſahedron; ſo is the greater ſegment C L, the ſide of the Icoſahedron, to the leſſer ſegment E D, the ſide of the Dodecahedron [c]. Wherefore, The ſide, &c. Which was required to be proved.

a) 3 C. 14. 15
b) 2 C. 13. 15
c) 3. def. 6.

PROP. 11. THEOR. 11.

The ſide of a Pyramis, is in power Octodecuple (that is, as 18 *to* 1,) *to the ſide of the Cube inſcribed in it.*

Demonſtration. FOr by what hath been demonſtrated [a], the ſide of the Pyramis is triple to the diameter of the baſe of the Cube inſcribed in it; and therefore it is in power noncuple to the ſame diameter [b]. But the diameter is in power double to the ſide of the Cube [c]. And the double of noncuple maketh Octodecuple. Wherefore, The ſide, &c. Which was required to be proved.

a) 18. 15.
b) 20. 6.
c) 47. 1.

PROP. 11. THEOR. 11.

The ſide of a Pyramis is in power Octodecuple to that right line, whoſe greater ſegment is the ſide of the Dodecahedron inſcribed in the Pyramis.

Demonſtration. FOraſmuch as the Dodecahedron and the Cube inſcribed in it, are ſet in one and the ſame Pyramis [a], and the ſide of the Pyramis circumſcribed about the Cube, is in power Octodecuple to the ſide of the Cube inſcribed [b]. But the greater ſegment of the ſame ſide of the Cube is the ſide of the Dodecahedron which conteineth the Cube [c]. Wherefore, The ſide, &c. Which was required to be proved.

a) Co. 1. 16.
b) 11. 16.
c) C. 17. 13.

PROP.

PROP. 13. THEOR. 13.

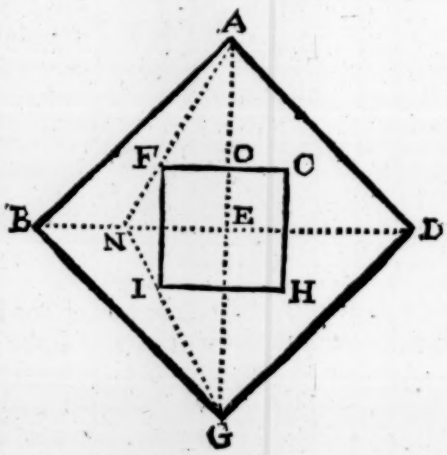
The ſide of an Icoſahedron inſcribed in an Octohedron, is in power double to the leſſer ſegment of the ſide of the ſame Octohedron.

Demonſtration FOraſmuch as [a] it was proved, that the ſide of an Icoſahedron inſcribed in an Octohedron, coupleth together the two ſections (which are produced by an extream and mean proportion) of the ſide of the Octohedron, which make a right angle, and that right angle is conteined under the leſſer ſegments of the ſides of the Octohedron, and is ſubtended of the ſide of the Icoſahedron inſcribed: It followeth therefore that the ſide of the Icoſahedron which ſubtendeth the right angle, being in power equal to the two lines which contein the ſaid angle [b], is in power double to each of the leſſer ſegments of the Octohedron which contein a right angle. Wherefore, The ſide of an Icoſahedron, &c. Which was to be demonſtrated.

a) 17. 15.

b) 47. 1.

PROP. 14. THEOR. 14.



The ſides AB, *of the Octohedron* ABGDE, *and* FI, *of the Cube* FC HI, *inſcribed in it, are in power to one another in quadrupla ſeſquialtera proportion; (that is as* 9 *to* 2.)

Conſtruction LEt there be drawn to BE, the baſe of the triangle ABE, a perpendicular AN; and again, let there be drawn to the ſame baſe in the triangle GBE, the perpendicular GN, which AN and GN ſhall paſſe by the centers F and I.

Demonſtration THe line AF is double to the line FN [a]: Wherefore the line AO is double to the line OE [b]; for the lines FO and NE are parallels; and therefore the diameter AG is triple to the line FI. Wherefore the power of AG is noncuple (that is, as 18 to 2, or 9 to 1,) to the power of FI. But the line AG is in power double to the ſide AB [c]. Wherefore the ſquare of the line AB being the half of the ſquare of the line AG, which is noncuple to the ſquare of the line FI, is quadruple ſeſquialter to the ſquare of the line FI. The ſides therefore, &c. Which was required to be proved.

a) C. 12. 13.
b) 2. 6.
c) 14. 13.

PROP.

PROP. 15. THEOR. 15.

The ſide of the Octohedron is in power quadruple ſeſquialter to that right line, whoſe greater ſegment is the ſide of the Dodecahedron inſcribed in the ſame Octohedron.

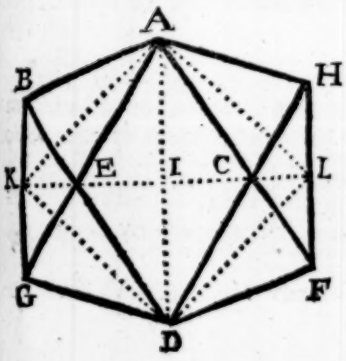
Demonſtration FOraſmuch as [a] it was proved, that the ſide of the Octohedron is in power quadruple ſeſquialter to the ſide of the Cube inſcribed in it; but the ſide of the Cube being cut by an extream and mean proportion, maketh the greater ſegment the ſide of the Dodecahedron circumſcribed about it [b]: Therefore the ſide of the Octohedron is in power quadruple ſeſquialter to that right line (to wit, to the ſide of the Cube) whoſe greater ſegment is the ſide of the Dodecahedron inſcribed in the Cube. But the Dodecahedron and the Cube inſcribed one within another, are inſcribed in one and the ſame Octohedron [c]. Therefore, The ſide, &c. Which was required to be proved.

a) 14. 16.

b) 3 C. 13. 15

c) Co. 1. 16.

PROP. 16. THEOR. 16.



The ſide BG, *or* EC, *of an Icoſahedron* ABGDFHEC, *is the greater ſegment of that right* BH, *or* KL, *which is in power double to the ſide* AL, *of the Octohedron* AKDL, *inſcribed in the ſame Icoſahedron.*

Demonſtration FOraſmuch as figures inſcribed and circumſcribed have one and the ſame center [a], let the ſame be the point I. Now right lines drawn by that center to the middle ſections of the oppoſite ſides, to wit, the lines AID and KIL, do in the point I cut one another into two equal parts, and perpendicularly [b]; and foraſmuch as they couple the middle ſections of the oppoſite lines BG and HF; therefore they cut them perpendicularly: Wherefore alſo the lines BG and HF are parallels [c]. Now then draw a line from B to H; and the ſaid line BH ſhall be equal and parallel to the line KL [d]. But the line BH ſubtendeth two ſides of the Pentagon which is compoſed of the ſides of the Icoſahedron, to wit, the ſides BA and AH: Wherefore the line BH being cut by an extream and mean proportion, maketh the greater ſegment the ſide of the Pentagon [e]; which ſide is alſo the ſide of the Icoſahedron, to wit, EC. And unto the line BH, the line KL is equal; and the line KL is in power double to AL, the ſide of the Octohedron [f]; for in the ſquare AKDL, the angle KAL is a right angle. Wherefore EC the ſide of the Icoſahedron, is the greater ſegment of the line BH or KL, &c. Which was required to be proved.

a) C. 21. 15.

b) C. 14. 15.

c) 4 C. 14. 15

d) 33. 1.

e) 8. 13.

f) 47. 1.

PROP. 17. THEOR. 17.

The ſide of a Cube is to the ſide of a Dodecahedron inſcribed in it, in double proportion of an extream and mean proportion.

Demonſtration FOr it was manifeſt [a] that the ſide of a Cube divided by an extream and mean proportion, maketh the leſſer ſegment the ſide of the Dodecahedron inſcribed in it; but the whole is to the leſſer ſegment in double proportion of that in which it is to the greater [b], for the whole, the greater ſegment, and the leſſer, are lines in continual proportion [c]: Wherefore the whole, to wit, the ſide of the Cube, is to the ſide of the Dodecahedron inſcribed in it, to wit, to his leſſer ſegment, in double proportion of an extream and mean proportion, to wit, of that which the whole hath to the greater ſegment [d].

a) 2 C. 13. 15.
b) 10. def. 5.
c) 3. def. 6.
d) 2. 14.

PROP. 18. THEOR. 18.

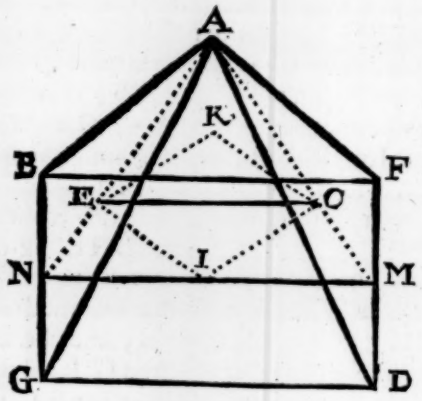
The ſide of a Dodecahedron, is to the ſide of the Cube inſcribed in it, in converſe proportion of an extream and mean proportion.

Demonſtration IT was proved [a] that the ſide of a Dodecahedron circumſcribed about a Cube, is the greater ſegment of the ſide of the ſame Cube. Wherefore the whole ſide of the Cube inſcribed is to the greater ſegment, to wit, to the ſide of the Dodecahedron circumſcribed, in an extream and mean proportion: Wherefore by converſion, the greater ſegment, that is, the ſide of the Dodecahedron, is to the whole, to wit, to the ſide of the Cube inſcribed [b], &c. Which was required to be proved.

a) 3 C. 13. 15
b) 13. def. 5.

PROP. 19. THEOR. 19.

The ſide GD, *of an Octohedron* ABGDF, *is ſeſquialter to the ſide* EC, *of a Pyramis inſcribed in it.*



Conſtruction FOr [a] the Octohedron is cut into two quadrilateral Pyramids, one of which let be ABGDF; and let the centers of the circles which contein the four baſes of the Octohedron be K, E, I, and C, and draw theſe right lines KE, EI, IC, CK, and EC. Wherefore KEIC is a ſquare, and one of the baſes of the Cube inſcribed in the Octohedron [b]. And foraſmuch as the

a) C. 14. 13.
b) 4. 15.

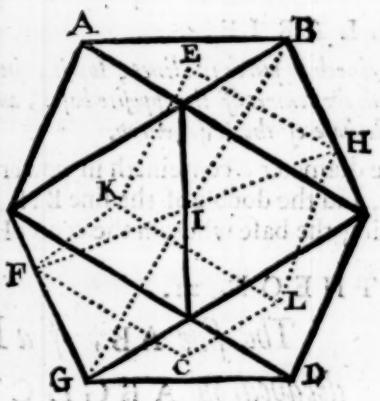
the angles of a Cube, and of the Pyramis inſcribed in it, are ſet in the centers of the baſes of the Octohedron circumſcribed about the Cube [c]; and the ſide of the Pyramis coupleth the oppoſite angles of the baſe of the Cube [d]: It is manifeſt that the line E C is the ſide of the Pyramis inſcribed in the Octohedron A B G D F. I ſay then that G D the ſide of the Octohedron, is ſeſquialter to E C, the ſide of the Pyramis inſcribed in it. From the point A, draw to the baſes B G and F D, perpendiculars A N and A M, which [e] ſhall paſſe by the centers E and C, and draw the line N M.

c) 6. 15.
d) 1. 15.
e) C. 12. 13.

Demonſtration FOraſmuch as B G D F is a ſquare [f], the lines N G and M D, ſhall be parallels and equal. For the lines B G and F D, are by the perpendiculars cut into two equal parts, in the points N and M [g]: Wherefore the lines N M and G D ſhall be parallels and equal [h]. And foraſmuch as the lines A N and A M, which are the perpendiculars of equal and like triangles, are cut alike in the points E and C, the lines E C and N M ſhall be parallels [i]. And therefore [k] the triangles A E C and A N M ſhall be like. Wherefore as the line A N is to the line A E, ſo is the line N M to the line E C [l]. But the line A N is ſeſquialter to the line A E, for the line A E is double to the line E N [m]. Wherefore the line N M or the line G D, which is equal unto it, is ſeſquialter to the line E C. Wherefore, &c. Which was required to be proved.

f) 14. 13.
g) 3. 3.
h) 33. 1.
i) 2. 6.
k) Cor. 2. 6.
l) 4. 6.
m) C. 12. 13.

PROP. 20. THEOR. 20.



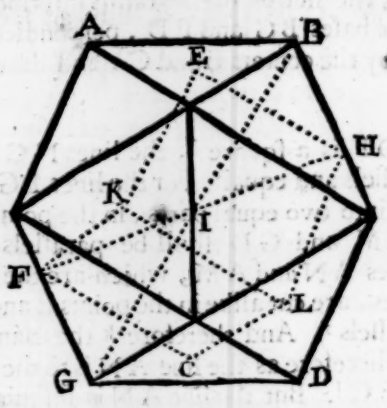
If from the power of the diameter B G, *of an Icoſahedron* A B G D, *be taken away the power tripled of the ſide* E H, *of the Cube inſcribed in the Icoſahedron, the power remaining ſhall be ſeſquitertia to the power of the ſide* A B, *of the Icoſahedron.*

Conſtruction LEt the two baſes of the Cube be E H K L, and L I K F C joyned; and let the diameter of the Cube be F H.

Demonſtration FOraſmuch as the centers of inſcribed and circumſcribed figures are in one and the ſame point [a], the diameters B G and F H, ſhall in one and the ſame point cut one another into two equal parts; for by the ſame Corollary it hath been taught that the tops of equal and like Pyramids, do in that point concurre; let the point be the center I. Now the angles of the Cube which are at the points F and H, are ſet at the centers of the baſes of the Icoſahedron [b]: Wherefore the line F H ſhall be perpendicular to both the baſes of the Icoſahedron.

a) C. 21. 15.
b) 11. 15.

c) 47. 1. Wherefore the line I B conteineth in power the two lines I H and H B c. But the line H B is drawn from the center of the circle which conteineth the baſe of the Icoſahedron, to wit, the angle B, is placed in the circumference, and the point H is the center. Wherefore the whole line B G conteineth in power the whole lines F H, and the diameter of the circle (to wit, the double of the line B H) d. But the diameter which is double to the line H B, is in power ſeſquitertia to the ſide of the equilateral triangle inſcribed in the ſame circle e; for it is in proportion to the ſide, as the ſide is to the perpendicular f. And F H the diameter of the Cube, is in power triple to E H, the ſide of the ſame Cube g: If therefore from the power of the diameter B G, be taken away the power tripled of E H, the ſide of the Cube inſcribed; that is, the power of the line F H; the reſidue (to wit, the power of the diameter of the circle, which is double to the line H B) ſhall be ſeſquitertia to the ſide of the triangle inſcribed in that circle, which ſide is A B the ſide of the Icoſahedron: If therefore, &c. Which was required to be proved.



d) 15. 5.

e) C. 12. 13.

f) Cor. 8. 6.

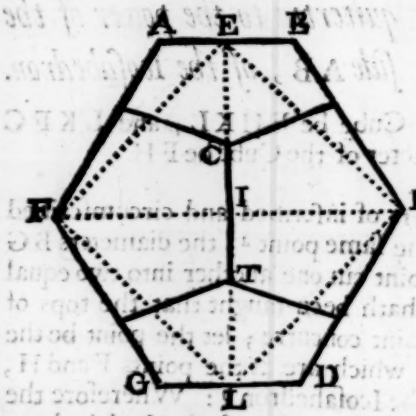
g) 15. 13.

COROLLARIE.

The Diameter of the Icoſahedron, conteineth in power two lines, to wit, the Diameter of the Cube inſcribed, which coupleth the centers of the oppoſite baſes, and the Diameter of the Circle which conteineth the baſe of the Icoſahedron.

For it was manifeſt, that B G the diameter, conteineth in power the line F H, which coupleth the centers, and the double of the line B H, that is, the diameter of the circle conteining the baſe wherein the center H is.

PROP. 21. THEOR. 21.



The ſide A B, *of a Dodecahedron* A B G D C T, *is the leſſer ſegment of that right line which is in power double to the ſide* E F, *of the Octohedron* E L F K I, *inſcribed in the ſame Dodecahedron.*

Conſtruction. DRaw the diameters E L and F K, of

of the Octohedron. Now they couple the middle sections of the opposite sides of the Dodecahedron AB and GD [a]; and all those diameters being divided by an extream and mean proportion, do make the lesser segment the side of the Dodecahedron [b]: Wherefore the side AB is the lesser segment of the line FK. But the line FK conteineth in power the two equal lines EF and EK [c]; for the angle FEK is a right angle of the square FEKL, of the Octohedron. Wherefore the line FK is in power double to the line EF. Wherefore the line AB (the side of the Dodecahedron) is the lesser segment of the line FK, which is in power double to EF, the side of the Octohedron: Therefore, The side of a Dodecahedron, &c. Which was required to be proved.

a) 9. 15. C. 3. 17. 13.
b) 4 Cor. of 17. 13.
c) 47. 1.

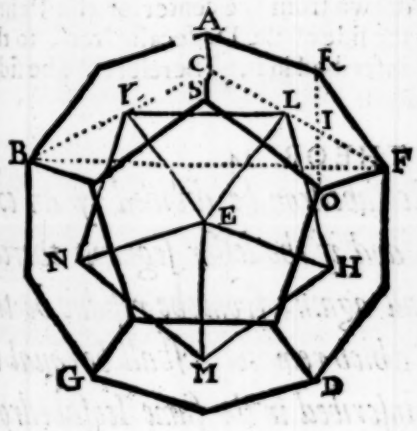
PROP. 22. THEOR. 22.

The Diameter of an Icosahedron is in power sesquitertia to the side of the saue Icosahedron, and also is in power sesquialter to the side of the Pyramis inscribed in the Icosahedron.

Demonstration. FOrasmuch as it hath been proved [a] that if from the power of the diameter of the Icosahedron, be taken away the triple of the power of the side of the Cube inscribed in it, there shall be left a square, sesquitertia to the square of the side of the Icosahedron: But the power of the side of the Cube tripled, is the diameter of the same Cube [b]. And the Cube and the Pyramis inscribed in it, are conteined in one and the same sphere [c], and in one and the same Icosahedron [d]: Wherefore one and the same Diameter of the Cube, or of the Sphere which conteineth the Cube and the Pyramis, is in power sesquialter to the side of the Pyramis [e]. Wherefore it followeth that if from the Diameter of the Icosahedron be taken away the triple power of the side of the Cube, or the sesquialter power of the side of the Icosahedron: The diameter therefore, &c. Which was required to be proved.

a) 20. 16.
b) 15. 13.
c) 1. 16.
d) Co. 1. 16.
e) 13. 13.

PROP. 23. THEOR. 23.

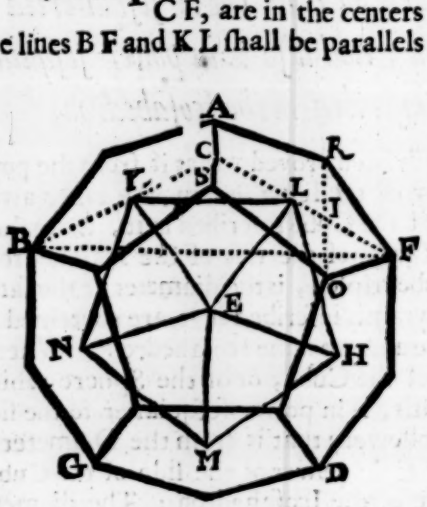


The side AS, *or* SO, *of a Dodecahedron* ABG DFSO, *is to the side* KL, *of an Icosahedron* KL HMNE, *inscribed in it, as the lesser segment* IF, *of the perpendicular* CF, *of the Pentagon, is to that line* LC, *which is drawn from the center to the side* AS, *of the same Pentagon.*

Demon-

Conſtruction FRom the two angles of the Pentagons B A S and F A S, of the Dodecahedron, to wit, from the angles B and F; let there be drawn to the common baſe A S, perpendicular lines B C and F C,
a) C. 10. 13. which ſhall paſſe by the centers K and L of the ſaid Pentagons [a]: Draw the lines B F and R O. Now foraſmuch as the line R O ſubtendeth the angle O F R, of the Pentagon of the Dodecahedron, it ſhall cut the line
b) 3. 16. F C by an extream and mean proportion [b]; let it cut it in the point I. And foraſmuch as the line K L is the ſide of the Icoſahedron inſcribed in the Dodecahedron, it coupleth the centers of the baſes of the Dodecahedron; for the angles of the Icoſahedron are ſet in the centers of the baſes
c) 7. 15. of the Dodecahedron [c].

Demonſtration FOraſmuch as in the triangle B C F, the two ſides C B and C F, are in the centers L and K, cut like proportionally,
d) 2. 6. the lines B F and K L ſhall be parallels [d]: Wherefore the triangles B C F and K C L, ſhall be equiangled [e]. Wherefore as the line C L is to the line K L, ſo is the line C F to the line B F [f]. But C F maketh the leſſer ſegment the line I F [g]; and the line B F maketh the leſſer ſegment the line S O, to wit, the ſide of the Dodecahedron [h]; for the line B F, which coupleth the angles B and F, of the baſes of the Dodecahedron, is equal to the ſide of the Cube which conteineth the Dodecahedron [i]: Wherefore as the whole line C F, is to the whole line B F, ſo is the leſſer ſegment I F, to the leſſer ſegment S O [k]. But as the line C F is to the line B F, ſo is the line C L to the line K L. Wherefore alternately, [l] as the line I F the leſſer ſegment of the perpendicular of the Pentagon F A S, is to the line L C, which is drawn from the center of the Pentagon, to the baſe; ſo is the line S O the ſide of the Dodecahedron; to the line K L the ſide of the Icoſahedron inſcribed in it. Therefore, The ſide, &c. Which was required to be proved.



e) Cor. 2. 6.
f) 4. 6.
g) 3. 16.
h) 2 C. 13. 15
i) 13. 15.
k) 2. 14.
l) 16. 5.

PROP. 24. THEOR. 24.

If half of the ſide of an Icoſahedron be divided by an extream and mean proportion, and if the leſſer ſegment thereof be taken from the whole ſide; and again, from the reſidue be taken away the third part, that which remaineth ſhall be equal to the ſide of the Dodecahedron inſcribed in the ſame Icoſahedron.

Conſtruction SUppoſe A B G D F be a Pentagon, conteining five ſides of the Icoſahedron [a], and let it be inſcribed in a circle, whoſe
a) 16. 13. center

center let be E. And upon the ſides of the Pentagon let there be raiſed triangles, making a ſolid angle of the Icoſahedron at the point I [b]. And in the circle A B D, inſcribe an equilateral triangle A H K. From the center E, draw to H K the ſide of the triangle, and G D the ſide of the Pentagon, a perpendicular line, which let be E C N M; and draw the lines E G, E D, I G, and I D. And divide the line B G into two equal parts in the point T. And draw the lines I N, I T, T N, and E T. And foraſmuch as in the perpendiculars I T and I N, are the centers of the circles which contein the equilateral triangles I B G and I G D [c]: Let thoſe centers be the points S and O, and draw the line S O. Divide the line T B the half of B G, the ſide of the Icoſahedron, by an extream and mean proportion in the point R [d]; and let the leſſer ſegment thereof be R B. And foraſmuch as the line S O coupleth the centers of the triangles I B G and I G D, it is [e] the ſide of the Dodecahedron inſcribed in the Icoſahedron, whoſe ſide is the line B G, from the ſide B G take away B R, the leſſer ſegment of the half ſide. And from the reſidue G R, take away the third part G V [f]; then I ſay that the reſidue R V is equal to S O, the ſide of the Dodecahedron inſcribed.

b) 16. 13.

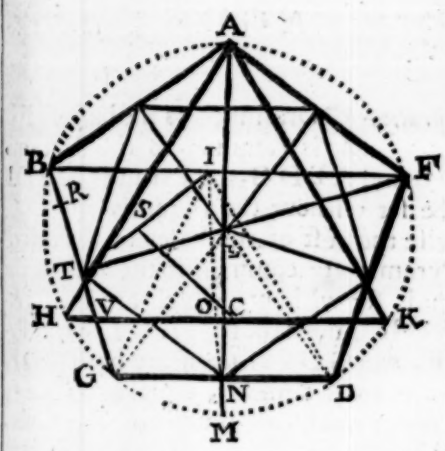
c) Cor. 1. 3.

d) 30. 6.

e) 5. 15.

f) 9. 6.

Demonſtration FOraſmuch as the perpendicular E N is in the point C divided by an extream and mean proportion [g], and the greater ſegment thereof is the line E C, and unto the line E C the line C M is equal [h]. Wherefore the line E C is to the line C N, as the line C N is to the ſame C N [i]. But as the line E C is to the line C N, ſo is the whole line E N, to the greater ſegment E C [k]. Wherefore [l] as the whole line E N is to the greater ſegment E C, ſo is the line C M to the line C N. Wherefore the line C M is divided by an extream and mean proportion in the point N, to wit, is divided like unto the line E N [m]. Wherefore the line E M exceedeth the line E N by the leſſer ſegment of his half, to wit, by M N. And foraſmuch as E G D is the triangle of an equilateral and equiangled Pentagon A B G D F. Therefore [n] the triangle E T N is like to the triangle E G D. Wherefore as the line E G is to the line E N, ſo [o] is the line G D to the line N T. Wherefore the line G D (or B G which is equal unto it) exceedeth the line N T by the leſſer ſegment of the halfe of B G; for the line E G did in like ſort exceed the line E N. But that leſſe ſegment is the line B R. Wherefore the reſidue R G is equal to the line T N. And foraſmuch as I B G is an equilateral triangle, the perpendicular S T ſhall be the half of the line S I, which is drawn from the center [p]. Wherefore the line I T exceedeth the line I S by his third part. And foraſmuch as the line S O, which coupleth the ſections, is a parallel to the line T N [q]; for the equal perpendiculars I T and I N, are cut like in the points S and O. Therefore the



g) Co. 1. 14.

h) C. 12. 13.

i) 7. 5.

k) 3. def. 6

l) 11. 5.

m) 2. 14.

n) 20. 6.

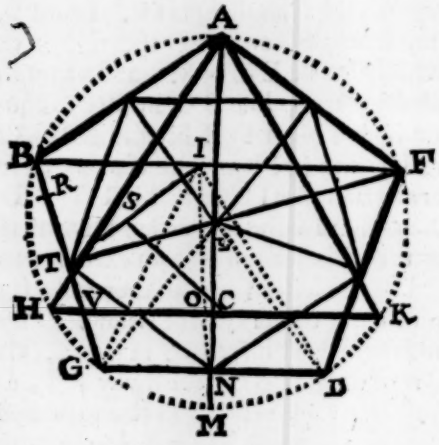
o) 4. 6.

p) C. 12. 13.

q) 2. 6.

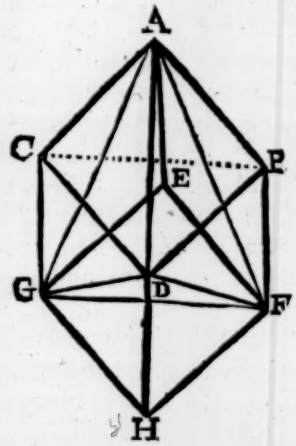
r) Cor. 2.6. s) 4.6.

the triangles ITN and ISO are like [r]. Wherefore as the line IT is to the line IS; so [s] is the line TN to the line SO. But the line IT exceedeth the line IS by a third part. Wherefore the line TN exceedeth the line SO by a third part. But the line TN is proved equal to the line RG. Wherefore the line RG exceedeth the line SO by a third part of it selfe which is GV. Wherefore the residue RV is equal to the line SO, which is the side of the Dodecahedron inscribed in the Icosahedron, whose side is the line BG. If therefore half of the side of an Icosahedron be divided by a[n] extream and mean proportion, &c. Which was required to be proved.



PROP. 25. THEOR. 25.

To prove that a given Cube AB CH *is triple to a trilateral equilateral Pyramis* AGDF, *inscribed in it.*



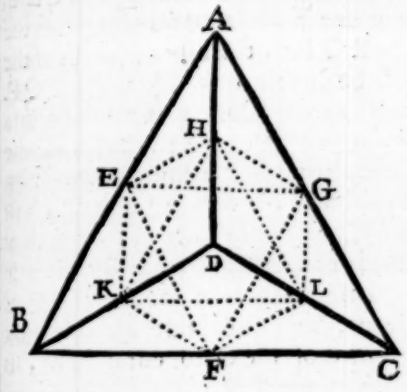
Demonstration For asmuch as the base AFD is common to the Pyramis AFDB and AFDG, the Pyramis AFDB shall be set without the Pyramis AFDC. Likewise the rest of the bases of the inscribed Pyramis are common to the rest of the Pyramids set without, which are the Pyramis AGDC upon the base AGD, the Pyramis AGFE upon the base AGF, and GDFH upon the base GDF, which Pyramids taken without, are four in number, equal and alike [a]; for every one of them is conteined under three half squares of the Cube, and one of the bases of the Pyramis inscribed. Wherefore each of them is conteined under the half base of the Cube, and the altitude of the Cube. As the Pyramis AEGF hath to his base half of the square EH, to wit, the triangle EGF, and hath to his altitude the altitude of the Cube, to wit, the line AE. Wherefore the said Pyramis is the sixth part of the Cube. For if the Cube be divided into two Prismes by the Plain CBFG, the prisme ACBGEF shall be triple to the Pyramis AEGF, having one and the same base with it EGF, and one and the same altitude EA [b]. Wherefore the said outward Pyramis AEGF is the sixth part of the whole Cube. Wherefore also the same Pyramis, together with the other three outward Pyramids AFDB, AGDC, and GDFH, shall contein two third parts of the Cube. Wherefore the re-

a) 8. de. 11.

b) 1 C. 7. 12.

reſidue, to wit, the Pyramis inſcribed A G D F, ſhall contein one third part of the Cube. And therefore converſely, the Cube ſhall be triple to it. Wherefore we have proved that a Cube, &c.

PROP. 26. THEOR. 26.

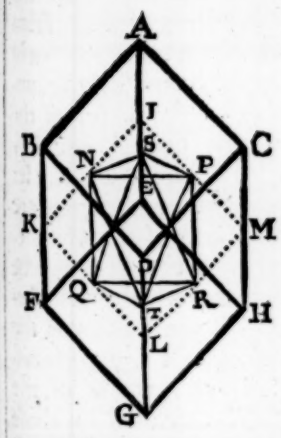


To prove that a trilateral Pyramis A B C D, *is double to an Octohedron inſcribed in it* E G L K H F.

Conſtruction LEt the ſides of the Pyramis be cut into two equal parts in the points E, K, F, G, L, and H, inſcribing thereby an Octohedron in the Pyramis [a]. Wherefore the Pyramids AEGH, BEFK, CFGL, and DKHL, fall without the Octohedron inſcribed [b]. But the outward Pyramis, to wit, A E G H, and the three other, are like unto the whole Pyramis [c]. For the baſes of the whole Pyramis are by parallel lines drawn in them, cut into like triangles [d], of which the foreſaid Pyramids are made. Wherefore the whole Pyramis is to every one of them in treble proportion of that in which the ſides of like proportion are [e]. But by Conſtruction, the proportion of the ſide A B to the ſide A E is double. Wherefore the whole Pyramis A B C D is octuple to the Pyramis A E G H, and ſo is it to each of the Pyramids which are equal to A E G H. For double proportion multiplied into it ſelfe twice, maketh octuple. Wherefore it followeth that A E G H, B E F K, C F G L, and D K H L, taken together, make the halfe of the whole Pyramis A B C D, Wherefore the reſidue, to wit, the Octohedron E G L K H F, is the other half of the Pyramis. Wherefore the Pyramis is double to the Octohedron. Wherefore we have proved that a trilateral equilateral Pyramis, &c.

a) 2. 15.
b) 2. 15.
c) 7. def. 11.
d) 2. 6.
e) 8. 12.

PROP. 27. THEOR. 27.



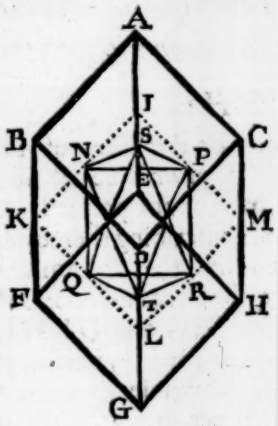
To prove that a Cube is ſextuple to an Octohedron inſcribed in it.

Conſtruction LEt the Cube A B C D, E F G H, whoſe four ſtanding lines A E, B F, C H, and D G, be cut into two equal parts in the points I, K, M, and L, and by thoſe points let there be extended a plain K L M I, which ſhall be a ſquare, and parallel to the ſquares B C and F H [a]. Wherefore in it ſhall be the baſe which is common to the two Pyramids of the Octohedron inſcribed in the Cube [b]. Let that baſe be N P R Q, coupling the centers of the baſes of the Cube, & upon that baſe let be ſet the

a) 15. 11.
b) 3. 15.

two Pyramids of the Octohedron, which let be N P Q R S and N P Q R T.

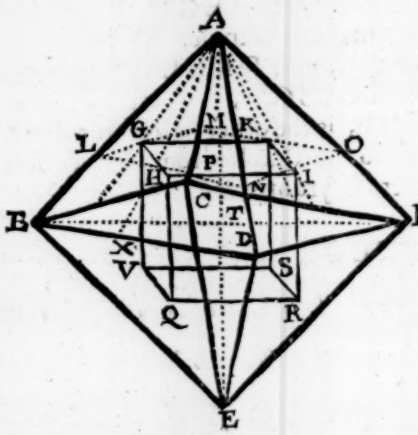
Demonſtration FOraſmuch as theſe two Pyramids taken together have their altitude equal with the altitude of the whole Cube, each of them apart hath to his altitude half the altitude of the Cube, to wit, half the ſide of the Cube, as the line K B. And foraſmuch as the
c) 47. 1. ſquare K L M I is double to the ſquare N R Q P [c], the other ſquares of the Cube ſhall alſo be double to the ſquare N R Q P.



d) ult. 15. And foraſmuch as the Cube [d] is reſolved into ſix Pyramids, whoſe baſes are the baſes of the Cube, and the altitudes the lines drawn from the center to the baſes, which are equal to half the ſide of the Cube, it followeth that each of the ſix Pyramids of the Cube, having his baſe double to the baſe of each of the Pyramids of the Octohedron, and the ſame altitude that the ſaid Pyramids of the Octohedron have, is
e) 6. 12. double to either of the Pyramids of the Octohedron [e]. And foraſmuch as each of the Pyramids of the Cube is equal to the two Pyramids of the Octohedron, the ſix Pyramids of the Cube ſhall be ſextuple to the whole Octohedron. Wherefore we have proved that a Cube is ſextuple to an Octohedron inſcribed in it.

PROP. 28. THEOR. 28.

To prove that an Octohedron ABCDEF, *is quadruple ſeſquialter to a Cube* GHIK, VQRS, *inſcribed in it.*



Demonſtration FOraſmuch as the lines drawn from the center of the Octohedron, or of the Sphere which conteineth it, unto the centers of the baſes of the Octohedron are proved equal [a]; and the angles of the Cube are ſet
a) 21. 15. in the centers of theſe baſes [b]: It followeth that the ſame right lines are drawn from one and the ſame center of the Cube, and of the Octohedron; for they have each one and the ſame center [c]. Let that center be the point T. Wherefore the baſe B D F C, which cutteth the Octohedron into two equal and quadrilateral Pyramids [d], ſhall alſo cut the Cube into two equal parts [e]. For it paſſeth by the center T [f]. And foraſmuch as the baſe of the Cube is in the four centers G, H, I, and K, of the baſes of the Pyramis A B D F C, a plain L N O M extended by thoſe points, ſhall be parallel to the plain B D F C [g], and ſhall cut the Pyramis

b) 4. 15.
c) C. 21. 15.
d) C. 14. 13.
e) C. 39. 11.
f) 14. 13.
g) 4. 15.

in

in the points L, N, O, and M, and the lines L N, B D, and N O, D F, ſhall be parallels, ſo alſo ſhall the lines O M, F C, and L M, B C, and the ſquare G H I K, of the Cube, ſhall be inſcribed in the ſquare L N O M. Wherefore the ſquare L N O M is double to the ſquare G H I K [h]. From the ſolid angle A, let there be drawn to the plain Superficies B D C F, a perpendicular, which let fall upon it, in the point T; and let the ſame pependicular be A T, cutting the plain L N O M in the point P. And it ſhall alſo be a perpendicular to the plain L N O M [i]. Again, from the angle B A D of the triangle A D B, let there be drawn by the center H, of the triangle, to the baſe, a line A H X. Wherefore the line A X is ſeſquialter to the line A H [k]. Wherefore the line A H is double to the line H X. But the other lines A B, A D, A F, and A C, and the perpendicular A P T, are cut like unto the line A H X [l]. Wherefore the line A P is double to the line P T. Wherefore the line A P is the altitude of the Cube; for the line P T is the half thereof. And foraſmuch as upon the baſe G H I K of the Cube, and under the altitude A P, of the ſame Cube, is ſet the Pyramis A G H I K, the ſaid Pyramis is the third part of the Cube [m]. But unto the Pyramis A G H I K the Pyramis A L N O M is double [n]; for the baſe of the one is double to the baſe of the other. Wherefore the Pyramis A L N O M is two third parts of the Cube. And foraſmuch as the Pyramids A L N O M and A B D F C, are like [o]; therefore they are in triple proportion of that in which the ſides of like proportion A H to A X, or A L to A B are [p]. But the ſide A B is proved to be ſeſquialter to the ſide A L. Wherefore the Pyramis A B C D F is to the Pyramis A L N O M, as 27 is to 8 (which is ſeſquialter proportion tripled; for the quantity or denomination of ſeſquialter proportion, to wit, $1\frac{1}{2}$, multiplied into it ſelfe once, it maketh $2\frac{1}{4}$, which again multiplied by $1\frac{1}{2}$, maketh $3\frac{3}{8}$; that is 27 to 8. But of what parts the Pyramis A L N O M conteineth 8, of the ſame the Cube conteineth 12, of the ſame the whole Octohedron (which is double to the Pyramis A B D F C) conteineth 54, which 54 hath to 12 quadruple ſeſquialter proportion: Wherefore the whole Octohedron is to the Cube inſcribed in it in quadruple ſeſquialter proportion. Wherefore we have proved, &c.

h) 47. 1.
i) Co. 14. 11.
k) C. 12. 13.
l) 17. 11.
m) C. 7. 12.
n) 6. 12.
o) 7. def. 11.
p) Co. 8. 12.

COROLLARIE.

An Octohedron is to a Cube inſcribed in it, in that proportion that the ſquares of their ſides are.

For [q] the ſide of the Octohedron is in power quadruple ſeſquialter to the ſide of the Cube inſcribed in it.

q) 14. 16.

PROP. 29. THEOR. 29.

To prove that a given Octohedron A B, *is tredecuple ſeſquialter (that is, as* $13\frac{1}{2}$ *to* 1) *to a trilateral equilateral Pyramis* F E G D, *inſcribed in it.*

Conſtruction LEt there be inſcribed in the given Octohedron a Cube F C E D [a], and in the Cube a Pyramis [b].

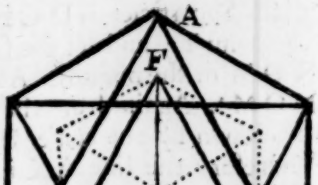
a) 4. 15.
b) 1. 15.

Demonſtration FOraſmuch as the angles of the Pyramis are [c] ſet in the angles of the Cube, and the angles of the Cube are ſet in

c) 1. 15.

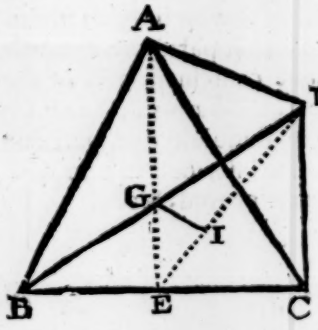
the centers of the baſe of the Octohedron, to wit, in the points F, E, C, D, and G [d]. Wherefore the angles of the Pyramis are ſet in the centers F C and E D of the Octohedron. Wherefore the Pyramis F E D G is inſcribed in the Octohedron [e]. And foraſmuch as the Octohedron A B is to to the Cube F C E D inſcribed in it, quadruple ſeſquialter (by the former Prop.) and the Cube C D E F is to the Pyramis F E D G inſcribed in it triple [f]. Wherefore three Magnitudes being given, to wit, the Octohedron, the Cube, and the Pyramis, the proportion of the extreames (to wit, of the Octohedron to the Pyramis) is made of the proportion of the meanes (to wit, of the Octohedron to the Cube, and of the Cube to the Pyramis [g].) Now then multiply the quantities or denominations of the proportions (to wit, of the Octohedron to the Cube, which is $4\frac{1}{2}$, and of the Cube to the Pyramis, which is 3 [h],) there ſhall be produced $13\frac{1}{2}$, to wit, the proportion of the Octohedron to the Pyramis inſcribed in it. For $4\frac{1}{2}$ multiplyed by 3, produce $13\frac{1}{2}$. Wherefore the Octohedron is to the Pyramis inſcribed in it in tredecuple ſeſquialter proportion. Wherefore we have proved, &c.

d) 4. 15.
e) 6. 15.
f) 25. 16.
g) 10. de. 5.
h) def. 6.



PROP. 30. THEOR. 30.

To prove that a trilateral equilateral Pyramis is nonuple to a Cube inſcribed in it.



Conſtruction LEt the given pyramis be ABCD, whoſe two baſes let be ABC and DBC, and let their centers be G and I, and from the angle A, draw unto the baſe BC, a perpendicular AE, alſo from the angle D, draw unto the ſame baſe BC, a perpendicular DE, and they ſhall meet in the ſection E [a], and in them ſhall be the centers G and I [b].

a) 3. 3.
b) Co. 1. 3.

Demonſtration FOraſmuch as the line GI coupleth the centers of the baſes of the pyramis, the ſaid line GI ſhall be the diameter of the baſe of the Cube inſcribed in the pyramis [c]. And foraſmuch as the line AG is double to the line GE [d], the whole line AE ſhall be triple to the line GE, and ſo is alſo the line DE to the line IE. Wherefore the lines AD and GI are parallels [e]. And therefore the triangles AED and GEI are like [f]. And foraſmuch as the triangles AED and GEI are like, the line AD ſhall be triple to the line GI [g]. But the line AD is the diameter of the baſe of the Cube circumſcribed about the pyramis ABCD, and the line GI is the diameter of the baſe of the Cube inſcribed in the pyramis ABCD; but the diameters of the baſes are

c) 18. 15.
d) C. 12. 13.
e) 2. 6.
f) Cor. 2. 6.
g) 4. 6.

are equimultiplices to the ſides (to wit, are in power double.) Wherefore the ſide of the Cube circumſcribed about the pyramis A B C D, is triple to the ſide of the Cube inſcribed in the ſame pyramis [h]. But like Cubes are in triple proportion to one another of that in which their ſides are [i], and the ſides are in triple proportion to one another. Wherefore triple taken three times, bring forth twenty ſeven cnple, which is 27 to 1; for the four terms 27, 9, 3, and 1, being ſet in triple proportion, the proportion of the firſt to the fourth (to wit, of 27 to 1,) ſhall be triple to the proportion of the firſt to the ſecond, to wit, of 27 to 9 [k]. Which proportion of 27 to 1, is the proportion of the ſides tripled, which proportion alſo is found in like ſolids. Wherefore of what parts the Cube circumſcribed conteineth 27, of the ſame the Cube inſcribed conteines 1. But of what parts the Cube circumſcribed conteineth 27, of the ſame the pyramis inſcribed in it conteineth 9 [l]. Wherefore of what parts the pyramis A B C D conteineth 9, of the ſame the Cube inſcribed in the Pyramis conteineth 1. Wherefore we have proved, &c.

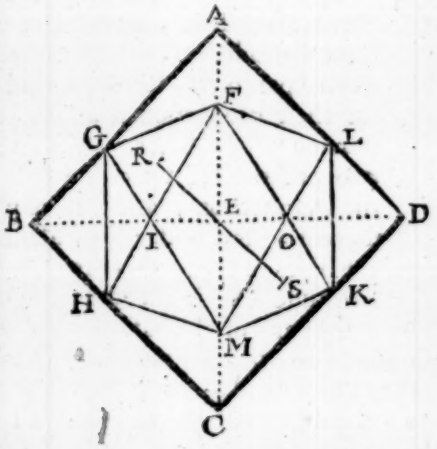
h) 15. 5.

i) 33. 11.

k) 10. de. 5.

l) 25. 6.

PROP. 31. THEOR. 31.



An Octobedron **ABCD**, *hath to an Icoſahedron* **F G HMKLIO**, *inſcribed in it, that proportion which two baſes of the Octohedron have to five baſes of the Icoſahedron.*

Demonſtration FOraſmuch as the ſolid of the Octohedron conſiſteth of eight pyramids, ſet upon the baſes of the Octohedron, and having to their altitude a perpendicular line drawn from the center to the baſe. Let that perpendicular be E R or E S, being drawn from the center E (which center is common to either of the ſolids [a]) to the centers of the baſes, to wit, to the points R and S. Wherefore ſeeing that three pyramids are equal and like, they ſhall be equal to a priſme ſet upon the ſame baſe, and under the ſame altitude [b]. But to this priſme is double that priſme which is ſet upon the ſame baſe, and hath his altitude double, to wit, the whole line R S [c]; for it is equal to the two equal and like priſmes whereof it is compoſed. Wherefore the priſme ſet upon the baſe of the Octohedron, and having to his altitude the line R S, is equal to ſix piramids, ſet upon ſix baſes of the Octohedron, and having to their altitude the line E R, ſo there remain two pyramids (for in the Octohedron are eight baſes) which ſhall be equal to the priſme which is ſet upon the third part of the baſe of the Octohedron, and under the altitude R S, for priſmes under one and the ſame altitude, are in proportion to one another as are their baſes [d]. Wherefors the two priſmes which are ſet upon the baſe of the

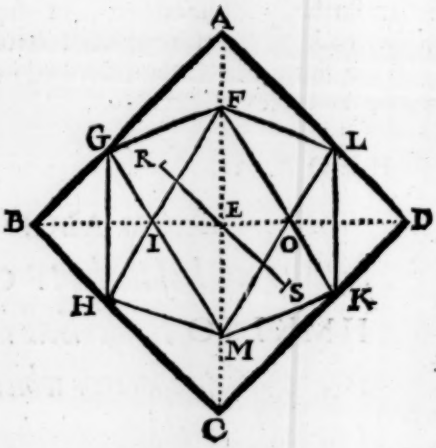
a) C. 21. 15.

b) Co. 7. 12.

c) C. 25. 11.

d) Co. 7. 12.

the Octohedron, and upon a third part thereof, and under the altitude R S, are equal to the eight pyramids of the Octohedron, or to the whole Solid of the Octohedron. And forasmuch as the Icosahedron inscribed in
c) 17. 15. the Octohedron, hath his bases set in the bases of the Octohedron [c]; it followeth that the pyramids set upon the bases of the Icosahedron, and having to their tops one and the same center E, are conteined under the same altitude that the pyramids of the Octohedron are conteined under, to wit, under the line E R or E S; and therefore a prisme set upon the base of the Icosahedron, and having his altitude double to the altitude of the Pyramis, to wit, the whole line R S, is equal to the six pyramids set upon the base of the Icosahedron, and under the altitude E R or E S, as we have proved in the Octohedron. Wherefore the twenty pyramids set upon the twenty bases of the Icosahedron, are equal to three prismes set upon the base of the Icosahedron, and under the altitude R S; and morover, to another prisme set upon a third part of the base of the Icosahedron, and under the same altitude R S, which prisme is

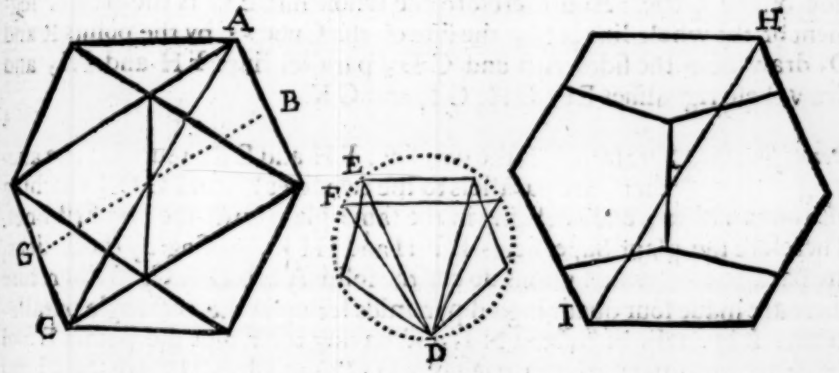


f) Co. 7. 12. a third part of the former prisme [f]; for their proportion is as the proportion of the bases. Wherefore two prismes set upon the base of the Octohedron, and a third part thereof, and under the altitude R S, is to four prismes set upon the three bases of the Icosahedron, and a third part thereof, and under the same altitude R S, in the same proportion that the bases are, that is, as four third parts of the base of the Octohedron (which are equal to one base and $\frac{1}{3}$) to 10 third parts of the base of the Icosahedron (which are equal to 3 bases and $\frac{3}{3}$) or as 2 third parts of the base of the Octohedron are to 5 third parts of the base of the Icosahedron. But 2 third parts of the base of the Octohedron, are to 5 third
g) 15. 5. parts of the base of the Icosahedron, as two bases are to 5 bases [g] (for they are parts of equimultiplices.) And two prismes of the Octohedron are to four prismes of the Icosahedron, as the solid of the Octohedron is to the solid of the Icosahedron, when as each are equal to each of the
h) 11. 5. solids. Wherefore [h] the solid of the Octohedron is to the solid of the Icosahedron inscribed in it, as two bases of the Octohedron are to five bases of the Icosahedron. Therefore, &c. Which was required to be proved.

PROP. 32. THEOR. 32.

The proportion of the solid of an Icosahedron ABGC, *to the solid of a Dodecahedron inscribed in it, consisteth of the proportion of the side* DF *of the Icosahedron, to the side* DE *of the Cube, conteined in the same Sphere, and of the proportion*

tion tripled of the Diameter A C, *to the line* B G, *which coupleth the centers of the oppoſite baſes of the Icoſahedron.*



Demonſtration FOraſmuch as the ſolid of the Icoſahedron A B G C is to the ſolid of the Dodecahedron H I, being conteined in one and the ſame ſphere, as D F is to D E [a]. But the Dodecahedron whoſe diameter is H I, is to the Dodecahedron, whoſe diameter is B G, in triple proportion of that in which the diameter H I is to the diameter B G [b]; and the lines H I and A C are equal by ſuppoſition (to wit, the diameters of one and the ſame ſphere.) Wherefore as H I is to B G, ſo is A C to B G. Wherefore the proportion of the extreames (to wit, of the Icoſahedron A B G C, to the Dodecahedron ſet upon the diameter B G, which coupleth the centers, conſiſteth [c] of the proportions of the means, to wit, of the proportion of A B G C to the Dodecahedron H I, (which is one and the ſame with the proportion of D F to D E) and of the proportion of the ſame H I to the other Dodecahedron ſet upon the diameter B G, inſcribed in the ſame Icoſahedron A B G C [d]; which proportion is triple to the proportion of the line H I (or the line A C) to G B, which coupleth the centers of the oppoſite baſes of the Icoſahedron. The proportion therefore, &c. Which was required to be proved.

a) 8. 14.
b) C. 17. 12.
c) 5. def. 6.
d) 5. 15.

PROP. 33. THEOR. 33.

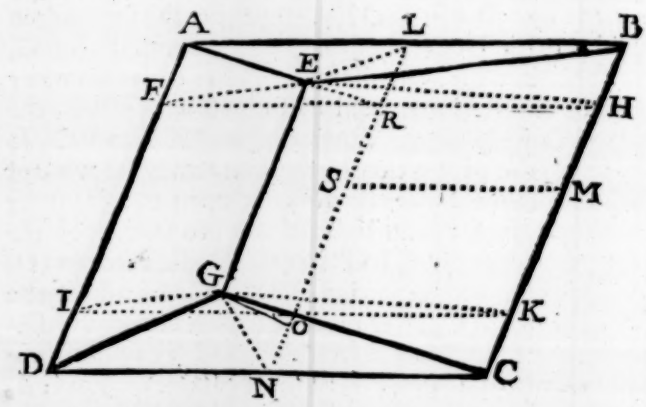
The ſolid of a Dodecahedron exceedeth the ſolid of a Cube inſcribed in it, by a Parallelepipedon, whoſe baſe wanteth of the baſe of the Cube by a third part of the leſſer ſegment, and whoſe altitude wanteth of the altitude of the Cube, by the leſſer ſegment of the leſſer ſegment of half the ſide of the Cube.

Conſtruction FOraſmuch as [a] it was manifeſt that the baſe of a Cube inſcribed in a Dodecahedron, doth with his ſides ſubtend the angles of four Pentagons, meeting at one and the ſame ſide of the Dodecahedron: Let that baſe of the Cube be A B C D; and let the ſide whereat four baſes of the Dodecahedron circumſcribed meet, be E G, which ſhall contein a ſolid A E B D G C, ſet upon the baſe A B C D. Divide the

a) 17. 13. 18. 15.

the ſides AB and DC into two equal parts in the points L and N, and
b) 17. 13. draw the line LN, which is a paralſel to the ſide EG [b]. The perpendiculars alſo ER and GO, which couple thoſe parallels, are each equal to halfe of the ſide EG, and each is the greater ſegment of half the ſide of the Cube. And therefore the whole line EG is the greater ſeg-
c) 17. 13. ment of the whole line LN, the ſide of the Cube [c]; by the points R and O, draw unto the ſides AB and CD, parallel lines FH and IK, and draw theſe right lines EF, EH, GI, and GK.

Demonſtration FOraſmuch as the two lines FH and ER, touching one another, are parallels to the two lines IK and GO, touching alſo one another, and not being in the ſame plain with the two firſt lines. Therefore the plain Superficies EFH and GIK, paſſing by thoſe lines,
d) 15. 11. are parallels [d]; which plains do cut the ſolid AEBDGC. Wherefore there are made four quadrangled pyramids ſet upon the rectangle parallegrams LH, LF, NK, and NI, and having their tops the points E and G. And foraſmuch as the triangles GOK and ERH, are equal and
c) 4. 1. like [e]; to wit, they contein equal angles comprehended under equal ſides, and they are parallels by conſtruction, being ſet in the plains GIK and EFH, the figures GKHE, OKHR, and GORE, ſhall be Parallelograms, by the definition of Parallelograms; and therefore the ſolid GOKERH is a priſme [f]. And



f) 11. de. 11.

by the ſame reaſon may the ſolid GOIERF be proved to be a priſme. And foraſmuch as upon equal baſes NOKC and RLBH, and under equal altitudes OG and RE are ſet pyramids, theſe pyramids ſhall be equal to that pyramis which is ſet upon the baſe CKID (which is double to either of the baſes NOKG and RLBH) and under the ſame altitude
g) 6. 12. OG [g]. And foraſmuch as the ſide GE is the greater ſegment of the line
h) 33. 1. CB, the line KH which [h] is equal to the line GE, ſhall be the greater
i) 2. 14. ſegment of the ſame line CB [i]. Wherefore the reſidues CK and HB, ſhall make the leſſer ſegment of the whole line CB. But as the greater ſegment KH is to the two lines CK and HB, the leſſer ſegment, ſo is the rectangle Parallelogram OH, to the two rectangle Parallelograms OC
k) 1. 6. HL [k]. Wherefore the pyramis ſet upon the baſe OKMS, conteineth
l) Cor. 7. 12. two third parts of the priſme ſet upon the ſame baſe [l]. Wherefore the priſme which is ſet upon two third parts of the baſe OKMS, is equal to the two pyramids NOKCG and RLBHE. For the ſections of a
m) Cor. of 25. 11. priſme are to one another, as the ſections of the baſe are [m]. But the ſec-
n) 1. 6. tions of the baſe are as the ſections of the line CB or KM [n]. Wherefore adde the two pyramids NOKCG and RLBHE, unto the priſme GOKEFH,

GOKERH, two third parts of the prisme set upon the base OKMS. And forasmuch as the line KM is the lesser segment of the whole line BC (for it is equal to the two lines CK and HB) and the prisme set upon the base OKHR is cut like unto the line KM, to wit, in each are taken two thirds, as hath been proved, the prisme equal to the two pyramids, shall adde unto the prisme GOKERH, which is set upon the greater segment KH, two thirds of the lesser segment. Wherefore in the line BC there shall remain one third part of the lesser segment; and therefore in the rectangle Parallelogram NB, which is half the base of the Cube, there shall remain the same third part of the lesser segment. And by the same reason may we prove that in the other pyramids ONDIG and RLAFE, and in the prisme GOIERF is left the same excesse of the base LAND, to wit, the third part of the lesser segment. Wherefore the whole prisme contained between the triangles IGK and FEH, and under the length of the greater segment, and two third parts of the lesser segment of BC the side of the Cube, is equal to the solid composed of four bases of the Dodecahedron, and set upon the base of the Cube. Wherefore the base of that prisme wanteth of the whole base of the Cube, only a third part of the lesser segment, and the altitude of that prisme was the line GO, which is the greater segment of half the side of the Cube. And forasmuch as unto the triangle IGK, is double the rectangle Parallelogram set upon the same base IK (the side of the Cube) and under the altitude GO. *It followeth that three rectangle parallelograms set upon the same IK, the side of the Cube, and under the altitude OG, the greater segment of half the side of the Cube, are sextuple to the triangle IGK.* Wherefore these three rectangle parallelograms do make one rectangle Parallelogram set upon the base IK, and under the altitude of the line GO tripled. But [p] there are six prismes equal and like unto the foresaid prisme, being set upon each of the six bases of the Cube; which prismes are in proportion to one another as their bases are [q]. Wherefore the solid composed of these six Prismes, shall want of the base ABCD the third part of the lesser segment, and taking his altitude of the foresaid rectangle Parallelogram, the said altitude shall be equal to three greater segments (one of which is GO) of halfe the side of the Cube.

o) 41. 1.

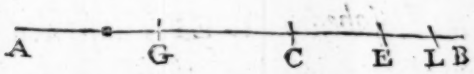
p) 7. def. 11.

q) 3 C. 7. 12.

Now resteth to prove that these three segments want of the side of the Cube by the lesser segment of the lesser segment of half the side of the Cube.

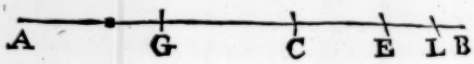
Suppose that AB the side of the Cube, be divided into the greater segment AC, and into the lesser segment CB [r]; divide into two equal parts the line AC in the point G, and the line CB in the point E. And

r) 30. 6.



unto the line CG, put the line CL equal. Now forasmuch as the lines AG and GC, are the greater segments of half the line AB, for each of them is the half of the greater segment of the whole line AB, the lines EB and EC shall be the lesser segments of half the line AB. Wherefore the whole line CL is the greater segment, and the line CE is the lesser

ſegment. But as the line C L is to the line C E, ſo is the line C E to the reſidue E L. Wherefore the line E L is the greater ſegment of the line C E, or of the line E B, equal thereto. Wherefore the reſidue L B is the leſſer ſegment of the ſame E B (which is the leſſer ſegment of half the ſide of the Cube.) But the lines A G, G C, and C L, are three greater ſegments of the half of the whole line A B, which three greater ſegments



make the altitude of the foreſaid ſolid. Wherefore the altitude of the ſaid ſolid wanteth of A B the ſide of the Cube by the line L B, which is the leſſer ſegment of the line B E, which line B E again is the leſſer ſegment of half the ſide A B of the Cube. Wherefore the foreſaid ſolid conſiſting of ſix ſolids, whereby the Dodecahedron exceedeth the Cube inſcribed in it, is ſet upon a baſe which wanteth of the baſe of the Cube, by a third part of the leſſer ſegment, and is under an altitude wanting of the ſide of the Cube by the leſſer ſegment of the leſſer ſegment of half the ſide of the Cube. Therefore, &c. Which was required to be proved.

COROLLARIE.

A Dodecahedron is double to a Cube inſcribed in it, taking away the third part of the leſſer ſegment of the Cube, and moreover, the leſſer ſegment of the leſſer ſegment of half of that exceſſe.

For if there be given a Cube, from which is cut off a ſolid ſet upon a third part of the leſſer ſegment of the baſe, and under one and the ſame altitude with the Cube, that ſolid taken away, hath to the whole ſolid the proportion of the ſection of the baſe to the baſe [s]. Wherefore from the Cube is taken away a third part of the leſſer ſegment. Again, foraſmuch as the reſidue wanteth of the altitude of the Cube, by the leſſer ſegment of the leſſer ſegment of half the altitude or ſide, and that reſidue is a Parallelepipedon, if it be cut by a plain ſuperficies parallel to the oppoſite plain ſuperficies, cutting the altitude of the Cube by a point, it ſhall take away from that Parallelepipedon a ſolid, having to the whole the proportion of the ſection to the altitude [t]. Wherefore the exceſſe wanteth of the ſame Cube, by the third part of the leſſer ſegment, and moreover, by the leſſer ſegment of the leſſer ſegment of half of that exceſſe.

s) 32. 11.

t) 3 Cor. of 25. 11.

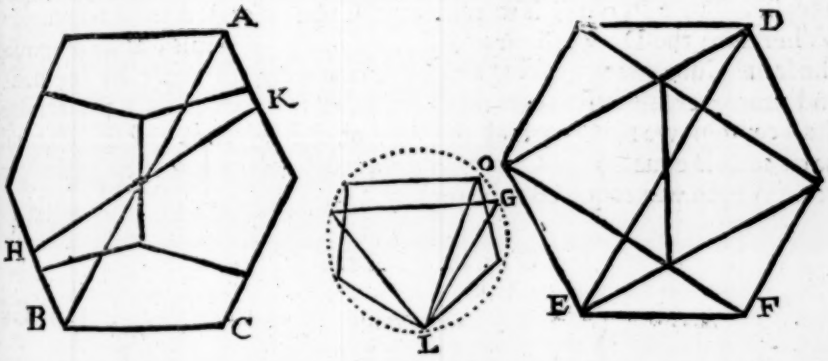
PROP. 34. THEOR. 34.

The proportion of the ſolid of a Dodecahedron A H B C K, *to the ſolid of an Icoſahedron* D E F, *inſcribed in it, conſiſteth of the proportion tripled of the diameter* A B, *to that line* K H, *which coupleth the oppoſite baſes of the Dodecahedron, and of the proportion of the ſide* I O *of the Cube, to the ſide* I G, *of the Icoſahedron inſcribed in one and the ſame Sphere.*

Con-

Conſtruction LEt A B be the diameter of the Dodecahedron, and let the line which coupleth the centers of the oppoſite baſes be K H; and let the diameter of the Icoſahedron be D E. Now foraſmuch as one and the ſame circle conteineth the Pentagon of a Dodecahedron, and the triangle of an Icoſahedron deſcribed in one and the ſame Sphere [a]: Let that circle be I G O. Wherefore I O is the ſide of the Cube, and I G the ſide of the Icoſahedron. I ſay then that the proportion of the Dodecahedron A H B C K, to the Icoſahedron D E F inſcribed in it, conſiſteth of the proportion tripled of the line A B to the line K H, and of the proportion of the line I O to the line I G.

a) 4. 14.



Demonſtration FOraſmuch as the Icoſahedron D E F is inſcribed in the Dodecahedron A B C, by ſuppoſition, the diameter D E ſhall be equal to the line K H [b]. Wherefore the Dodecahedron ſet upon the diameter K H ſhall be inſcribed in the ſame ſphere, wherein the Icoſahedron D E F is inſcribed: But the Dodecahedron A H B C K is to the Dodecahedron upon the diameter K H, in triple proportion of that in which the diameter A B is to the diameter K H [c], and the ſame Dodecahedron which is ſet upon the diameter K H, hath to the Icoſahedron D E F (which is ſet upon the ſame diameter, or upon a diameter equal to it, to wit, D E,) that proportion which I O the ſide of the Cube, hath to I G, the ſide of the Icoſahedron, inſcribed in one and the ſame ſphere [d]. Wherefore the proportion of the Dodecahedron A H B C K, to the Icoſahedron D E F inſcribed in it, conſiſteth of the proportion tripled of the diameter A B, to the line K H, which coupleth the centers of the oppoſite baſes of the Dodecahedron (which proportion is that which the Dodecahedron A H B C K hath to the Dodecahedron ſet upon the diameter K H) and of the proportion of I O the ſide of the Cube, to I G the ſide of the Icoſahedron (which is the proportion of the Dodecahedron ſet upon the diameter K H, to the Icoſahedron D E F, deſcribed in one and the ſame ſphere [e]:) Therefore the proportion of the ſolid of a Dodecahedron, &c. Which was required to be proved.

b) 7. 15.

c) C. 17. 12.

d) 8. 14.

e) 5. def. 6.

PROP. 35. THEOR. 35.

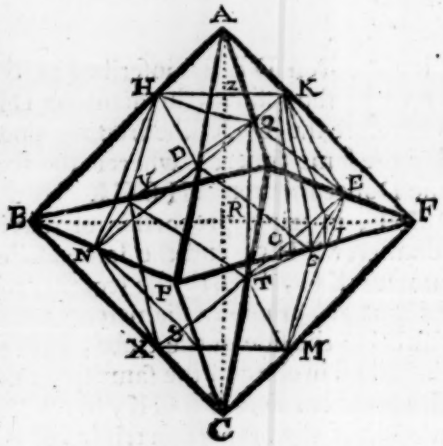
The ſolid of a Dodecahedron conteineth of a Pyramis circumſcribed about it, two ninth parts, taking away a third

part of one ninth part of the leſſer ſegment (of a line divided by an extream and mean proportion) and moreover the leſſer ſegment of the leſſer ſegment of half the reſidue.

Demonſtration IT hath been proved that the Dodecahedron, together with the Cube inſcribed in it, is conteined in one and the ſame Pyramis [a]. And it is manifeſt [b] that the Dodecahedron is double to the ſame Cube, taking away the third part of the leſſer ſegment; and moreover, the leſſer ſegment of the leſſer ſegment of half the reſidue, or of this exceſſe. But a Pyramis is to the ſame Cube inſcribed in it noncuple [c]. Wherefore the Dodecahedron inſcribed in the Pyramis, and conteining the ſame Cube twice, taking away the ſame third of the leſſer ſegment; and moreover, the leſſer ſegment of the leſſer ſegment of half the reſidue, ſhall contein two ninth parts of the ſolid of the Pyramis (of which ninth parts each is equal to the Cube) taking away the ſame exceſſe. Therefore, &c. Which was required to be proved.

a) Co. 1. 16
b) C. 33. 16.
c) 30. 16.

PROP. 36. THEOR. 36.



An Octohedron ABCF PL, *exceedeth an Icoſahedron* HKEGMXNVDSQT, *by a Parallelepipedon ſet upon the ſquare of the ſide of the Icoſahedron, and having to his altitude the line which is the greater ſegment of half the ſemidiameter of the Octohedron.*

Conſtruction DRaw the diameters AZRC and BROIF, and the perpendicular KO parallel to the line AZR.

Demonſtration FOraſmuch as [a] the triangles KDG and KEQ are deſcribed in the baſes APF and ALF, of the Octohedron. Therefore about the ſolid angle there remain upon the baſe FEG three triangles KEG, KFE, and KFG, which contein a Pyramis KEFG, unto which Pyramis ſhall be equal and like the oppoſite Pyramis MEFG, ſet upon the ſame baſe FEG [b]. And by the ſame reaſon ſhall there of every ſolid angle of the Octohedron remain two Pyramids, equal and like; to wit, two upon the baſe AHK, two upon the baſe BNV, two upon the baſe DPS, and two upon the baſe QLT. Now then there ſhall be made twelve Pyramids, ſet upon a baſe conteined of the ſide of the

a) 16. 15.
b) 8. def. 11.

the Icoſahedron, and under two leſſer ſegments of the ſide of the Octohedron conteining a right angle, as for example, the baſe G E F. And foraſmuch as the ſide G E ſubtending a right angle, is [c] in power double to either of the lines E F and F G, and ſo the ſide K H is in power double to either of the ſides A H and A K, and either of the lines A H, A K, or E F, F G, is in power double to either of the lines A Z or Z K, which contein a right angle, made in the triangle or baſe A H K, by the perpendicular A Z. Wherefore it follows that the ſide G E or H K, is in power quadruple to the triangle E F G or A H K. But the Pyramis K E F G, having his baſe E F G in the plain F L B P, of the Octohedron, ſhall have to his altitude the perpendicular K O [d], which is the greater ſegment of the ſemidiameter of the Octohedron [e]. Wherefore three Pyramids ſet under the ſame altitude, and upon equal baſes ſhall be equal to one Priſme ſet upon the ſame baſe, and under the ſame altitude [f]. Wherefore four priſmes ſet upon the baſe G E F quadrupled (which is equal to the ſquare of the ſide G E) and under the altitude K O (or R Z the greater ſegment (which is equal to K O) ſhall contein a ſolid equal to the twelve Pyramids, which twelve Pyramids make the exceſſe of the Octohedron above the Icoſahedron inſcribed in it. Therefore an Octohedron, &c. Which was required to be proved.

c) 47. 1.

d) 4. def. 6.
e) 16. 15.
f) 1 C. 7. 12.

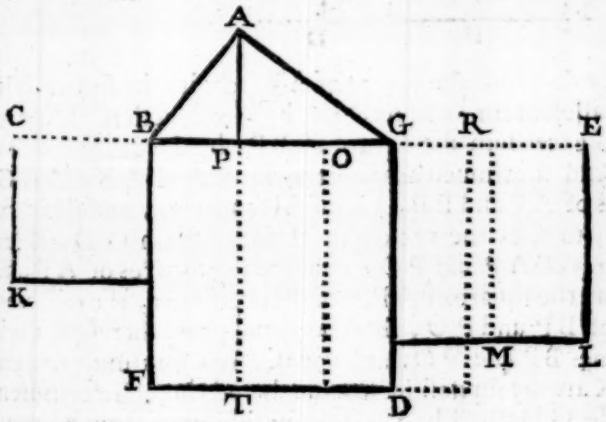
COROLLARIE.

A Pyramis exceedeth the double of an Icoſahedron inſcribed in it, by a ſolid ſet upon the ſquare of the ſide of the Icoſahedron inſcribed in it, and having to his altitude that whole line of which the ſide of the Icoſahedron is the greater ſegment.

For [g] it is manifeſt that an Octohedron and an Icoſahedron inſcribed in it, are inſcribed in one and the ſame Pyramis. It hath been proved likewiſe [h] that a Pyramis is double to an Octohedron inſcribed in it. Wherefore the two exceſſes of the two Octohedrons (unto which the Pyramis is equal) above the two Icoſahedrons (inſcribed in the ſaid two Octohedrons) being brought into a ſolid, the ſaid ſolid ſhall be ſet upon the ſame ſquare of the ſide of the Icoſahedron, and ſhall have to his altitude the perpendicular K O doubled, whoſe double coupling the oppoſite ſides H K and X M, maketh the greater ſegment the ſame ſide of the Icoſahedron [i].

g) 19. 15.
h) 26. 16.
i) 1, 2. Cor. 14. 15.

PROP. 37. THEOR. 37.



If in a triangle A B G, *having to his baſe a rational line ſet* B G, *the ſides* A B *and* A G *be commenſurable in power to the baſe, and from the top* A *be drawn to the baſe a perpendicular*

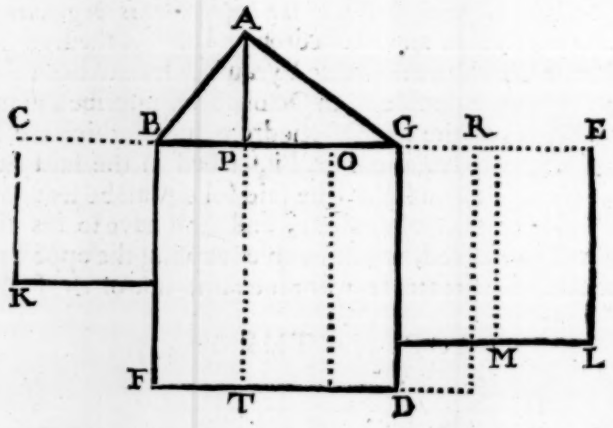
dicular line A P, *cutting the baſe, the ſections of the baſe ſhall be commenſurable in length to the whole baſe, and the perpendicular ſhall be commenſurable in power to the ſaid whole baſe.*

Conſtruction PRoduce on either ſide the line B G, to the points C and E, and unto the line A G put the line G E equal, and unto the line A B put the line B C equal. And upon the lines C B, B G, and G E, deſcribe the ſquares B K, B D, and G L, and from the greater of the ſquares of the lines A B or A G, which let be G L, cut off a Parallelogram E M, equal to the leſſer ſquare B K [a], and [b] unto the reſidue G M, let there be applyed upon the line G D, an equal rectangle Parallelogram O D.

a) 45. 1.
b) 45. 1.

Demonſtration FOraſmuch as the angles A P B and A P G are right angles; therefore [c] the line A G conteineth in power the two lines A P and P G, and the line A B the two lines A P and P B. Wherefore how much the line A G conteineth in power more than the line A B, ſo much alſo doth the line P G contein in power more than the line B P, to wit, taking away the common ſquare of A P, there is left the exceſſe of the ſquare P G, above the ſquare of B P. But the ſquare of

c) 47. 1.



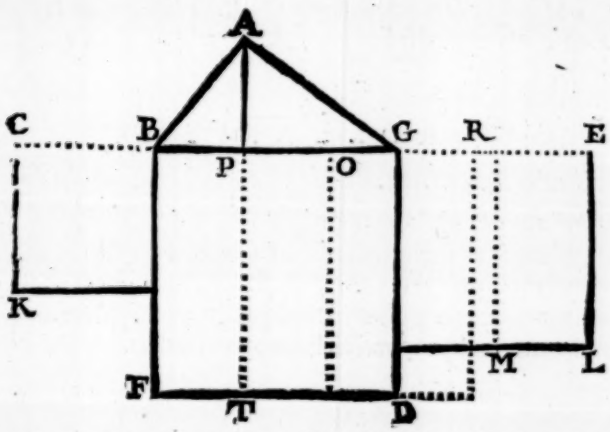
A G (which is G L) exceedeth the ſquare of A B (to wit, the ſquare B K) by the rectangle Parallelogram G M or O D, by Conſtruction. Wherefore the ſquare of P G exceedeth the ſquare of B P, by the rectangle Parallelogram O D. And foraſmuch as unto the ſquares of A B and A G, are equal the ſquares of A P and P B, and of A P and P G, and their exceſſe is taken away, to wit, the rectangle Parallelogram O D, there ſhall be left the ſquares of A P and P O, equal to the ſquares of A P and P B, and taking away the ſquare of A P, which is common, the reſidues, to wit, the ſquares of B P and P O, ſhall be equal; and therefore their ſides (to wit, the lines B P and P O) are equal. And foraſmuch as the ſquares G L and B K are (by ſuppoſition) rational, and therefore commenſurable, their exceſſe O D ſhall be commenſurable unto them [d]; and therefore it is rational [e]. Wherefore the rational Parallelogram O D being applyed upon the rational line G D (or B G) maketh the breadth O G

d) 15. 10.
e) 9. def. 10.

rati-

rational and commenſurable in length to the whole line B G [f]. But if the whole line B G be commenſurable to one of the parts O G, the lines B O, O G, and B G, ſhall be commenſurable [g]. Wherefore alſo the line O G ſhall be commenſurable to the half of the line B O; to wit, to the line P O, or P B [h]. And foraſmuch as the two lines P O and O G are commenſurable, the whole line P G ſhall be commenſurable to the line P O, or to the line P B [i]. Wherefore either of the lines P G and P B ſhall be commenſurable unto the whole line G B [k]. Wherefore the lines B G, P B, and P G, have to one another that proportion which numbers have [l]. Wherefore the ſections P B and P G, of the baſe B G, are commenſurable in length to the ſame baſe [m].

f) 20. 10.
g) 15. 1.0
h) 12. 10.
i) 15. 10.
k) 15. 10.
l) 5. 13.
m) 6. 10.



And now that the perpendicular A P is commenſurable in power to the baſe B G, is thus proved. Foraſmuch as the ſquare of A B is by ſuppoſition commenſurable to the ſquare of B G, and unto the rational ſquare of A B, is commenſurable the rational ſquare of P B [n]. Wherefore the reſidue, to wit, the ſquare of P A, is commenſurable to the ſame ſquare of B P [o]. Wherefore [p] the ſquare of P A is commenſurable to the whole ſquare of B G. Wherefore the perpendicular A P is commenſurable in power to the baſe B G [q]. Which was required to be proved.

n) 12. 11.
o) 15. 11.
p) 12. 10.
q) 3. de. 10.

In demonſtrating of this we made no mention at all of the length of the ſides A B and A G, but only of the length of the baſe B G, for that the line B G is the rational line firſt ſet, and the other lines A B and A G, are ſuppoſed to be commenſurable in power only to the line B G. Wherefore if that be plainly demonſtrated, when the ſides are commenſurable in power only to the baſe, much more eaſily will it follow, if the ſame ſides be ſuppoſed to be commenſurable both in length and in power to the baſe, that is, if their lengths be expreſſed by the Roots of ſquare numbers.

COROLLARIE I.

By the former things demonſtrated, it is manifeſt that if from the powers of the baſe, and of one of the ſides be taken away the power of the other ſide, and if the half of the power remaining be applyed upon the whole baſe, it ſhall make the breadth that ſection of the baſe which is coupled to the firſt ſide.

For from the powers of the baſe B G, and of one of the ſides A G, that

that is, from the ſquares B D and G L, the power of the other ſide A B, to wit, the ſquare B K (or the Parallelogram E M is taken away.) And of the reſidue (to wit, of the ſquare B D, and of the Parallelogram O D or D R, which by ſuppoſition is equal to O D) the half (to wit, of the whole F R, which is P D; for the lines G R and P B are equal to the lines G O and P O) is applyed to the whole line B G or G D, and maketh the breadth the line P G, the ſection of the baſe B G, which ſection is coupled to the firſt ſide A G. And by the ſame reaſon in the other ſide, if from the ſquares B D and B K, be taken away the ſquare G L, there ſhall remain the rectangle Parallelogram F O; for the Parallelogram E M is equal to the ſquare B K, and the Parallelogram G M to the Parallelogram G M to the Parallelogram O D. Wherefore E P the half of the reſidue F O, maketh the breadth B P, which is coupled to the firſt ſide taken A B.

COROLLARIE II.

If a Perpendicular drawn from an angle of a triangle do cut the baſe, the ſections are to the other ſides in power proportional by an Arithmetical proportion.

For it was proved that the exceſſe of the powers of the lines A G and A B, is one and the ſame with the exceſſe of the powers of the lines P G and P B: If therefore the powers do equally exceed one another, they ſhall by an Arithmetical proportion be proportional.

The End of the Sixteenth Element of EUCLIDE, *added by* FLUSSAS.

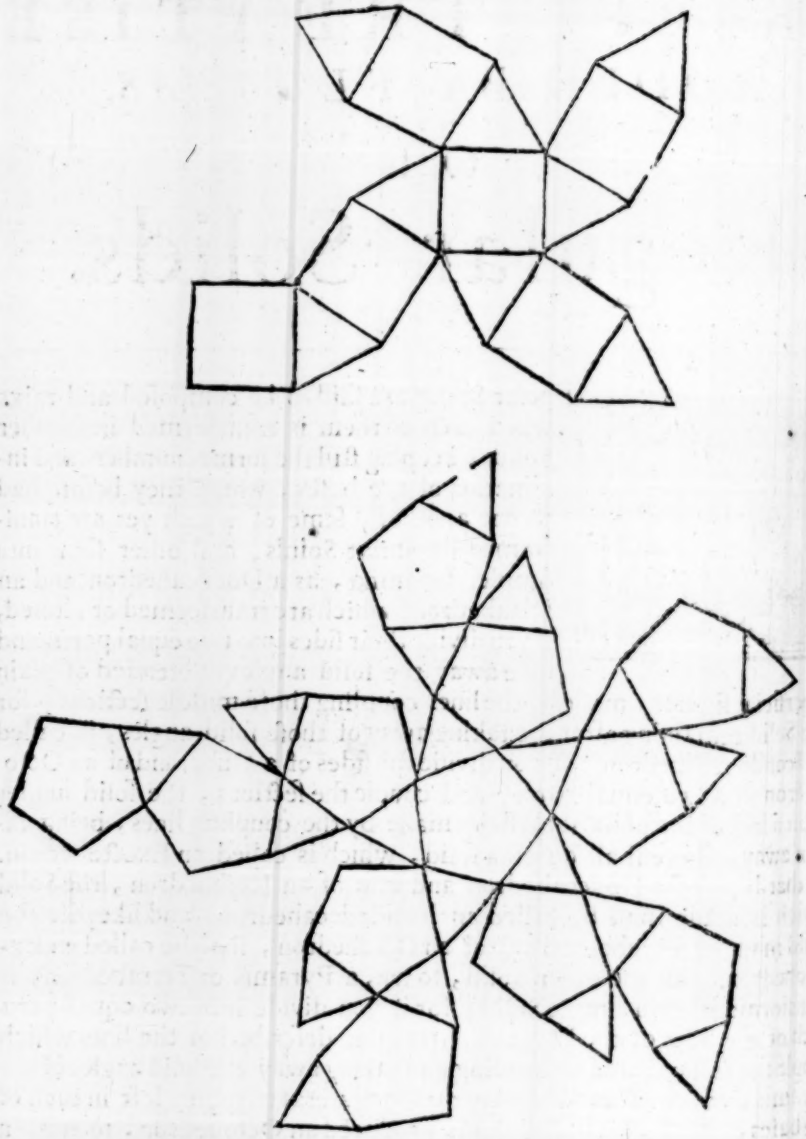
A BRIEFE TREATISE ADDED BY *FLUSSAS*, OF Regular Solids.

REgular Solids are ſaid to be compoſed and mixt when each of them is transformed into other Solids, keeping ſtill the forme, number, and inclination of the baſes, which they before had to one another; ſome of which yet are tranſformed into mixt Solids, and other ſome into ſimple. Into mixt, as a Dodecahedron and an Icoſahedron, which are transformed or altered, if you divide their ſides into two equal parts, and take away the ſolid angles ſubtended of plain ſuperficial figures, made by the lines coupling thoſe middle ſections; for the Solid remaining after the taking away of thoſe ſolid angles, is called an Icoſidodecahedron. If you divide the ſides of a Cube and of an Octohedron into two equal parts, and couple the ſections, the ſolid angles ſubtended of the plain ſuperficies made by the coupling lines, being taken away, there ſhall be left a ſolid, which is called an Exoctohedron. So that both of a Dodecahedron and alſo of an Icoſahedron, the Solid which is made ſhall be called an Icoſidodecahedron; and likewiſe the Solid made of a Cube, and alſo of an Octohedron, ſhall be called an Exoctohedron. But the other Solid, to wit, a Pyramis or Tetrahedron, is transformed into a ſimple Solid; for if you divide into two equal parts each of the ſides of the Pyramis, triangles deſcribed of the lines which couple the ſections, and ſubtending and taking away the ſolid angles of the Pyramis, are equal and like unto the equilateral triangles left in each of the baſes, of all which triangles is produced an Octohedron, to wit, a ſimple, and not a compoſed Solid. For the Octohedron hath four baſes,

like in number, form, and mutual inclination with the baſes of the Pyramis, and hath the other four baſes with like ſituation oppoſite and parallel to the former. Wherefore the application of the Pyramis taken twice, maketh a ſimple Octohedron, as the other Solids make a mixt compound Solid.

DEFINITIONS.

1 *An Exoctohedron is a ſolid figure conteined of ſix equal ſquares, and eight equilateral and equal triangles.*



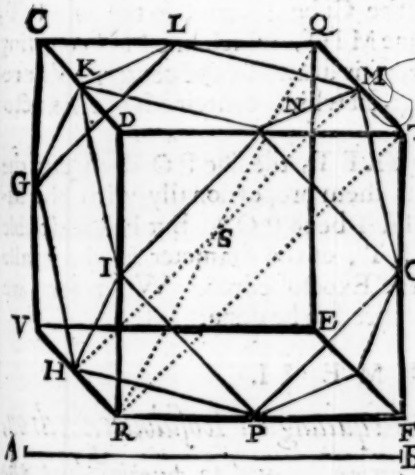
2 *An Icosidodecahedron is a solid figure conteined under twelve equilateral, equal, and equiangled Pentagons, and twenty equal and equilateral triangles.*

For the better understanding of the two former Definitions, and also of the two Propositions following, I have here set two figures, whose figures if you first describe upon pasted paper or such like matter, and then cut them and fold them accordingly, they will represent unto you the perfect formes of an Exoctohedron, and of an Icosidodecahedron.

PROBLEME 1.

To describe an equilateral and equiangled Exoctohedron, and to contein it in a given Sphere, and to prove that the Diameter of the Sphere is double to the side of the said Exoctohedron.

Construction Suppose a Sphere, whose diameter let be AB, and about the diameter AB let there be described a square [a], and upon the square let there be described a Cube [b], which let be CDEF QTVR; and let the diameter thereof be QR, and the center S. Divide the sides of the Cube into two equal parts in the points G, H, I, K, L, M, N, O, P, &c. and couple the middle sections by the right lines IN, NO, OP, PI, and such like, which subtend the angles of the squares or bases of the Cube; and they are equal [c], and contein right angles, as the angle NIP. For the angle NID, which is at the base of the Isosceles triangle NDI, is the half of a right angle, and so likewise is the opposite angle RIP. Wherefore the residue NIP is a right angle, and so the rest. Wherefore NIPO is a square. And by the same reason shall the rest NMLK, KGHI, &c. inscribed in the bases of the Cube, be squares, and they shall be six in number, according to the number of the bases of the Cube. Again, forasmuch as the triangle KIN subtendeth the solid angle D of the Cube, and likewise the triangle KGL the solid angle C, and so the rest which subtend the right solid angles of the Cube, and these triangles are equal and equilateral (to wit) being made of equal sides, and they are the limits or borders of the squares, and the squares the limits or borders of them; as hath been before proved. Wherefore

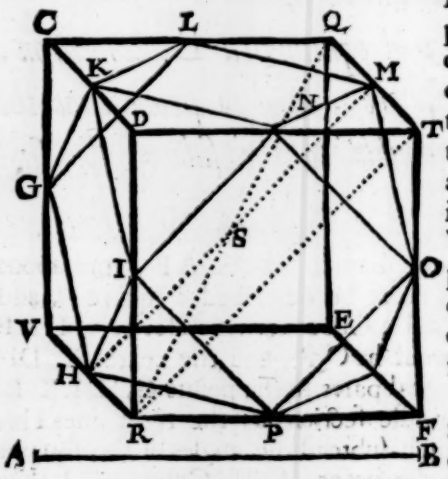


a) 6. 4.
b) 15. 13.
c) 4. 1.

LMNOPHGK is an Exoctohedron by the definition, and is equilateral; for it is conteined of equal ſubtendant lines, it is alſo equiangled; for every ſolid angle thereof is conteined under two ſuperficial angles of two ſquares, and two ſuperficial angles of two equilateral triangles.

Demonſtration FOraſmuch as the oppoſite ſides and diameters of the baſes of the Cube are parallels, the plain extended by the right lines Q T and V R, ſhall be a Parallelogram. And for that alſo in that plain lyeth Q R, the diameter of the Cube, and in the ſame plain alſo is the line M H, which divideth the ſaid plain into two equal parts, and alſo coupleth the oppoſite angles of the Exoctohedron; this line M H therefore divideth the diameter into two equal parts [d]; and alſo divideth it ſelfe in the ſame point, which let be S, into two equal parts [e].

d) Co.34.1.
e) 4. 1. Ex.



And by the ſame reaſon may we prove that the reſt of the lines which couple the oppoſite angles of the Exoctohedron, do in S the center of the Cube, divide one another into two equal parts, for each of the anangles of the Exoctohedron are ſet in each of the baſes of the Cube. Wherefore making the center the point S, with the diſtance S H or S M deſcribe a Sphere, and it ſhall touch every one of the angles equidiſtant from the point S.

And foraſmuch as A B the diameter of the ſphere given, is put equal to the diameter of the baſe of the Cube, to wit, to the line R T, and the ſame line R T is equal to the line M H [f], which line M H coupling the oppoſite angles of the Exoctohedron, is drawn by the center. Wherefore it is the diameter of the Sphere given which conteineth the Exoctohedron.

f) 33. 1.

Laſtly, foraſmuch as in the triangle R F T, the line P O doth cut the ſides into two equal parts, it ſhall cut them proportionally with the baſes, to wit, as F R is to F P, ſo ſhall R T be to P O [g]. But F R is double to F P by ſuppoſition: Wherefore R T, or the diameter H M, is alſo double to the line P O, the ſide of the Exoctohedron. Wherefore we have deſcribed, &c. Which was required to be done.

g) 2. 6.

PROBLEME II.

To deſcribe an equilateral and equiangled Icoſidodecahedron, and to comprehend it in a ſphere given, and to prove that the Diameter being divided by an extream and mean proportion, maketh the greater ſegment double to the ſide of the Icoſidodecahedron.

Con-

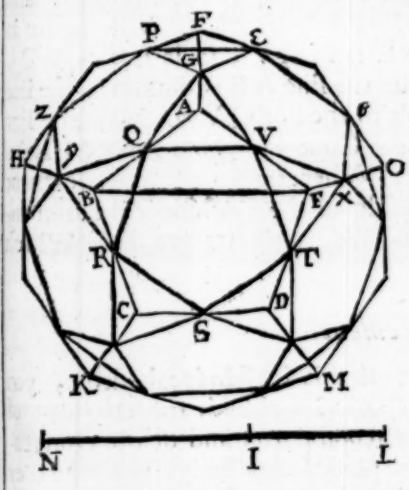
Conſtruction Suppoſe that the diameter of the ſphere given be NL, and [a] divide the line N L, by an extream and mean proportion in the point I, and the greater ſegment thereof let be N I; and upon the line NI deſcribe a Cube [b]; and about this Cube let there be circumſcribed a Dodecahedron [c]; and let the ſame be A B C D E F H K M O, and divide each of the ſides into two equal parts in the points Q, R, S, T, V, X, Y, Z, P, ε, ϑ, G, &c. and couple the ſections with right lines, which ſhall ſubtend the angles of the Pentagons, as the lines P G, G V, V Q, QY, YR, R Q, V T, T X, X V, and ſo the reſt.

a) 30. 6.
b) 15. 13.
c) 17. 13.

Demonſtration FOraſmuch as theſe lines ſubtend equal angles of the Pentagons, and thoſe equal angles are conteined of equal ſides to wit, of the halves of the ſides of the Pentagons; therefore thoſe ſubtending lines are equal [d]. Wherefore the triangles G QV, Y QR, and VXT, and the reſt, which take away ſolid angles of the Dodecahedron, are equilateral.

d) 4. 1.

Again, foraſmuch as in every Pentegon are deſcribed five equal right lines, coupling the middle ſections of the ſides, as are the lines QV, VT, TS, SR, and RQ, they deſcribe a Pentagon in the plain of the Pentagon of the Dodecahedron. And the ſaid Pentagon is conteined in a circle, to wit, whoſe center is the center of a Pentagon of the Dodecahedron. For the lines drawn from that center to the angles of this Pentagon are equal, for that they are perpendiculars upon the baſes cut [e]. Wherefore the Pentagon QRSTV, is equiangled [f]. And by the ſame reaſon may the reſt of the Pentagons deſcribed in the baſes of the Dodecahedron, be proved equal and like.



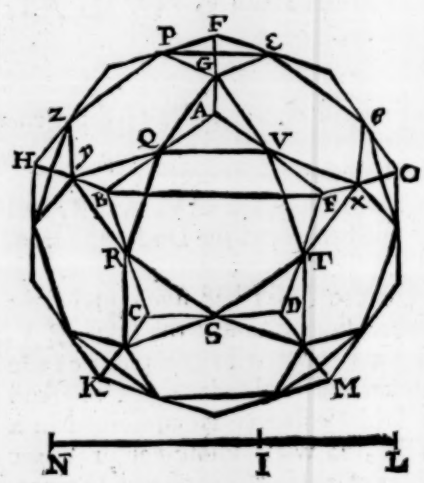
e) 12. 4.
f) 11. 4.

Wherefore thoſe Pentagons are twelve in number: And foraſmuch as the equal and like triangles do ſubtend and take away twenty ſolid angles of the Dodecahedron; therefore the ſaid triangles ſhall be twenty in number. Wherefore we have deſcribed an Icoſidodecahedron by the definition, which Icoſidodecahedron is equilateral; for that all the ſides of the triangles are equal and common with the Pentagons; and it is alſo equiangled. For each of the ſolid angles is made of two ſuperficial angles of an equilateral Pentagon, and of two ſuperficial angles of an equilateral triangle

Now let us prove that it is conteined in the given ſphere whoſe Diameter is N L. Foraſmuch as perpendiculars drawn from the centers of the Dodecahedron, to the middle ſections of his ſides, are the halves of the lines which couple the oppoſite middle ſections of the ſides of the Dodecahedron [g]; which lines alſo [h] do in the center divide one another into two equal parts. Therefore right lines drawn from that point to the angles

g) 3 Cor. of 17. 13.
h) idem.

angles of the Icoſidodecahedron (which are ſet in thoſe middle ſections) are equal; which lines are 30 in number, according to the number of the ſides of the Dodecahedron; for each of the angles of the Icoſidodecahedron are ſet in the middle ſections of each of the ſides of the Dodecahedron. Wherefore making the center the center of the Dodecahedron, and the ſpace any one of the lines drawn from the center to the middle ſections, deſcribe a ſphere, and it ſhall paſſe by all the angles of the Icoſidodecahedron, and ſhall contein it.



i) 4C. 17. 13

And foraſmuch as the diameter of this ſolid, is that right line whoſe greater ſegment is the ſide of the Cube inſcribed in the Dodecahedron [i], which ſide is N I by ſuppoſition. Wherefore that ſolid is contein-ed in the ſphere given, whoſe diameter is put to be the line N L.

Now let us prove that the greater ſegment of the diameter is double to Q V the ſide of the ſolid. Foraſmuch as the ſides of the triangle A E B, are in the points Q and V divided into two equal parts, the lines Q V and B E are parallels [k]. Wherefore as A E is to A V, ſo is E B to V Q [l]. But the line A E is double to the line A V. Wherefore the line B E is double to the line Q V [m]. Now the line B E is equal to N I, or to the ſide of the Cube [n]; which line N I is the greater ſegment of the diameter N L. Wherefore the greater ſegment of the diameter given is double to the ſide of the Icoſidodecahedron inſcribed in the given ſphere. Wherefore, We have deſcribed, &c. Which was required to be done.

k) Co. 39. 1.
l) 2. 6.
m) 4. 6.
n) 2 Cor. of 17. 13.

Advertiſement.

To the underſtanding of the nature of this Icoſidodecahedron, you muſt well conceive the paſſions and proprieties of both theſe ſolids, of whoſe baſes it conſiſteth, to wit, of the Icoſahedron and of the Dodecahedron. And although in it the baſes are placed oppoſitely, yet have they to one another one and the ſame inclination. By reaſon whereof there lie hidden in it the actions and paſſions of the other Regular Solids. And I would have thought it not impertinent to the purpoſe to have ſet forth the inſcriptions and circumſcriptions of this Solid, if want of time had not hindred. But to the end the Reader may the better attein to the underſtanding thereof, I have here following briefly ſet forth, how it may in or about every one of the five Regular Solids be inſcribed or circumſcribed; by the help whereof he may, with ſmall travail or rather none at all, having well poiſed and conſidered the Demonſtrations apperteining to the foreſaid five Regular Solids, demonſtrate both the inſcription of the ſaid Solids in it, and the inſcription of it in the ſaid Solids.

Of

Of the inſcriptions and circumſcriptions of an Icoſidodecahedron.

AN Icoſidodecahedron may contein the other five regular bodies. For it will receive the angles of a Dodecahedron in the centers of the triangles which ſubtend the ſolid angles of the Dodecahedron, which ſolid angles are twenty in number, and are placed in the ſame order in which the ſolid angles of the Dodecahedron taken away, or ſubtended by them, are. And by that reaſon it ſhall receive a Cube and a Pyramis conteined in the Dodecahedron, when as the angles of the one, are ſet in the angles of the other.

An Icoſidodecahedron receiveth an Octohedron, in the angles cutting the ſix oppoſite ſections of the Dodecahedron, even as if it were a ſimple Dodecahedron.

And it conteineth an Icoſahedron, placing the twelve angles of the Icoſahedron in the ſame centers of the twelve Pentagons.

It may alſo by the ſame reaſon be inſcribed in each of the five regular bodies; to wit, In a Pyramis, if you place four triangular baſes concentrical with four baſes of the Pyramis, after the ſame manner that you inſcribed an Icoſahedron in a Pyramis; ſo likewiſe may it be inſcribed in an Octohedron, if you make eight baſes thereof concentrical with the eight baſes of the Octohedron. It ſhall alſo be inſcribed in a Cube, if you place the angles which receive the Octohedron in it, in the centers of the baſes of the Cube. Again, you ſhall inſcribe it in an Icoſahedron, when the triangles compaſſed in of the Pentagon baſes, are concentrical with the triangles which make a ſolid angle of the Icoſahedron.

Laſtly, it ſhall be inſcribed in a Dodecahedron, if you place each of the angles thereof in the middle ſections of the ſides of the Dodecahedron, according to the order of the Conſtruction thereof.

The oppoſite plain ſuperficieces alſo of this ſolid are parallels. For the oppoſite ſolid angles are ſubtended of parallel plain ſuperficieces, as well in the angles of the Dodecahedron ſubtended by triangles, as in the angles of the Icoſahedron ſubtended of Pentagons, which thing may eaſily be demonſtrated. Moreover in this ſolid are infinite properties and paſſions, ſpringing of the ſolids whereof it is compoſed.

Wherefore it is manifeſt that a Dodecahedron and an Icoſahedron, mixed, are transformed into one and the ſelf ſame ſolid of an Icoſidodecahedron. A Cube alſo and an Octohedron are mixed and altered into another ſolid, to wit, into one and the ſame Exoctohedron. But a Pyramis is transformed into a ſimple and perfect ſolid, to wit, into an Octohedron.

If we will frame theſe two ſolids joyned together into one ſolid, this onely muſt we obſerve.

In the Pentagon of a Dodecahedron inſcribe a like Pentagon, and let its angles be ſet in the middle ſections of the Pentagon circumſcribed, and then upon the ſaid Pentagon inſcribed let there be ſet a ſolid angle of an Icoſahedron, and ſo obſerve the ſame order in each of the baſes of the Dodecahedron, and the ſolid angles of the Icoſahedron ſet upon theſe Pentagons ſhall produce a ſolid conſiſting of the whole Dodecahedron, and whole

whole Icoſahedron. In like ſort, if in every baſe of the Icoſahedron, the ſides being divided into two equal parts, be inſcribed an equilateral triangle, and upon each of thoſe equilateral triangles be ſet a ſolid angle of a Dodecahedron, there ſhall be produced the ſame ſolid conſiſting of the whole Icoſahedron, and of the whole Dodecahedron.

And after the ſame order, if in the baſes of a Cube be inſcribed ſquares ſubtending the ſolid angles of an Octohedron, or in the baſes of an Octohedron be inſcribed equilateral triangles ſubtending the ſolid angles of a Cube, there ſhall be produced a ſolid conſiſting of either of the whole ſolids, to wit, of the whole Cube, and of the whole Octohedron.

But equilateral triangles inſcribed in the baſes of a Pyramis, having their angles ſet in the middle ſections of the ſides of the Pyramis, and the ſolid angles of a Pyramis, ſet upon the ſaid equilateral triangles, there ſhall be produced a ſolid conſiſting of two equal and like Pyramids.

And now if in theſe ſolids thus compoſed, you take away the ſolid angles, there ſhall be reſtored again the firſt compoſed ſolids, to wit, the ſolid angles taken away from a Dodecahedron and an Icoſahedron compoſed into one, there ſhall be left an Icoſidodecahedron, the ſolid angles taken away from a Cube and an Octohedron compoſed into one ſolid, there ſhall be left an Exoctohedron. Moreover, the ſolid angles taken away from two Pyramids compoſed into one ſolid, there ſhall be left an Octohedron.

Of the nature of a trilateral and equilateral Pyramis.

1 A trilateral equilateral Pyramis is divided into two equal parts, by three equal ſquares, which in the center of the Pyramis cut one another into two equal parts, and perpendicularly, and whoſe angles are ſet in the middle ſections of the ſides of the Pyramis.

2 From a Pyramis are taken away 4 Pyramids like unto the whole, which utterly take away the ſides of the Pyramis, and that which is left is an Octohedron inſcribed in the Pyramis in which all the ſolids inſcribed in the Pyramis are conteined.

3 A perpendicular drawn from the angle of the Pyramis to the baſe, is double to the diameter of the Cube inſcribed in it.

4 And a right line coupling the middle ſections of the oppoſite ſides of the Pyramis is triple to the ſide of the ſame Cube.

5 The ſide alſo of a Pyramis is triple to the diameter of the baſe of the Cube.

6 Wherefore the ſame ſide of the Pyramis is in power double to the right line which coupleth the middle ſections of the oppoſite ſides.

7 And it is in power ſeſquialter to the perpendicular which is drawn from the angle to the baſe.

8 Wherefore the perpendicular is in power ſeſquitertia to the line which coupleth the middle ſections of the oppoſite ſides.

9 A Pyramis and an Octohedron inſcribed in it, alſo an Icoſahedron inſcribed in the ſame Octohedron, do contein one and the ſame ſphere.

Of the nature of an Octohedron.

1 Four perpendiculars of an Octohedron, drawn in four baſes thereof from two oppoſite angles of the ſaid Octohedron, and coupled together by

by those 4 bases, describe a Rhombus, or Diamond figure; one of whose diameters is in power double to the other diameter.

2 For it hath the same proportion that the diameter of the Octohedron hath to the side of the Octohedron.

3 An Octohedron and an Icosahedron inscribed in it, do contein one and the same sphere.

4 The diameter of the solid of the Octohedron is in power sesquialter to the diameter of the circle which conteineth the base, and is in power duple superbipartiens tertias (that is, as 8 to 3,) to the perpendicular or side of the foresaid Rhombus; and moreover is in length triple to the line which coupleth the centers of the next bases.

5 The angle of the inclination of the bases of the Octohedron, doth with the angle of the inclination of the bases of the Pyramis, make angles equal to two right angles.

Of the nature of a Cube.

1 The diameter of a Cube is in power sesquialter to the diameter of his base.

2 And is in power triple to his side.

3 And unto the line which coupleth the centers of the next bases, it is in power sextuple.

4 Again, the side of the Cube, is to the side of the Icosahedron inscribed in it, as the whole is to the greater segment.

5 Unto the side of the Dodecahedron, it is as the whole is to the lesser segment.

6 Unto the side of the Octohedron it is in power duple.

7 Unto the side of the Pyramis it is in power subduple.

8 Again, the Cube is triple to the Pyramis, but to the Cube the Dodecahedron is in a manner double. Wherefore the same Dodecahedron is in a manner sextuple to the said Pyramis.

Of the nature of the Icosahedron.

1 Five triangles of an Icosahedron, do make a solid angle, the bases of which triangles make a Pentagon. If therefore from the opposite bases of the Icosahedron be taken the other Pentagon by them described, these Pentagons shall in such sort cut the diameter of the Icosahedron which coupleth the foresaid opposite angles, that that part which is conteined between the plaines of these two Pentagons shall be the greater segment, and the residue which is drawn from the plain to the angle, shall be the lesser segment.

2 If the opposite angles of two bases joyned together, be coupled by a right line, the greater segment of that right line is the side of the Icosahedron.

3 A line drawn from the center of the Icosahedron to the angles, is in power quintuple to half that line, which is taken between the Pentagons, or of the half of that line, which is drawn from the center of the circle which conteineth the foresaid Pentagon, which two lines are therefore equal.

4 The side of the Icosahedron conteineth in power either of them, and also the lesser segment, to wit, the line which falleth from the solid angle to the Pentagon.

5 The diameter of the Icosahedron conteineth in power the whole line,

which coupleth the oppoſite angles of the baſes joyned together, and the greater ſegment thereof, to wit the ſide of the Icoſahedron.

6 The diameter alſo is in power quintuple to the line which was taken between the Pentagons, or to the line which is drawn from the center to the circumference of the circle which conteineth the Pentagon compoſed of the ſides of the Icoſahedron.

7 The dimetient conteineth in power the right line which coupleth the centers of the oppoſite baſes of the Icoſahedron, and the diameter of the circle which conteineth the baſe.

8 Again, the ſaid dimetient conteineth in power the diameter of the circle which conteineth the Pentagon, and alſo the line which is drawn from the center of the ſame circle to the circumference: That is, it is quintuple to the line drawn from the center to the circumference.

9 The line which coupleth the centers of the oppoſite baſes, conteineth in power the line which coupleth the centers of the next baſes, and alſo the reſt of that line of which the ſide of the Cube inſcribed in the Icoſahedron is the greater ſegment.

10 The line which coupleth the middle ſections of the oppoſite ſides, is triple to the ſide of the Dodecahedron inſcribed in it.

11 Wherefore if the ſide of the Icoſahedron, and the greater ſegment thereof be made one line, the third part of the whole is the ſide of the Dodecahedron inſcribed in the Icoſahedron.

Of the Dodecahedron.

1 The diameter of a Dodecahedron conteineth in power the ſide of the Dodecahedron, and alſo that right line to which the ſide of the Dodecahedron is the leſſer ſegment, and the ſide of the Cube inſcribed in it is the greater ſegment, which line is that which ſubtendeth the angle of the inclination of the baſes, conteined under two perpendiculars of the baſes of the Dodecahedron.

2 If there be taken two baſes of the Dodecahedron, diſtant from one another by the length of one of the ſides, a right line coupling their centers being divided by an extream and mean proportion, maketh the greater ſegment the right line which coupleth the centers of the next baſes.

3 If by the centers of five baſes ſet upon one baſe, be drawn a plain ſuperficies, and by the centers of the baſes which are ſet upon the oppoſite baſe, be drawn alſo a plain ſuperficies, and then be drawn a right line, coupling the centers of the oppoſite baſes, that right line is ſo cut, that each of his parts ſet without the plain ſuperficies, is the greater ſegment of that part which is conteined between the plaines.

4 The ſide of the Dodecahedron is the greater ſegment of the line which ſubtendeth the angle of the Pentagon.

5 A perpendicular line drawn from the center of the Dodecahedron to one of the baſes, is in power quintuple to half the line which is between the plains.

6 And therefore the whole line which coupleth the centers of the oppoſite baſes is in power quintuple to the whole line which is between the ſaid plains.

7 The line which ſubtendeth the angle of the baſe of the Dodecahedron, together with the ſide of the baſe are in power the quintuple to the line which is drawn from the center of the circle which conteineth the baſe, to the circumference.

8 A Section of a ſphere conteining 3 baſes of the Dodecahedron, taketh a third part of the diameter of the ſaid ſphere.

9 The ſide of the Dodecahedron and the line which ſubtendeth the angle of the Pentagon, are equal to the right line which coupleth the middle ſections of the oppoſite ſides of the Dodecahedron.

THE END.

Euclides Data.

A Commentary or Preface written by the Phyloſopher MARINUS, *on* EUCLIDES DATA.

IN the firſt place we ought to ſet down (as a Foundation) what that is, which we call *DATUM* or GIVEN; then to conſider the profit and utility thereof; and in the third place, to what Art or Science this Tract doth appertein.

The Word *DATUM* therefore is diverſly defined, for the Ancients have defined it after one manner, and later Writers after another, whence it follows that it ſeemeth a difficult thing to give a true explication thereof; for ſome of them have not delivered the Definition of the Word; but have with much labour and trouble ſought certain proprieties thereof, and ſome others collecting and mingling what hath been delivered by others before, have endevoured to define the Word *DATUM*; but not ſo exquiſitely but that they have contradicted themſelves; although what hath been ſaid by all of them, ſeemes to be grounded on one and the ſame notion and ſuppoſition; for they all take the Word *DATUM* to be a thing compriſed; and therefore among ſuch as have endevoured to deſcribe it moſt ſimply, and with ſome ſimple difference, ſome of them have taken the Word *DATUM* to be the ſame with *ORDINATUM*, and ſo *Appollonius* underſtands it in his Tract of Inclinations, and in his univerſal Tract; and ſo others, as *Diodorus*, takes it to be *COGNITUM* KNOWN; for in this ſignification he takes the right line and the angles to be given, and all that may arrive to our Knowledge, although we may not be able well to expreſſe it. But others have believed that it hath the ſame ſignification as the Word [*Effabile*] that may be declared, and ſo *Ptolomie* would have it, who calls thoſe things GIVEN, whoſe meaſure is known whether preciſely, or neer the matter. Others alſo have thought the Word *DATUM* to be what is

granted us by the propoſer in the Hypotheſis; being that in the Firſt Elements, a point given, and a right line given, is diverſly taken (that is to ſay, that who ſo would give and determine the quantity of a right line) all which things ſignifie ſome COMPREHENSION; and therefore of all theſe Definitions, thoſe are moſt agreeable, which do moſt openly declare the COMPREHENSION, as we ſhall make evident by what follows.

Let us now unfold the diverſe opinions of thoſe, who writing the nature of *DATUM* GIVEN, have not taken one ſimple Marke, or only Character for its Definition; and let us reduce it as in a Summary or Epitomy, to the end we may with the more eaſe know or number all their differences. Some of them then have defined *DATUM* to be *Ordinatum* and *Porimon* together, and others *Ordinatum* and *Cognitum* together, and others *Porimon* and *Cognitum* together. Wherefore all ſeem to have ſo defined it as to have have had regard to the *Comprehenſion*, or *Aſſuming* and *Invention*, of the thing given; and to the end that we may the better conceive their opinions, and that from the ſaying of many we may be able to draw a true Definition of what is propoſed, we will take notice in the firſt place of the ſignification of all the ſimple terms which they make uſe of, as alſo of the terms oppoſed to them, to wit, *Inordinatum*, and *Incognitum*, *Aporon*, and *Irrational*; for thoſe things appertein to this Geometrical buſineſſe, to natural things, and to Mathematical Diſcipline.

Now we may call that *Ordinatum* (or Regulated) which doth alwayes keep and obſerve that for which it is ſaid to be ordered, whether you regard its Magnitude or Species, or touching ſome other ſuch like thing: It is alſo thus defined, *Ordinatum* is that which cannot be done in divers manners, but in one only manner, and in ſome determined place: As for Example, A right line drawn by two given points, is ſaid to be ordered, by reaſon it cannot be otherwiſe done, nor in divers manners. But an angle paſſing by two points is ſaid to be *Inordinatum* (or diſordinate and irregular) for that it is made in infinite and divers manners by a great or ſmall circle deſcribed by two points *ad infinitum*. Contrariwiſe, an angle conſtituted by three points, is ſaid to be *Ordinatum*, as alſo thoſe things which follow, are ſaid to be *Ordinatum*, as to conſtitute an equilateral triangle on a right line, for it cannot be diverſly made, but unchangeably, on both the extremities of the line. Again, To divide a given right line according to a given proportion, for that cannot be done but in one certain point. The things *Inordinatum* are ſuch as are done contrary to thoſe laſt mentioned, as to conſtitute a Scalene triangle, and to divide a right line indefinitely. Wherefore the Problem is ordered, is propoſed in the determination, conſidering that a certain thing may be in one manner ſaid to be *Ordinatum*, and *Inordinatum* in another, as an equilateral triangle, conſidering the equality of the ſides, it is *Ordinatum*, but conſidering its Magnitude it is *Inordinatum*, being in no wiſe determined.

But we call that *Cognitum* which is notorious, as clear and comprehended of us, and *Incognitum* that which is not known or comprehended of us, as the length of a way is ſaid to be known, when we know how many Miles it conteins; alſo that the three lines of a Rectiline triangle are equal to two right angles; and in like manner, that the Binomial is Irrational, ſuch things are known, as alſo that it is only one right line that can touch a ſpiral line from a given point without it, from both parts; for if there were yet another line, two right lines would encloſe a ſpace, which is im-

impoſſible. Again, Irrational things are not ſaid to be unknown, but ſuch of them only which are neither known nor comprehended of us.

Porimon is that which we may make and conſtitute, that is to ſay, bring to our underſtanding. Again, it is defined thus, *Porimon* is that which may be exhibited by Demonſtration, or which is apparent without Demonſtration, as to deſcribe a circle from a center and with a ſpace, as alſo to conſtitute not only an equilateral triangle, but alſo a Scalene; or to find a *Binomium*, or to find two right lines rational, commenſurable in power only, and other things which are known infinitely, are *Porimon*; as to deſcribe a circle by two points.

Aporon is wholly oppoſite to *Porimon*, as for Example, the Quadrature of a circle, for that it hath not as yet been found; although it be certainly known that it may be: Nevertheleſſe the manner of finding it out hath not been to this preſent comprehended. But we ſpeak here of that which is already known, which is called *Porimon principale*; for what hath not been as yet made, and yet nevertheleſſe is poſſible, is called *Poriſton*, (or feaſible) although the Conſtruction be yet unknown. But *Aporon* as hath been afore ſaid, is oppoſite to *Porimon*, and is that whoſe Nature is not as yet decided, nor well determined.

Effabile, that is to ſay, rational (or ſpeakable and explicable) is that whoſe Magnitude, Species, and Poſition, we may be able to declare; but this Definition is a little too general, for properly, and according to it ſelfe, *Effabile* is that which is known by certain things, and according to a meaſure given by poſition, as of a ſpan, or a fingers breadth, &c.

Theſe things then being thus unfolded, we may eaſily perceive in what all thoſe things that we have afore ſpoken of do agree together, and wherein they do differ; and firſt of all how *Ordinatum* and *Cognitum* do agree together, and likewiſe their oppoſites the one to the other, for it cannot be ſaid that any one of thoſe things by counterchanging is the other, nor yet that the one hath not more extent than the other, although they agree in many things, as to deſcribe a right line by two points, and to conſtitute an equilateral triangle by three Circles. But to ſquare a Circle, that is indeed *Ordinatum*, yet nevertheleſſe, *Incognitum*. Alſo that at a point of a ſpiral line there is but one touch line, that is, of the kind we call *Ordinatum*, and cannot be otherwiſe done, yet nevertheleſſe the Demonſtration and Conſtruction thereof is not yet known. Again, the indefinite ſection, and the Conſtruction of the Scalenum is *Cognitum*; but is not *Ordinatum*; in ſo much as it is manifeſt that amongſt thoſe things which are *Ordinatum*, there are ſome that are *Cognitum*, and others that are *Incognitum*; and contrariwiſe, that amongſt the things that are *Cognitum*, there are ſome that are *Ordinatum*, and others that are *Inordinatum*; and therefore thoſe things anſwer one another, as among Living Creatures, that which hath reaſon, with that which hath Feet, for there is no equality amongſt them, neither doth the one extend more than the other.

In like manner, *Ordinatum* and *Inordinatum* agree together, reſpecting *Porimon* and *Aporon*; ſeeing that between them there is a very great reſemblance, and becauſe that they do differ only in the manner before expreſſed; for in truth the ſpiral line is *Ordinatum*; but it was not *Porimon* before *Archimedes*; and by the ſame reaſon thoſe things that are inordinate and known by an infinity of wayes and meanes are *Porimon*, if any one ſhall undertake to invent their Conſtitution and Conſtruction. Yet nevertheleſſe

theleſſe they are not ordinate, as to conſtitute a Scalenum triangle, it being no difficult thing to make known the conſtitution thereof by an equilateral triangle, yea it is moſt eaſie, although it be inordinate, and known by an infinity of wayes.

And in the ſame manner do agree *Ordinatum* and *Inordinatum*, together with *Effabile* and *Irrationale*; for they agree together in many things, differing nevertheleſſe by the fore-going reaſon, ſeeing thoſe things there mentioned bear no equality to each other, neither doth one thing contein the other; for all Binomiums and ſuch as are taken as Irrationals, are indeed ordinate, but yet they are not *Effabile*, or expreſſible, or to be unfolded, as the diameter of the ſquare is in reſpect of its ſide. Now touching *Effabile*, there are divers inordinate; becauſe they are diverſly known, and indeterminately, for a Scalenum triangle may be meaſured by a defined and propoſed meaſure, as explicable, although it be inordinate.

Now it is eaſie to ſee the agreement that there is between *Cognitum* and *Porimon*, but it is a difficult thing to expreſſe or unfold their difference; foraſmuch as in their natures they come ſo neer to one another as that there ſeems to be an equality between them: Nevertheleſſe, there will ſome difference appear to him that ſhall conſider it more ſtrictly; for let it be conſented to that there can be only one line that can touch a ſpiral line in a certain point, that is *Cognitum*, yet notwithſtanding the Problem is not *Porimon*, it being not as yet comprehended; ſo as that all that which is *Cognitum* is not therefore *Porimon*. But all that which is *Porimon* is alſo *Cognitum*, and therefore *Cognitum* appeares to be of a greater extent than *Porimon*.

Now *Cognitum*, *Porimon*, and *Effabile*, do agree in ſome certain things, and do differ in other things by the ſame reaſon before alledged; for thoſe lines which are there called Irrationals are in truth known; and yet nevertheleſſe are not *Effabile* or explicable. Contrariwiſe, every number is indeed *Effabile*, and yet every number is not *Cognitum*. But *Effabile* is alwayes of its own nature expreſſable, although that ſome lengths may be now *Effabile*, and at another time not, if it be examined with ſome other according to one and the ſame meaſure. But alſo that ſame length is ſometimes known, and other times not, though they wholly agree with one another. Now it is a difficult thing to find ſome thing that ſhall be *Effabile* and *Incognitum*, for *Cognitum* ſeemes to be of a greater extent than *Effabile*, and by theſe things it is manifeſt that *Porimon* and *Aporon* do differ from RATIONAL or *Effabile*, and from IRRATIONAL, for of IRRATIONAL ſome of them may be *Porimon*; but of RATIONAL none of them can be Irrationals; and therefore it is very eaſie to perceive in what the before expreſſed things agree. Notwithſtanding they ſeem to agree together, in ſuch ſort as that *Porimon* ſeemes to be of a greater extent than *Effabile*.

Now by theſe things we may come to know the difference of thoſe things that have been before ſpoken of, for in truth *Effabile* and *Irrationale* are ſo termed in reſpect of meaſure, which notwithſtanding is not as yet arrived to our underſtanding, ſeeing that ſome thing, that is rational, may be as yet unknown to us, and in like manner may be rational, and yet may never be comprehended ſo to be. But *Ordinatum* and *Inordinatum* is ſo termed according to it ſelf, and according to the proper nature of the thing on which we contemplate, although it be not comprehended by us. As *Archimedes* had perceived ſome things to be ordinate from

from the nature of the things, the which *Serenus* had before contemplated. But *Cognitum* and *Incognitum* is ſpoken in reſpect of us, ſo as the things before mentioned do differ among themſelves; for theſe have reſpect to us, the others, ſome of them to their proper nature, and the reſt to meaſure.

Having then explained the agreements and differences of the things that have been propoſed, it remaines now that we conſider what is meant by the Word *DATUM*. for of all thoſe that believe the Word *DATUM* to be that which is conſented to by the Propoſer in the Hypotheſis, are wide from what is ſought; becauſe that all the Elements of the things GIVEN are not compoſed of this ſort of GIVEN, which is according to the Hypotheſis, as may be ſeen in thoſe Tracts which have been made on this ſubject GIVEN. Wherefore waving this opinion, let us judg of the Definitions of others.

Then, that which is conſented to or granted in the Hypotheſis, is ſome thing which is conſequently known by the principles; but ſuch as make uſe of Definitions of one only Word, do define it and remarke it by ſome one of the before mentioned, as hath been ſpoken in the beginning, ſo as that almoſt all ſeem to have had this common notion of GIVEN, to wit, that it is comprehended even as the Word *DATUM*, doth alſo manifeſt it to be; and amongſt thoſe, theſe are the chief that do define it by the Hypotheſis or Suppoſition; and others have had regard to what is conſented to or granted. But we making uſe of the ſaid things as of a Rule and Direction to judge rightly, we may be able to find out a perfect Definition of *DATUM*; for it is certain that it ought to equal and be convertible with the thing defined, which is one thing proper to good Definitions. Now ſuch ſeemes to be the Definition of the thing propoſed, which among the moſt ſimple and plain Expoſitors is defined *Porimon*, and amongſt the more acute, that which defineth it to be *Porimon* and *Cognitum* together; but all the reſt are imperfect; for that which defineth it *Ordinatum* is not ſufficient for the comprehenſion and knowledge of *DATUM*; becauſe that neither wholly ordinate, nor alone ordinate, is not compriſed, ſeeing that there are things inordinate that have the ſame condition, as hath been ſhewn. Again, that reaſon gives not ſatisfaction neither, which deſcribes it to be *Cognitum*; for all that is known is not comprehended, although that alone *Cognitum* be comprehended. Moreover, that alſo is not perfect which defineth it to be *Effabile*, for *Effabile* is not alone comprehended; ſeeing that ſome of the Irrationals are alſo comprehended. In like manner, all *Effabile* is not comprehended, as hath been before declared. Now amongſt the Definitions which expound it by one only Word, there remains that which defineth it to be *Porimon*, which ſeemeth greatly to manifeſt the comprehenſion; for whole *Porimon* and alone *Porimon* is comprehended. Wherefore *EUCLIDE* himſelf uſeth ſuch a Definition in a Deſcription of all the kinds of GIVEN by him conceived and regarded. But amongſt ſuch Definitions as are compounded, that is a perfect Definition which defineth *DATUM* to be *Cognitum* and *Porimon* together, having *Cognitum* for analogical kind, and *Porimon* for difference; but that is imperfect which hath *Ordinatum* and *Porimon* together; for thoſe things which are ſuch, are not alone GIVEN, and that which defineth it *Ordinatum* and *Effabile* together, comprehendeth likewiſe the GIVEN, with the defect or want. But that of *Cognitum* and *Ordinatum* together, is not to be received or admitted, becauſe

cauſe it doth exceed what is defined, for ſuch is not given alone: Therefore thoſe only which have declared that *DATUM* is *Cognitum* and *Porimon* together, ſeem to have atteined the notion of GIVEN, for that which is ſuch is all, and alone comprehended, which two things ought to be in thoſe Definitions that are well given. But the former come neer to thoſe which have thus defined it: *DATUM* is that to which we may find an equal, according to thoſe things we have propoſed in the firſt principles and Hypotheſis, of which number *EUCLIDE* is one, making uſe throughout of the Verb πορισάδαι, which ſignifies to exhibit or invent, although he leaves *Cognitum* as a conſequent of *Porimon*; ſome one nevertheleſſe might reprove him, for that in the firſt place he hath not defined *DATUM* in general; but immediately ſome of the kinds of GIVEN, although in his Elements of GEOMETRY; he hath defined the line ſimple before the Species or Kinds.

What is the Utility and Profit that ariſeth from this Tract of DATA, *or things GIVEN.*

AFter having explained univerſally, and according to what ſeemed neceſſary for our preſent uſe, what this Word *DATUM* ſignifieth: It followes to ſhew the Utility of this Tract. Now this Tract is ſuch, as that it is not only ordained and inſtituted for its own reſpect, but for ſome other thing; for it is very neceſſary to a place which is called Reſolved, and we have already declared elſewhere how much ſtrength a reſolved place doth obtein in Mathematical Diſciplines, as alſo in Opticks and Cannons, which come very neer to them, as well for that Reſolve is an invention of the Demonſtration, as for that in ſuch like things it ſerves us much for the invention of the Demonſtration, or for that it is much more excellent to meet with a Reſolutive power, then to enjoy divers particular Demonſtrations.

To what Art or Science this Tract is referred.

NOw ſeeing the conſideration of GIVEN is uſeful and profitable in all theſe kinds of Arts, for that it ſerveth much to RESOLUTION, it may well be ſaid to be recalled, not only to one only Science, but to the Mathematicks univerſally which treat of Numbers, Time, Swiftneſſe, and ſuch like things, which treateth likewiſe of Reaſons, as alſo of proportions, and in a word of all *Medietez*: Wherefore for the perfect and demonſtrative Knowledge of things GIVEN, being of ſo great Utility, *EUCLIDE* hath taken pains to frame this Book of things GIVEN, which Author amongſt all ſuch as have compoſed the Elements of Geometry, hath juſtly deſerved the firſt place and rank, and who having invented the Elements, or rather the Introductions almoſt of all Mathematical Diſciplines, to wit, of all Geometry in 13 Books, of Aſtronomy, of Muſick, and Opticks, he hath left in writing the Elements RESOLUTIVE, in this Treatiſe of things GIVEN; but as he was a Geometrician, he hath particularly accommodated to Magnitudes, that, that was of the GIVEN; yet nevertheleſſe common in other things, which method hath alſo been obſerved by him, when treating univerſally of Reaſons and Proportions, he appropriates them to the Magnitudes mentioned in his Fifth Book of Plains.

Now

Now it hath been declared in general what is the meaning of *DATUM*, to what Science it apperteineth, and how profitable the Contemplation thereof is. We will add to what hath been said, the Description of this Science which treats of things GIVEN; seeing that it is (as appeares by what hath been said) a comprehension in all manners of things GIVEN; and of their accidents and proprieties. But having respect to the proposed Book, we shall declare it to be an Elementary Doctrine of the whole Knowledge of things GIVEN, whence it follows that it will be very profitable, as also the things therein conteined, seeing they refer to things GIVEN.

Now this Book is divided according to the Species or Kinds of the things GIVEN, and in the first section are conteined those things which are given by reason. Secondly, such as are given by position: and lastly, such as are given by Species or Kind; for that which is given by Magnitude is simple, and particularly conteined in the others, and principally in those things given by Specis or Kind. Now he hath begun with those things given by Reason and Position, forasmuch as those that are given by Species are constituted of them. *EUCLIDE* gives yet another division to this Book, for that he divides it into universal Magnitudes, Lines, and Superficies, and into Circular Theoremes, which order he hath also observed in the Definitions and Suppositions of this Book. Moreover, he useth a certain way of instructing, which proceeds not by Composition, but by Resolution, as *Pappus* hath amply set down in his Commentaries on this Book.

THE END OF THE PREFACE OF MARINUS.

DEFINITIONS.

1 PLains or Spaces, Lines, and Angles, to which we may find others equal, are ſaid to be given by Magnitude.

2 A Reaſon is ſaid to be given, when we may find one of the ſame or equal thereto.

3 Rectiline figures, whoſe angles are given, and alſo the reaſon of the ſides to one another, are ſaid to be given by Species or Kind.

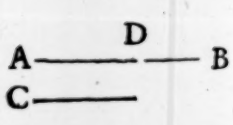
4 Points, Lines, and Angles, which have and keep alwayes one and the ſame place and ſituation, are ſaid to be given by Poſition or Situation.

5 A Circle is ſaid to be given by Magnitude, when the Semidiameter thereof is given by Magnitude.

6 A Circle is ſaid to be given by Poſition, and by Magnitude when the center thereof is given by Poſition, and the Semidiameter by Magnitude.

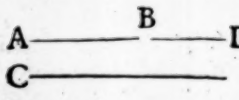
7 Segments of Circles, whoſe angles and baſes are given by Magnitude, are ſaid to be given by Magnitude.

8 Segments of a Circle, whoſe angles are given by Magnitude, and the baſes of the ſegments by Poſition and Magnitude, are ſaid to be given by Poſition and by Magnitude.

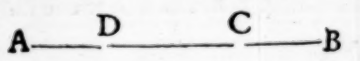


9 A Magnitude AB, is greater than another Magnitude C, by a given Magnitude BD, when having taken away the given Magnitude DB, the reſt AD, is equal to the other Magnitude C.

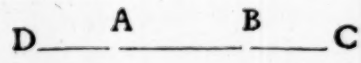
10 *A Magnitude* A B, *is leſſe than another Magnitude* C, *by a given magnitude* B D, *when having added thereto the given magnitude* B D, *the whole* A D, *is equal to the other magnitude* C.



11 *A Magnitude* A B, *is ſaid to be greater than another magnitude* C B, *by a given magnitude* A D, *and in reaſon, when taking from the ſame magnitude the given magnitnde* A D, *the reſt* D B, *hath to the other magnitude* C B, *a given reaſon.*



12 *A magnitude* A B *is ſaid to be leſſe than another magnitude* B C, *by a given magnitude* A D, *and in reaſon, when the given magnitnde* A D *being added thereto, the whole* D B *hath to the other magnitude* B C, *a given reaſon.*



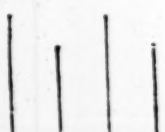
13 *A right line is ſaid to be drawn down from a given point, unto a right line given in poſition, the right line being drawn in a given angle.*

14 *A right line is ſaid to be drawn up from a given point, to a right line given in poſition, the right line being drawn in a given angle.*

15 *A right line is againſt another right line in poſition, when it is drawn parallel thereto thorough a given point.*

PROPOSITION 1.

Two magnitudes A *and* B, *being given, the reaſon they have to one another* A *to* B, *is alſo given.*

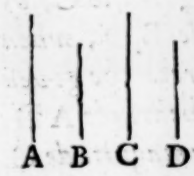


Demonſtration FOr ſeeing that the magnitude A is given [a], we may find one equal thereto, which let be C. Again, foraſmuch as the magnitude B is given, we may alſo find one equal to that, and let that be D. Therefore ſeeing that A is equal to C, and B to D, as A is to C [b], ſo is B to D, and by permutation, [c] as A ſhall be to B, ſo C ſhall be to D. Therefore [d] the reaſon of A to B is given, for it is the ſame reaſon as of C to D, as we have found, and which ought to be demonſtrated.

a) 1 def.
b) 7. 5.
c) 16. 5.
d) 2 def.

PROP. 2.

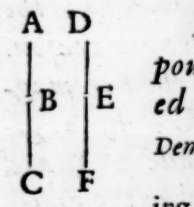
If a given magnitude A, *hath to ſome other magnitude* B, *a given reaſon, that other magnitude* B, *is alſo given by magnitude.*



Demonſtration FOr ſeeing that A is given, we may find one equal thereto, which let be C: And foraſmuch as the reaſon of A to B, is alſo given, we may find [a] one of the ſame. Let it be found, and let the reaſon be of C to D. Now ſeeing that as A is to B, ſo C is to D; and by permutation, as A is to C, ſo B is to D: But A is equal to C, Therefore [b] B ſhall be alſo equal to D. Therefore [c] the magnitude B is given, ſeeing that thereto there hath been found one equal, to wit, D.

a) 2. def.
b) 14. 5.
c) 1. def.

PROP. 3.

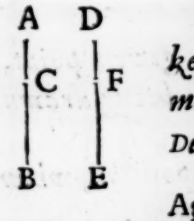


If given magnitudes A B *and* B C, *are compounded, that magnitude* A C, *that is compounded of them ſhall be alſo given.*

Demonſtration FOr ſeeing that A B is given, we may find one equal to it, which let be D E. Again, ſeeing that B C is given, we may alſo find one equal to that, which let be E F. Wherefore ſeeing that D E is equal A B, and E F is equal to B C, the whole A C [a] is equal to the whole D F. Therefore A C is given, ſeeing that D F is propoſed equal thereto.

a) 2 ax. 1.

PROP. 4.

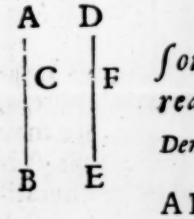


If from a given magnitude A B, *there be taken away a given magnitude* A C, *the remaining magnitude* C B, *is alſo given.*

Demonſtration FOraſmuch as A B is given, we may find one equal thereto, which let be D E. Again, ſeeing that A C is given, we may alſo find one equal to it, which let be D F. Seeing then that the Magnitude A B is equal to the magnitude D E, and the magnitude A C to the magnitude D F; the reſt C B [a] ſhall be equal to the reſt F E. Wherefore C B is given, for to it there hath been found an equal, to wit, F E.

a) 3 ax. 1.

PROP. 5.



If a magnitude A B, *hath a given reaſon to ſome part thereof* A C, *it will have alſo a given reaſon to the part remaining* C B.

Demonſtration LEt D E be expoſed as a given magnitude, and ſeeing that the reaſon of the magnitude A B, to the magnitude A C, is given, [a] we may find one of the ſame, which let be D E to D F; therefore the reaſon of the ſame DE to D F is given. But D E being given, ſo is [b] alſo its part D F; and conſequently, [c] the reſt F E: Therefore [d] ſeeing that D E and F E are given, the reaſon of the ſame D E to F E, is alſo given. And foraſmuch as D E is to D F, as A B is to A C, and by converſion, as D E to F E, ſo A B is to C B. But the reaſon of D E to F E is given, as hath been demonſtrated; therefore the reaſon of A B to C B is alſo given.

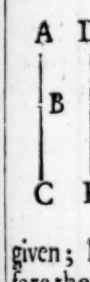
a) 2. def.
b) 2. prop
c) 4. prop.
d) 1. prop.
19. 5.

SCHO-

SCHOLIUM.

From this it is evident that if a Magnitude hath to ſome part thereof a given reaſon, by diviſion, the reaſon that one part hath to the other, ſhall be alſo given. For ſeeing that as DE *is to* FE, *ſo is* AB *to* CB; *by diviſion, as* DF *to* FE, *ſo* AC *to* CB. *But it hath been demonſtrated that the parts* DF *and* FE *are given, and conſequently, their reaſon is alſo given: In like manner, therefore the reaſon of* AC *to* CB *is given.*

PROP. 6.



If two magnitudes AB *and* BC, *having to one another a given reaſon, are compounded, the magnitude* AC, *compounded of them, ſhall alſo have a given reaſon to each of them* AB *and* BC.

Demonſtration LEt the given magnitude DE be expoſed, and ſeeing that the reaſon of AB to BC is given; let there be made one and the ſame of the ſaid DE to EF; therefore the reaſon of the ſame DE to EF is given; and therefore [a] the magnitude DE being given, both the one and the other of them DE and FE, is given. Wherefore [b] the whole DF ſhall be alſo given. Therefore [c] the reaſon of the ſame DF to each of them DE and EF, ſhall be given. And foraſmuch then as AB is to BC, ſo is DE to EF; in compounding, [d] as AC is to BC, ſo is DF to EF: Therefore by converſion, as AC to AB, ſo is DF to DE. Therefore as the whole DF is to each of the other magnitudes DE and EF, ſo the whole AC is to each of the Magnitudes AB and BC: Therefore [e] the reaſon of the ſame AC to each of the magnitudes AB and BC, is given.

a) 2. p.
b) 3. p.
c) 1. p.
d) 18. 5.
e) 2. def.

PROP. 7.

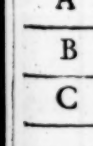


If a given magnitude AB, *be divided according to a given reaſon* AC *to* CB, *each ſegment* AC *and* CB *is given.*

Demonſtration FOr ſeeing the reaſon of AC to CB is given, the reaſon of [a] AB to each of them (AC and CB) is alſo given. But AB is given: Therefore [b] each of the ſegments AC and CB, is alſo given.

a) 6. p.
b) 2. p.

PROP. 8.



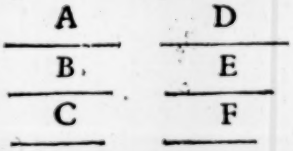
Magnitudes A *and* C, *which have to one and the ſame a given reaſon* B, *ſhall be to one another in a given reaſon,* A *to* C.

Demonſtration FOr let the given magnitude D be expoſed, and ſeeing that the reaſon of A to B is given, Let the ſame be done of the ſaid D to E. Now ſeeing that D is given, [a] E is alſo given. Again, ſeeing that the reaſon of B to C is given, let the ſame be done of E to F. But E is given, and therefore F is alſo given. But ſeeing that D is given, [b] the reaſon of the ſame D to F is given; and ſeeing that as A to B, ſo D to E, and

a) 2. p.
b) 1. p.

c) 22. 5. and as B to C, ſo is E to F; in reaſon of equality, [c] as A is to C, ſo is D to F; but the reaſon of D to F is given. Therefore the reaſon of A to C is alſo given.

PROP. 9.



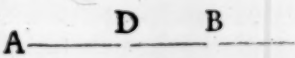
If two or more magnitudes A, B, *and* C, *are to one another in a given reaſon, and that the ſame magnitudes* A, B, *and* C, *have to other magnitudes* D, E, *and* F, *given reaſons, although they be not the ſame, thoſe other magnitudes* D, E, *and* F, *ſhall be alſo to one another in given reaſons.*

Demonſtration FOraſmuch as the reaſon of A to B is given, as alſo that of A to D, the reaſon of D to B ſhall be given: But the reaſon of B to E is alſo given; therefore the reaſon of the ſame D to E ſhall be in like manner given. Again, ſeeing that the reaſon of B to C is given, and alſo that of B to E, the reaſon of E to C ſhall be given. But the reaſon of C to F is alſo given. Therefore [a] the reaſon of E to F ſhall be given. But it hath been demonſtrated that the reaſon of D to E is alſo given; and therefore [b] the reaſon of D to F ſhall be given. Therefore the magnitudes D, E, and F, are to one another in given reaſons.

a) 8. p.

b) 8. p.

PROP. 10.



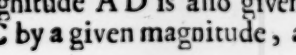
If a magnitude AB, *be greater than another magnitude* BC, *by a given magnitude, and in reaſon, the magnitude* AC *compounded of both, ſhall be alſo greater then that ſame magnitude, by a given magnitude, and in reaſon: But if that compounded magnitude be greater then the ſame magnitude, by a given magnitude, and in reaſon; either the remainder ſhall be alſo greater then that ſame by a given magnitude, and in reaſon; or elſe the ſame remainder is given with the following, to which the other magnitude hath a given reaſon.*

Demonſtration FOr ſeeing that AB is greater than BC by a given magnitude, and in reaſon, let the given magnitude AD be taken away. Therefore [a] the reaſon of the remainder DB to BC is given; and in compounding, [b] the reaſon of DC to BC is alſo given. But the magnitude AD is alſo given; Therefore AC is greater than the ſame BC by a given magnitude, and in reaſon.

a) 11 def.

b) 6. p.

Again, Let the magnitude AC be greater than the magnitude BC, by a given magnitude, and in reaſon: I ſay that the reſt AB, is either greater then the ſame BC by a given magnitude and in reaſon, or that the ſame AB, with that which followeth, to which BC hath a given reaſon, is given.

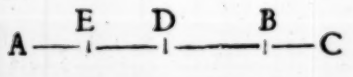


For

Foraſmuch as the magnitude A C is greater than the magnitude B C, by a given magnitude, and in reaſon, cut off from it the given magnitude: Now the ſame given magnitude is either leſſe than the magnitude A B, or greater: Let it in the firſt place be leſſe, and let it be A D. Therefore the reaſon of the remainder D C to C B is given. Wherefore by diviſion, the reaſon of D B to B C is given. But the magnitude A D is alſo given; therefore the magnitude A B is greater [c] then the magnitude B C by a given magnitude, and in reaſon. Now let the given magnitude be greater than the magnitude A B, and let A E be put equal thereto; therefore [d] the reaſon of the remainder E C to C B is given; and by converſion, [e] the reaſon of the ſame B C to B E, is alſo given. But the ſame E B with B A is given, for that the whole A E is given: Therefore there is given A B, with that which followes, B E, to which B C hath a given reaſon.

c) 11 def.
d) 11 def.
e) 5. p.

PROP. 11.



If a magnitude A B, *be greater than a magnitude* B C, *by a given magnitude, and in reaſon, the ſame magnitude* A B, *ſhall be alſo greater than the magnitude compounded of them by a given magnitude, and in reaſon, and if the ſame magnitude be greater then the two others together by a given magnitude, and in reaſon, that ſame magnitude ſhall be alſo greater then the reſt by a given magnitude, and in reaſon.*

Demonſtration FOr ſeeing that the magnitude A B is greater then B C by a given magnitude, and in reaſon; let there be taken from it a given magnitude A D: Therefore [a] the reaſon of the reſt D B to D C, is given, and therefore [b] the reaſon of D C to B D ſhall be alſo given: Let the ſame be done of A D to D E, therefore the reaſon of the ſame A D to D E is given. But A D is given, therefore [c] D E is alſo given, and conſequently, [d] the reſt A E, is alſo given. But ſeeing that as A D is to D E, ſo is D C to B D; by permutation, [e] as A D is to D C, ſo is D E to D B: Therefore by compounding, [f] as A C is to C D, ſo is E B to D B; and by permutation, [g] as A C is to E B, ſo is D C to D B. But the reaſon of D C to D B is given: Therefore alſo is A C to E B, and conſequently, that of E B to A C. But it hath been demonſtrated that A E is given, Therefore [h] A B is greater then A C by a given magnitude, and in reaſon.

a) 11 def.
b) 6. p.
c) 2. p.
d) 4. p.
e) 16. 5.
f) 18. 5.
g) 16. 5.
h) 11 def.

But now let A B be greater then A C by a given magnitude, and in reaſon: I ſay that the ſame A B is alſo greater then the reſt B C, by a given magnitude, and in reaſon.

For ſeeing that A B is greater than A C by a given magnitude, and in reaſon, Let the given magnitude A B be cut off there-from: Therefore [i] the reaſon of the remainder E B to A C is given, and conſequently, alſo ſhall be given that of A C to E B. Let the ſame be done of A D to D E, Therefore the reaſon of A D to D E is given; and by converſion, [k] the reaſon of A D to A E ſhall be alſo given, and conſequently that of A E to A D. Now A E is given, Therefore the whole A D [l] ſhall be alſo given; and ſeeing that as the whole A C is to whole E B, ſo the part cut off A D,

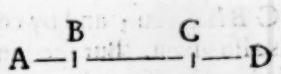
i) 11 def.
k) 5. p.
l) 2. p.

m) 19. 5. AD, is to the part cut off ED, so also shall be [m] the remainder DC to the remainder DB. But the reason of AC to EB is given: Therefore also shall

n) Schol. 5. p. be given that of DC to DB. Wherefore by division, [n] the reason of BC to DB is given; and consequently also shall be given that of DB to BC. But

o) 11 def. it hath been demonstrated that AD is given: Therefore [o] AB is greater then the same BC by a given magnitude, and in reason.

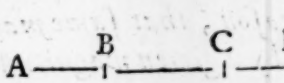
PROP. 12.



If there be three magnitudes AB, BC, *and* CD, *and that the first* AB, *with the second* BC, *to wit* AC, *be given. But the second* BC, *with the third* CD, (*to wit,* BD, *be also given: Either the first* AB *shall be equal to the third* CD, *or the one shall be greater then the other by a given magnitude.*

Demonstration. FOrasmuch as each of the magnitudes AC and BD are given, the given magnitudes are either equal to one another, or inequal. Let them be first equal: Therefore AC is equal to BD, take

a) 3 ax. 1. away the common magnitude BC, and there will remain [a] AB, equal to CD. But suppose them to be inequal, as in this second figure, and let



BD be greater then AC: Let then BE be put equal to AC. Now seeing that AC is given, BE is also given. But the whole BD is also given,

b) 4. p. the rest ED [b] shall be so also; and forasmuch as BE is equal to

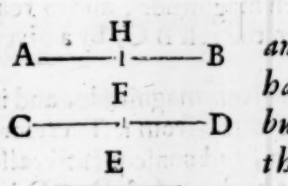
c) 3 ax. 1. AC, taking away the common magnitude BC [c], there will remain AB equal to CE. But ED is given: Therefore CD is greater then AB by the given magnitude ED.

SCHOLIUM.

And if the first with the second, to wit, AC, *were greater then the second with the third, to wit,* BD, *as in the other figure,* CE *would be made equal to the same* BD, *and by the same reasons as was above demonstrated, that* AE *is given and equal to* CD; *and therefore* AB *greater then* CD *by a given magnitude.*



PROP. 13.



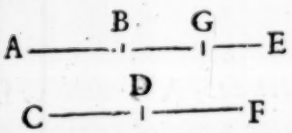
If there be three magnitudes AB, CD, *and* E, *and that the first of them* AB, *hath a given reason to the second* CD; *but the second* CD, *is greater then the third* E, *by a given magnitude, and in reason, also the first* AB, *shall be greater then the third* E, *by a given magnitude, and in reason.*

Demonstration. FOr seeing that CD is greater then E by a given magnitude, and in reason, let the given magnitude CF be taken there-

there-from: Therefore the reaſon of the reſt F D to E is given. And foraſmuch as the reaſon of A B to C D is given, let the ſame be done of A H to C F. Therefore the reaſon of the ſame A H to C F is given. But C F is given: Therefore [a] A H is alſo given. And ſeeing that as the whole A B is to the whole C D, ſo the part cut off A H is to the part cut off C F, and ſo [b] alſo the reſt H B is to the reſt F D, the reaſon of the ſame H B to F D is alſo given. But the reaſon of F D to E is alſo given: Therefore [c] the reaſon of H B to E is given. But it hath been demonſtrated that A H is given: Therefore [d] A B is greater then the ſaid E by a given magnitude, and in reaſon.

a) 2. p.
b) 19. 5.
c) 8. p.
d) 11 def.

PROP. 14.



If two magnitudes A B *and* C D, *have to one another a given reaſon, and that to each of them there be added a given magnitude, to wit,* B E *and* D F; *either the whole* A E, *and* C F, *ſhall have to one another a given reaſon, or the one ſhall be greater then the other by a given magnitude, and in reaſon.*

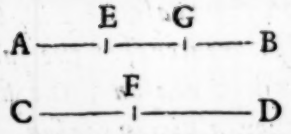
Demonſtration FOr ſeeing that each of thoſe magnitudes B E and D F, is given, [a] the reaſon of the ſaid B E and D F is alſo given; and if that reaſon be the ſame with that of A B to C D, that of the whole A E to the whole C F, [b] ſhall be the ſame; and therefore the reaſon of the ſaid A E to C F is given.

a) 1. p.
b) 12. 5.

Now let the reaſon of B E to D F be not the ſame, with that of A B to C D, and let it be as A B to C D, ſo B G to D F. Therefore the reaſon of the ſaid B G to D F is given. But the magnitude D F is given, therefore [c] B G is alſo given; and ſeeing that the whole B E is given, [d] the reſt G E ſhall be alſo given. But foraſmuch as A B is to C D, as B G is to D F, [e] ſo alſo is the whole A G to the whole C F; and therefore the reaſon of the ſaid A G to C F is given: But the magnitude G E is given: Therefore [f] the magnitude A E is greater then the magnitude C F by a given magnitude, and in reaſon.

c) 2. p.
d) 4. p.
e) 12. 5.
f) 11 def.

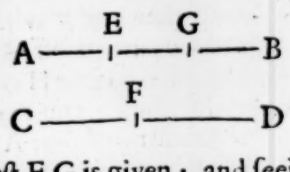
PROP. 15.



If two magnitudes A B *and* C D, *have to one another a given reaſon, and that from each of them be taken away a given magnitude (to wit, from the magnitude* A B *the magnitude* A E, *and from the magnitude* C D *the magnitude* C F) *the remaining magnitudes* E B *and* F D, *either ſhall have to one another a given reaſon, or the one of them ſhall be greater then the other by a given magnitude, and in reaſon.*

Demonſtration FOr ſeeing that each magnitude A E and C F is given, the reaſon of A E to C F is given; and if it be the ſame with

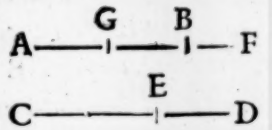
that of A B to C D, that of the remainder E B to the remainder F D,
a) 19. 5. [a] ſhall be alſo the ſame; and therefore the reaſon of the ſaid E B to F D ſhall be alſo given. But if it be not the ſame,



Let it be as A B to C D, ſo A C to C F. Now the reaſon of A B to C D is given, therefore alſo that of A G to C F ſhall be given. But C F is given, therefore [b] A G is given. But A E is alſo given, therefore [c] the reſt E G is given; and ſeeing that as A B is to C D, ſo the part cut off A G is to the part cut off C F, and ſo alſo is [d] the reſt G B to the reſt F D; the reaſon of the ſaid G B to F D is alſo given. Therefore ſeeing that E G is given, E B is greater then F D [e] by a given magnitude, and in reaſon.

b) 2. p.
c) 4. p.
d) 4. p.
e) 11 def.

PROP. 16.



If two magnitudes A B *and* C D, *have to one another a given reaſon, and that from one of them, to wit,* C D, *there be taken away a given magnitude* D E, *and to the other* A B *there be added a given magnitude* B F, *the whole* A F *ſhall be greater then the reſt* C E, *by a given magnitude, and in reaſon.*

Demonſtration FOr ſeeing that the reaſon of A B to C D is given, let the ſame be made of B O to D E: Therefore [a] the reaſon of the ſaid B G to D E is given. But D E is given, therefore [b] B G is alſo given. But B F is alſo given, therefore [c] the whole G F is given. And ſeeing that as A B is to C D, ſo the part cut off B G, is to the part cut off D E; and [d] ſo alſo is the remainder A G to the remainder C E; the reaſon of the ſaid A G to C E is given: But G F is given, Therefore the magnitude A F is greater then the magnitude C E by a given magnitude, and in reaſon.

a) 2. def.
b) 2. p.
c) 3. p.
d) 19. 5.

PROP. 17.



If there be three magnitudes A B, E, *and* C D, *and that the firſt* A B *be greater then the ſecond* E, *by a given magnitude, and in reaſon. But the third* C D *be alſo greater then the ſame ſecond* E, *by a given magnitude, and in reaſon; the firſt* A B *ſhall have to the third* C D, *either a given reaſon, or elſe the one ſhall be greater then the other by a given magnitude, and in reaſon.*

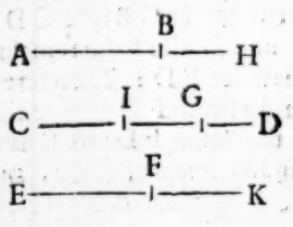
Demonſtration FOr ſeeing that A B is greater then E by a given magnitude, and in reaſon, let the magnitude A F be taken away: Therefore the reaſon of the remainder F B to E is given. Again, ſeeing that C D is greater then the ſaid E by a given magnitude, and in reaſon, let the given magnitude C G be cut off there-from; and the reaſon of the remainder G D to E ſhall be given: Therefore [a] the reaſon of

a) 8. p.

F B

F B to G D ſhall be alſo given. But to the ſaid F B and G D are added the given magnitudes A F and C G: Therefore the whole A B and C D [b] ſhall either have to one another a given reaſon, or the one ſhall be greater then the other by a given magnitude, and in reaſon. b) 14. p.

PROP. 18.



If there be three magnitudes A B, C D, *and* E F, *and that the one of them, to wit,* C D, *be greater then either of the other* A B *or* E F, *by a given magnitude, and in reaſon; either the two others* A B *and* E F, *ſhall have to one another a given reaſon, or the one ſhall be greater then the other by a given magnitude, and in reaſon.*

Demonſtration FOraſmuch as the magnitude C D is greater then the magnitude A B by a given magnitude, and in reaſon, let the given magnitude D G be taken there-from: Therefore the reaſon of the remainder C G to A B is given. Let the ſame be made of G D to B H, Therefore the reaſon of the ſaid D G to B H is given. But D G is given, Therefore [a] B H is alſo given. And ſeeing that as C G is to A B, ſo is G D to B H, [b] ſo alſo is the whole C D to the whole A H, the reaſon of the ſaid C D to A H ſhall be alſo given. a) 2. p. b) 12. 5.

Again, ſeeing that the ſame C D is greater then E F by a given magnitude, and in reaſon; let the magnitude D I be cut off there-from: Therefore the reaſon of the remainder C I to E F is given: Let the ſame be made of D I to F K. Therefore the reaſon of the ſaid D I to F K ſhall be alſo given. But D I is given, Therefore F K is alſo given. And ſeeing that as C I is to E F, ſo is I D to F K; ſo alſo is the whole [c] C D to the whole E K; the reaſon of the ſaid C D to E K ſhall be given. But the reaſon of the ſame C D to A H is alſo given: Therefore [d] the reaſon of the ſaid A H to E K ſhall be given. And ſeeing that from the ſaid A H and E K, the given magnitudes B H and F K are cut off, the magnitudes A B and E F [e] are either in a given reaſon to one another, or the one is greater then the other by a given magnitude, and in reaſon. c) 12. 5. d) 8. p. e) 15. p.

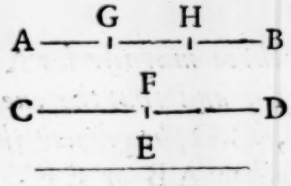
PROP. 19.



If there be three magnitudes A B, C D, *and* E, *and that the firſt* A B, *be greater then the ſecond* C D, *by a given magnitude, and in reaſon; and that the ſecond* C D *be greater then the third* E, *by a given magnitude, and in reaſon; alſo the firſt magnitude* A B *ſhall be greater then the third* E, *by a given magnitude, and in reaſon.*

Demonſtration FOr ſeeing that C D is greater then E by a given magnitude, and in reaſon; let the given magnitude C F be taken there-from: Therefore the reaſon of the remainder F D to E is given.

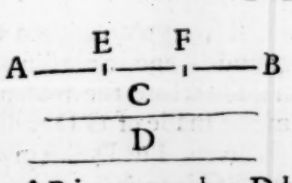
Again, ſeeing that A B is greater then the ſame C D by a given magnitude, and in reaſon: Let the magnitude A G be taken there-from: Therefore the reaſon of the remainder G B to C D is given: Let the ſame be made of G H to C F: Therefore the reaſon of the ſaid G H to C F is given. But C F is given: Therefore alſo G H is given, and then A G is alſo given, the whole [a] A H ſhall be alſo given. But as G B is to C D, ſo is G H to C F, and ſo alſo [b] the remainder H B to the remainder F D: Therefore the reaſon of the ſaid H B to F D is given. But the reaſon of the ſame F D to E is alſo given: Therefore the reaſon of H B to E is in like manner given, and ſo is alſo the magnitude A E: Wherefore the magnitude A B [c] is greater then E by a given magnitude, and in reaſon.



a) 3. p.
b) 19. 5.
c) 11 def.

OTHERWISE.

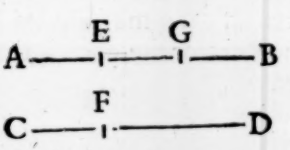
Conſtruction LEt there be three magnitudes A B, C, and D, and let A B be greater then C by a given magnitude, and in reaſon; but let C be alſo greater then D, by a given magnitude, and in reaſon: I ſay that A B is greater then D by a given magnitude, and in reaſon.



Demonſtration FOraſmuch as A B is greater then C by a given magnitude, and in reaſon, let the given magnitude A E be cut off therefrom: Therefore the reaſon of the remainder E B to C is given. But the magnitude C is greater then the magnitude D by a given magnitude and in reaſon; therefore [d] E B is greater then D by a given magnitude, and in reaſon: Wherefore let the given magnitude E F be cut off therefrom; and the reaſon of the remainder F B to D ſhall be given. But A F [e] is given. Therefore [f] A B is greater then D by a given magnitude, and in reaſon.

d) 13. p.
e) 3. p.
f) 11 def.

PROP. 20.



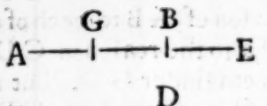
If there be two given magnitudes, A B *and* C D, *and that from them there be taken magnitudes* A E *and* C F, *having to one another a given reaſon; either the remaining magnitudes* E B *and* F D, *ſhall have to one another given reaſons, or elſe the one ſhall be greater then the other by a given magnitude, and in reaſon.*

Demonſtration FOr ſeeing that both the magnitudes A B and C D, are given, the reaſon of the ſaid A B to C D is [a] alſo given; and if it be the ſame as of A E to C F, that of the remainder E B to the remainder F D ſhall be [b] alſo the ſame; and therefore the reaſon of the ſaid E B to F D ſhall be alſo given. But if it be not the ſame, let it be ſo as that A E be to C F, as A G to C D. Now the reaſon of the ſaid A E to C F is given: Therefore the reaſon of the ſaid A G to C D is given. But C D is given, Therefore [c] A G is alſo given. But the whole A B is likewiſe given, Therefore [d] the remainder B G is given. And ſeeing that as A E

a) 1. p.
b) 19. 5.
c) 3. p.
d) 4. p.

AE is to CF, so is AG to CD, and also the remainder EG to the remainder FD, the reason of the said EG to FD is given. But GB is also given: Therefore the magnitude EB is greater [c] then the magnitude BD by a given magnitude, and in reason. c) 11 def.

PROP. 21.

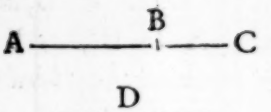


If there be two magnitudes given AB *and* CD, *and to them be added other magnitudes* BE *and* DF, *having to one another a given reason: Either the whole* AE *and* CF *shall have to one another a given reason, or else the one shall be greater then the other by a given magnitude, and in reason.*

Demonstration FOr seeing that both the magnitudes AB and CD are given their reason [a] is also given; and if it be the same reason as of BE to DF, the reason of the whole AE to the whole CF shall be also given; for it shall be [b] the same. But if it be not the same, Let it be as BE is to DF, so BG to CD: Therefore the reason of the said BG to CD is given. But CD is given, Therefore [c] also BG shall be given. But the whole AB is given, Therefore also the [d] remainder AG shall be given. And seeing that as BE is to DF, so is BG to CD, and and also [e] the whole GE to the whole CF, the reason of the said GE to CF shall be likewise given. But AG is given, Therefore the magnitude AE is greater then the magnitude CF by a given magnitede, and in reason.

a) 1. p.
b) 12, 5.
c) 2. p.
d) 4. p.
e) 12. 5.

PROP. 22.

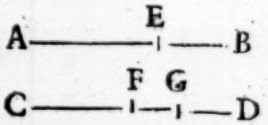


If two magnitudes AB *and* BC, *have to some other magnitude* D, *a given reason, also their compound magnitude* AC, *shall have to the same magnitude* D, *a given reason.*

Demonstration FOr seeing that each magnitude AB and BC, hath a given reason to D, the reason [a] of AB to BC is given; and by compounding, [b] the reason of AC to BC is given. But that of BC to D is also given, Therefore [c] the reason of the said AC to D shall be likewise given.

a) 8. p.
b) 6. p.
c) 8. p.

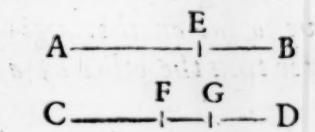
PROP. 23.



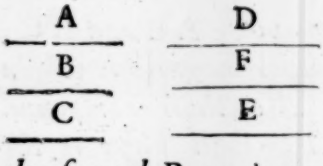
If the whole AB *be to the whole* CD *in a given reason, and that the parts* AE *and* EB, *be to the parts* CF *and* FD *in given reasons, although they be not the same, the whole (to wit,* AB, AE, *and* BE,) *shall be to the whole (to wit,* CD, CF, *and* FD,) *in given reasons.*

Demon-

Demonſtration FOr ſeeing that A E is to C F in a given reaſon, let the ſame be made of A B to C G; therefore the reaſon of the ſaid
a) 19. 5. A B to C G is given; and conſequently, alſo that [a] of the reſt E B to the reſt F G. But the reaſon of F D to the ſame E B is alſo given: Therefore
b) 8. p. the reaſon of F D to F G [b] is likewiſe given; and therefore [c] that of F D
c) 5. p. to the remainder G D is alſo given. But the reaſon of A B to each of the
d) 8. p. magnitudes C D and C G is given: Therefore [d] alſo the reaſon of C D to
e) 5. p. C G is given, and again [e] that of C D to the remainder G D. But the
f) 8. p. reaſon of F D to D G is given, Therefore alſo [f] that of the ſame C D to
g) 5. p. F D, and conſequently that of [g] C D to the remainder F C; and therefore alſo the reaſon of C F to F D ſhall be given. But the reaſon of E B to F D is propoſed to be given; therefore the reaſon of C F to E B ſhall be given. Again, for that the reaſon of A B to C D is given; and alſo that of C D to each of thoſe F C and F D, the reaſon of
h) 8. p. the ſame A B to each of the ſaid F C and F D [h], ſhall be likewiſe given. But the reaſon of the ſaid F D to E B is given: Therefore the reaſon of A B to B E ſhall be alſo given, and conſequently A B to the remainder
i) 5. p. [i] A E. Wherefore by diviſion [k] the reaſon of A E to E B ſhall be likewiſe
k) Sch. 5. p. given. But the reaſon of E B to F D is given. Therefore alſo that of A E to F D. In like manner, ſeeing that the reaſon of C D to A B is given; and that of A B to each of his parts A E and E B; alſo the reaſon of the ſaid
l) 8. p. C D to each of the ſaid A E and E B, [l] ſhall be given: Wherefore each of the magnitudes A B, C D, A E, E B, C F, and F D, is to each of the others in a given reaſon.



PROP. 24.



If of three right lines A, B, *and* C, *proportional*, A *to* B, *as* B *to* C, *the firſt* A *hath to the third* C *a given reaſon, it will alſo have to the ſecond* B *a given reaſon.*

Demonſtration FOr, Let there be expoſed another right line D, and ſeeing that the reaſon of A to C is given: Let the ſame be made of D to F; therefore the reaſon of D to F is given. But D is given, therefore F is alſo given; betwixt the two right lines D and F, let there
a) 13. 6. be taken [a] a mean proportional E. Therefore the rectangle made
b) 17. 6. under D and F is equal [b] to the ſquare of E. But the ſame rectangle of D
c) 3 def. and F is [c] given: (for all the angles of that rectangle are given, being right angles, and the reaſons that the ſides have to one another are alſo given;) therefore the ſquare of E is given, and conſequently the ſame
d) 14. 2. right line E is alſo given (for one equal thereto may be found [d] ſeeing
e) 1. p. that the rectangle of D and F is given.) But D is given, therefore [e] the reaſon of D to E is given, and as A is to C, ſo D is to F. But as A is to C [f] ſo
f) 1. 6. the ſquare of A is to the rectangle of A and C, and alſo as D is to F, ſo the ſquare of D is to the rectangle of D and F. Therefore as the ſquare of A is to the rectangle of A and C, ſo the ſquare of D is to the rectangle of D and F. But the rectangle of A and C is equal to the ſquare of B, (ſeeing that

that A, B, and C, are proportional) and that of D and F to the ſquare of E, Therefore as the ſquare of A is to the ſquare of B, ſo the ſquare of D is to the ſquare of E: Wherefore g as A is to B, ſo D is to E. But the reaſon of D to E is given, Therefore h alſo the reaſon of A to B is given. g) 12. 6. h) 2. def.

OTHERWISE.

Demonſtration FOraſmuch as the reaſon of A to C is given, and that as A is to C, ſo the ſquare of A and C, the reaſon of the ſaid ſquare of A to the rectangle made of A and C, is alſo given. But to that rectangle made of A and C the ſquare of B is equal (ſeeing that A, B, and C, are proportional;) therefore the reaſon of the ſquare of A to the ſquare of B is given; and by conſequence, the reaſon of the line A to the line B is given; for to each of them A and B, we have exhibited an equal to the proper ſquare of each one.

A
B
C

PROP. 25.



If two lines AB *and* CD, *given by poſition do interſect, the point* E *in which they interſect one another, is given by poſition.*

Demonſtration FOr if it change its place, the one or the other of the lines AB and CD, would change its poſition: But ſo it is that by Suppoſition it changeth not: Therefore a the point E is given by poſition. a) 4. def.

PROP. 26.

A——B

If the extremities A *and* B, *of a right line* AB, *be given by poſition, that ſame right line* AB *is given by poſition and by magnitude.*

Demonſtration FOr if the point A remaining in its place, the poſition, or the magnitude of the right line AB ſhall change, the point B will fall elſwhere. But ſo it is, that by Suppoſition it doth not fall elſwhere. Therefore the right line AB is given by poſition, and by magnitude.

PROP. 27.

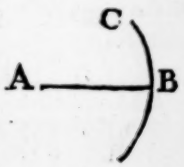
A——B

If one of the extremities A *of a right line* AB, *given by poſition and magnitude be given, the other extremity* B *ſhall be alſo given.*

Demonſtration FOr if the point A remaining in its place, the point B ſhall change and fall in ſome other place, either the poſition of the right line AB, or its magnitude would change: But ſo it is that according to the Suppoſition, neither the one nor the other doth change. Therefore the point B is given.

OTHER-

OTHERWISE.

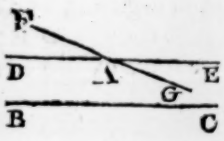


Conſtruction ON the center A, with the diſtance A B, deſcribe the circumference B C.

a) 6 def. b) 25. p.

Demonſtration THerefore [a] that circumference B C is given by poſition. But the right line A B is alſo given by poſition; therefore the point [b] B is given.

PROP. 28.

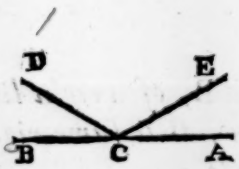


If by a given point A, *there be drawn a right line* D A E, *againſt another right line* B C, *given by poſition, the right line* D A E *ſo drawn, is given by poſition.*

Demonſtration FOr if it be not given, the point A remaining in its place, the poſition of the right line D A E may change: Let it then change if it be poſſible, and fall elſwhere, remaining parallel to B C, and let it be the line F A G: Therefore B C is parallel to the ſaid line F A G. But [a] the ſame B C is alſo parallel to D A E: Therefore [b] D A E is parallel to the ſaid line F A G, which is abſurd; ſeeing they joyn together and meet in A: Therefore the poſition of the right line D A E falls not elſwhere. Wherefore the ſaid line D A E is given by poſition.

a) 13 def. b) 30. 1.

PROP. 29.



If to a right line A B, *given by poſition, and to a point* C *given therein, there be drawn a right line* C D, *which ſhall make a given angle* A C D, *the line drawn* C D, *is given by poſition.*

Demonſtration FOr if it be not given by poſition, the point C remaining in its place, the poſition of the line C D obſerving the magnitude of the angle A C D, will fall elſwhere. Let it fall elſwhere then if it be poſſible, and let it be C E. Therefore the angle A C D is equal to the angle A C E, the greater to the leſſer, which is abſurd. Therefore the poſition of the right line C D, ſhall not fall elſwhere; and therefore the ſaid line C D is given by poſition.

PROP. 30.



If from a given point A, *be drawn to a right line* B C, *given by poſition, a right line* A D, *making a given angle* A D B, *the line drawn* A D *is given by poſition.*

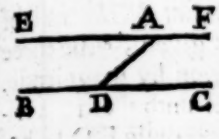
Demonſtration FOr if it be not given, the point A remaining in its place, the poſition of the right line A D keeping the magnitude of the angle A D B will change.

change. Let it change then, and let it be the right line A E: Therefore the angle A D B is equal to the angle A E B, the greater [a] to the leſſer, which is abſurd. Therefore the poſition of the right line A D doth not change; and therefore the ſaid line A D is given by poſition. a) 16. 1.

OTHERWISE.

Conſtruction BY the point A let there be drawn the line E A F, parallel to the right line B C.

Demonſtration THen ſeeing that by the given point A, and againſt the right line B C, given by poſition, there is drawn the right line E F, thoſe lines E F and B C are parallels. But on the ſame lines doth alſo fall the right line A D. Therefore [b] the angle F A D is equal to the given angle A D B; and therefore it is alſo given. Wherefore to the right line E F given by poſition, and to the given point A therein, there is drawn the right line A D, making the given angle F A D. Therefore [c] the ſaid line A D is given by poſition. b) 29. 1. c) 29. p.



OTHERWISE.

Conſtruction IN the line B C E, let there be taken the given point C, and by the ſame let there be drawn the line C F, parallel to the ſaid D A.

Demonſtration FOraſmuch as A D and F C are parallels, and that on them there doth fall the right line B C E, the angle F C B is equal [d] to the given angle A D B; and therefore it is alſo given. And ſeeing that the right line B C is given by poſition, and that to a given point C therein, there is drawn the right line F C, making the given angle F C B, that ſame line F C [e] is given by poſition. But by the given point A, oppoſite to the line F C given by poſition, there is drawn the line A D. Therefore the ſaid line [f] A D is given by poſition. d) 29. 1. e) 29. p. f) 28. p.



OTHERWISE.

Conſtruction IN the right line B C aſſume ſome point at F, and draw A F.

Demonſtration FOraſmuch as each point A and F is given, the right line A F is given [g] by poſition. But the line B C is alſo given by poſition. Therefore * the angle A F D is given. But by Suppoſition, the angle A D F is given: Therefore D A F (which is the reſidue [h] of two right angles) is given; and ſeeing that to the right line A F given by poſition, and to the given point therein A there is drawn the right line D A, making the given angle D A F, [i] that ſame line D A is given by poſition. g) 26. p. h) 32. 1. i) 29. p.



SCHOLIUM.

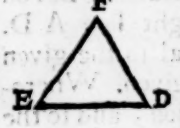
* *EUCLIDE ſuppoſeth here that two right lines being given by poſition, and inclining to one another do make a given angle, which ſome do demonſtrate after this manner.*

Demonſtration FOraſmuch as the two right lines given by poſition, do incline to one another, the inclination of thoſe lines is given. But the angle is the inclination of the lines: Therefore the angle which makes the right lines given by poſition, and inclining to one another, is given.

Another thus demonſtrateth it.



Conſtruction LEt there be two right lines inclining to one another, as A B and C B, given by poſition, and in the line A B let there be taken a given point A, and in B C alſo ſome point, as C; and let the right line A C be drawn.



Demonſtration SEeing that as well the point B, as each of the points A and C, is given, [k] the three right lines A B, B C, and A C, are given by magnitude. Wherefore of three direct lines equal unto them, a triangle may be conſtituted: Let there then be made the triangle F D E, having the ſide F D equal to the ſide A B, the ſide F E equal to the ſide A C, and the baſe D E equal to the baſe B C.

Seeing then the angles comprized of equal right lines are equal, we have found the angle F D E equal to the angle A B C; and therefore the ſame [l] angle A B C is given.

k) 26. p.

l) 1 def.

PROP. 31.



If from a given point A *there be drawn to a right line given by poſition* B C, *a right line* A D, *given by magnitude, that line* A D *ſhall be alſo given by poſition.*

Conſtruction FRom the center A, with the diſtance A D, let the circle D E F be deſcribed.

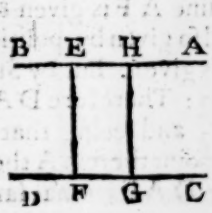
Demonſtration FOraſmuch as the center A is given by poſition, and the ſemidiameter A D by magnitude, the circle D E F [a] is given by poſition. But the right line B C is alſo given by poſition: Therefore the point of interſection D [b] is given, and ſeeing that the point A is alſo given: [c] the right line A D is given by poſition.

a) 6 def.

b) 25. p.

c) 26. p.

PROP. 32.



If unto parallel right lines A B *and* C D, *given by poſition, there be drawn a right line* E F, *making the given angles* B E F *and* E F D, *the line drawn* E F *ſhall be given by magnitude.*

Conſtruction FOr let there be taken in the line C D a given point G, and from that point let be drawn G H parallel to F E.

Demonſtration FOraſmuch as the lines E F and H G are parallels, and that on them doth fall the line C D; [a] the angle E F D is equal

a) 29. 1.

equal to the angle F G H. But the angle E F D is given, therefore the angle F G H is alſo given. And foraſmuch as to the right line C D given by poſition, and to the point G given in the ſame, there is drawn the right line G H, making the given angle F G H, [b] the ſaid line G H is given by poſition. But A B is alſo given by poſition, Therefore [c] the point H is given. But the point G is alſo given: Therefore [d] the line G H is given by magnitude, and is [e] equal to E F. Wherefore [f] the ſaid line E F is given by magnitude.

b) 29. p.
c) 25. p.
d) 26. p.
e) 34. 1.
f) 1 def.

PROP. 33.



If unto parallel right lines A B *and* C D, *given by poſition, there be drawn a right line* E F, *given by magnitude, that line* E F *ſhall make the given angles* B E F *and* D F E.

Conſtruction FOr let there be taken in the right line A B the point G, and by that point let there be drawn the line G H, parallel to E F.

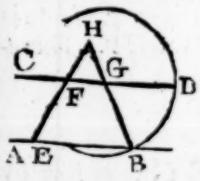
Demonſtration THerefore E F is equal to the ſaid [a] G H. But E F is given by magnitude, Therefore G H is alſo given by magnitude. But the point G is given; and therefore if on that point, with the diſtance G H, there be deſcribed a circle, [b] that circle ſhall be given by poſition: Let it be then deſcribed, and let it be H K L, the ſaid circle H K L is therefore given by poſition. But the line C D which doth cut the circumference K H L in H, is alſo given by poſition. Therefore the ſaid point of interſection H [c] is given. But the point G is given: Therefore [d] the right line G H is given by poſition. But the right line C D is alſo given by poſition: Therefore [e] the angle G H F is given. But to that angle [f] the angle F E D is equal: Therefore the angle E F D is given; and therefore alſo the angle B E F; for that it is the reſidue of the ſumme of two [g] right angles,

a) 34. 1.
b) 6 def.
c) 25. p.
d) 26. p.
e) Sch. 30. p.
f) 29. 1.
g) 29. 1.

OTHERWISE.

Conſtruction LEt there be taken in the right line C D the point G, and let G D be put equal to E F, then from the center G, with the diſtance G D, let there be deſcribed the circle D B H, and draw G B.

Demonſtration FOraſmuch as the center D is given by poſition, and the ſemidiameter G D by magnitude, the circle B D H [h] is given by poſition. But the line A B is alſo given by poſition: Therefore [i] the point B is given. But the point G is alſo given, Therefore [k] the right line G B is given by poſition. But the right line C D is alſo given by poſition: Therefore [l] the angle B G D is given. Wherefore if E F be parallel to B G, the angle E F D [m] ſhall be given, and conſequently alſo the other angle B E F. But the right lines B G and E F being not parallels, let them meet in the point H. Foraſmuch as E B is parallel to F G, and E F is equal to G D, that is to ſay to B G; alſo F H [n] ſhall be equal to G H (for E H and B H being cut proportionally [o] by the parallel F G, as E F is to F H, ſo is B G to G H; and by permutation, as E F is to B G, ſo



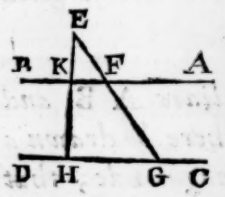
h) 6 def.
i) 25. p.
k) 26. p.
l) Sch. 30. p.
m) 29. 1.
n) 14. 5.
o) 2, 6.

is F H to G H :) Therefore p the angle H F G is equal to the angle H G F, but the ſaid angle H G F is given (for that it is equal q to the given angle B G D :) Therefore the angle H F G is alſo given. But to that angle the angle B E F is equal; and therefore is given, as alſo the remaining angle E F G.

p) 5. 1.
q) 15. 1.

PROP. 34.

If from a given point E, *there be drawn unto parallel right lines* A B *and* C D, *given by poſition, a right line* E F G, *that right line* E F G *ſhall be divided in a given reaſon* (*to wit, as* E F *to* F G.



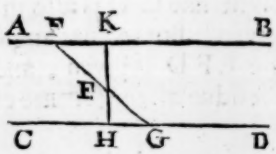
Conſtruction FOr from the point E let there be drawn the line E H, perpendicular to the line C D.

Demonſtration FOraſmuch as from the given point E there is drawn to the line C D the right line E H, making the given angle E H G a the ſaid line E H is given by poſition, but both the one and the other lines A B and C D is alſo given by poſition. Therefore b the points of interſection K and H, are given. But the point E is alſo given: Therefore c each line E K and K H is given. Wherefore d the reaſon of the ſaid E K to K H is given. But as E K is to K H, ſo is E F to F G; (for in the triangle G E H the line K F being parallel to H G, the ſides E H and E G are cut proportionally:) Therefore the reaſon of the ſaid E F to F G is given.

a) 30. p.
b) 25. p.
c) 26. p.
d) 1. p.

OTHERWISE.

Conſtruction TO the parallel right lines given by poſition, A B and C D, let there be drawn from the point E the right line F E G: I ſay that the reaſon of G E to E F is given.



Demonſtration FOr from the point E let there be drawn to C D the perpendicular E H, and produced to the point K; ſeeing therefore that from the point E to the right line C D, given by poſition, there is drawn the line E H, making the given angle E H G, a the ſaid line E H is given by poſition. But each line A B and C D is alſo given by poſition: Therefore b each point of interſection H and K is given. But the point E is alſo given, Therefore c each of the lines E H and E K is given by magnitude; and therefore d the reaſon of the ſaid E H to E K is given. But e as E H is to E K, ſo is E G to E F (for the oppoſite angles at the point E being equal, and the lines A B and C D parallels, the triangles E H G and E K F are equiangled; and therefore as E H is to E G, ſo is E K to E F; and by permutation as E H to E K, ſo is E G to E F.) Therefore the reaſon of the ſaid lines E G to E F is given.

a) 30. p.
b) 25. p.
c) 26. p.
d) 1. p.
e) 4. 6.

PROP. 35.

If from a given point A, *to a right line* B C, *given by poſition, there be drawn a right line* A D, *which let be divided in* E, *in a given reaſon* (*to wit, as* A E *to* E D, *and that*

that by the point of ſection E, *there be drawn a right line* F E G, *oppoſite to the right* B C, *given by poſition, the line* F G *drawn ſhall be given by poſition.*

Conſtruction FOr from the point A, let there be drawn the line A H, perpendicular to the line B C.

Demonſtration FOr ſeeing that from the given point A there is drawn to B C given by poſition, the right line A H, making the given angle A H D, [a] the ſaid line A H is given by poſition. But B C is alſo given by poſition: Therefore [b] the point H is given. But the point A is alſo given: Therefore [c] the line A H is given by magnitude and by poſition. And ſeeing that [d] as A E is to E D, ſo is A K to K H, and that the reaſon of A E to E D is given, alſo the reaſon of A K to K H is given; and by compounding, [e] the reaſon of A H to A K is given. But A H is given by magnitude: Therefore [f] alſo A K is given by magnitude. But A K is alſo given by poſition, and the point A is given: Therefore [g] the point K is alſo given, and ſeeing that by the ſaid given point K there is drawn the line F G, oppoſite to the right line B C given by poſition; the ſaid line F G [h] is given by poſition.



a) 30. p.
b) 25. p.
c) 26. p.
d) 2. 6.
e) 6. p.
f) 2. p.
g) 27. p.
h) 28. p.

PROP. 36.



If from a given point A, *there be drawn to a right line* B C *given by poſition, a right line* A D, *and to it be added a right line* A E, *having to the ſame* A D *a given reaſon, and that by the extremity* E *of the added line* A E, *there be drawn a right line* F E K, *oppoſite to the line* B C, *given by poſition, that ſame line* F E K *ſhall be given by poſition.*

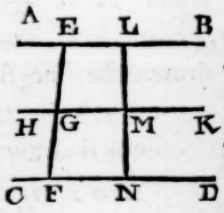
Conſtruction FOr from the point E let there be drawn to the line B C, the perpendicular A L, and let it be prolonged to the point G.

Demonſtration FOraſmuch as from the given point A, there is drawn to the right line B C, given by poſition, the right G L, which makes the given angled G L D, [a] that line G L is given by poſition. But B C is alſo given by poſition, Therefore [b] the point L is given; and ſeeing that the point A is alſo given the line [c] A L is given. But foraſmuch as the reaſon of A E to A D is given, and that [d] as the ſaid A E is to A D, ſo is A G to A L; (becauſe the triangles A L D and A G E are equiangled) the reaſon of A G to A L is alſo given. But A L is is given by magnitude: Therefore [e] A G is given by magnitude. But it is alſo given by poſition, and the point A is given: Therefore [f] the point G is alſo given. And ſeeing that by the ſame given point G, there is drawn the line F K, oppoſite to the right line B C, given by poſition, [g] the ſaid line F K is given by poſition.

a) 30. p.
b) 26. p.
c) 26. p.
d) 4. 6.
e) 2. p.
f) 27. p.
g) 28. p.

PROP.

PROP. 37.



If unto parallel right lines A B *and* C D, *given by poſition, there be drawn a right right line* E F, *divided in the point* E, *in a given reaſon (to wit, of* E G *to* G F;) *but if by the point of ſection* G, *there be drawn oppoſite to the right lines* A B *or* C D, *given by poſition, a right line* H G K, *that line drawn ſhall be given by poſition.*

Conſtruction. FOr let there be taken in the line A B the given point L, and from that point let there be drawn the line L N, perpendicular to C D.

Demonſtration. SEeing that from the given point L, there is drawn to the right line C D, the line L N, making the given angle L N D, the ſaid L N [a] is given by poſition. But C D is alſo given by poſition: Therefore the point N [b] is given. But the point L is alſo given: Therefore [c] the line L N is given; and ſeeing that the reaſon of F G to G E is given, and that * as F G is to G E, ſo is N M to M L, the reaſon of the ſaid M N to M L is given; and in compounding, [d] the reaſon of L N to L M is alſo given. But L N is given by magnitude, therefore M L is [e] given by magnitude. But it is alſo given by poſition, and the point L is given: Therefore the point M [f] is alſo given. And conſidering that by the ſaid point M there is drawn the right line K H, oppoſite to the right line C D, given by poſition, the ſaid line K H is alſo given by poſition.

a) 30. p.
b) 25. p.
c) 26. p.
d) 6. p.
e) 2. p.
f) 27. p.

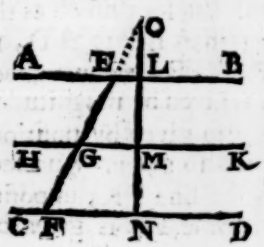
SCHOLIUM.

* *EUCLIDE ſuppoſeth here, that as* F G *is to* G E, *ſo* N M *is to* M L; *but by another it is thus demonſtrated.*

The lines E F *and* L N *are parallels or not parallels: Let them in the firſt place be parallels, and foraſmuch as by Conſtruction the lines* E L, F N, E F, *and* L N, *are parallels,* E N *ſhall be a parallelogram; and therefore the ſide* E F *is equal to the ſide* L N. *Again, ſeeing that* M G *is parallel to* F N, *and* G F *to* M N, G N *ſhall be alſo a parallelogram; and therefore the ſide* G F *is equal to the ſide* M N. *Wherefore the equal ſides* E F *and* L N, *ſhall have to the equal ſides* F G *and* M N, [g] *one and the ſame reaſon. Therefore as* E F *is to* F G, *ſo is* L N *to* M N; *and in dividing,* [h] *as* G E *to* G F, *ſo is* L M *to* M N.

g) 7. 5.
h) 17. 5.

Now ſuppoſe that the lines E F *and* L N *be not parallels, but that they meet in the point* O. *Foraſmuch as in the triangle* O F N *there is drawn* H K, *parallel to* F N *one of the ſides;* [i] *the ſides* O F *and* O N *are divided reciprocally; and therefore as* F G *is to* G O, *ſo is* N M *to* M O. *Again, ſeeing that in the triangle* O G M *there is drawn* E L, *parallel to the ſide* G M, *the ſides* O G *and* O M *are divided proportionally: Wherefore* [k] *as* O E *is to* E G, *ſo is* O L *to* L M; *and by compounding,* [l] *as* O G *is to* E G, *ſo is* O M *to* L M; *but it hath been demonſtrated that as* F G *is to* G O; *ſo is* N M *to* M O; *therefore in reaſon of equality,* [m] *as* F G *is to* G E, *ſo is* N M *to* M L.



i) 2. 6.
k) 2. 6.
l) 18. 5.
m) 22. 5.

PROP.

PROP. 38.



If unto parallel right lines A B *and* C D, *there be drawn a right line* E F, *and that to it there be added ſome other right line* E G, *which hath a given reaſon to the ſame* E F, *but if by the extremity* G, *of the added line* E G, *there be drawn a right line* H K, *againſt the parallels given by poſition* A B *and* C D, *the line drawn* H K *ſhall be alſo given by poſition.*

Conſtruction FOr, Let there be taken in the line A B, the given point N, and from thence let there be drawn to C D the perpendicular N M, and let it be prolonged to the point L.

Demonſtration FOraſmuch as from the given point N there is drawn to the right line C D, given by poſition, the right line N M making a given angle N M F, the ſaid angle N M F [a] is given by poſition. But the line C D is alſo given by poſition: Therefore [b] the point M is given. But the point N is alſo given: Therefore [c] the line N M is given, and for that the reaſon of E G to E F is given, and that [d] as E G is to E F, ſo is L N to N M, the reaſon of L N to N M is alſo given: But N M is given, therefore L N is [e] alſo given. But the point N is given: Therefore [f] the point L is alſo given. Seeing then that by the given point L there is drawn the right line H K, oppoſite to the line A B given by poſition, [g] the ſaid line A K is alſo given by poſition.

a) 30. p.
b) 25. p.
c) 26. p.
d) Sch. 37. p.
e) 2. p.
f) 27. p.
g) 28. p.

PROP. 39.



If all the ſides of a triangle A B C *are given by magnitude, the triangle is given by Kind.*

Conſtruction FOr, Let there be expoſed the right line D G given by poſition, ending in the point D; but being infinite towards the other part G, and therein let there be taken D E, equal to A B.

Demonſtration NOw ſeeing the ſaid A B is given by magnitude, D E is ſo alſo; but the ſame D E is alſo given by poſition, and the point D is given: Therefore [a] the point E is given.

a) 27. p.

Again, Let E F be put equal to B C; and ſeeing that B C is given by magnitude, E F ſhall be ſo alſo. But the ſaid E F is in like manner given by poſition, and the point E is given: Therefore [b] the point F is given.

b) 27. p.

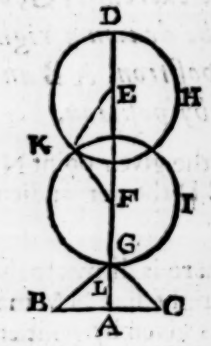
Furthermore, Let F G be taken equal to A C. Now foraſmuch as the ſaid A C is given by magnitude, F G is ſo alſo. But F G is alſo given by poſition, and the point F is given: Therefore the point G is alſo given. Now from the center E, with the diſtance E D, let there be deſcribed the circle D H K, [c] and that circle ſhall be given by poſition. Again, on the center F, and diſtance F G, let there be deſcribed the circle G L K.

c) 6. def.

There-

d) 6 def. e) 25. p. f) 26. p. Therefore [d] the ſaid circle G L K is given by poſition; and therefore [e] the point of interſection K is given. But each of the points E and F is given: Therefore each line [f] E K, E F, and F K, is given by poſition and magnitude. Therefore the triangle E K F is given * by kind; but it is equal and alike to the triangle A B C; and therefore the triangle A B C is alſo given by kind.

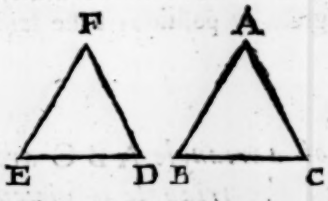
SCHOLIUM.



g) 1. p. h) Sch. 30. p. i) 5 def. * *EUCLIDE ſuppoſeth here that a triangle whoſe ſides are given by magnitude and poſition, is given by kind; but the antient Interpreters demonſtrate it in a manner thus. Foraſmuch as the right lines* K E *and* E F *are given,* [g] *the reaſon which they have to one another is given. Alſo the right lines* E F *and* F K *being given, their reaſon is alſo given; and in like manner, the reaſon of the ſaid* E K *and* F K *is given. Again, ſeeing that the ſame lines* K E *and* E F *are given by poſition,* [h] *the angle* K E F *is given by magnitude: Moreover, the right lines* E F *and* F K *being given by poſition, the angle* E F K *is given by magnitude, as is alſo the reſidue* E K F, *and ſo in the triangle* E K F *are all the angles given, and alſo the reaſons of the ſides: Therefore* [i] *the ſaid triangle* E K F *is given by kind.*

PROP. 40.

If the angles of a triangle A B C, *are given by magnitude, the triangle is given by kind.*



Conſtruction LEt there be expoſed the right line D E, given by poſition and by magnitude; and let there be conſtituted at the point D the angle E D F, equal to the angle C B A; but in the point E the angle D E F, equal to the angle B C A; therefore the third angle B A C is equal to the third angle D F E.

Demonſtration FOr each of the angles conſtituted in the points A, B, and C, is given: Therefore each of thoſe which are poſited in the points D, F, and E, is alſo given; and ſeeing that to the right line D E given by poſition, and to the point D given therein, there is drawn the right line D F, which makes the given angle E D F, [a] the line D F is given by poſition; and by the ſame reaſon, the line E F is given by poſition: Therefore [b] the point F is given by poſition. But each of the point D and E is given: Therefore [c] each of the lines D F, D E, and E F, is given by magnitude. Wherefore the triangle D F E is given by kind; and is alike to the triangle A B C: Therefore the triangle A B C is given by kind.

a) 29. p. b) 25. p. c) 26. p.

PROP. 41.

If a triangle A B C, *hath one angle* B A C *given, and that the two ſides* B A *and* A C, *which do conſtitute it, have*

to

to one another a given reaſon, the triangle is given by kind.

Conſtruction FOr, Let there be expoſed the right line D F given by magnitude and poſition. But thereon and at the given point F, let there be conſtituted the angle D F E equal to the angle B A C.

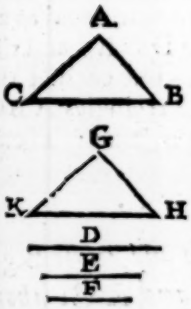
Demonſtration NOw the angle B A C is given: Therefore alſo the angle D F E is given, And ſeeing that to the right line D F given by poſition, and from the give point F therein, is drawn a right line F E, making the given angle D F E, [a] the ſaid line F E is given by poſition. But ſeeing that the reaſon of A B to A C is given, let the ſome be made of D F to F E, then let D E be drawn. Therefore the reaſon of D F to F E is given. But D F is given: Therefore [b] F E is given by magnitude. But the ſame F E is alſo given by poſition, and the point F is given. Therefore [c] the point E is alſo given. But each of the points D and F is given: Therefore [d] each of the right lines D F, F E, and D E, is given by poſition and magnitude. Wherefore [e] the triangle D E F is given by kind. And ſeeing that the two triangles A B C and D E F have an angle equal to an angle, that is to ſay, the angle B A C to the angle D F E, and the ſides which conſtitute thoſe equal angles, proportional; [f] the triangle A B C is alike to the triangle D E F. But the triangle D E F is given by kind: Therefore the triangle A B C is given by kind.



a) 29. p.
b) 2. p.
c) 27. p.
d) 26. p.
e) 39. p.
f) 6. 6.

PROP. 42.

If the ſides of a triangle A B C, *be to one another in given reaſons, the triangle* A B C *is given by kind.*



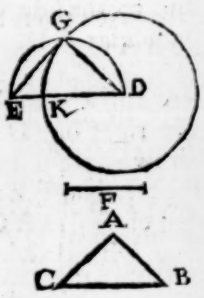
Conſtruction FOr, Let there be expoſed the right line D, given by magnitude, and ſeeing that the reaſon of B C to A C is given, let the ſame be made of D to E.

Demonſtration NOw D is given, therefore [a] E is alſo given. Again, ſeeing that the reaſon of A C to A B is given, let the ſame be made of E to F. Now E is given, therefore [b] F is alſo given. Now of three right lines, equal to the three given right lines D, E, and F, (and of which three lines, two of them, in what manner ſoever they be taken, are greater then the other:) Let there be conſtituted the triangle G H K, in ſuch ſort as D may be equal to H K, but E is equal to K G, and G H equal to F; therefore each of the ſaid lines H K, K G, and G H, is given by magnitude: Wherefore [c] the triangle H G K is given by kind. And ſeeing that as B C is to C A, ſo is D to E, and that D is equal to H K, and E to K G, as B C is to C A, ſo H K is to K G. Again, ſeeing that as C A is to A B, ſo is E to F, and that E is equal to K G, and F to G H; as C A is to A B, ſo is K G to G H. But it hath been demonſtrated that as B C is to C A, ſo is H K to K G: Therefore by reaſon of equality, as B C is to A B, ſo is H K to G H. Therefore [d] the triangle A B C is alſo given by kind.

a) 2. p.
b) 2. p.
c) 39. p.
d) 5. 6.

PROP. 43.

If the ſides B C *and* D A, *about one of the acute angles of a rectangled triangle* A B C, *have to one another a given reaſon, that triangle is given by kind.*



Conſtruction LEt there be expoſed the right line DE given by magnitude and poſition, and on it let there be deſcribed the ſemicircle D G E: Therefore [a] the ſemicircle D G E is given by poſition.

a) 6 def.

Demonſtration FOr the line D E being given, and divided in two equal parts, the center of the ſaid circle is given by poſition, and the ſemidiameter by magnitude. And foraſmuch as the reaſon of B C to B A is given, let the ſame be made of D E to F: Therefore the reaſon of DE to F is given. But D E is given, therefore F [b] is alſo given. Now B C is greater then [c] A B: Therefore E D is [d] alſo greater then F. Let D G be fitted equal to F, and let E G be drawn; then on the center D, with the diſtance D G, let the circle G K be deſcribed. Now that circle [e] is given by poſition, ſeeing that the center D is given, and the ſemidiameter D G alſo given by magnitude. But the ſemicircle D G E is alſo given by poſition: Therefore [f] the point of interſection G is given. But the points D and E are alſo given, therefore [g] each of the right lines D E, D G, and E G, is given by poſition and magnitude. Wherefore [h] the triangle D G E is given by kind. And ſeeing that the triangles A B C and D G E have an angle equal to an angle, to wit, the right angle B A C to the right angle [i] D G E, and the ſides about the angles C B A and E D G proportional. But each of the others A C B and D E G leſſe then a right angle: Thoſe triangles A B C and D E G [k] are alike. But the triangle D G E is given by kind: Therefore the triangle A B C is alſo given by kind.

b) 2. p.
c) 19. 1.
d) 14. 5.
e) 6 def.
f) 25. p.
g) 26. p.
h) 39. p.
i) 31. 3.
k) 7. 6.

PROP. 44.

If a triangle A B C, *hath one angle* B *given, and that the ſides* B A *and* A C, *about another angle* B A C, *have to one another a given reaſon, the triangle* A B C *is given by kind.*



Conſtruction NOw the given angle B is either acute or obtuſe, (for it was a right angle in the fore-going prop.) Let it be in the firſt place acute, and from the point A let A D be drawn perpendicular to B C.

Demonſtration THerefore the angle A D B is given: But the angle B is alſo given; and therefore the third angle B A D is given: Wherefore [a] the triangle A B D is given by kind; and therefore [b] the reaſon of B A to A D is given. But the reaſon of the ſame B A to A C is alſo given: Therefore [c] the reaſon of A D to A C is given, and the angle A D C is a right angle: Wherefore the triangle [d] A C D is given by kind: Therefore the

a) 40. p.
b) 3 def.
c) 8. p.
d) 43. p.

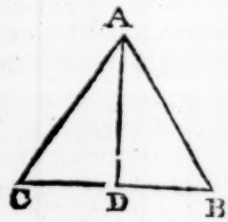
c the angle C is given. But the angle B is alſo given; and therefore the other angle B A C is given: Therefore f the triangle A B C is given by kind. c) 3. def. f) 40. p.

Conſtruction NOw let the angle A B C be obtuſe, and on the ſide C B prolonged, let there be drawn the perpendicular A D.

Demonſtration FOraſmuch as the angle A B C is given, the angle A B D which followes it, ſhall be given. But the angle A B D is alſo given: Therefore the third angle D A B is given. Wherefore g the angle A B D is given by kind; and therefore h the reaſon of D A to A B is given. But the reaſon of A B to A C is alſo given: Therefore i the reaſon of D A to A C is given, and the angle D is a right angle: Therefore the triangle D A C is given by kind, and therefore the angle A C B is given. But the angle A B C is alſo given: Therefore the third angle B A C is given. Wherefore the triangle A B C is given by kind. g) 40. p. h) 3 def. i) 8. p.



PROP. 45.



If a triangle A B C, *hath one angle* B A C *given, and that the line compounded of the two ſides* A B *and* A C, *about the ſaid given angle* B A C, *hath to the other ſide* B C *a given reaſon, the triangle* A B C *is given by kind.*

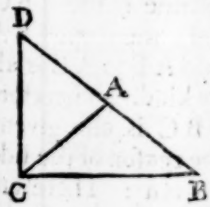
Conſtruction FOr, Let the angle B A C be divided into two equal parts by the line A D, therefore a the angle C A D is given. a) 7. p.

Demonſtration SEeing that as A B is to A C, ſo b is B D to C D; by compounding, c as the line compounded of C A B is to C A, ſo is B C to C D, and by permutatiou, as the line compounded of C A B is to C B, ſo is C A to C D. But the reaſon of the line compounded of C A B to B C is given; therefore the reaſon of C A to C D is alſo given, and the angle C A D is given. Therefore d the triangle A C D is given by kind, and therefore the angle C is given. But the angle B A C is alſo given: Therefore the third angle B is given: Wherefore e the triangle A B C is given by kind. b) 3. 6. c) 18. 5. d) 44. p. e) 40. p.

OTHERWISE.

Conſtruction LEt B A be prolonged directly unto the point D, in ſuch ſort as that A D may be equal to A C, and let C D be joyned.

Demonſtration FOraſmuch as the reaſon of the line compounded of C A B to C B is given, and that A D is equal to A C, the reaſon of the whole line B D to B C is given. But the angle A D C is alſo given, for it is the half of the the given angle B A C (for that the ſaid angle B A C f is equal to the two internal angles A C D and A D C, which are g equal to one another, being the ſides A C and A D are equal:) Wherefore the triangle B D C h is given by kind, and therefore the angle B is given. But f) 32. 1. g) 5. 1. h) 44. p.



the angle B A C is alſo given: Therefore the remaining angle A C B is given: Wherefore [i] the triangle A B C is given by kind.

i) 40. p.

PROP. 46.



If a triangle A B C *hath one angle* B *given, and that the line* C A B *compounded of the two ſides* A C *and* A B, *about another angle* B A C, *hath to the other ſide* B C *a given reaſon, the triangle* A B C *is given by kind.*

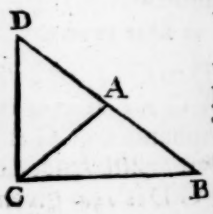
Conſtruction FOr let the angle B A C be divided into two equal parts, by the line A D.

Demonſtration THerefore (as hath been ſhewn in the foregoing Prop.) the compound line C A B is to C B, as A B is to B D. But the reaſon of the ſaid compound line C A B to C B is given: Therefore alſo the reaſon of A B to B D is given. But the angle B is alſo given: Therefore the triangle A B D [a] is given by kind; and therefore [b] the angle B A D is given. But the angle B A C is double to that of B A D; and therefore it is alſo given. Therefore the third angle C is given. Wherefore the triangle A B C is given by kind.

a) 41. p.
b) 7. def.

OTHERWISE.

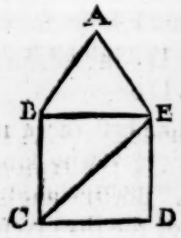
Conſtruction LEt B A be prolonged directly, and let A D be put equal to A C, and let C D be joyned.



Demonſtration FOraſmuch as the reaſon of the line compounded of C A B to C B is given, and that A D is equal to A C, the reaſon of B D to B C is given; and the angle B is alſo given: Therefore the triangle C D B [c] is given by kind; and therefore [d] the angle D is given: Therefore the angle B A C which is double to B D C, is alſo given: Wherefore the other angle A C B is given; and therefore the triangle A B C is given by kind.

c) 41. p.
d) 3 def.

PROP. 47.



Rectiline figures as A B C D E, *given by kind, are divided into triangles given by kind.*

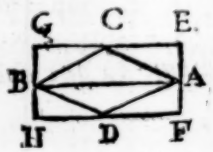
Conſtruction FOr let the right lines E B and E C be drawn.

Demonſtration FOraſmuch as the rectline figure A B C D E is given by kind, the angle [a] B A E is given, and the reaſon of the ſide A B to A E is alſo given: Therefore [b] the triangle B A E is given by kind. Wherefore the angle A B E is given. But the whole angle A B C is alſo given: Therefore [c] the remaining angle E B C is given. But the reaſon of the ſide A B to the ſide B E, and alſo that of A B to B C is given: Therefore the

a) 3. def.
b) 41. p.
c) 4. p.

[a] the reason of BC to BE is given, and the angle CBE is also given: Therefore [c] the triangle BCE is given by kind. By the same discourse it may be demonstrated that the triangle CDE is given by kind. Therefore rectiline figures given by kind divide themselves into triangles given by kind. d) 8. p. e) 41. p.

PROP. 48.

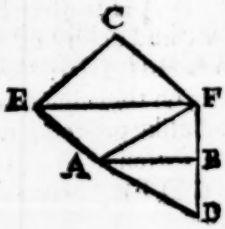


If on one and the same right line AB, *are described triangles as* ACB *and* ABD, *given by position, those triangles shall have to one another a given reason, as* ACB *to* ABD.

Construction FOr from the points A and B, let there be drawn at right angles on the line AB, the lines AE and BG, and prolonged unto the points F and H; but by the points C and D, let there be drawn the lines ECG and FDH, parallel to AB.

Demonstration FOrasmuch then as the triangle ABC is given by kind, [a] the reason of CA to BA is given, and the angle CAB also given; but the angle BAE is given: Therefore the remaining angle CAE is also given; but the angle CAE is given; and therefore the other angle ACE is also given. Wherefore [b] the triangle AEC is given by kind. Now the reason of EA to AB [c] is given; (for [d] the reason of EA to AC, and that of AC to AB is given;) and in like manner, the reason of FA to AB is given. Therefore [e] the reason of EA to AF is given; but as AE is to AF, so [f] the parallelogram AH to the parallelogram AG; but ACB is [g] the half of AH, and ADB the half of AG; therefore the reason of the triangle ACB to the triangle ADB is given; for it is the same reason with that of AH to AG [h]; that is to say, of EA to AF, which is given. a) 3. def. b) 40. p. c) 8. p. d) 3 def. e) 8. p. f) 1. 6. g) 41. 1. h) 15. 5.

PROP. 49.



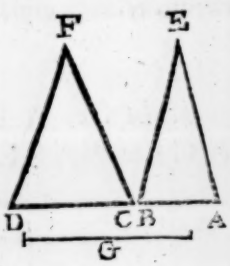
If on one and the same right line AB *there be described any two rectiline figures* AECFB *and* ADB, *given by kind, they shall have to one another a given reason (to wit,* AECFB *to* ADB.)

Construction FOr let the lines FA and FE be drawn: Therefore each of the triangles [a] ABF AFE, and ECF is given by kind. a) 47. p.

Demonstration SEeing that on one and the same right line EF there are described the triangles ECF and EAF, given by kind; the reason of ECF to EAF [b] is given. Therefore, by compounding, [c] the reason of AECF to EAF is given. But the reason of the said EAF to FAB is given, [d] being they are triangles given by kind, described on one and the same right line AF: Therefore [e] the reason of AECF to FAB is given. Wherefore by compounding, [f] the reason of AECFB to FAB is given. But the reason of the same FAB to ABD [g] is given: Therefore [h] the reason of AECFB to ABD is also given. b) 48. p. c) 6. p. d) 48. p. e) 8. p. f) 6. p. g) 48. p. h) 8. p.

PROP.

PROP. 50.

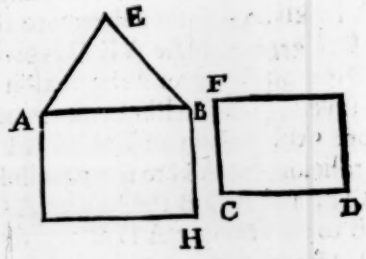


If two right lines A B *and* C D, *have to one another a given reason, and that on those lines there be described rectiline figures* A E B *and* C F D, *alike, and alike posited, they will have to one another a given reason.*

Demonstration TO the two lines A B and C D, let there be taken a third proportional: Therefore as A B is to C D, so is C D to G. But the reason of A B to C D is given: Therefore the reason of C D to G is also given: Wherefore [a] the reason of A B to G is given. But [b] as A B is to G, so is A E B to C F D: Therefore the reason of the same A E B to C F D is given.

a) 8. p.
b) Cor. 19, 20. 6.

PROP. 51.



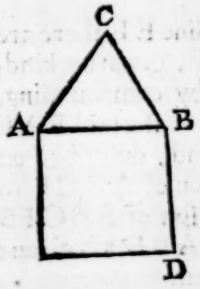
If two right lines A B *and* C D, *have to one another a given reason, and that upon them there be described any rectiline figures* A E B *and* C F D, *given by kind, they will have to one another a given reason, (to wit, that of* A E B *to* C F D.)

Construction FOr on A B, let the rectangled figure A H be described alike and alike posited to D F.

Demonstration NOw D F is given by kind: Therefore also A H is given by kind. But A E B is also given by kind, and described on the same line A B: Therefore [a] the reason of A E B to A H is given: And seeing that the reason of A B to C D is given, and that on those lines are described the rectiline figures A H and D F alike, and alike posited, the reason [b] of the said line A H to D F is given. But the reason of A E B to A H is also given: Therefore the reason [c] of A E B to D F is given.

a) 49. p.
b) 50. p.
c) 8. p.

PROP. 52.



If on a right line A B, *given by magnitude, there be described a figure* A C B, *given by kind, that figure* A C D *is given by magnitude.*

Construction FOr on the same line A B, let the square A D be described. Therefore A D is given by kind * and by magnitude.

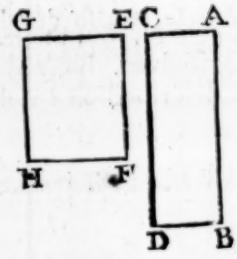
Demonstration SEeing that on the right line A B, are described the two rectiline figures A C B

A C B and A D, given by kind, [a] the reaſon of A C B to A D is given: Therefore [b] A C B is given by magnitude. a) 29. p. b) 2. p.

SCHOLIUM.

* *The ancient Interpreter hath noted here that every ſquare is given by kind; for that all the angles thereof are given; being all equal and right angles: But alſo the reaſons of the ſides are given; for thoſe ſides being all equal, their reaſons are alſo equal. Moreover, whenſoever a ſquare is expoſed, a ſquare equal thereto may be exhibited; and therefore the ſquare is given by magnitude, as alſo each ſide thereof.*

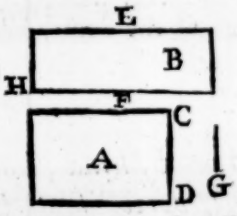
PROP. 53.



If there be two figures A D *and* E H, *given by kind, and that one ſide* B D, *of the one, hath to a ſide* F H *of the other, a given reaſon; the other ſides ſhall have alſo to the other ſides given reaſons.*

Demonſtration. FOr ſeeing that the reaſon of B D to F H is given, and alſo that [a] of B D to B A, [b] the reaſon of the ſaid A B to F H is given. But the reaſon of the ſame F H to F E [c] is alſo given: Therefore [d] the reaſon of A B to E F is given. In like manner alſo the reaſons of the other ſides to the other ſides are given. a) 3. def. b) 8. p. c) 3 def. d) 8. p.

PROP. 54.



If two figures A *and* B, *given by kind, have to one another a given reaſon, alſo their ſides ſhall be to one another in a given reaſon.*

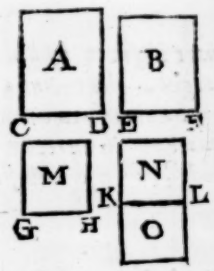
Conſtruction. FOr either the figure A is alike and alike poſited to B, or is not: Let it in the firſt place be alike and alike poſited; and let there be taken the line G, a third proportional to the lines C D and E F.

Demonſtration. AS C D is to G, [a] ſo is A to B. But the reaſon of A to B is given; therefore alſo the reaſon of C D to G is given. And ſeeing that C D, E F, and G, are proportional, [b] alſo the reaſon of C D to E F is given. But A and B are given by kind: Therefore [c] the other ſides ſhall have given reaſons to the other ſides. b) Cor. 19, 20. 6. b) 24. p. c) 53. p.

Now let the figure A be not alike to the figure B, and let there be deſcribed on E F the figure E H, alike and alike poſited to A: Therefore the figure E H is given by kind; but the figure B is alſo given by kind: Therefore [d] the reaſon of B to E H is given; and therefore the reaſon of A to the ſame E H [e] is alſo given: But A is alike to E H: Therefore (by what is aboveſaid) the reaſon of C D to E F is given; and in like manner the reaſon of the other ſides to the other ſides is given. d) 49. p. e) 8. p.

OTHER-

OTHERWISE.



Conſtruction LEt there be expoſed the given line G H: Now either the figure A is alike to the figure B, or not. Let it in the firſt place be alike, and let it be as C D is to E F, ſo is G H to L K; then on G H and L K let the figures M and N be deſcribed alike and alike poſited to the ſaid A and B, which figures M and N ſhall be conſequently given by kind.

Demonſtration THerefore ſeeing that as C D is to E F, ſo is G H to L K, and that on thoſe lines C D, E F, G H, and L K, are deſcribed the figures A, B, M, and N, alike and alike poſited; [f] as A is to B, ſo is M to N. But the reaſon of A to B is given: Therefore the reaſon of M to N is given. But [g] M is given, conſidering that it is a figure given by kind, deſcribed on a right line given by magnitude; therefore N is alſo given.

f) 22. 6.

g) 52. p.

Conſtruction 2. NOw, on L K let the ſquare O be deſcribed. Therefore [h] the figure O is given by kind.

h) Sch. 52. p.

Demonſtration 2. WHerefore the reaſon of K to N is given. But N is given: Therefore K is given; and conſequently, [i] alſo K L. But G H is given: Therefore [k] the reaſon of G H to K L is given. But as G H is to L K, ſo is C D to E F. Therefore the reaſon of C D to E F is given; and therefore the figures A and B being given by kind, [l] the other ſides of the ſame figures ſhall alſo have to the other ſides given reaſons. But if the figures be not alike, the later part of the demonſtration here above muſt be obſerved.

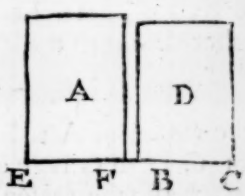
i) Sch. 52. p.

k) 3. p.

l) 53. p.

PROP. 55.

If a Space A, *be given by kind, and by magnitude, the ſides thereof ſhall be given by magninude.*



Conſtruction FOr, Let the right line B C given by poſition and by magnitude, be expoſed; and thereon let there be deſcribed the ſpace D, alike and alike poſited to A; therefore the ſaid ſpace D is given by kind.

Demonſtration FOr that it is deſcribed on the line B C, given by magnitude, it is alſo [a] given by magnitude. But the figure A is alſo given: Therefore [b] the reaſon of A to D is given. But thoſe figures A and D are given by kind: Therefore [c] the reaſon of the line E F to the line B C is given. But B C is given: Therefore [d] E F is alſo given. But the reaſon of the ſame E F to F G is given: Therefore [e] F G is given. And by the ſame reaſons it may be demonſtrated that each of the other ſides are given by magnitude.

a) 52. p.

b) 3. p.

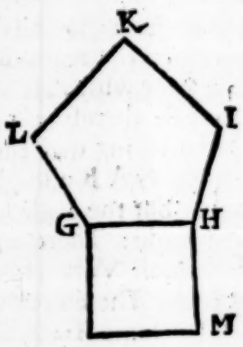
c) 54. p.

d) 3. def.

e) 2. p.

OTHER-

OTHERWISE.



Conſtruction LEt the ſpace GHIKL be given by kind and by magnitude: I ſay that the ſides thereof are given by magnitude. For on the right line GH let there be deſcribed the ſquare GM; therefore [f] GM is given by kind. f) Sch. 52. p.

Demonſtration BUt the ſpace GHIKL is alſo given by kind: Therefore [g] the reaſon of the ſame ſpace GK to GM is given. But GK is given by magnitude: Therefore [h] GM is alſo given by magnitude; and ſeeing that GM is the ſquare of the line GH, [i] that line GH is given by magnitude. Wherefore in like manner, each of the other lines HI, IK, KL, and LG, is given. g) 49. p. h) 2. p. i) Sch. 52. p.

PROP. 56.



If two equiangled Parallelograms A *and* B, *have to one another a given reaſon, as one ſide* CD *of the firſt* A, *is to one ſide* FG, *of the ſecond* B; *ſo the other ſide* GE, *of the ſecond* B, *is to that to which* DH *the other ſide of the firſt* A, *hath the given reaſon that the Parallelogram* A *hath to the Parallelogram* B.

Conſtruction FOr let HD be prolonged directly to L, ſo that as CD is to FG, ſo HD may be to DL; and finiſh the Paralellogram DK.

Demonſtration SEeing that as CD is to FG, ſo HD is to DL, and [a] that CD is equal to KL; as LK is to FG, ſo is GE to DL; and thus the ſides about the equal angles DLK and EGF are reciprocally proportional: Wherefore [b] DK is equal to B; and therefore ſeeing the reaſon of A to B is given, and that B is equal to DK, the reaſon of A to DK is given. But as [c] A is to DK (that is to B) ſo is HD to DL; therefore the reaſon of HD to DL is alſo given, and ſeeing that as CD is to FG, ſo GE is to DL, and that the right line HD hath to DL a given reaſon; to wit, that which the ſpace A hath to the ſpace B; as CD is to FG, ſo GE is to that to which HD hath the given reaſon that the ſpace A hath to the ſpace B, that is to ſay, the reaſon of HD to DL. a) 34. 1. b) 14. 6. c) 1. 6.

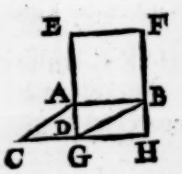
PROP. 57.

If a given ſpace AD *be applyed to a given right line* AB, *in a given angle* CAB, *the breadth* CA *of the application is given.*

Conſtruction FOr on AB, let there be deſcribed the ſquare AF; therefore [a] the ſame AF is given: Let the lines EA, FB, and CD, be prolonged to the points G and H. a) Sch. 52. p

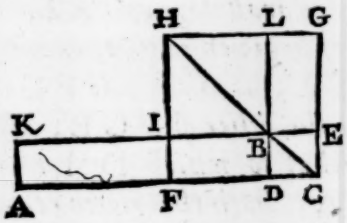
Demonſtration SEeing therefore that each ſpace A D and A F is given, their reaſon is alſo given. But b A D is equal to A H: Therefore the reaſon of A F to A H is given: Wherefore the reaſon of E A to A G is given. (for c it is the ſame with that of A F to A H.) But E A is equal to A B; therefore the reaſon of A B to A G is given. Now ſeeing that the angle C A B is given, and the angle G A B alſo given, the reſidue C A G is given. But the angle C G A is alſo given, being a right angle: Therefore the remaining angle A C G is given. Wherefore the triangle d C A G is given by kind. Therefore the reaſon of C A to A G is given. But the reaſon of A B to the ſame A G is alſo given: Therefore the reaſon of C A to A B is given; and the ſaid A B is given: Wherefore C A is alſo given.

b) 36. 1.
c) 1. 6.
d) 40. p.



PROP. 58.

If a given ſpace A B, *be applyed to a given right line* A C, *wanting by a figure* D E, *given by kind, the breadths of the defects are given.*



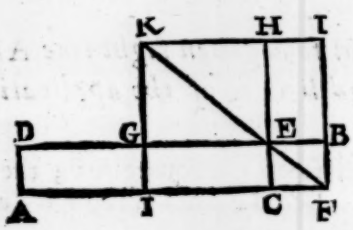
Conſtruction FOr let A C be divided in two equal parts in the point F: Therefore as well A F as F C is given. On the ſaid line F C let there be deſcribed the rectangle figure F G alike and alike poſited to D E. Therefore F G is given by kind.

Demonſtration SEeing the figure F G is deſcribed on the right line F C given by magnitude, the ſaid rectiline F G is a alſo given by magnitude. But F G is equal to A B and I L; (for b A I and F E being equal, and c F B and B G alſo equal, the Gnomon I C L is equal to A B; and therefore their added figure I L, common to both, F G ſhall be equal to A B anp I L:) Therefore the figures A B and I L together are given by magnitude. But A B is given by magnitude: Therefore d the remaining figure I L is alſo given by magnitude. But it is alſo given by kind, ſeeing it is e alike to D E: Therefore f the ſides of the ſame I L are given: Wherefore I B is given; and ſeeing that it is equal g to F D, the ſame F D is alſo given. But F C is given, therefore the remainder D C h is given; and i in a given reaſon to B D, and therefore k B D is given.

a) 52. p.
b) 36. 1.
c) 43. 1.
d) 4. p.
e) 24. 6.
f) 55. p.
g) 34. 1.
h) 4. p.
i) 3. def.
k) 2. p.

PROP. 59.

If a given ſpace A B *be applyed according to a given right line* A C, *exceeding it by a figure* C B *given by kind, the breadths of the exceſſes* C E *and* C F *are given.*



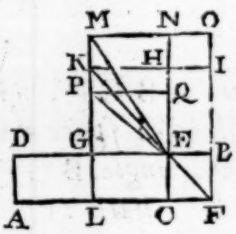
Conſtruction FOr D E being divided into two equal parts in G, let there

there be described on G E the rectiline figure G H, alike and alike posited to C B.

Demonstration NOw seeing that C B is alike to G H, those figures C B and G H * are about one and the same diameter, and G H is given by kind, as is C B. But it is described on the given line G E: Therefore [a] the same G H is also given by magnitude. But A B is given: Therefore A B and G H are given by magnitude. Now those figures A B and G H, are equal to L I, (for A G, L E, and E I, being equal, the Gnomon G F H is equal to A B; and therefore adding G H common to both, L I shall be equal to A B and G H;) therefore L I is given by magnitude; but it is also given by kind, being it is [b] alike to C B. Therefore [c] the sides of the said L I are given, seeing it is equal to G E: Therefore [d] the remainder C F is given, and in a given reason [e] to C E. Wherefore [f] C E is given.

a) 32. p.
b) 24. 6.
c) 55. p.
d) 4. p.
e) 3 def.
f) 2. p.

SCHOLIUM.

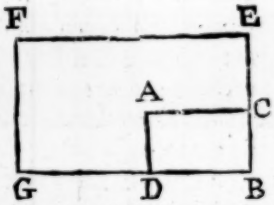


* *EUCLIDE supposeth here that* C B *and* G H *are about one and the same diameter, but we shall thus demonstrate it: Let* C B *and* G H *be two alike Parallelograms disposed as above, that is to say, that the equal angles joyn together in* E, *the side* C E *meets directly with his homologal side* E H, *and the side* B E, *his correspondent side* E G; *and let the diameter* F E *be drawn I say that the said diameter* F E *prolonged, will passe by the point* K; *that is to say, the Paralellograms* G H *and* C B, *consist about one and the same diameter. For if it be denied, the diameter* E F *being produced, will passe above the point* K, *or below it. Let it in the first place passe above it, and let it cut* G K, *prolonged in the point* M, *and by the point* M, *let there be drawn* M N, *parallel to* K H, *which shall meet* E H, *prolonged in the point* N, *and* F B *in* O.

Demonstration FOrasmuch as the Paralellograms G N and C B are with the Parallelogram L O about one and the same diameter, they are [g] alike to one another. Wherefore as F C is to C E, so is E G to G M. In like maner, seeing the parallelograms C B and G H are alike, as F C is to C E, so is E G to G K: Therefore [h] as E G is to G M, so is E G to G K. Wherefore [i] G M and G K are equal, a part to the whole, which is absurd: By the same reasons it may be demonstrated, that the diameter prolonged will not fall below the point K: Therefore the Paralelograms C B and G E consist about one and the same diameter.

g) 24. 6.
h) 11. 5.
i) 9. 5.

PROP. 60.

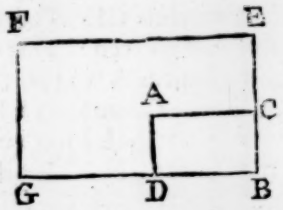


If a Paralellogram A B, *given by kind and by magnitude, be augmented or diminished by a Gnomon* C F D, *the breadths of the Gnomon (consisting of the lines* C E *and* D G) *are given.*

Demonstration FOr seeing that A B is given, and the Gnomon C F D also given, the whole Parallelogram B F is given: But it is also given by kind, seeing it is alike to B A: Therefore [a] the sides of the same

a) 55. p.

same BF are given; and therefore each of the lines BE and BG is given. But each of the lines BC and BD is given; therefore each of the remaining lines CE and DG is also given.

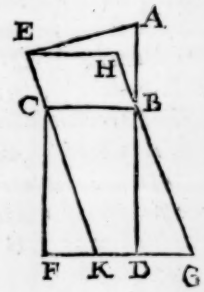


Conſtruction NOw let the Paralellogram BF, given by kind and by magnitude, be diminiſhed by the given Gnomon CFD: I ſay that each of the lines CE and DG is given.

Demonſtration 2. FOr ſeeing that BF is given, and the Gnomon CFD given, the remaining figure AB is alſo given. But it is alſo given by kind, b) 55. p. ſeeing it is alike to BF: Therefore b the ſides of the ſaid AB are given, and therefore each of the lines CB and BD is given. But each of the lines BE and BG is given: Therefore alſo each of the remaining lines CE and DG is given.

PROP. 61.

If to one ſide of a figure ABCE, *given by kind, there be applyed a ſpace Parallelogram* CD, *in a given angle* BCF, *and that the given figure* AC *hath to the Parallelogram* CD *a given reaſon, the Parallelogram* CD *is given by kind.*



Conſtruction FOr by the point B, let BH be drawn parallel to CE, and by the point E let EH be drawn parallel to CB, and let EC and HB be prolonged to the points K and G.

Demonſtration FOraſmuch as the angle BCE is given, and the reaſon of EC to CB, a) 3 def. a the parallelogram CH is given * by kind. But the figure ABCE is alſo given by kind, and is deſcribed on the ſame b) 49. p. line BC, as the Parallelogram CH given by kind is: Therefore b the reaſon of the figure ABCE to the Parallelogram CH is given. But by Suppoſition, the reaſon of the ſaid figure ABCE to the Paralielogram c) 36. 1. CD is alſo given; and CD is c equal to CG: Therefore d the reaſon of d) 8. p. CH to CG is given. Wherefore the reaſon of the line EC to the line e) 1. 6. CK is given; (for e as CH is to CG, ſo is EC to CK.) But the reaſon f) 8. p. of EC to CB is alſo given: Therefore f the reaſon of the ſaid CB to CK is given. And ſeeing that the angle ECB is given, alſo the follow- g) 13. 1. ing angle BCK g is given. But the angle BCF is propoſed given; and 4. p. therefore the remaining angle FCK is given. Alſo the angle CKF is gi- h) 29. 1. ven, for that h it is equal to the angle BCK: Therefore the other angle i) 40. p. CFK is given: Wherefore i the triangle FCK is given by kind; and therefore the reaſon of FC to CK is given. But the reaſon of CB to the k) 8. p. ſame CK is alſo given: Therefore k the reaſon of FC to CB is given; and the angle BCF is alſo given. Wherefore the Parallelogram CD is given by kind.

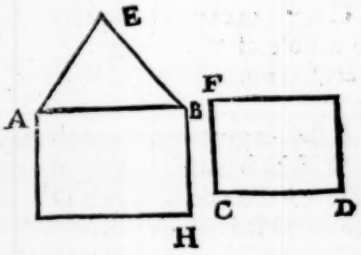
SCHO-

SCHOLIUM.

* *Although it be manifeſt that a Parallelogrrm that hath one angle given, and the reaſon of the ſides about the ſame angle alſo given, is given by kind,* as Euclide *doth here declare, ſo it is notwithſtanding that the ancient Interpreter doth thus demonſtrate it.*

Seeing that in the Parallelogram C H *the angle* E C B *is given, the angle* C E H *is alſo given; for the right line* E C *falling on the Parallels* E H *and* C B, *doth make the two internal angles on the ſame part equal to two right angles. And therefore ſeeing that the angle* E C B *is given, the other angles are given; and ſeeing that the reaſon of* E C *to* C B *is given, and that* B H *is equal to* C E, *and* E H *to* B C, *the reaſon of the ſides to one another is alſo given.*

PROP. 62.



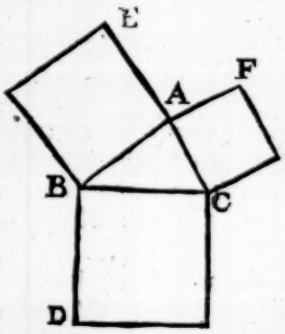
If two right lines A B *and* C D, *have to one another a given reaſon, and that on one of them* A B, *there be deſcribed a figure* A E B, *given by kind; but on the other* C D, *a ſpace Parallelogram* D F, *in a given angle* D C F, *and that the figure* A E B *hath to the Parallelogram* D F *a given reaſon, the Parallelogram* D F *is given by kind.*

Conſtruction FOr on the line A B let there be deſcribed the Parallelogram A H, alike and alike poſited to D F.

Demonſtration SEeing then that the reaſon of A B to C D is given, and that on thoſe lines are deſcribed the rectiline figures A H and F D, alike and alike poſited, [a] the reaſon of A H to F D is given. But the reaſon of F D to A E B is alſo given: Therefore [b] the reaſon of A H to A E B is given. But the angle A B H is alſo given, being equal to the angle F C D, and ſo the figure A E B is given by kind; and to A B one of the ſides thereof, the Parallelogram A H is applyed in a given angle A B H, and the reaſon of the ſaid figure A E B to the ſaid Parallelogram A H is given: Therefore [c] the Parallelogram A H is given by kind; and therefore E D which is alike thereto, is alſo given by kind.

a) 50. p.
b) 8. p.
c) 61. p.

PROP. 63.



If a triangle A B C *be given by kind, the ſquare* B E, C D, *and* C F, *which is deſcribed on each of the ſides, ſhall have a given reaſon to the triangle* A B C.

Demonſtration FOr ſeeing that on one and the ſame right line B C, there are deſcribed the two rectiline figures A B C and C D,

a) 49. p. CD given by kind, [a] the reaſon of the ſame ABC to CD is given; and therefore the reaſon of the ſquares BE and CF, to the triangle ABC, is alſo given.

PROP. 64.



If a triangle ABC, *hath an obtuſe angle* ABC *given, that ſpace of which the ſide* AC *ſubtending the obtuſe angle* ABC, *is more in power then the ſides* AB *and* BC, *that comprehend the ſaid angle, ſhall have a given reaſon to the triangle* ABC.

Conſtruction LEt the line CB be prolonged directly, and from the point A let the perpendicular AD be drawn: I ſay that the ſpace of which the ſquare of the line AC doth exceed the ſquares of the
a) 12. 2. lines AB and BC, that is to ſay, [a] the double of the rectangle conteined under CB and BD, ſhall have a given reaſon to the triangle ABC.

Demonſtration FOr ſeeing that the angle ABC is given, the angle ABD is alſo given. But the angle ADB is alſo given; therefore
b) 40. p. the other angle BAD is given: Wherefore [b] the triangle ABD is given
c) 3. def. by kind; therefore [c] the reaſon of AD to DB is given. But as AD to
d) 1. 6. DB, ſo [d] the rectangle of AD and BC is to the rectangle of BC and BD. But the reaſon of AD to BD is given: Therefore alſo is the reaſon of the rectangle of AD and BC to the rectangle of BC and BD given: Wherefore the reaſon of the double of the ſaid rectangle BC and BD to the rectangle of AD and BC is alſo given. But the ſaid rectangle of AD and BC hath alſo a given reaſon to the triangle ABC (to wit, double
e) 41. 1. reaſon; for the rectangle is [e] double to the triangle) therefore the reaſon of the double of the rectangle of BC and BD [f] to the triangle ABC is given.
f) 8. p. But the ſame double of the rectangle of CB and BD is that ſpace of which the ſquare of the line AC doth exceed the ſquares of the lines AB and BC: Therefore the ſame ſpace hath a given reaſon to the triangle ABC.

PROP. 65.



If a triangle ABC, *hath one acute angle* ACB *given, that ſpace, of which the ſide ſubtending the ſaid acute angle is leſſe in power then the ſides comprehending the ſame acute angle, ſhall have a given reaſon to the triangle.*

Conſtruction FRom the point A let there be drawn the line AD, perpendicular to BC: I ſay that ſpace of which the ſquare of the
a) 13. 2. line AB is leſſe then the ſquares of the lines AC and CB, that is to ſay, [a] the double of the rectangle of BC and CD, hath a given reaſon to the triangle ABC.

Demonſtration FOr ſeeing that the angle C is given, and the angle ADC alſo given, the other angle DAC is given: Wherefore the

the triangle [b] A D C is given by kind; and therefore the reason of A D to D C is given, and consequently also [c] that of the rectangle of B C and C D to the rectangle of B C and A D: Therefore the reason of the double of the rectangle of B C and C D to the rectangle of B C and A D is given. But the reason of the same rectangle of B C and A D to the triangle A B C is given: (for [d] the rectangle is double to the triangle) Therefore [e] the reason of the double of the rectangle of B C and C D to the triangle A B C is given. And seeing that the same double of the rectangle of B C and C D is that whereof the square of the line A B is lesse then the squares of the lines A C and B C, that space of which the square of the liue A B is lesse then the squares of the lines A C and B C, shall have a given reason to the triangle A B C.

b) 40. p.
c) 1. 6.
d) 41. 1.
e) 8. p.

PROP. 66.



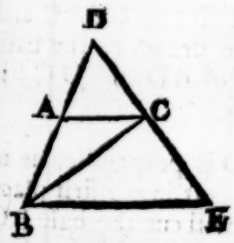
If a triangle A C B, *hath one angle* B *given, the rectangle made of the lines* A B *and* B C, *conteining the same angle, shall have a given reason to the triangle.*

Construction FOr from the point A let A D be drawn perpendicular to C B.

Demonstration THerefore seeing that the angle B is given, and also the angle A C B; the other angle C A D is likewise given. Wherefore the triangle A C B [a] is given by kind; and consequently the reason of A C to A D is given. But as A B is to A D, [b] so the rectangle of A C and C B is to the rectangle of C B and A D: Therefore the reason of the rectangle of A C and C B to the rectangle of C B and A D is given. But the reason of the said rectangle of C B and A D to the triangle A C B is also given; (for that it is double reason, the rectangle being double [c] to the triangle:) Therefore [d] the reason of the rectangle of A C and C B to the triangle A B D is given.

a) 40. p.
b) 1. 6.
c) 41. p.
d) 8. p.

PROP. 67.



If a triangle A B C, *hath one angle* B A C *given, that Space by which the square of the line compounded of the two sides* B A *and* A C, *that contein the same given angle* B A C *doth exceed the square of the other side, it shall have a given reason to the triangle* A B C.

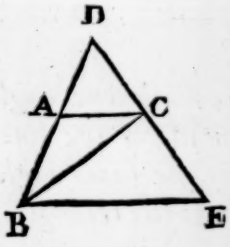
Construction FOr let B A be prolonged in such sort as that A D may be equal to A C, then having drawn D C E infinitely, from the point B let B E be drawn parallel to A C, meeting the said D E in the point E.

Demonstration FOrasmuch as A D is equal to A C, [a] D B is equal to B E; (for the two triangles A D C and B D E are alike) and from the top B is drawn to the base D E, the right line B C: Therefore

a) 4. 6. and 14. 5.

fore * the rectangle of D C and C E, with the ſquare of B C, is equal to the ſquare of B D; but the ſame B D is compounded of B A and A C; therefore the ſquare of the compound of A B and A C is greater then the ſquare of B C, of the rectangle of D C and C E.

Now I ſay that the rectangle of D C and C E hath a given reaſon to the triangle A B C: Foraſmuch as the angle B A C is given, the angle D A C is alſo given. But each of the angles A D C and A C D is given,
b) 40. p. it being the half of the angle B A C which is given. Therefore [b] the triangle A D C is given by kind; and therefore the reaſon of D A to D C
c) 50. p. is given. Therefore [c] the reaſon of the ſquare of the ſaid D A to the ſquare of D C is alſo given. And ſeeing that as
d) 2. 6. B A is to A D, [d] ſo is E C to C D, and alſo as
e) 1. 6. B A is to A D, [e] ſo is the rectangle of B A and A D to the ſquare of A D; and as E C is to C D,
f) 1. 6. [f] ſo alſo is the rectangle of E C and C D to the ſquare of C D; by permutation, as the rectangle of B A and A D is to the rectangle of E C and C D, ſo is the ſquare of A D to the ſquare of D C. But the reaſon of the ſaid ſquare of A D to the ſquare of D C is given: Therefore the reaſon of the rectangle of B A and A D to the rectangle of E C and C D is alſo given. But A D is equal to A C: Therefore the reaſon of the rectangle of B A and A C to the rectangle of E C and C D is given. But the reaſon of
g. 66. p. the rectangle of B A and A C to the triangle A B C [g] is given, becauſe
h) 8. p. the angle B A C is given: Therefore [h] the reaſon of the rectangle E C and C D to the triangle A B C is given. But the rectangle of E C and C D is that whereof the ſquare of the line compounded of B A and A C is greater then the ſquare of B C: Therefore that ſpace to which the ſquare of the line compounded of B A and A C is greater then the ſquare of B C, ſhall have a given reaſon to the triangle A B C.



SCHOLIUM.

* *EUCLIDE ſuppoſeth in this place that when in an Iſoſceles triangle a right line is drawn from the top to the baſe, the ſquare of that line, with the rectangle conteined under the ſegments of the baſes, is equal to the ſquare of either of the other legges, which the ancient Interpreter doth thus demonſtrate.*

Conſtruction LEt A B C be an Iſoſceles triangle, whoſe legges are A B and A C; and from the top A let A D be drawn to the baſe B C: I ſay that the ſquare of A D with the rectangle of B D and D C, is equal to the ſquare of either of the legges A B or A C.



Demonſtration NOw the line A D is perpendicular to B D, or not: Let it in the firſt place be perpendicular: Therefore it will cut the baſe B C into two equal parts in the point D; and therefore the rectangle conteined under B D and D C is equal to the ſquare of the ſaid B D, and adding to them the common ſquare of A D, the rectangle of B D and D C with the ſquare of A D, ſhall be equal to the ſquares of D B and A D. But to thoſe ſquares of
i) 47. 1. A D and D B [i] the ſquare of A B is equal: Therefore the

the ſquare of AB is equal to the rectangle of BD and DC, and the ſquare of AD together.

Now ſuppoſe AD not to be perpendicular, but that from the point A there doth fall on BC the perpendicular AE, that being ſo, BC ſhall be cut into two parts equally in the point E, and unequally in D. Wherefore the rectangle of BD and DC, with the ſquare of DE, [k] is equal to the ſquare of BE; and adding the common ſquare of AE, the rectangle of BD and DC, with the ſquares of DE and AE, ſhall be equal to the ſquares of BE and AE. But [l] the ſquare of AD is equal to the two ſquares of DE and AE: Therefore the rectangle of BD and DC, with the ſquare of AD is equal to the ſquares of BE and AE. But to theſe ſquares of BE and AE the ſquare of AB is equal: Therefore the ſquare of AD, with the rectangle of BD and DC, is equal to the ſquare of AB.

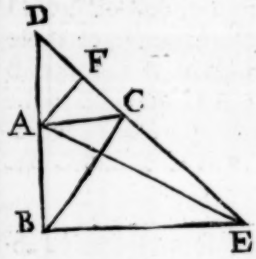
k) 5. 2.

l) 47. 1.

OTHERWISE.

Conſtruction HAving done, as in the foregoing Demonſtration, from the point A, let AF be drawn perpendicular to CD, and let AE be drawn.

Dmonſtration FOraſmuch as the angle BAC is given, the half thereof ACF ſhall be alſo given. But the angle AFC is given; and therefore the triangle AFC is given by kind: Therefore the reaſon of AF to FC is given. But the reaſon of CD to the ſame FC is alſo given, ſeeing that CD is double to FC: Therefore [m] the reaſon of CD to AF is given; and therefore alſo the reaſon of the rectangle of CD and EC, to the rectangle of AF and EC, is given; (for it is the ſame reaſon [n] as that of CD to AF.) But the reaſon of the rectangle of AF and FC to the triangle ACE is given; ſeeing it is double [o] to the ſame triangle. Therefore the reaſon of the rectangle of CD and CE to the triangle ACE is alſo given. But the triangle ACE is equal to the triangle ABC [p], they being both conſtituted on one and the ſame baſe AC, and between the ſame parallels AC and BE: Therefore [q] the reaſon of the rectangle of CE and CD to the triangle ABC is given. But the ſaid rectangle of CE and CD is the ſpace by which the ſquare of the line compounded of AB and AC, is greater then the ſquare of BC: Therefore that ſpace by which the ſquare of the line compounded of AB and AC is greater then the ſquare of BC, hath a given reaſon to the triangle ABC.



m) 8. p.

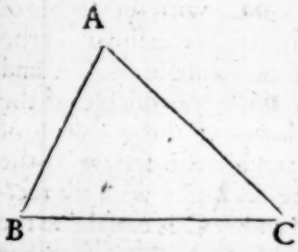
n) 1. 6.

o) 41. 1.

p) 37. 1.

q) 8. p.

OTHERWISE.



FOr the given angle A is either a right, acute, or obtuſe angle: Let it in the firſt place be ſuppoſed a right angle: Therefore the ſquare of the line compounded of BAC, is greater then the ſquare of BC, by twice the rectangle of BA and AC; (ſeeing that [r] the ſquare of BC is equal to the ſquares of BA and AC; and the ſquare of the line compounded of BAC [s] is equal to thoſe two

r) 47. 1.

s) 5. 2.

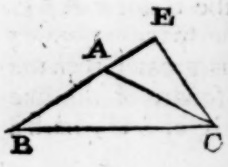
ſquares of BA and AC, and twice the rectangle of the ſaid BA and AC:) Wherefore the reaſon of double the rectangle of BA and AC to the triangle ABC is given.

Conſtruction NOw let the angle C be ſuppoſed acute, and from the point A let there be drawn on CB the perpendicular AD.

Demonſtration FOraſmuch as the triangle CAB is an Oxigonium triangle, and the perpendicular AD being drawn, the ſquare of
t) 13. 5. CA and CB are equal [t] to the ſquare of AB with twice the rectangle of CB and CD; adding therefore the common double rectangle of CA and CB, the ſquares of CA and CB, with the double rectangle of the ſaid CA and CB, that
u) 4. 2. is to ſay, [u] the alone ſquare of the line compounded of ACB, are equal to the ſquare of AB, with the double of the rectangle of CD and CB, and over and above the double of the rectangle of AC and CB, that is to ſay, the double of the rectangle conteined under the compound line of ACD and and CB (for the



w) 1. 2. rectangle of ACD and CB is [w] equal to the rectangles of AC and CB, and of CD and CB:) Therefore the ſquare of the line compounded of ACB is greater then the ſquare of AC, by double the rectangle of ACD and CB. And ſeeing that the angle ACB is given, and the an-
x) 40. p. gle BDA alſo given, the other angle CAD is given: Therefore [x] the triangle CAD is given by kind, and therefore the reaſon of CD to CA is given; and by conſequence the reaſon of the line compounded of ACD
y) 6. p. to CA [y] is alſo given. Wherefore the reaſon of the rectangle of thoſe
z) 1. 6. lines compounded of ACD and CB [z] to the rectangle of AC and CB is alſo given. But the reaſon of the ſaid rectangle of AC and CB to the
a) 66. p. triangle CAB [a] is given, ſeeing the angle C is given; therefore the reaſon of double the rectangle of the line compounded of ACD and CB to the triangle CAB is given.



Laſtly, Let the angle BAC be ſuppoſed to be obtuſe, and having prolonged BA from the point C, let the perpendicular CE be drawn on the ſaid line BA prolonged; and let AF be propoſed to be equal to AE.

Demonſtration FOraſmuch as the angle BAC is obtuſe, and the perpendicular CE being drawn, the ſquares of AB and AC, and the dou-
b) 12. 2. ble of the rectangle under BA and AE, or AF, are all alike equal [b] to the ſquare of BC, and adding the common double rectangle of
c) 4. 2. BA and AC, the ſquares of the ſaid AB and AC, with the double of the rectangle of the ſame AB and AC, that is to ſay, [c] the ſquare of the line compounded of BAC, and the double of the rectangle of BA and AF are together equal to the ſquare of BC, with the double of the rectangle of BA and AC. Let the common double of the rectangle of BA and AF be taken away, and there will remain the ſquare of the line compounded of BAC, equal to the ſquare of BC, with the rect-
d) 1. 2. angle of AB and CF; (for the rectangle of AB and AC is equal [d] to the two rectangles of AB and AE, and of AB and CF:) Therefore the

ſquare

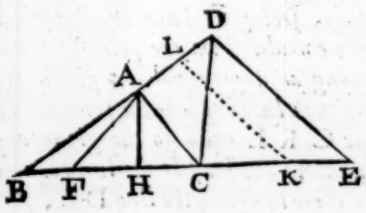
ſquare of the line compounded of BAC is greater then the ſquare of BC by the double of the rectangle of AB and CF. And foraſmuch as the angle BAC is given, the angle CAE [c] is given. But the angle AEC is alſo given; therefore the other angle ACE is given: Wherefore [f] the triangle ACE is given by kind, and therefore the reaſon of CA to AE, that is to ſay, to AF is given. Therefore [g] the reaſon of the ſaid CA to FG is alſo given. But the reaſon of the ſame CA to CE is given; therefore [h] the reaſon of CE to CF is alſo given. Wherefore the reaſon of the rectangle of EC and AB to the rectangle of FC and AB is given; (for the rectangle is to the rectangle [i] as CE is to CF) and alſo that of the rectangle of AC and AB, to the rectangle of EC and AB. Therefore [k] the reaſon of the rectangle of FC and AB to the rectangle of AC and AB is given. But the reaſon of the rectangle of AC and AB to the triangle ABC [l] is given: Therefore alſo the reaſon of the double of the rectangle of FC and AB, to the triangle ABC is given. But the ſame double of the rectangle of FC and AB is that, whereof the ſquare of the line compounded of BAC is greater then the ſquare of BC, whereof that ſpace of which the ſquare of the line compounded of BAC is greater then the ſquare of BC, hath a given reaſon to the triangle ABC.

c) 13. 1.
f) 40. p.
g) 5. p.
h) 8. p.
i) 2. 6.
k) 8. p.
l) 66. p.

OTHERWISE.

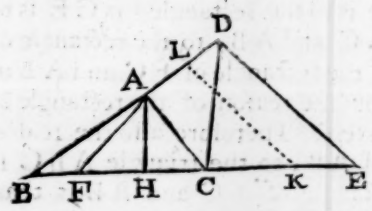
Conſtruction LEt the line BA be prolonged to the point D, in ſuch ſort as AD may be equal to AC, and let CD be drawn.

Demonſtration FOraſmuch as the angle BAC is given, each of the angles ADC and ACD, which is the half thereof ſhall be alſo given; and therefore the other angle DAC is a alſo given: Therefore [m] the triangle ACD is given by kind. Wherefore the reaſon of AC to CD is given. And foraſmuch as the angle ADC is given: Let each of the angles DEC and AFC be made † equal to the ſaid ADC: Therefore ſeeing that the angle BDC is equal to the angle DEC, and the angle DBE is common to the triangles DBE and DBC, the other angle BDE is equal to the other angle BCD; and therefore the triangle BDE is equiangled to the triangle BDC. Therefore [n] as EB is to BD, ſo is BD to CB: Wherefore the rectangle of EB and CB, that is to ſay, [o] the rectangle of EC and CB, [p] with the ſquare of CB is equal, [q] to the ſquare of BD, that is to ſay, to the ſquare of the line compounded of BAC; for AD is equal to AC; and therefore the rectangle of EC and CB, with the ſquare of CB, that is to ſay, the ſquare of the line compounded of BAC is greater then the ſquare of the rectangle of BC and CE: I ſay therefore that the reaſon of the ſaid rectangle of BC and CE to the triangle ABC is given. Foraſmuch as the angle BDE is equal to the angle BCD, and the angle ADC equal to the angle ACD, the other angle CDE is equal to the other angle ACB: But the angle DEC is alſo equal to the angle AFC; therefore the remaining



m) 40. p.
n) 4. 6.
o) 5. 2.
p) 5. 2.
q) 17. 6.

angle C A F is equal to the remaining angle D C E. Wherefore the triangle A F C is equiangled to the triangle D C E; and therefore [r] as C A is to A F, ſo is C D to C E; and by permutation, as A C is to C D, ſo is A F to C E. But the reaſon of A C to C D is given: Therefore alſo the reaſon of A F to C E is given. From the point A let A H be drawn perpendicular to B C: Foraſmuch as the angle A F C is given, and the angle A H F alſo given, the third angle H A F is given: Wherefore [ſ] the triangle A H F is given by kind; and by conſequence the reaſon of A F to A H is given. But the reaſon of A F to C E is alſo given: Therefore [t] the reaſon of A H to C E is given; and therefore the reaſon of the rectangle of A H and B C [u] to the rectangle of B C and C E is alſo given. But the reaſon of the rectangle of A H and B C to the triangle A B C is likewiſe given; (for the rectangle [w] is double to the triangle) and the rectangle of B C and C E is that whereof the ſquare of the line compounded of B A C is greater then the ſquare of B C. Therefore that ſpace of which the ſquare of the line compounded of B A C is greater then the ſquare of B C by a given reaſon to the triangle A D C.



r) 4. 6.

ſ) 40. p.

t) 8. p.

u) 1. 6.

w) 41. 1.

SCHOLIUM.

† *The ancient Interpreter pretending to ſhew the conſtruction of the angle* D E B *equal to the angle* A D C, *ſaith that on the line* B D *and in the point* D, *the angle* B D E *ought to be made, equal to the angle* B C D, *and that the right lines* B C *and* D E *be drawn until they interſect in* E, *in ſuch ſort as he ſuppoſeth the angle* B C D, *to be given, but it is not.*

The ſame Interpreter afterward ſhews how there may univerſally from a given point, be drawn a right line given by poſition to a right line, making an angle equal to a given angle. But we will alſo reject this way, ſeeing we have elſ-where ſhewn another more brief and eaſie. For example, if we would from the point D *draw to the line* B C *given by poſition a right line, making an angle equal to a given angle* A D C, *as is here required, we have no more to do but to aſſume the point* K *in the ſaid line* B C, *and there make the triangle* C K L *equal to the given angle* A D C: *If the line* K L *doth meet with the point* D, *it ſhall be the line required. But if it meet not with it, from the point* D *let there be drawn the line* D E, *parallel to the ſaid* K L, *cutting* B C *prolonged in* E, *and the angle* D E C *ſhall be equal to the given angle* A D C, *for on the two parallel lines* L K *and* D E, *there doth fall the line* B E; *and therefore the angle* D E C [x] *is equal to the angle* L K C, *which hath been made equal to the given angle* A D C; *and by conſequence the ſame angle* D E C *is alſo equal to* A D C.

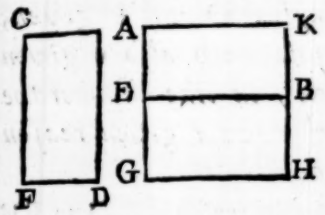
x) 29. 1.

PROP. 68.

If two Parallelograms A B *and* C D, *have to one another a given reaſon, and that a ſide hath alſo a given reaſon to a ſide, the other ſide ſhall have likewiſe a given reaſon to the other ſide.*

Con-

Conſtruction LEt the reaſon of BE to FD be given: I ſay the reaſon of AE to FC is alſo given. For to the right line EB let there be applyed the Parallelogram FH, equal to the Parallelogram CD, and conſtituted in ſuch ſort as AE and EG may make one right line: † Therefore KB and BH will alſo make one right line.



Demonſtration FOraſmuch as the reaſon of AB to CD is given, and that EH is equal to the ſaid CD; the reaſon of AB to EH is given; and therefore the reaſon of AE to EG is alſo given. Seeing therefore that EH is equal and equiangled to CD, as [b]EB is to FD, ſo is FC to EG. But the reaſon of EB to FD is given: Therefore alſo the reaſon of FC to EG is given. But the reaſon of AE to the ſame EG is alſo given: Therefore the reaſon of AE to FC is given.

a) 1. 6.

b) 14. 6.

SCHOLIUM.

† *EUCLIDE having poſited* AE *and* EG *directly in one right line, preſently concludeth that* KB *and* BH *ſhall alſo make a right line; but we ſhall demonſtrate it thus. Seeing the lines* AE *and* EG *are poſited directly, the angles* AEB *and* BEG [b] *are equal to two right angles; and ſeeing that* AB *is a Parallelogram, the lines* AK *and* EB *are parallels, on which the line* AE *doth fall; and therefore the two internal angles* A *and* BEA [c] *are alſo equal to two right angles, and taking away the common angle* BEA, *there will remain the angle* A, *equal to the angle* BEG; *and conſequently their oppoſite angles* EBK *and* H *are alſo equal to one another: Again, ſeeing that* BG *is a Parallelogram, the two lines* BE *and* HG *are Parallels, on which* BH *doth fall; and therefore the two internal angles* H *and* EBH [c] *are equal to two right angles. But it hath been demonſtrated that* H *is equal to* EBK: *Therefore the two angles* EBK *and* EBH *are alſo equal to two right angles; and therefore* [d] *the two lines* KB *and* BH *do meet directly according to* EUCLIDE.

b) 13. 1.

c) 29. 1.

d) 14. 1.

OTHERWISE.

Conſtruction LEt the given right line K be expoſed, and ſeeing that the reaſon of A to B is given, let the ſame be made of K to L; therefore the reaſon of K to L is alſo given.

Demonſtration BUt K is given; therefore [e] L is alſo given. Again, ſeeing that the reaſon of CD to EF is given, let the ſame be made of K to M: Therefore the reaſon of K to M is given. But K is given, therefore [f] M is alſo given; and therefore the reaſon of L to M is given. Now ſeeing that A is equiangled to B, [g] the reaſon of the ſaid A to B is compounded of that of the ſides, that is to ſay of CD to EF, and of CG to EH. But alſo the reaſon of K to L is compounded of K to M, and of M to L; therefore the reaſon compounded of CD to EF, and of CG to EH, is the ſame with that which is compounded of K



c) 2. p.

f) 1. p.

g) 23. 6.

to

to M, and of M to L (the reaſon of K to L being the ſame as of A to B:) But the reaſon of C D to E F is the ſame as of K to M: Therefore the other reaſon of C G to E H is alſo the ſame as of M to L. But the ſaid reaſon of M to L is given: Therefore alſo the reaſon of C G to E H is given.

PROP. 69.



If two Parallelograms C B *and* EH, *having the angles* D *and* F *given, and that a ſide hath alſo a given reaſon to a ſide; in like manner the other ſide ſhall have a given reaſon to the other ſide.*

Conſtruction LEt the reaſon of B D to F H be alſo given: I ſay that the reaſon of A B to E F is given. For if C B be equiangled to H E, it is manifeſt by the precedent Propoſition; but if it be not equiangled thereto, let the right line D B be conſtituted, and in the given point B therein, let the angle D B K be made equal to the angle E F H, and finiſh the Parallelogram D K.

Demonſtration FOraſmuch as each of the angles B K L and B A K is given, † the other angle K B A is given: Wherefore the triangle
a) 40. p. [a] A B K is given by kind; and therefore the reaſon of A B to B K is given. But
b) 35. p. reaſon of C B to E H is ſuppoſed to be given, and [b] C B is equal to D K; therefore the reaſon of D K to E H is given; and ſeeing that D K is equiangled to E H, and the reaſon of the ſaid D K to E H is given, as alſo
c) 68. p. that of D B to F H, [c] the reaſon of B K to F E is given. But the reaſon of
d) 29. 1. the ſaid B K to B A is alſo given: Therefore [d] the reaſon of A B to F E is given.

SCHOLIUM.

†*EUCLIDE ſuppoſeth there that a Parallelogram having one angle given, all the other angles are alſo given, and as well the Ancient Interpreters as others, do give the reaſons why, the angle* F *being given, the other angle* E *ſhall be alſo given, it being the remainder of two right angles, for that on the parallel lines* E G
e) 29. 1. *and* F H *there doth fall the line* E F, *which makes* [e] *the two internal angles (of the*
f) 34. 1 *ſame part)* F *and* G, *equal to two right angles. But to thoſe angles* [f] *the oppoſite angles* G *and* H *are equal, and therefore they are alſo given.*

From whence it follows that the angles B D C *and* F *being given by ſuppoſition, all the other angles of the two Parallelograms* C B *and* E H, *are alſo given: Therefore the angle* D B K *having been made equal to the angle* F, *the angle* K *ſhall be equal to the angle* E, *and given as that is: But the angle* B A L *which is oppoſite to the given angle* B D C, *is alſo given; and therefore* B A K *which is the remainder of two right angles, ſhall be alſo given; in ſuch ſort as in the triangle* A B K, *the two angles* B A K *and* B K A *are given, as EUCLIDE doth declare in this place.*

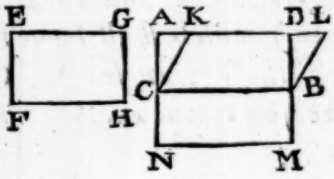
PROP. 70.

If of two Parallelograms A B *and* E H, *the ſides about the equal angles, or about the unequal angles (yet nevertheleſſe given*

given angles) have to one another a given reaſon, to wit (A C *to* E F, *and* C B *to* F H) *alſo the ſame Parallelograms* A B *and* E H *ſhall have to one another a given reaſon.*

Conſtruction FOr let A B be prolonged to E H, and on the right line C B let the Parallelogram C M be applyed equal to the Parallelogram E H, in ſuch ſort as A C may be direct to C N; that is to ſay, that A C and C N make one right line; and by conſequence D B ſhall be [a] directly with B M. a) Sch. 68. p.

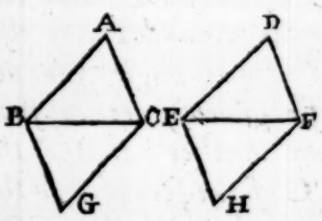
Demonſtration FOraſmuch then as C M is equiangled and equal to E H, the ſides about the equal angles ſhall be reciprocally [b] proportional: Wherefore as B C is to H F, ſo is F E to N C. But the reaſon of B C to H F is given: Therefore the reaſon of F E to N C is alſo given. But the reaſon of A C to the ſame E F is given: Therefore [c] the reaſon of A C to N C is alſo given. Wherefore the reaſon of A B to C M is given; (for it is the ſame [d] as of A C to C N.) But C M is equal to E H: Therefore the reaſon of A B to E H is given. b) 14. 6. c) 8. p. d) 1. 6.



Conſtruction NOw ſuppoſe A B not to be equiangled to E H, and on the right line C B, and in the given point C therein: Let there be conſtituted the angle B C K, equal to the given angle F, and ſo finiſh the Parallelogram C L.

Demonſtration FOraſmuch as the angle A C B is given, and the angle B C K alſo given, the remaining angle A C K is given: Therefore the triangle A C K [e] is given by kind; and therefore the reaſon of A C to C K is given: But the reaſon of A C to E F is alſo given: Therefore the reaſon of C K to E F is given. But the reaſon of B C to H F is alſo given, and the angle B C K is equal to the angle F; therefore (by the firſt part of this Prop.) the reaſon of C L to E H is given. But to the ſaid C L, A B is equal: Therefore the reaſon of A B to E H is given. e) 40. p.

PROP. 71.



If of two triangles A B C *and* D E F, *the ſides about the equal angles* A *and* D, *or elſe about the unequal angles (yet nevertheleſſe given angles) have to one another a given reaſon* (*to wit* A B *to* D E *and* A C *to* D F) *the ſame triangles ſhall have alſo to one another a given reaſon* A B C *to* D E F.

Conſtruction Let the Parallelogram A G and D H be finiſhed.

Demonſtration SEeing that the two Parallelograms A G and D H, have the ſides about the equal angles A and D, or elſe about the unequal angles (nevertheleſſe given) have a given reaſon to one another, the

a) 70. p. b) 34. p. the reaſon [a] of the Parallelogram A G to the Parallelogram D H is given. But the triangle A B C is the half of the Parallelogram A G [b] and the triangle D E F the half of the Parallelogram D H. Therefore the reaſon of the triangle A B C to the triangle D E F is given.

PROP. 72.



If of two triangles A B C *and* D E F, *the baſes* B C *and* E F, *are in a given reaſon,* B C *to* E F, *and that from the angles* A *and* D, *there be drawn to thoſe baſes the right lines* A G *and* D H, *making the equal angles* A G C *and* D H F, *or elſe unequal (yet nevertheleſſe given) which ſhall have to one another given reaſons* A G *to* D H, *thoſe triangles* A B C *and* D E F *ſhall have alſo a given reaſon to one another, to wit,* A B C *to* D E F.

Conſtruction For let the Parallelogram K C and L F be finiſhed.

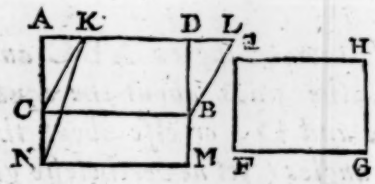
Demonſtration FOraſmuch as the angles A G C and D H F are equal, or unequal (yet given) and that the angle A G C [a] is equal to the angle K B C. But the angle D H F equal to the angle L E F, the angles at the points B and E are equal, or elſe unequal (yet given) and for that the reaſon of A G to D H is given, and A G is equal to K B. But D H is equal to L E, alſo the reaſon of K B to L E is given. But the reaſon of B C to E F is alſo given, and the angles at the points B and E are equal, or elſe unequal (yet given:) Therefore [b] the reaſon of the Parallelogram K C to the Parallelogram L F is given; and therefore the reaſon of the triangle A B C to the triangle D E F is given, ſeeing thoſe triangles [c] are the one half of the Parallelograms.

a) 29. 1.

b) 70. p.

c) 41. 1.

PROP. 73.



If of two Parallelograms A B *and* E G, *the ſides about the equal angles* C *and* F, *or elſe about the unequal angles (but nevertheleſſe given) are in ſuch ſort to one another, that as the ſide* C B *of the firſt, is to the ſide* F G *of the ſecond; ſo the other ſide* E F *of the ſecond, is to ſome other right line* C N. *But that the other ſide* A C *hath alſo to the ſame right line* C N *a given reaſon, thoſe Parallelograms will have alſo to one another a given reaſon* A B *to* E G.

Conſtruction FOr in the firſt place, Let the Parallelogram A B be equiangled to E G, and having placed C N directly to A C: Let the Parallelogram C M be finiſhed.

Demon-

Demonſtration FOraſmuch then, as C B or N M its equal, is to F G, ſo is E F to C N, and that the angles N and F are equal (for N is equal to the angle A C B, which is put equal to F) the Parallelograms C M and E G [a] are equal: But as A C to C N, ſo [b] the Parallelogram A B is to the Parallelogram C M or E G: Therefore ſeeing that the reaſon of A C to C N is given, the reaſon of A B to E G is alſo given. a) 14. 6. b) 1. 6.

Conſtruction 2. NOw ſuppoſe the Parallelogram A B not to be equiangled to the Parallelogram E G, and let there be conſtituted at the given point C in the line C B, the angle B C K, equal to the angle E E G, and ſo finiſh the Parallelogram C L.

Demonſtration 2. SEeing that each of the angles A C B and K C B is given, the remaining angle A C K is alſo given. But [c] the angle C A K is given, as alſo the remaining angle A K C: Therefore [d] the triangle A C K is given by kind; and therefore the reaſon of A C to C K is given. But the reaſon of the ſame A C to C N is alſo given: Therefore [e] the reaſon of C K to C N is given. And ſeeing that as C B is to F G, ſo is E F to the right line C N, to which the other ſide K C hath a given reaſon, and that the angle B C K is equal to the angle F, the reaſon of the Parallelogram C L to the Parallelogram E G is given (by the firſt part of this Prop.) but the Parallelogram C L is equal to the Parallelogram A B: Therefore the reaſon of the Parallelogram A B to the Parallelogram E G is given. c) Sch. 69 p. d) 40. p. e) 8. p.

PROP. 74.

If two Parallelograms (as in the former figure) A B and E G, in equal angles C and F, or elſe in unequal angles (yet nevertheleſſe given angles) have a given reaſon to one another, as one ſide C B of the firſt ſhall be to one ſide F G of the ſecond, ſo the other ſide E F of the ſecond, ſhall be to that to the which the other ſide A C of the firſt hath a given reaſon.

Conſtruction FOr either A B is equiangled or not; ſuppoſe it in the firſt place to be equiangled; and to the right line B C let there be applyed the Parallelogram C M, equal to the Parallelogram E G, and ſo poſited, as that A C and C N may be direct: Therefore [a] D B and B M ſhall be alſo direct (that is as one right line.) a) Sch. 68. p.

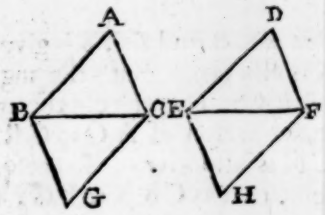
Demonſtration SEeing that the reaſon of A B to E G is given, and that C M is equal to E G, the reaſon of A B to C M is alſo given; and therefore the reaſon of A C to C N is given (ſeeing A B is to C M, [b] as A C is to C N;) and for that C M is equal and equiangled to E G, the ſides about the equal angles of the Parallelograms C M and E G, [c] are reciprocally proportional; and therefore as C B is to F G, ſo is E F to C N. But the reaſon of A C to C N is given: Therefore as C B is to F G, ſo is E F to that to which A C hath a given reaſon. b) 1. 6. c) 14. 6.

Conſtruction 2. NOw ſuppoſe A B not to be equiangled to E G, and in the given point C of the line C B, let there be conſtituted the

angle B C K equal to the angle E F G, and finish the Parallelogram C L.

Demonstration 2. SEeing then that the reason of A B to E G is given, and
d) 36. 1. [d] that A B is equal to C L, also the reason of C L to
E G is given, and the angle B C K is equal to the angle F, and therefore
e) Sch. 69 p C L [e] is equiangled to E G: Therefore (by the first part of this Prop.) as
C B is to F G, so is E F to that to which C K hath a given reason.
But the reason of A C to C K is given; (as appears by what hath been demonstrated in the latter part of the precedent Prop.) Therefore as C B is to F G, so is E F to that to which A C hath a given reason.

PROP. 75.



If two triangles A B C *and* D E F, *in equal angles* A *and* D, *or else unequal (yet nevertheless given) have to one another a given reason, as the side* A B *of the first, shall be to the side* D E *of the second, so the other side* D F *of the second, shall be to that right line to the which the other side* A C *of the first hath a given reason.*

Construction For let the Parallelograms A G and D H be finished.

Demonstration FOrasmuch as the reason of the triangle A B C to the triangle D E F is given, also the reason of the Parallelogram A G to the Parallelogram D H is given.

Seeing therefore that the two Parallelograms A G and D H in equal angles, or unequal angles (nevertheless given) have to one another a given
a) 74. p. reason; as [a] A B is to D E, so is D F to that to which A C hath a given reason.

PROP. 76.



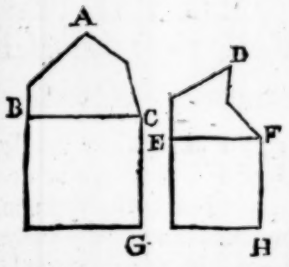
If from the top A *of a triangle* A B C, *given by kind, there be drawn to the base* B C, *a perpendicular line* A D, *that line* A D *shall have to the base* B C *a given reason.*

Demonstration FOr seeing that the triangle A B C is given by kind, the reason of A B to B C is given; and the angle B is also given. But the angle A D B is given; therefore the other angle B A D is given. Where-
a) 40. p. fore [a] the triangle A B D is given by kind; and therefore the reason of A B to A D is given. But the reason of A B to B C is given: Therefore
b) 8. p. [b] the reason of A D to B C is given.

PROP. 77.

If two figures A B C *and* D E F, *given by kind, have to another a given reason, the reason also shall be given of which you*

you pleaſe of the ſides of one of the figures, to which you pleaſe of the ſides of the other figure.

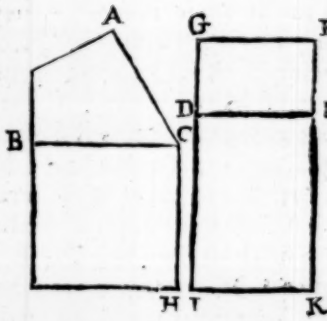


Conſtruction FOr on the right lines B C and E F, let there be deſcribed the ſquares B G and E H.

Demonſtration FOraſmuch as on one and the ſame right line B C, are deſcribed two figures A B C and B G, given by kind, [a] the reaſon of the ſaid A B C to B G is given: In like manner, the reaſon of D E F to E H is given; and ſeeing that the reaſon of A B C to D E F is given, and alſo that of the ſame figure A B C to B G; and again the reaſon of D E F to E H: [b] the reaſon of B G to E H is given; and therefore the reaſon of B C to E F is alſo given. a) 49. p. b) 8. p.

PROP. 78.

If a given figure A B C, *hath a given reaſon to ſome rectangled figure* D F, *and that one ſide* B C *hath a given reaſon to one ſide* D E, *the rectangled figure* D F *is given by kind.*



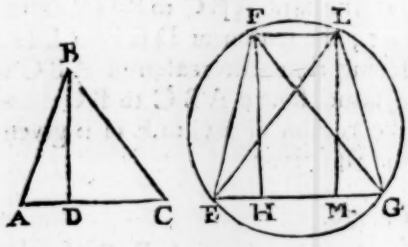
Conſtruction FOr on the right line B C let the ſquare B H be deſcribed, and to the right line D E, let the Parallelogram D K be applyed equal to B H, in ſuch manner as that G D and D I may be placed directly, [a] and by conſequence F E and E K alſo directly. a) Sch. 68. p

Demonſtration THerefore ſeeing that on one and the ſame right line B C are deſcribed the two rectiline figures A B C and B H, given by kind; [b] the reaſon of A B C to B H is given. But the reaſon of the ſaid A B C to D F is alſo given: Therefore [c] the reaſon of B H to D F is given. But B H is equal to D K: Therefore the reaſon of D K to D F is alſo given. And ſeeing that B H is equal and equiangled to D K, both the one and the other being rectangles, [d] the ſides of thoſe figures are reciprocally proportional; and as B C is to D E, ſo is D I to C H. But by ſuppoſition, the reaſon of B C to D E is given; therefore alſo the reaſon of D I to C H is given; but the reaſon of D I to D G is alſo given: (for D I is to D G [e] as D K to D F:) Therefore [f] the reaſon of D G to C H is given. But C H is equal to B C, ſeeing that B H is a ſquare; therefore the reaſon of B C to D G is given. But the reaſon of the ſame B C to D E is alſo given; therefore the reaſon of D E to D G is given, and the angle at D is a right angle: Therefore [g] D F is given by kind. b) 49. p. c) 8. p. d) 14. 6. e) 1. 6. f) 8. p. g) Sch. 61. p.

PROP. 79.

If two triangles A B C *and* E F G, *have an angle* B *equal*

to an angle F. *But from the equal angles* B *and* F *there be drawn perpendiculars* BD *and* FH, *to the baſes* AC *and* EG; *and that as the baſe* AC *of the firſt triangle* ABC, *is to the perpendicular* BD, *ſo alſo the baſe* EG *of the other triangle* EFG, *is to the perpendicular* FH, *thoſe triangles* ABC *and* EFG *are equiangled.*



Conſtruction FOr about the triangle EFG let there be deſcribed the circle EFLG, then on the right line EG, and in the point E given therein, let there be made the angle GEL, equal to the angle C, and let FL and LG be drawn, and the perpendicular LM.

Demonſtration SEeing then that the angle GEL is equal

a) 21. 3. to the angle C, and the angle ELG is equal to the angle EFG, [a] they being in one and the ſame ſegment of the circle; the third angle EGL is equal to the third angle A. Wherefore the triangle ABC is alike to the triangle ELG, and the perpendiculars BD and LM are drawn: Therefore † as AC is to BD, ſo is EG to LM; but by ſuppoſition as AC is to

b) 7. 5. BD; ſo is EG to FH: Therefore [b] LM is equal to FH. But the ſaid LM

c) 28. 1. is [c] parallel to FH: Therefore [d] FL is alſo parallel to EG; and there-

d) 33. 1. fore the angle FLE [e] is equal to the angle LEG. But the angle C is alſo

e) 29. 1. equal to the ſaid angle LEG, and the angle FLE to the angle FGE [f]:

f) 21. 3. Therefore alſo the angle C is equal to the angle FGE. But by ſuppoſition the angle ABC is equal to the angle EFG: Therefore the third angle BAC is equal to the third angle FEG: Wherefore the triangle ABC is equiangled to the triangle EFG.

SCHOLIUM.

† *Now that as* AC *is to* BD, *ſo* EG *is to* LM, *it is by ſome thus demonſtrated. For aſmuch as the angle* C *is equal to the angle* GEL, *and the angle* BDC *to the angle* LME, *each being a right angle, the other angle* CBD *is equal to the other*

g) 4. 6. *angle* ELM: *Therefore* [g] *as* EM *is to* ML, *ſo is* CD *to* DB. *Again, ſeeing the angle* ABC *is equal to the angle* ELG, *and the angle* CBD *to the angle* ELM, *the remaining angle* ABD *is equal to the remaining angle* MLG; *but the angle* ADB *is alſo equal to the angle* LMG; *and therefore the third angle* A

h) 4. 6. *is equal to the third angle* LGM: *Therefore* [h] *as* AD *is to* DB, *ſo is* GM *to* ML. *But it hath been demonſtrated that as* CD *is to* DB, *ſo is* EM *to* ML: *Thereſore*

i) 14. 5. [i] *as* AC *is to* BD, *ſo is* EG *to* LM.

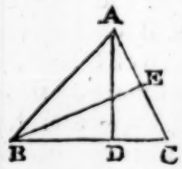
PROP. 80.

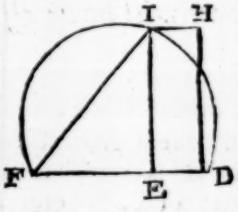
If a triangle ABC *hath one angle* A *given, and that the rectangle conteined under the ſides* AB *and* AC, *compriſing the given angle* A, *hath a given reaſon to the ſquare of the other ſide* BC, *the triangle* ABC *is given by kind.*

Con-

Conſtruction FOr from the points A and B, let there be drawn the perpendiculars A D and B E.

Demonſtration FOraſmuch as the angle B A E is given, and alſo the angle A E B, the triangle A B E is given [a] by kind; and therefore the reaſon of A B to B E is given: Therefore the reaſon of the rectangle of A B and A C to the rectangle of B E and A C is alſo given (for it is the ſame reaſon [b] as of A B to B E.) But the rectangle of A C and B E is equal to the rectangle of B C and A D; for that each of thoſe rectangles is [c] double to the triangle A B C. Therefore the reaſon of the rectangle of A B and A C to the rectangle of B C and A D is alſo given. But the reaſon of the rectangle of A B and A C to the ſquare of B C is given: Therefore [d] alſo the reaſon of the rectangle of B C and A D to the ſquare of B C is given; and therefore the reaſon of the right line B C to the right line A D is given. (For that [e] the rectangle is to the ſquare as A D to B C.) Now let the right line F G given by poſition and magnitude, be expoſed; and thereon let there be deſcribed the ſegment of a circle F I G, capable of an angle equal to the angle A. And ſeeing the ſaid angle A is given, alſo the angle in the ſegment F L G ſhall be given; and therefore [f] the ſame ſegment is given by poſition. From the point G let there be erected at right angles on the line F G, the line G H, which [g] is given by poſition: Let it be ſo made, that as B C is to A D, ſo F G may be to G H; and ſeeing that the reaſon of B C to A D is given, alſo that of F G to G H is given. But F G is given: Therefore [h] G H is given by magnitude. But it is alſo given by poſition, and the point G is given: Therefore the point H is [i] alſo given. Now by the point H let there be drawn H I, parallel to F G, and that line H I ſhall be given by [k] poſition. But the ſegment of the circle F I G is alſo given by poſition. Therefore [l] the point I is given. Let the right lines I F and I G be drawn, and the perpendicular I K: Therefore I K is given by poſition. But the point I is given, as alſo each of the points F and G: Therefore [m] each of the lines F G, F I, and I G is given by poſition and magnitude: Wherefore [n] the triangle F I G is given by kind; and ſeeing that as B C is to A E, ſo is F G to G H, and [o] that to G H, I K is equal, as B C is to A E, ſo is F G to I K, and the angle A is equal to the angle F I G: Therefore [p] the triangle A B C is equiangled to the triangle F I G. But F I G is given by kind: Therefore alſo the triangle A B C is given by kind.





a) 40. p.
b) 1. 6.
c) 41. 1.
d) 8. p.
e) 1. 6.
f) 8. def.
g) 4. def.
h) 2. p.
i) 27. p.
k) 28. p.
l) 25. p.
m) 26. p.
n) 39. p.
o) 34. p.
p) 7). p.

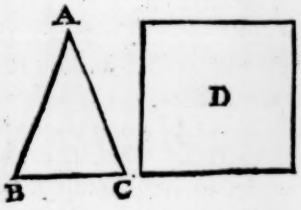
OTHERWISE.

Conſtruction LEt the triangle A B C, whoſe angle A is given, and the reaſon of the rectangle contained under A B and A C, to the ſquare of B C be given: I ſay that the triangle A B C is given by kind.

Demonſtration FOr ſeeing the angle A is given, that ſpace of which the ſquare of the line compounded of B A C is greater then the ſquare of B C, [q] hath a given reaſon to the triangle A B C. Now let that ſpace be D: Therefore the reaſon of D to the triangle A B C is given

q) 67. p.

given. But the reaſon of the triangle A B C to the rectangle of A B and A C is given; r ſeeing the angle A is given: Therefore s the reaſon of the ſpace D to the rectangle of A B and A C is given. But the reaſon of the rectangle of A B and A C to the ſquare of B C is alſo given: Therefore t the reaſon of the ſpace D to the ſquare of B C is given. Wherefore by compounding, t the reaſon of the ſpace D, with the ſquare of B C to the ſaid ſquare of B C is given: Therefore the reaſon of the ſquare of the line compounded of B A C, to the ſquare of B C is given; (for that the ſpace D with the ſquare of B C is equal to the ſquare of the line compounded of B A C;) and therefore v the reaſon of the ſaid line compounded of B A C to B C is given. But the angle A is alſo given: Therefore w the triangle A B C is given by kind.



r) 66. p.
s) 8. p.
t) 6. p.
v) Sch. 52. p
w) 46. p.

PROP. 81.

A / B / C — D / E / F

If of three right lines A, B, *and* C, *proportional to three other proportional right lines* D, E, *and* F, *the extreams* A *and* D, C *and* F, *are in a given reaſon* (*to wit, as* A *to* D, *and* C *to* F,) *alſo the meanes* B *and* E *ſhall be in a given reaſon, and if one extream hath a given reaſon to an extream, and the mean to the mean, the other will have alſo a given reaſon to the other.*

Demonſtration FOraſmuch as the reaſon of A to D, and of C to F is given, the rectangle of A and D a ſhall have a given reaſon to the rectangle of C and F. But the rectangle of A and D is equal b to the ſquare of B; and the rectangle of C and F to the ſquare of E. Therefore the reaſon of the ſquare of B to the ſquare of E is given; and therefore c the reaſon of the line B to the line E is alſo given.

Again, Let the reaſon of A to D, and B to E, be given: I ſay that the reaſon of C to F is alſo given. For ſeeing that the reaſon of A to D, and of B to E is given, alſo the reaſon of the ſquare of B d to the ſquare of E is given. But the ſquare of B is equal to the rectangle of A and C, and the ſquare of E to the rectangle of D and F: Therefore the reaſon of the rectangle of A and C to the rectangle of D and F is given. But the reaſon of a ſide A to a ſide D is given: Therefore e the reaſon of the other ſide C to the other ſide F is alſo given.

a) 70. p.
b) 17. 6.
c) Sch. 52. p
d) 50. p.
e) 68 p.

PROP. 82.

A
B
C
D
E
F

If there be four right lines A, B, C, *and* D, *proportional, as the firſt* A, *ſhall be to that line to which the ſecond* B *hath a given reaſon, ſo the third* C, *ſhall be to that to which the fourth* D *hath a given reaſon.*

Con-

Conſtruction LEt E be the line to which B hath a given reaſon, and let it be ſo as that B may be to E, as D is to F.

Demonſtration NOw the reaſon of B to E is given, Therefore alſo the reaſon of D to F is given. And ſeeing that as A is to B, ſo is C to D. And again, as B is to E, ſo is D to F, by reaſon of equality as A is to E, ſo is C to F. But E is that line to which B hath a given reaſon, and F that to which D alſo hath a given reaſon: Therefore as A is to that to which B hath a given reaſon, ſo C is to that to which D hath a given reaſon.

PROP. 83.

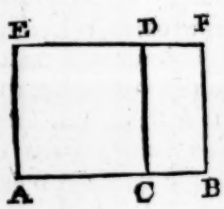
A
B
C
D
E

If four right lines A, B, C, *and* D, *are in ſuch ſort to one another, that of any three of them* A, B, *and* C, *and a fourth* E, *taken proportional, to which that line* D, *which remaines of the four lines, hath a given reaſon, the four lines* A, B, C, *and* E, *are proportional; as the fourth* D *is to the third* C, *ſo the ſecond* B *ſhall be to that to which the firſt* A *hath a given reaſon.*

Demonſtration FOraſmuch as A is to B as C is to E, the rectangle conteined under A and E [a] is equal to the rectangle conteined under B and C; and ſeeing that the reaſon of D to E is given, alſo ſhall be given the reaſon of the rectangle of A and D to the rectangle of A and E (for [b] it is the ſame reaſon as of D to E.) But the rectangle of A and E is equal to the rectangle of B and C: Therefore the reaſon of the rectangle of A and D to the rectangle of B and C is given. Wherefore [c] as D is to C, ſo is B to that to which A hath a given reaſon.

a) 16. 6.
b) 1. 6.
c) 74. p.

PROP. 84.



If two right lines A B *and* A E *comprehending a given ſpace* A F *in a given angle* B A E, *and that the one* A B *be greater then the other* A E *by a given line* C B, *alſo each of the lines* A B *and* A E *is given.*

Demonſtration FOr ſeeing that A B is greater then A E by the given line C B, the remainder A C is equal to A E: Finiſh the Parallelogram A D. Therefore ſeeing that A E is equal to A C, the reaſon of A E to A C is given, and the angle A is alſo given: Therefore [a] A D is given by kind. Wherefore the given ſpace A F is applyed to the given right line C B, exceeding it by the given figure A D given by kind; and therefore [b] the breadth of the exceſſe is given. Therefore A C is given. But C B is alſo given: Therefore the whole A B is given. But A E is alſo given: Therefore each of the right lines A B and A E is given.

a) Sch. 61. p.
b) 59. p.

PROP.

PROP. 85.

If two right lines A C *and* C D, *do comprehend a given ſpace* A D *in a given angle* A C D, *the line compounded of thoſe lines* A C *and* C D *is given, alſo each of thoſe lines* A C *and* C D *is given.*

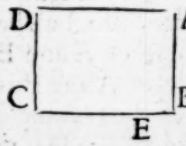


Conſtruction FOr let A C be prolonged to the point B, and let C B be put equal to C D, then by the point B let B F be drawn parallel to C D, and ſo finiſh the Parallelogram C F.

Demonſtration SEeing then that C B is equal to C D, and the angle D C B is given; for that angle that followes is the given angle; and therefore [a] the Parallelogram D B is given by kind; and again, ſeeing that the line compounded of A C D is given, and C B is equal to C D, alſo A B is given. And thus to the right line A B there is applyed the given ſpace A D, deficient by the figure D B given by kind; and therefore [b] the breadths of the defects are alſo given: Therefore the right lines D C and C B are given. But the compounded line A C D is alſo given: Therefore [c] each of the lines A C and C D is given.

a) Sch. 61. p.
b) 58. p.
c) 4. p.

PROP. 86.



If two right lines A B *and* B C, *do comprehend a given ſpace* A C, *in a given angle* A B C, *the ſquare of the one* B C *is greater then the ſquare of the other* A B, *by a given ſpace (yet in a given reaſon) alſo each of thoſe lines* A B *and* B C *ſhall be given.*

Demonſtration FOr ſeeing that the ſquare of B C is greater then the ſquare of A B by a given ſpace (yet in a certain reaſon.) Let the given ſpace be taken away, that is to ſay, the rectangle conteined under C B and B E: Therefore [a] the reaſon of the remainder, [b] which is the rectangle conteined under B C and C E to the ſquare of A B is given. And foraſmuch as the rectangle † under A B and B C is given, and alſo that of C B and B E, their [c] reaſon is given. But as the rectangle under A B and B C is to the rectangle under C B and E B, [d] ſo A B is to B E; and therefore the reaſon of A B to B E is given: Wherefore [e] the reaſon of the ſquare of A B to the ſquare of B E is alſo given. But the reaſon of the ſquare of A B to the rectangle under B C and C E is given: Therefore [f] alſo the reaſon of the rectangle under B C and C E to the ſquare of B E is given. Wherefore the reaſon of four times the rectangle under B C and C E to the ſquare of B E is given; and by compounding, [g] the reaſon of four times the rectangle under B C and C E, with the ſquare of B E to the ſquare of B E is given. But four times the rectangle of B C and C E, with the ſquare of B E, [h] is the ſquare of the compound line B C E: Therefore the reaſon of the ſquare of the compound line B C E to the ſquare of B E is given: Wherefore [i] the reaſon of the line compounded of

a) 11 def.
b) 2. 2.
c) 1. p.
d) 1. 6.
e) 50. p.
f) 8. p.
g) 8. p.
h) 8. 2.
i) 54. p.

of BC and CE to BE is given, and by compounding, [k] the reaſon of the compound of the lines BC, CE, and BE, that is to ſay, the double of BC to BE is given; and therefore the reaſon of the onely line BC to BE is alſo given. But as BC is to BE, [l] ſo the rectangle under BC and BE is to the ſquare of BE: Therefore the reaſon of the rectangle under BC and BE to the ſquare of BE is given. But the rectangle of BC and BE is given: Therefore [m] the ſquare of BE is alſo given, and conſequently the line BE is given. Wherefore BC is alſo given, ſeeing that the reaſon of BE to BC is given. But the ſpace AC is given, and alſo the angle B: Therefore [n] AB is given. Wherefore each of the lines AB and BC is given.

k) 6. p.

l) 1. 6.

m) 2. p.

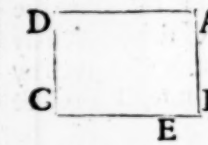
n) 57. p.

SCHOLIUM.

† *Inſtead of ſaying in this place* [what is under, &c.] *we have uſed this Word Rectangle, it being manifeſt by what followes that ſuch was the intention of* EUCLIDE, *ſeeing he makes uſe in the ſaid Demonſtration of the 2d. and 8th. Prop. of the 12th. Element; and alſo that the ſpace or Parallelogram given being not rectangled, it may be reduced thereto, making on* BC, *and in the given point* B, *a right angle* CBA, *ſo as that there will be two Parallelograms conſtituted on one and the ſame baſe* BC, *and between the ſame Parallels, as in the 69th. Prop. by meanes whereof this concluſion is drawn.*

☾ Note this ſerves alſo for the next Propoſition.

PROP. 87.



If two right lines AB *and* BC, *do comprehend a given ſpace* AC, *in a given angle* B, *the ſquare of the one* BC, *is greater then the ſquare of the other* AB, *by a given ſpace, alſo each of thoſe lines* AB *and* BC *ſhall be given.*

Demonſtration FOr ſeeing that the ſquare of BC is greater then the ſquare of AB by a given ſpace: Let the given ſpace be taken away, and let the rectangle be conteined under BC and BE: Therefore the remainder, [a] which is the rectangle of BC and CE, is equal to the ſquare of AB. And ſeeing that the rectangle of BC and BE is given, and alſo the ſpace or rectangle AC; the reaſon of the ſaid rectangle of BC and BE to AC is given. But as [b] the rectangle of BC and BE is to the rectangle of AB and BC, ſo is BE to AB: Therefore the reaſon of BE to AB is given, and therefore [c] the reaſon of the ſquare of the ſaid DE to the ſquare of AB is alſo given. But to that ſquare of AB the rectangle of BC and CE is equal: Therefore the reaſon of the ſaid rectangle of BC and CE to the ſquare of BE is given; and therefore the reaſon of the quadruple of the ſaid rectangle of BC and CE to the ſquare of BE is alſo given; and by compounding, [d] the reaſon of four times the rectangle of BC and CE, with the ſquare of BE, to the ſaid ſquare of BE is given. But four times the rectangle of BC and CE, with the ſquare of BE, [e] is the ſquare of the compound line BCE: Therefore the reaſon of the ſquare of that compound line BCE to the ſquare of BE is alſo given; and therefore the reaſon [f] of the compound line BCE to BE is given. Wherefore by compounding,

a) 2. 2.

b) 1. 6.

c) 50. p.

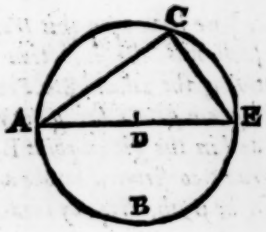
d) 6. p.

e) 8. 2.

f) 54. p.

g) 6. p. pounding, g the reaſon of the ſaid compound line BCE and EB, that is to ſay, twice BC to BE is alſo given; therefore the reaſon of the only line BC to BE is given. But the reaſon of the ſame BE to AB is alſo given: Therefore h) 8. p. h the reaſon of AB to BC is given. And ſeeing that the reaſon of BC to BE i) 1. 6. is given, and that as the ſaid BC is to BE, ſo the ſquare of BC i to the rectangle of BC and BE, the reaſon of the ſquare of BC to the rectangle of BC and BE is alſo given. But the ſaid rectangle of BC and BE is given, it being that which was taken away, and which was given. Therefore k) 2. p. the ſquare of BC k is given, and therefore the line BC is given. But the reaſon of the ſame BC to BA is given, therefore AB is alſo given.

PROP. 88.



If in a circle ABC, *given by magnitude, there be drawn a right line* AC, *which ſhall take away a ſegment* ABC, *which doth comprehend a given angle* AEC, *that line* AC *is given by magnitude.*

Conſtruction FOr let D be the center of the circle; and let the diameter thereof ADE be drawn, and let EC be joyned.

a) 31. 3. *Demonſtration* FOraſmuch as the angle ACE is given, for a it is a right angle. But the angle AEC is alſo given, and therefore the b) 40. p. other angle CAE is given. Wherefore the triangle ACE b is given by kind; and therefore the reaſon of EA to AC is given. But AE is given by magnitude, ſeeing that the circle ABC is given by magnitude. Therefore c) 2. p. c AC is alſo given by magnitude.

PROP. 89.

If in a circle ABC, *given by magnitude, there be drawn a right line* AC, *given by magnitude, that line* AC *will take away a ſegment* ABC, *comprehending a given angle.*

Conſtruction FOr having taken the point D for the center of the circle, Let the diameter ADE be drawn, as alſo the right line EC.

Demonſtration FOraſmuch as each of the right lines AE and AC are given, a) 1. p. the reaſon of the line AE to AC a is given; and the angle b) 43. p. ACE is a right angle: Therefore b the triangle ACE is given by kind, and therefore the angle AEC is given.

PROP. 90.

If in the circumference of a circle ABC *given by poſition, and by magnitude, there be taken a given point* B, *and that from that point* B, *to the circumference of the circle* ABC *there doth bend a right line* BAC, *making a given angle* BAC *the other extremity* C *of the bent line ſhall be given.*

Conſtruction FOr let the center of the circle be D, and let the right lines BD and BC be drawn.

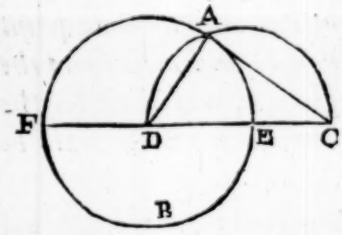
Demon-



Demonſtration FOraſmuch as each point B and D is given, the right line B D, [a] is given by poſition; and ſeeing that the angle B A C is given, the angle B D C is alſo given. Wherefore to the right line B D given by poſition, and in the point D given therein, there is drawn the right line C D; which makes the given angle B D C; and therefore [b] the line D C is given by poſition. But the circle A B C is given by poſition and magnitude: Therefore [c] the right line D C is given by poſition and by magnitude. But the point D is given: Therefore [d] the point C is alſo given.

a) 26. p.
b) 29. p.
c) 6. def.
d) 27. p.

PROP. 91.



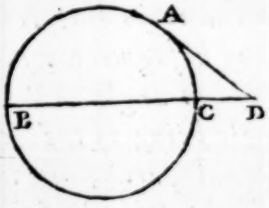
If from a given point C, *there be drawn a right line* C A, *which ſhall touch a circle* A B, *given by poſition; that line* C A *is given by poſition and by magnitude.*

Conſtruction FOr having taken the point D for the center of the circle, let the right lines D A and D C be drawn.

Demonſtration FOraſmuch as each point C and D is given, the right line C D [a] is given by poſition and by magnitude. But the angle C A D [b] is a right angle; and therefore the ſemicircle deſcribed on C D ſhall paſſe by the point A: Let it then paſſe by that point, and let the ſemicircle be D A C: Foraſmuch as the ſame D A C [c] is given by poſition and alſo the circle A B E, [d] the point A is given. But the point C is alſo given: Therefore [e] the right line A C is given by poſition and by magnitude.

a) 26. p.
b) 18. 3.
c) 6. def.
d) 25. p.
e) 26. p.

PROP. 92.



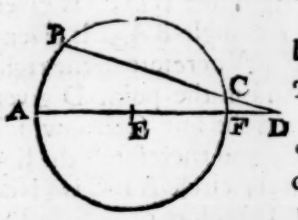
If without a circle A B C, *given by poſition, there be taken ſome point* D, *and from that given point there be drawn a right line* D B, *cutting the circle, the rectangle compriſed under the whole line* B D, *and the part* D C, *between the point* D *and the circumference convex* A C *ſhall be given.*

Conſtruction FOr from the point D let the right line D A be drawn, which ſhall touch the circle in the point A.

Demonſtration THerefore D A [a] is given by poſition and magnitude; and therefore the ſquare of the ſaid D A is [b] given. But the ſaid ſquare of D A is equal [c] to the rectangle of B D and D C: Therefore the ſaid rectangle of B D and D C is alſo given.

a) 91. p.
b) 52. p.
c) 36. 3.

OTHERWISE.



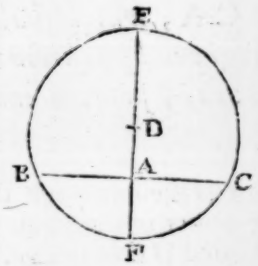
Conſtruction LEt E be the center of the cirele, and by the ſame center let there be drawn from the point D the right line D A.

Demonſtration FOraſmuch as each point D and E is given, the right line D E is [d] given by poſition and by magnitude. But the circle A B C is given by poſition and by magnitude: Therefore each point A and F [e] is given, and the point D is alſo given; and therefore [d] each line A D and F D is given. Wherefore the rectangle of the lines A D and D F is alſo given. But the ſaid rectangle of A D and D F is equal to the rectangle of D B and D C: Therefore the rectangle of D B and D C is given.

d) 26. p.

e) 25. p.

PROP. 93.



If in a circle given by poſition there be taken a given point A, *and by that point* A *there be drawn a right line* B C *to the circle, the rectangle compriſed under the ſegments of the ſame line* B C *ſhall be given.*

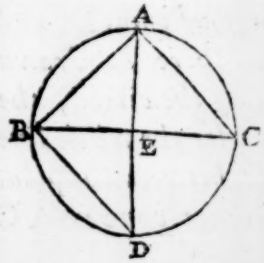
Conſtruction FOr let D be taken for the center of the circle, and having drawn the right line A D prolong it to the points E and F.

Demonſtration FOraſmuch as each point A and D is given, the right line A D [a] is given by poſition. But the circle B E C is alſo given by poſition: Therefore each point E and F is alſo given by poſition, and the point A is given. Wherefore each line [b] A E and A F is given: Therefore the rectangle of the ſame lines A E and A F is given; and is equal to the rectangle [b] of A B and A C: Therefore the ſaid rectangle of A B and A C is given.

a) 26. p.

b) 35. 3.

PROP. 94.



If in a circle A B C, *given by magnitude, there be drawn a right line* B C, *which doth take away a ſegment which doth comprehend a given angle* A B C, *and that the ſaid angle being in the ſegment is cut into two equal parts, the line componnded of the right lines* B A *and* A C, *which comprehend the given angle* B A C *ſhall have a given reaſon to the line* A D, *which doth divide that angle into two equal parts; and the rectangle conteined under the line compounded of thoſe lines* B A *and* A C, *comprehending the given angle* B A C, *and that part* E D *of the interſecting line which is below the ſegment between the baſe* B C *and the circumference, ſhall be given.*

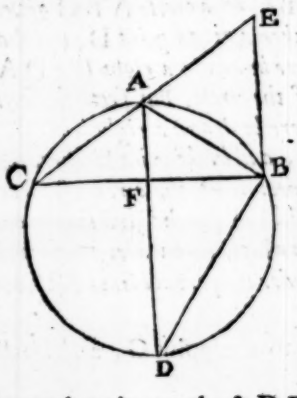
Con-

Construction Let B D be drawn.

Demonstration FOraſmuch as in the circle A B C given by magnitude, there is drawn the right line B C, which takes away the ſegment B A C comprehending the given angle B A C, that line B C [a] is given; and therefore B D is alſo given: Therefore the reaſon of B C to B D [b] is given. And ſeeing that the given angle B A C is cut in two equal parts by the right line A D, as [c] B A is to C A, ſo is B E to C E; and by compounding, as B A C is to C A, ſo is B C to C E; and by permutation, as B A C is to B C, ſo is C A to C E. And ſeeing that the angle B A E is equal to the angle C A E, and the angle A C E [d] to the angle B D E, the other angle A E C is equal to the other angle A B D; and therefore the triangle A C E is equiangled to the triangle A B D: Therefore [e] as A C is to C E, ſo is A D to B D. But as A C is to C E, ſo the line compounded of B A and A C is to B C: Therefore as the compound line B A C is to B C, ſo is A D to B D; and by permutation, as the compound line B A C is to A D, ſo is B C to B D. But the reaſon of B C to B D is given: Therefore the reaſon of the compound line B A C to A D is alſo given. Moreover, I ſay that the rectangle under the compound line B A C and E D is given. For ſeeing that the triangle A E C is equiangled to the triangle B D E, (for the angle A C E [d] is equal to the angle B D E, and the angle A E C [f] to the angle B E D) as B D is to D E, ſo is A C to C E. But as A C is to C E, ſo is alſo the compound line B A C to B C: Therefore as the compound line B A C is to B C, ſo is B D to D E. Wherefore the rectangle of the compound line B A C and D E [g] is equal to the rectangle of B C and B D. But the rectangle of B C and B D is given, (for that thoſe lines B C and B D are given:) Therefore the rectangle under the compound line B A C and E D is alſo given.

a) 88. p.
b) 1. p.
c) 3. 6.
d) 21. 3.
e) 4. 6.
f) 15. 1.
g) 16. 6.

OTHERWISE.



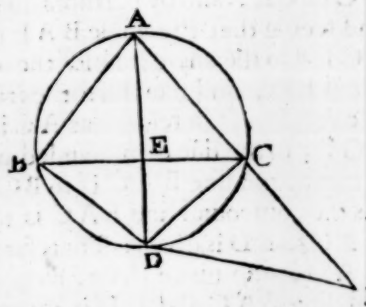
Construction LEt C A be prolonged to the point E, and let A E be put equal to B A, and let B E and B D be joyned.

Demonstration FOraſmuch as the angle B A C is double to each of the angles C A D and A E B (for the angle B A C is cut into two equal parts by the line A D, and equal [h] to the two angles A B E and A E B, which [i] are equal) the angle A B E is equal to the angle C A D, that is to ſay, [k] to the angle C B D; adding therefore the common angle A B C, the whole angle A B D ſhall be equal to the whole angle F B E. But the angle A C B is [k] equal to the angle A D B: Therefore the third angle A E B is equal to the third angle B A D; and therefore the triangle C E B is equiangled to the triangle A B D: Wherefore as C E is to C B, ſo is A D to B D. But the right line C E is compounded of the two lines C A and A B: Therefore as the compound line B A C is to C B, ſo is A D to B D; and by permutation, as the compound line B A C is to A D, ſo is C B to B D. But the reaſon of C B to B D is given, ſeeing that each of thoſe lines is given: Therefore the reaſon of the compound line B A C to A D is alſo given. And ſeeing that the triangle C E B is equiangled to the triangle F B D (for the angle A F C is equal [l] to the angle B F D, and the angle E C B [m] to the angle A D B) as E C is to C B, ſo is B D to D F. But E C is equal to the compound

h) 32. 1.
i) 5. 1.
k) 21. 3.
m) 21. 3.
n) 16. 6.

pound line B A C: Therefore as the compound line B A C is to C B, so is
n) 16.6. B D to D F. Wherefore n the rectangle of the compound line B A C and D F is equal to the rectangle of C B and B D. But the rectangle of C B and B D is given, considering that each of the lines C B and B D is given: Therefore the rectangle of the compound line B A C and D F is given.

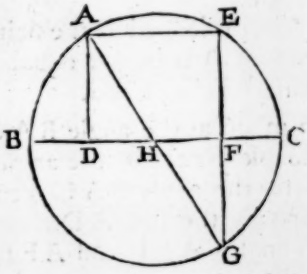
OTHERWISE.



Construction LEt A C be prolonged to F, and let C F be put equal to A B, and let the right lines B D and D F be drawn.

Demonstration FOrasmuch as B A is equal to C F, and o B D
o) 26,29.3. to D C, the two sides A B and B D are equal to the two sides C D and D F, each to his corresponding side, and the angle A B D is equal to the angle D C F,
p) 22.3. 13. 1. p seeing that the four sided figure A B D C is within the circle: Therefore the base A D is q equal to the base D F,
q) 4. 1. and the angle D A B to the angle D F C. But the angle B A D is given, (being the half of the given angle B A C.) Therefore the angle D F C is so also But D A F is also given: Therefore the triangle A D F is given by kind. Wherefore the reason of F A to A D is given. But A F is the compound of B A and A C, for that C F is equal to A B: Therefore the reason of the compound line B A C to A D is given: The same demonstration will serve to shew that the rectangle conteined under the compound line B A C and E D is given also.

PROP. 55.



If in the diameter B C *of a circle* A B C *given by position, there be taken a given point* D, *and that from that point* D *there be drawn a right line* D A, *to the circumference of the circle. But from the section of the said line there be drawn a right line* A E, *perpendicular thereto, and by the point* E *where that perpendicular doth meet with the circumference, there be drawn a parallel* E F, *to the first line drawn* A D, *that point* F *in which the parallel meets with the diameter, is given; and the rectangle conteined under the parallel lines* A D *and* E F *is also given.*

Construction LEt the right line E F be prolonged to the point G, and let the right line A G be drawn.

Demonstration FOrasmuch as the angle A E G is a right angle, the right line A G is the diameter of the circle. But B C is also the diameter: Therefore the point H is the center of the circle. Now the point D is given; and therefore
a) 26. p. *a* the line D H is given by magnitude. But seeing that A D is parallel to E G,
b) 26. p. and A H equal to G H; *b* D H is equal to F H, and A D to F G; (for the angles
c) 15. 1. A H D and F H G *c* are equal, and D A H and F G H *d* are also equal.) But the
d) 29. 1. line D H is given: Therefore F H is also given. But each of those lines D H and
e) 27. p H F is also given by position, and the point H is given: Therefore *e* the point F is also given And seeing that in the circle A B C given by position, is taken the given point F, and through the same is drawn the right line E F G; the rectangle under
f) 93. p. E F and F G *f* is given. But F G is equal to A D. Therefore the rectangle comprehended under A D and E F is given. Which Was to be demonstrated.

THE END OF EUCLIDES DATA.

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TO THE

Moſt Illuſtrious and Moſt Excellent

Franciſcus Maria II:

PRINCE OF

URBIN.

WHen John Dee *of* London, *a man of excellent wit, and ſingular learning, (Moſt Illuſtrious Prince) going away, left with me this little Book of the* Diviſions of Superficies, *as a token or teſtimony of his affection towards me, he added, that nothing could happen more welcome to him from me, than that it might by my means come into the hands of thoſe that are deſirous of learning, eſpecially of the* Mathematicks. *Therefore I being moved both with the honeſt minde and intent of the friendly and moſt learned man, and alſo much taken with the marvelous benefit of the Book, for that I knew there was not any ſuch thing extant among us, I have now moſt willingly endeavoured to ſatisfie his deſire, and as he requeſted me, have not ſuffered this Treatiſe to conſiſt in a Pentagonal diviſion: for what things the Author of the Book hath at large comprehended in many Problems, I have compen-*

 diouſly

dionſly compriſed and diſpatched in two only; yet ſo, as that from them it may clearly appear, in what ſort thoſe ſections might in other Figures be infinitely produced. In which thing truly, (if I miſtake not) very profitable, I have both obeyed the requeſt of a Friend, and alſo promoted their Studies, who are delighted with this moſt Noble and Excellent kinde of Learning. For it can hardly be conceived, how great a help and Ornament, this attempt will bring to him, that endeavours to be a Geometrician, *ſo as he refuſe not to beſtow good pains therein. Therefore this product of common induſtry (ſuch as it is) I reſolved ſhould come forth adorned with your moſt Excellent Name, until I carefully trim up for you ſome more conſiderable teſtimonies of my engagements to you; both for that I have conſecrated all my endeavours to your bounty (to which I am exceedingly obliged) and alſo becauſe I conceived that would be moſt acceptable to Dr.* Dee, *who by the renown of your famous Court was drawn hither, with the pains of a tedious Journey. Farewell, and be pleaſed courteouſly to continue in your defending and cheriſhing of Learning and its Friends.*

Fredericus Commandinus.

TO

TO
FREDERIC COMMANDINE
OF
URBIN.
Iohn Dee of London
Wiſheth Health and all Happineſſe.



HAving now for many years ſet my ſelfe chiefly (my moſt learned *Frederick*) how to preſerve from utter ruine, the moſt famous moniments of our Anceſtors (ſuch as I could) in all the more curious or elegant kinds of Philoſophy; leaſt that either ſo worthy men ſhould be robbed of their due renown, or we longer want the moſt abundant benefit of ſuch Books. I ſay that I ſo beſtowing my pains among other moſt ancient writings of Philoſophers, did at length happen upon this ſmall Book, written, indeed, in a very blinde ilfavoured Character, and alſo by reaſon of its age, hard to be read. But that I might the better ſee, I uſed helps to my ſight: and by often ſtudy and exerciſe therein, I got the knack of reading it. Whereupon being hereby better perſwaded of the excellencie and worth of the Book, I earneſtly wiſhed that the ſame might forthwith be communicated to the Society of Philoſophers. But while I was pondering this in my minde, I found none more worthy than your ſelf (my *Commandine*) in this our age, to enjoy theſe our Labours; who have alſo your own ſelfe revived certain moſt excellent Works of *Archimedes*

and *Ptolemee* almoſt loſt, and have brought them forth into pnblick view in a moſt magnificent dreſſe. Therefore this little Book, as a perpetual pledge, even of the affection wherewith I ever embrace you, I commit to you and your truſt; and do earneſtly beſeech you, that you will not ſuffer this our common labour to go forth into the World deſtitute of that adornment wherewith you are wont to ſend abroad others. Yea, I ſurely hope (if I well know you and your endeavours) that you will ſome time or other ſo enrich this ſubject, as that you will neither permit it to reſt in a Pentagonal form; not ſuffer the *Solids* themſelves long to want the like Sections by Plains. Verily, if you would but a little put forward theſe things, they will by themſelves go on to the other kindes of Superficies. But that they may be applyed to *Solids* they will require your ſound knowledge, and more than ordinary pains in the Mathematicks. As for the Authors name, I would have you underſtand, that to the very old Copy from whence I writ it, the name of *MACHOMET BAGDEDINE* was put in Ziphers or Characters, (as they call them) who whether he were that *Albategnus* whom *Copernicus* often cites as a very conſiderable Author in Aſtronomie; or that *Machomet* who is ſaid to have been *Alkindus*'s Scholar, and is reported to have written ſomewhat of the art of Demonſtration, I am not yet certain of: or rather that this may be deemed a Book of our *Euclide*, all whoſe Books were long ſince turned out of the Greeke into the Syriack and Arabick Tongues. Whereupon, It being found ſometime or other to want its Title with the *Arabians* or *Syrians*, was eaſily attributed by the tranſcribers to that moſt famous Mathematician among them, *Machomet*: Which I am able to prove by many teſtimonies, to be often done in many Moniments of the Ancients; and certain friends of mine doe know (that I may bring one of many) that we have by this means reſtored one ſmall Book, incomparable in occult and myſticall Philoſophie, of that moſt ancient and excellent Philoſopher *Anaxagoras*, every where row through many ages enobled with the name of *Ariſtotle* to *Anaxagoras* himſelfe, and that by moſt ſure proofs. Yea further, we could not yet perceive ſo great acuteneſſe of any *Machomet* in the Mathematicks, from their moniments which we enjoy, aſeverywhere

rywhere appears in theſe Problems. Moreover, that *Euclid* alſo himſelfe wrote one Book περὶ διαιρέσεων, that is to ſay, *of Diviſions*, as may be evidenced from *Proclus*'s Commentaries upon his firſt of *Elements*: and we know none other extant under this title, nor can we finde any, which for the excellencie of its treatment, may more rightfully or worthily be aſcribed to *Euclid*. Finally, I remember that in a certain very ancient piece of Geometry, I have read a place cited out of this little Book in expreſſe words, even as from amoſt certain work of *Euclid*. Therefore we have thus briefly declared our opinions for the preſent, which we deſire may carry with them ſo much weight, as they have truth in them. And if any man ſhall object to me, that that title of Diviſions doth not denote the ſections of Magnitudes into their parts, but the diviſions of *Genus's* by their differences into *Species*, like as the methodicall diviſions of Points, Lines, Angles, Figures, and the like; ſuch as we have in our demonſtrated work of Mathematicall *Acribologie* ſhewed above five huudred: truly I confeſſe, that this alſo may in probability be ſaid; but yet how truly, is not more manifeſt yet to me, than our conjecture is to him. But whatſoever that Book of *Euclid* was concerning Diviſions, certainly this is ſuch an one as may be both very profitable for the ſtudies of many, and alſo bring much honour and renown to every moſt noble ancient Mathematician; for the moſt excellent acuteneſſe of the invention, and the moſt accurate diſcuſſing of all the Caſes in each Probleme: and ſo much for this, I will now direct my diſcourſe to you, who are herein to be greatly intreated by me, that you would with all poſſible diligence advance your moſt weighty and uſefull labours, which you did yeſterday moſt courteouſly ſhew to me in your Study. For ſo you will make the faireſt way to perpetuate your fame, who have in ſo few years, put forth many Books, ſo happily, ſo neatly, and ſo many of your own: who alone in our time doſt adorn the moſt excellent Princes of Mathematicians, *Archimedes*, *Apollonius*, and *Ptolomeus*, with their due luſtre. So you will reſtore a new and wonderfull livelyneſſe to Mathematicall Learning much decayed: and ſo you ſhall now at laſt oblige me wholly to your ſelfe by many engagements. And I intreat you, that ſo ſoon as this

this Book is printed, you will be pleaſed to ſend a copie or two thereof to the moſt Noble man, and ſingular patron of all good Arts, and ſpecially of the Mathematicall, Sir *William Pickering* Knight, my exceeding great friend, living at *London*. For then they will moſt readily be conveyed to my Library. Now my purpoſe of Travell calls me away, leſt I ſhould be put to undergo a greater trouble of this ſcorching Seaſon round about us, before I can ſhelter my ſelfe in the Roman ſhades. Farewel, therefore, the honour and renown of Mathematicians; farewel my moſt courteous *Commandine*. And I very earneſtly beſeech the great God, that he will vouchſafe to bring your excellent attempts by his ſpecial favour, to their deſired ends.

URBIN.

TO THE READER.

I Am here to advertiſe thee (kinde Reader) that this Author which we preſent to thee, made uſe of Euclid *tranſlated into the Arabick Tongue, whom afterwards* Campanus *made to ſpeake Latine. This I thought fit to tell thee, that ſo in ſearching or examining the Propoſitions which are cited by him, thou mighteſt not ſometime or other trouble thy ſelfe in vain,* FAREWEL.

A BOOK OF THE DIVISION OF SUPERFICIES.

PROP. 1. PROBL. 1.

By a line drawn from an angle of a triangle, to divide that triangle according to a proportion given.



LEt the triangle be A B C, and let it be required to divide the same by a line descending from the angle A in proporiton as E to F.

Divide B C in D in proportion as E to F; and [a] having drawn the line A D, that line A D divides the triangle as is required [b].

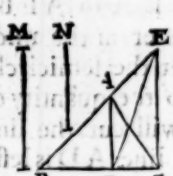
a) 12. 6.

b) 1. 6.

PROP. 2. PROB. 2.

To divide a triangle given according to a proportion given, by a line drawn from a point assigned in the side of the triangle.

LEt the triangle be A B C, in whose side B C let the point D be assigned, from whence it is required to draw a line dividing the triangle according to the proportion of M to N, draw D A. From that end or extream of B C, towards which I would have the consequent term of the division, which for Examples sake let be the point C. Draw a parallel to D A till it meet in the point E with the line B A extended (that they will meet is manifest [a].) The proportion of M to N will be either equal to the proportion of B A to A E, or greater, or lesser. Let first the proportion be equal, therefore [b] the proportion of the triangle B A D to the triangle A D E, is as the proportion of M to N. But [c] the triangle A D E is equal to the triangle A D C: Therefore [d] the proportion of the triangle A B D to the triangle A D C, is as the proportion of M to N. Which was to be proved.



a) 29. 17. 1.

b) 1. 6.

c) 37. 1.

d) 7. 5.

Secondly, Let the proportion of M to N be lesser then the proportion of the line B A to the line A E; therefore divide the line B E according to the proportion of M to N, and let the division fall between B and A [e], it will fall in the point F; and let the line D F be drawn; which line will I say

e) 8. 5.



ſay divide the triangle according to the proportion of M to N.

Demonſtr. For having drawn the line D E, the triangle A D E [f] ſhall be equal to the triangle A D C. Putting therefore the triangle A F D common to both, the triangle F D E will be equal to the quadrilateral figure A F D C. Seeing that [g] the triangle B F D is to the triangle F E D, as B F is to F E; and by conſequence, as M to N; then alſo the triangle B F D is to the quadrangle A F D C as M to N, and the Propoſition is manifeſt.

f) 37. 1.

g) 1. 6.

Thirdly, Let the proportion of M to N be greater then the proportion of B A to A E. Divide therefore B E in the point F, which will be between A and E, according to the proportion of M to N; and draw F G parallel to C E, until it meet with the line A C in the point G; then joyn G D, I ſay the line G D will divide the triangle according to the given proportion.

For draw the lines D F and D E, then is the triangle A D E equal to the triangle A D C, and [h] the triangle A D F is equal to the triangle A D G; therefore [i] the remaining angles, to wit, the triangle F D E and the triangle G D C are alſo equal.

h) 37. 1.

i) 37. 1.



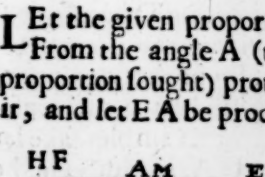
Putting alſo the triangle A B D common to both the equal triangles A F D and A G D, the triangle B F D will be equal to the quadrilateral figure B A G D. Therefore the triangle F B D is to the triangle F D E, as the quadrilateral figure B A G D is to the triangle G C D. But the triangle F B D is to the triangle F D E, as M is to N, by ſuppoſition [k]. Therefore the proportion of the quadrilateral figure B A G D to the triangle G D C is as the proportion of M to N, as was propoſed.

k) 1. 6.

PROP. 3. PROBL. 3.

By a line equidiſtant to a ſide aſſigned of a known triangle, to divide that triangle according to a proportion given.

LEt the given proportion be H K to K L, and the triangle A B C, &c. From the angle A (towards which I would have the antecedent in the proportion ſought) protract a line A E perpendicular to C A, and equal to it, and let E A be produced to F, until the proportion of E A to A F be as H L to H K, and making a center in the middeſt of the line F E, as at M, let the ſemicircle F D E be deſcribed, according to the quantity of the line M E, which ſemicircle will cut the line A C in the point D; becauſe the line A D is leſſe then the line A E, and the line A E is equal to the line A C; drawing then D G parallel to B C: I ſay that the proportion of the triangle A G D to the ſuperficies G B C D, is according to the proportion of H K to K L.



Demonſt. For the proportion of the triangle A B C to the triangle A G D, is as A C to A D, a duplicate proportion [a]. But A C is equal to A E; therefore the proportion of the triangle A B C to the triangle A G D, is as the proportion of A E to A D, duplicate; for the proportion of A E to A D duplicate, is as the proportion of A E to A F [b]. Therefore

a) 17. 6.

b) 30. 3.

fore the proportion of the triangle A B C to the triangle A G D, is as the proportion of E A to A F. But the proportion of E A to A F is as H L to H K: Therefore the proportion of A B C to A G D is as L H to H K: Therefore severally the proportion of the superficies G B C D to the triangle A G D, is as L K to K H: Therefore contrarily A G D to G B C D, is in proportion as H K to K L. Which was to be proved. 8. 6.

PROP. 4. PROBL. 4.

By a line parallel to a perpendicular line A D, *drawn from an angle of a triangle to the base, to divide the triangle according to a proportion given.*

LEt the given proportion be K L to L M, and according thereto I would divide the triangle A B C by a line parallel to the perpendicular A D. Divide the line K M according to the proportion of the line B D to the line D C. As for example, in the first place, Let the division fall in L, the proportion then of K L to L M is as B D to D C; and by consequence, as of the triangle A B D to the triangle A D C [a]: Therefore the line A D divideth the triangle according to the given proportion. a) 1. 6.



Secondly, Let the proportion of K G to G M, be as the proportion of B D to D C, so as that G be betwixt L and M: I will then divide the triangle A B D accordingly by a line F E, parallel to the side A D; according to the proportion of K L to L G: I say therefore that the proportion of the triangle F B E to the superficies A F E C, is as the proportion K L to L M.



Demonstr. For the proportion of the triangle A D C to the triangle A B D, is as the proportion of M G to G K. Therefore [b] joyntly, the proportion of the triangle A B C to the triangle A B D, is as the proportion of M K to K G. But the proportion of the triangle A B D to the triangle F B E, is as the proportion of K G to K L: Therefore according to equal proportionality, [c] the proportion of the triangle A B C to the triangle F B E, will be as the proportion of M K to K L: Therefore severally the proportion of the superficies A F E C to the triangle F B E, is as the proportion of M L to K L: Therefore on the contrary, the proportion of K L to L M, is as the proportion of the triangle F B E to the superficies A F E C. Which was to be proved. b) 18. 5. c) 22. 5.

Thirdly, Let the proportion of K H to H M, be as the proportion of B D to D C; and let H fall betwixt K and L, then divide (by the aforesaid) the triangle A D C according to the proportion of H L to L M, by the line N O parallel to the side A D: I say then that the proportion of the superficies N A B O to the triangle N O C, is as the proportion of K L to L M.



Demonst. For the proportion of the triangle A B D to the triangle A D C, is as K H to H M [d]. Therefore joyntly, [e] the proportion of the triangle A B C to the triangle A D C is as the proportion of K M to H M. But the proportion of the triangle A D C to the triangle N O C, is as the d) 1. 6. & 11. 5. e) 18. 5.

proportion of HM to LM: Therefore according to equal proportionality, the proportion of the triangle ABC to the triangle NOC, is as the proportion of KM to LM: Therefore ſeverally the proportion of the ſuperficies NABO to the triangle NOC, is as the proportion of KL to LM, which was required.

PROP. 5. PROBL. 5.

To divide a known triangle according to a proportion given by a line parallel to a line drawn from an angle thereof, which is neither parallel to any of the ſides, nor any of its perpendiculars.

THis concluſion might be proved as the former: It may alſo be thus otherwiſe proved.

First, Let the proportion given be as M to N, and let the triangle be ABC, which I would divide according to the proportion of M to N, by a line parallel to the line AD, which let fall from the angle A, and be neither perpendicular nor parallel to any ſide of the triangle: Therefore I divide BC according to the proportion of M to N; and let the diviſion fall for example ſake in D: I ſay that AD [a] doth divide the triangle according to the proportion of M to N.



a) 1. 6.

Secondly, Let the diviſion fall twixt B and D, in the point E, ſo as that the proportion of BE to EC be as the proportion of M to N: Then I put the line BF a mean proportional between BD and BE, and drawing FG parallel to AD: I ſay that line divideth the triangle according to the Propoſition.

Demonſtr. I draw the line AE: Therefore the proportion of the triangle ABD to the triangle GBF, is as BD to BF, a double proportion [b]; and therefore is as the proportion of BD to BE. But according to the proportion of BD to BE, is the proportion of the triangle ABD to the triangle ABE: Therefore the proportion is the ſame of the triangle ABD to the triangle GBF, and to the triangle ABE. Therefore the triangles GBF and ABE are equal: Therefore placing H in the ſection of the lines AE and GF; it is manifeſt that the triangles AGH and EFH are equal, to which adding the ſuperficies AHFC, the triangle AEC will be equal to the ſuperficies AGFC; there is therefore the ſame proportion of the triangle ABE to the triangle ABC, as of the triangle BFG to the ſuperficies AGFC. But the proportion of the triangle ABE to the triangle AEC is as the proportion of M to N given: Therefore the Propoſition is manifeſt.

b) 17. 6.



Let thirdly, the diviſion fall between D and C in the point E, ſo as that the proportion of BE to EC may be as M to N: I will then put CK a mean proportional between DC and EC: Then drawing KL parallel to AD: I ſay KL divides the triangle as was required.



For as before, the proportion of the triangle ADC to the triangle LKC, is as the proportion of DC to KC doubled;

doubled; and by conſequence, is as the proportion of DC to EC; and according to the ſame proportion is the triangle ADC to AEC: Therefore the triangles LKC and AEC are equal. Wherefore alſo the triangles AHL and KHE are equal: Therefore the ſuperfices LABK is equal to the triangle ABE: Therefore there is the ſame proportion of the ſuperficies LABK to the triangle LKC, as of the triangle ABE to the triangle AEC. But that proportion is as M to N: Therefore the propoſition is cleer.

¶ NOTE that in this manner the former concluſion may be proved; and this proof is eaſier then the former.

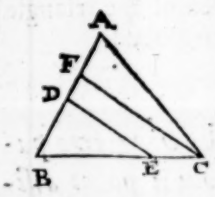
PROP. 6. PROBL. 6.

To divide a known triangle ABC, *according to a proportion given, by a line parallel to any line drawn therein, whether the ſaid line be drawn from an angle or not.*

FIrſt, if the line given be parallel to any ſide of the triangle, then work according to the third Probl. of this Book.

But ſecondly, if the given line deſcend from any angle, the Propoſition will be had by the aforeſaid.

But thirdly, if the aſſigned line neither deſcend from any angle of the triangle, nor is parallel to any ſide, as in the triangle ABC, let the line DE be aſſigned, which is not parallel to AC; but would meet with it on that ſide that C is, if it were drawn out at length, then from the angle on that ſide where that meeting would be, as here from the angle C, let CF be drawn parallel to the line aſſigned, *viz.* DE: Then (by the aforeſaid) let the triangle be divided by a line parallel to CF, according to the proportion given: It is manifeſt [a] that it is then divided by a line parallel to DE, and ſo the Propoſition is manifeſt.

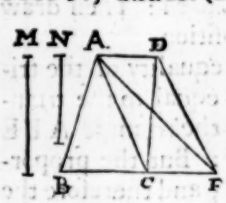


a) 30. 1.

PROP. 7. PROBL. 7.

To divide a known Quadrangle ABCD, *by a line drawn from an angle thereof* A, *according to a proportion given, as* M *to* N.

PRotract the diameter AC, and from the point D protract DF, parallel to AC, till it meet with BC in the point F: Then divide BF as M to N; and let (in the firſt place) the diviſion fall in C, ſo that there be the ſame proportion of BC to CF, as of M to N: I ſay that AC performes the Propoſition.



Demonſtr. For the triangle ADC is equal to the triangle AFC [a]. But the proportion of the triangle ABC to the triangle ACF is as M to N [b]. Therefore the proportion of the triangle ABC to the triangle ACD, is as the proportion of M to N.

a) 37. 1.
b) 1. 6.

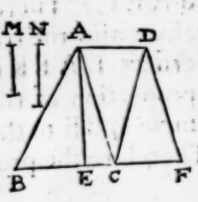
Secondly, Let the diviſion fall in E, between B and C; ſo as the proportion of BE to EF be as M to N: Then drawing AE, I ſay the proportion of the triangle ABE to the ſupeficies AECD, is as M to N.

De-

Demonſtr. Protracting A F, the triangle A D C will be equal to the triangle A F C [c]; and putting the triangle A E C common to both, the ſuperficies A E C D will be equal to the triangle A E F: Therefore there is the ſame proportion of the triangle A B E to the ſuperficies A E C D, and to the triangle A E F. Since therefore [d] the proportion of the triangle A B E to the triangle A E F, is as M to N; 'tis manifeſt that the proportion of the triangle A B E to the ſuperficies A E C D, is as M to N. Which was to be proved.

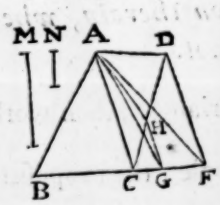
c) 37. 1.

d) 1. 6.



Thirdly, Let the diviſion fall betwixt C and F in G, ſo as that B G be to G F, as M to N: Draw G H parallel to D F, till it meet with D C in H: Then draw A H, ſo is the proportion of the ſuperficies A B C H to the triangle A D H, as M to N.

Demonſtr. Let A G be produced: Therefore the triangle A H C is equal to the triangle A G C. But alſo the whole triangle A D C is equal to the whole triangle A F C: Therefore the reſidue A D H is equal to the reſidue A F G. Putting therefore the triangle A B C common to both, the triangles A C H and A C G ſhall be equal; and the ſuperficies A B C H will be equal to the triangle A B G, the proportion therefore of the ſuperficies A B C H to the triangle A D H, is as the proportion of the triangle A B G to the triangle A G F. But the proportion of the triangle A B G to the triangle A G F, is as M to N, as hath been proved.



PROP. 8. PROBL. 8.

To divide a known quadrangle A B C D, *having two parallel ſides* A D *and* B C, *by a line drawn from a point aſſigned* E, *in one of the parallel ſides, according to a proportion given, viz. as* L *to* M.

EXtend B C to F, ſo that C F be equal to A D, and let A F cut the line D C in the point G; then [a] the triangles A D G and G C F are alike, and the ſides A D and C F are equal; and therefore thoſe triangles are equal one to another. Adding therefore A B C G, which is common to both, it is manifeſt that the Quadrangle A B C D is equal to the triangle A B F.

a) 37. 1.



Then divide B F according to the proportion of L to M, and let the diviſion fall in E; ſo as that the proportion of B E to B F, may be as L to M: Then draw E A, which performeth the Propoſition.

Demonſtr. For by reaſon of the equality of the triangles A D G and C G F, the ſuperficies A E C D is equal to the triangle A E F: Therefore there is the ſame proportion of the triangle A B E to the ſuperficies A E C D, and to the triangle A E F: But the proportion of A B E to A E F, is as the proportion of L to M; and therefore the proportion of A B E to the reſidue of the Quadrangle, is as the proportion of L to M. Which was propoſed.

Secondly, Let the diviſion fall between B and E in H, ſo as that B H be to B F, as L to M: Then draw H K parallel to A E, dividing A B in K:

Then

Then draw KE: I ſay that line KE divides the Quadrangle according to the Propoſition.

Demonſtr. For I will draw AH, and becauſe AE and KH are parallels, the triangles KAH and KEH ſhall be equal: Therefore adding KBH to them both, the triangle ABH will be equal to the triangle KBE.



But the triangle AKE is equal to the triangle AHE; Therefore adding AECD common to both, the ſuperficies AKECD will be equal to the Quadrangle AHCD. But the Quadrangle AHCD is equal to the triangle AHF, as is before ſhewn: Therefore there is the ſame proportion of the triangle KBE to the ſuperficies AKECD, as of the triangle ABH to the triangle AHF, and by conſequence as of L to M. Which was to be proved.

Thirdly, Let the diviſion fall between E and F, and having made the figure, I will cut off from EF a line EP, equal to DA: Then I will divide the line BF in proportion as L to M.



And firſt of all, let the diviſion fall in P, ſo as that the proportion of BP be to PF as L to M. Draw ED, which I ſay doth divide the Quadrangle according to the Propoſition.

Demonſtr. For draw the line PA, and becauſe EP is equal to AD, and parallel unto it, the triangle ADE will be equal to the triangle APE. Putting therefore the triangle ABE common, the Quadrangle ABED will be equal to the triangle ABP; and by conſequence, the remaining triangle DEC will be equal to the remaining triangle APF; becauſe (as hath been before proved) the Quadrangle ABCD is equal to the triangle ABF: Therefore the ſame proportion is of the Quadrangle ABED to the triangle DEC, as of the triangle ABP to the triangle APF [b]. But the proportion of ABP to APF is as L to M: Therefore the proportion of ABED to DEC, is as L to M. Which was to be proved.

b) 19. 5.

Secondly, Let the diviſion fall between E and P in the point Q, ſo as that the proportion of BQ to QF may be as L to M. From AD cut off AK, equal to EQ: Then draw ER, I ſay that that line divides the Quadrangle, according to the Propoſition.

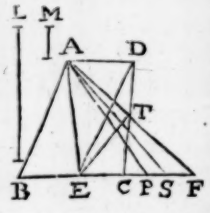


Demonſtr. Draw AQ, and becauſe the lines AR and EQ are equal and parallels, the triangles ARE and AQE are equal, to which adde the triangle ABE common, the Quadrangle ABER will be equal to the triangle ABQ. But it is proved that the whole Quadrangle ABCD was equal to the triangle ABF: Therefore the remaining Quadrangle RECD is equal to the remaining triangle AQF: Therefore there is the ſame proportion of the Quadrangle ABER to the Quadrangle RECD, as of the triangle ABQ to the triangle AQF; and by conſequence as L to M. Which was propoſed.

Thirdly, Let the diviſion fall between P and F in S, ſo as that the proportion of BS to SF, may be as L to M. Divide DC in T, according to the proportion of PS to SF; and draw ET: I ſay that that line divideth the Quadrangle according to the Propoſition.

Demonſtr. Draw AS, becauſe therefore the lines AD and EP are equal

and parallels, the triangles A D E and A P E are equal, and by conſequence, adding the common triangle A B E, the Quadrangle A B E D is equal to the triangle A B P. But the whole Quadrangle A B C D is equal to the whole triangle A B F; Therefore the triangle D E C is equal to the triangle P A F. But the proportion of the triangle D E T to the triangle T E C, is as P A S to S A F: Therefore the triangle D E T is equal to the triangle P A S, and the triangle T E C equal to the triangle S A F. It was proved that the Quadrangle A B E D is equal to the triangle A B P. Adding the triangle D E T to the firſt, and the triangle P A S equal to it, to the ſecond, the Pentagon A B E T D will be equal to the triangle A B S, But it was proved that the triangles T E C and S A F are equal: Therefore there is the ſame proportion of the Pentagon A B E T D to the triangle T E C, as of the triangle A B S to the triangle A S F; and by conſequence, as L to M. Which was propoſed.



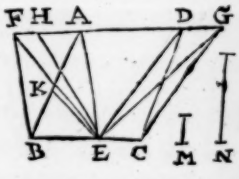
PROP. 9. PROB. 9.

To divide any known Quadrangle A B C D, *according to any proportion given* M *to* N, *by a line drawn from a point* E, *in one of the ſides aſſigned* B C, *not parallel.*

DRaw A E and E D, and extend D A at length on both ſides, then draw B F parallel to E A, to cut D A extended in F. Likewiſe draw D G parallel to E D; cutting D A extended in G, then divide F G in proportion as M to N.

And firſt let the diviſion fall between F and A in the point H, ſo as that the proportion of F A to H G, may be as M to N.

Divide alſo B A in proportion as F H to H A, and let the diviſion be in the point K, ſo as that the proportion of B K to K A, may be as F H to to H A: Then drawing K E, you have divided the Quadrangle as was required.



Demonſt. For having drawn E F and E G, the triangle A F E will be equal to the triangle A B E, a) 37. 1. and the triangle D G E equal to the triangle D C E. Then adding to both, the triangle A E D; the triangle F E G will be equal to the Quadrangle propoſed A B C D. And becauſe the triangle A F E is equal to the triangle A B E; and that there is the ſame b) 1. 6. proportion of F H to H A, as of B K to K A: Therefore b the triangle E H F is equal to the triangle E K B: Therefore alſo the reſidue is equal to the reſidue: Therefore the remaining triangle H E G is equal to the Pentagon A K E C D. There is therefore the ſame proportion of the triangle E K B to the Pentagon A K E C D, as of the triangle E H F to the triangle E G H; and therefore alſo as the line F H to the line H G; and by conſequence as M to N. Which was to be proved.

Secondly, Let the diviſion fall in the point A, ſo as that the proportion of F A to A G may be as M to N, then drawing E A: I ſay that that line divideth the Quadrangle according to the Propoſition.

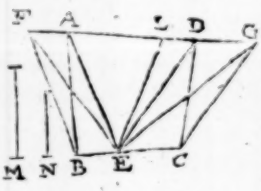
Demonſtr. For the triangle A F E is equal to the triangle A B E: Therefore the triangle A E G the reſidue, is equal to the Quadrangle A E

C D



C D the reſidue: Therefore there is the ſame proportion of the triangle A B E to the Quadrangle A E C D, as of the triangle A F E to the triaegle A E G. Therefore as the line F A to the line A G; and by conſequence, as M to N: As hath been proved.

Thirdly, Let the diviſion fall bewteen A and D in the point L, ſo as that the proportion of F L to L G, may be as M to N: Then I ſay that the line E L divides the Quadrangle, as is required.

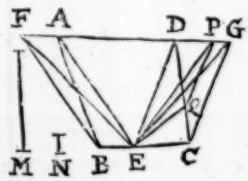


Demonſtr. For ſeeing that the triangles A F E and A B E are equal; and that if to both be added the triangle L E A: Then the triangle L F E will be equal to the Quadrangle A B E L: Therefore the triangle L E G the reſidue, is equal to the Quadrangle L E C D, alſo the reſidue: Therefore the Quadrangle A B E L is in proportion to the Quadrangle L E C D, as the triangle L F E to the triangle L E G; and by conſequence as M to N, as hath been proved.

Fourthly, Let the diviſion fall in the point D: Then becauſe the triangle D G F is equal to the triangle D C E, the triangle remaining D F E, will be equal to the remaining Quadrangle D A B E: Therefore the proportion of the Quadrangle A B E D, to the triangle D E C, is as the triangle D F E to the triangle D F G; and therefore as the line F D to the line D G; and by conſequence as M to N: Therefore D E divides the Quadrangle, as is required.



Fifthly, Let the diviſion fall between D and G, in the point P; ſo as that the proportion of F P to P G, may be as M to N: From P draw a line P Q, parallel to C G, until it meet with C D in Q; then having drawn E Q, the Quadrangle is divided as is required.



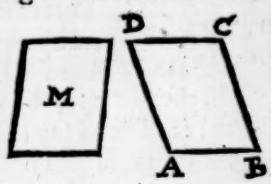
Demonſtr. For draw P E, then the triangle D E P will be equal to the triangle D E Q[c]: Putting therefore the triangle A E D common, the triangle will be equal to the Quadrangle A E Q D; alſo the triangle A F E will be equal to the triangle A B E: Therefore the triangle F E P is equal to the Pentagon A B E G D: Therefore the triangle P E G reſidue, is equal to the triangle Q E C, alſo reſidue: Therefore the proportion of the Pentagon A B E Q D, to the triangle Q E C, is as the triangle F E P to the triangle F E G: Therefore as F P to P G; and by conſequence as M to N, as was propoſed. c) 37. 1.

PROP. 10. PROBL. 10.

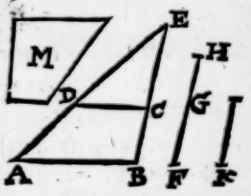
***On** a known line* A B, *and two lines* A D *and* B C, ***drawn** at any angle at pleaſure with it, and on the ſame ſide: **To make** a ſuperficies* A B D C *equal to a ſuperficies given* M, *ſo that the ſaid ſuperficies ſhall be included between the lines* A D *and* B C, *and between* A B *and a line* D C, *parallel to* A B.

The

THe two angles D A B and C B A are either equal to two right angles, or greater or leſſer: Let them in the firſt place be equal to two right
a) 44. 1. angles: Therefore A D will be parallel to B C. Then [a] upon A B make a ſuperficies of parallel ſides, whoſe angle let be equal to the angles D A B and C B A; and let the ſuperficies be equal to the ſuperficies M; and the Propoſition is manifeſt.

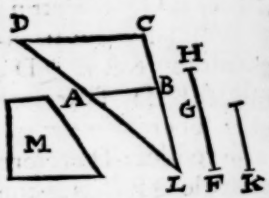


Secondly, Let the two angles D A B and C B A be leſſe then two right angles: Therefore A D and B C will meet on that ſide that C D is, to wit, in the point E. Unleſſe therefore the triangle E A B ſhall be found greater then the ſuperficies M: It is not poſſible on that ſide that D C is to conſtitute the ſuperficies required. Let therefore the triangle E A B be greater then the ſuperficies M; and let the proportion of the triangle E A B to the ſuperficies M, be as F H to F G; and let K be a mean proportional between F H and G H. Then cut off from E B the line E C, which let be in proportion to E B, as K is to F H: Then drawing C D parallel to B A: I ſay that the ſuperficies A B C D is equal to the ſuperficies M.



b) 17. 6. *Demonſtr.* For [b] the triangle B A E is in proportion to the triangle C D E, as B E to C E doubled; and therefore the proportion of F H to K doubled; and by conſequence, the proportion of the triangle B A E to the triangle C D E, is as F H to G H: Therefore the everſe proportion of the triangle B A E is to the Quadrangle B A D C, as F H is to F G. But F H is in proportion to F G, as the triangle B A E to the ſuperficies M. Therefore there is the ſame proportion of the triangle B A E to the ſuperficies M, and alſo to the Quadrangle B A D C: Wherefore the ſuperficies M and the Quadrangle B A C D, are equal. Which is the thing deſired.

Thirdly, Let the angles D A B and C B A be greater then two right angles, they will then meet on that ſide that A B is; and let it be in the point E: Then let the proportion of G H to G F be put as the proportion of the triangle A B E to the ſuperficies M; and let K be a mean proportional between F H and G H; and let the proportion of E C to E B, be as F H to K: Then drawing C D parallel to A B: I ſay the ſuperficies M is equal to the quadrangle A B C D.



Demonſtr. For the proportion of the triangle C D E to the triangle B A E is (as is before ſhewn) as the proportion of F H to G H: Therefore the everſe proportion of the triangle C D E is to the quadrangle C D A B as F H to F G: Therefore the disjunct proportion of the triangle A B E is in proportion to the quadrangle A B C D, as G H to G F; and by conſequence, as the ſame triangle A B E to the ſuperficies M: Therefore the quadrangle A B C D, and the ſuperficies M are equal. Which was to be demonſtrated.

PROP. 11. PROBL. 11.

To divide a Quadrangle A B C D, *of parallel ſides, by a line*

line F E, *parallel to one of its ſides* A B, *according to the proportion of* G *to* H.



DIvide the line B C in the point E, according to the proportion of G to H; and draw E F parallel to A B; and the Propoſition is done.

Demonſtr. For [a] there is the ſame proportion of the quadrangle A B E F to the quadrangle F E C D, as of the line B E to the line E C; and by conſequence, as of G to H. As was required. a) 1. 6.

PROP. 12. PROBL. 12.

To divide a Quadrangle A B C D, *having only two ſides parallel,* A D *and* B C, *by a line* G F, *parallel to* A D *and* B C, *according to a given proportion,* M *to* N.

FIrſt, The ſides A B and D C, muſt neceſſarily meet in the point E, and let the proportion of H O to L O, be as the triangle D A E to the triangle C B E: Therefore by converſion and diviſion, the proportion of the triangle C B E will be to the quadrangle D A B C, as L O to L H. Then divide K L in K, ſo as that the proportion of H K to K L may be as M to N; and let P be a mean proportional between K O and O L; and let the proportion of F E to C E, be as K O to P: Then draw F G parallel to D A. I ſay that line doth divide the quadrangle, as was required.



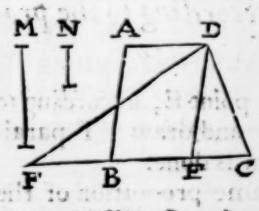
Demonſtr. For the proportion of the triangle F G E to the triangle C B E, is as F E to C E double proportion: Therefore alſo as K O to P double proportion. And by conſequence, the proportion of the triangle F G E to the triangle C B E, is as K O to L O: Therefore the everſe proportion of the quadrangle F G B C is in proportion to the triangle C B E, as K L to L O. But the triangle C B E is in proportion to the quadrangle A B C D (as is afore ſhewn) as L O to L H: Therefore by equal proportionality, the quadrangle F G B C is in proportion to the quadrangle A B C D, as K L to L H: Therefore the disjunct proportion of the quadrangle F G B C is in proportion to the quadrangle A G F D, as K L is to K H: Therefore contrarily, the quadrangle A G F D is to the quadrangle G B C F, as K H to K L; and by conſequence, as M to N. Which was propoſed.

PROP. 13. PROBL. 13.

To divide a Quadrangle A B C D, *having only two ſides parallel,* A D *and* B C, *by a line parallel to one ſide, not parallel* A B *according to any proportion, as* M *to* N.

FRom one of the angles C or D, draw a line D E within the quadrangle, parallel to A B: Then extend E B at length to F; until E F be equal to B E, and divide B F in proportion as M to N.

And in the firſt place, Let the diviſion fall in the point E, ſo as that the proportion of F E to E C may be as M to N: I ſay then that the line D E divides the quadrangle, as was required.



Demonſtr. For draw D F, then the proportion of the triangle F D E to the triangle E D C, is as F E to E C; and therefore as M to N. But [a] the quadrangle A B E D is equal to the triangle F D E: Therefore the quadrangle A B E D is in proportion to the triangle D E C, as M is to N. Which was propoſed.

a) 1.6. 41. 1.

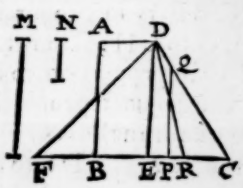
Secondly, Let the diviſion fall betwixt F and E, ſo as that the proportion of F E to E C be greater then the proportion of M to N: Therefore E C being equally divided in G, the proportion of B E to E G will be greater then that of M to N; by reaſon that B E is the halfe of F E, and E G the half of E C. Therefore dividing B G in proportion as M to N: Let the diviſion fall betwixt B and E in the point H, ſo as that the proportion of B H to H G, may be as M to N. Then drawing the line H K parallel to B A: I ſay the ſaid line H K doth divide the quadrangle, as is required.



Demonſtr. For let A D be produced to L, till it meet with G L, parallel to D E. Becauſe therefore E C is double to E G, the quadrangle D E G L will be equal to the triangle D E C. Adding therefore to both of them the quadrangle K H E D; then the quadrangle K H G L will be equal to the quadrangle K H C D. Therefore the quadrangle A B H K is in the ſame proportion to the quadrangle K H G L, and alſo to the quadrangle K H C D. But the proportion of the quadrangle A B H K, is to the quadrangle K H G L, as B H to H G; and by conſequence, as M to N. Therefore the proportion of the quadrangle A B H K is to the quadrangle K H C D as M to N. Which was propoſed.

Thirdly, Let the diviſion fall between F and C in the point R, ſo as that the proportion of F R to F C, may be as M to N: Draw D R, and [b] divide the triangle D E C, by the line P Q, according to the proportion of the triangle D E R to the triangle D E C; and let P Q be parallel to D E; and let the quadrangle D E P Q be equal to the triangle D E R; and alſo the triangle Q P C equal to the triangle D R C: Then I ſay that the line P Q divideth the quadrangle A B C D, as was required.

b) 3. of this.



Demonſt. For the triangle F D R is in proportion to the triangle R D C, as M is to N. But the quadrangle A B E D is equal to the triangle F D E, and the quadrangle D E P Q equal to the triangle D E R: Therefore the Pentagon A B P Q D is equal to the triangle F D R. But alſo the triangle D R C is equal to the triangle Q P C: Therefore the Pentagon A B Q P D is in proportion to the triangle Q P C, as the triangle F D R is to the triangle D R C; and by conſequence, as M to N. Which was propoſed.

In like manner muſt you work by a line parallel to the ſide D C, and ſo the whole which was propoſed, will be manifeſt.

PROP. 14. PROBL. 14.

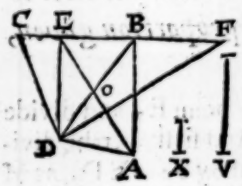
To divide a Quadrangle A B C D, *having one of his ſides parallel, according to a proportion given,* V *to* X, *by a line* D E, *parallel to one of its ſides* A B.

Draw

DRaw from one of his angles C or D, a line DE, within the quadrangle, parallel to AB: Then draw the lines EA and BD, cutting each other in the point O. Extend CB at length to F, until the proportion of FB to BE be as AO to OE; and draw FD, then divide FC as V to X.

And first of all, Let the division fall in E, so as that the proportion of FE to EC, be as V to X: I say DE divideth the quadrangle, as is required.

Demonstr. For the triangle ADO is in proportion to the triangle ODE, as AO to OE; also the triangle ABO is in proportion to the triangle OBE as AO to OE: Therefore as the aggregand proportion of the triangle BAD is to the triangle BED, so is AO to OE; and by consequence, as FB to BE; and according to the same proportion, the triangle FDB to the triangle BED. Therefore the triangle BAD is equal to the triangle FBD: Therefore adding the triangle BDE common to both, the triangle FDE will be equal to the quadrangle ABED. But the triangle FDE is to the triangle EDC as FE to EC; and by consequence, as V to X: Therefore the quadrangle ABED is to the triangle EDC, as V is to X. Which was proposed.

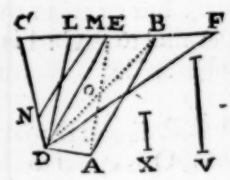


Secondly, Let the division fall between F and E (whether within or without the quadrangle, it matters not) as in the point G. Let FG be to GC as V to X, and draw GD: Therefore the triangle FGD to the triangle GDC, will be as V to X. Joyn therefore [a] to the line AB a superficies, equal to the triangle FDG, which may be conteined by the two angles ABC and BAD. Then separating them by the line HK, parallel to AB: I say that the line HK divideth the quadrangle, as was required. a) 10. prop.



Demonstr. For it will passe within the quadrangle ABED, because the triangle FDE is equal to the quadrangle ABED; and the triangle FDG is lesse then the triangle FDE. Since therefore the triangle FDE is equal to the quadrangle ABED, and the triangle FDG equal to the quadrangle ABHK, the triangle GDE must also be equal to the quadrangle KHED. Putting therefore the triangle EDC common to both, the triangle GDC will be equal to the quadrangle KHCD: Therefore the quadrangle ABHK is to the quadrangle HKCD, as the triangle FGD is to the triangle GDC; and by consequence, as V to X. Which was proposed.

Thirdly. Let the division fall between E and C, in the point L, so as that FL may be to LC, as V to X: Therefore the triangle FDL is to the triangle LDC as V to X: Then [b] cut off from the triangle DEC a triangle like to it, but equal to the triangle LDC, by the line MN, parallel to ED: I say that line, to wit, MN, divideth the quadrangle, as was required. b) 3. prop.



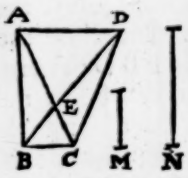
Demonstr. For the triangle FDE is equal to the quadrangle ABED, and the triangle EDL is equal to the quadrangle DEMN; because the triangle MNC is equal to the triangle LDC: Therefore the Pentagon ABMND is equal to the triangle

angle FDL: Therefore the Pentagon ABMND is to the triangle MNC, as the triangle FDL is to the triangle LDC; and by consequence as V to X. Which was proposed.

¶ As the quadrangle is divided according to a proportion given by a line parallel to its side AB, so may it also divided by a line parallel to any other of its sides, which soever, and the Proposition is manifest.

PROP. 15. PROBL. 15.

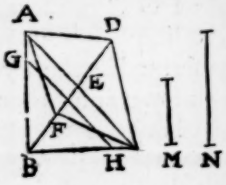
To divide any Quadrangle ABCD, *by a line parallel to one of his diameters* AC, *according to any proportion given,* M *to* N.



DRaw BD, cutting CA in the point E, and divide BD in proportion as M to N. And first, Let the division fall in the point E, so as that EB may be to ED, as M to N: I say then that the diameter AC doth divide the quadrangle ABCD in the proportion required.

Demonst. For ABE is to AED as BE is to ED; likewise BEC to EDC is as BE to ED, and by consequence as M to N.

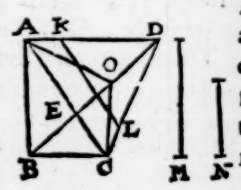
Secondly, Let the division fall between B and E, in the point F, so as that the proportion of BF may be to FD as M to N. Then draw the lines FA and FC, and there will be the same proportion of the two triangles ABF and CBF joyntly, to the quadrangle AFCD, as of BF to FD. Then from the triangle ABC take the triangle GBH, [a] like unto it, and equal to the two triangles ABF and CBF joyntly, by the line GH parallel to AC, which line GH doth divide the quadrangle, as is required.



a) 3. prop.

Demonstr. For the triangle GBH is equal to the superficies ABCF, the triangle AFC will be equal to the quadrangle AGHE: Therefore adding ADC common to both, the quadrangle AFCD will be equal to the Pentagon AGHCD, the proportion therefore of the triangle GBH to the Pentagon AGHCD, is as the superficies ABCF to the quadrangle AFCD; and by consequence as M to N. Which was proposed.

Thirdly, Let the division fall between E and D, in the point O, so as that BO may be to OD as M to N. Draw OA and AC: There will be the same proportion of the quadrangle ABCO to the superficies AOCD, as of BO to OD; and by consequence, as M to N: Therefore [b] from the triangle ACD cut off the triangle KLD, like to it, but equal to the superficies AOCD, by KL, parallel to AC: I say the line KL divideth the quadrangle, as was required.



b) 3. prop. of this Book.

Demonstr. For the triangle AOC is equal to the quadrangle ACLK: Therefore the quadrangle ABCO is equal to the Pentagon ABCLK, and the triangle KLD equal to the superficies AOCD: Therefore the proportion of the Pentagon ABCLK to the triangle KLD, is as the quadrangle ABCO to the superficies AOCD; and by consequence as M to N. Which was proposed. In

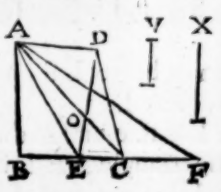
In like manner may the quadrangle A B C D be divided according to a proportion given, by a line parallel to the diameter B D.

PROP. 16. PROBL. 16.

To divide any quadrangle A B C D, *by a line parallel to a line aſſigned* A E, *within the quadrangle, which is neither parallel to any of its ſides, nor either of his diameters according to any proportion given, as* V *to* X.

DRaw the diameters A C and E D, cutting each other in O, produce B C at length to F, until there be the ſame proportion of E C to C F, as of E O to O D; and draw A F: then divide B F in proportion as V to X.

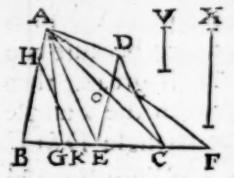
And in the firſt place, Let the diviſion fall in the point E, ſo as that B E may be to E F as V to X: I ſay that line divideth the quadrangle, as is required.



Demonſtr. For the proportion of the triangle A E C to the triangle A C D, is as E O to O D: Therefore as E C to C F; and by conſequence, as the triangle A E C to the triangle A C F: Therefore the triangles A C F and A C D are equal: Therefore the whole quadrangle A E C D is equal to the whole triangle A E F: There is therefore the ſame proportion of the triangle A B E to the quadrangle A E C D, as to the triangle A E F. But the proportion of the triangle A B E to the triangle A E F, is as V to X: Therefore the proportion of the triangle A B E to the quadrangle A E C D, is as V to X. Which was propoſed.

Secondly, Let the diviſion fall between B and E, in the point G, ſo as that B G may be to G F as V to X. Then draw A G, and [a] cut off from the triangle A B E the triangle H B K, like unto it, and equal to the triangle A B G, by the line H K, parallel to A E, then will the ſaid line H K divide the quadrangle, as is required.

a) 3. prop. of this.



Demonſtr. For the quadrangle A H K E, the reſidue of the triangle A B E, will be equal to the triangle A G E, the reſidue of the ſame triangle A B E. But the quadrangle A E C D is equal to the triangle A E F: Therefore the Pentagon A H K C D is equal to the triangle A G F: Therefore the ſame proportion is of the triangle H B K to the Pentagon A H K C D; as of the triangle A B G to the triangle A G F: Therefore as of B G to G F, and by conſequence as of V to X. Which was required.

Thirdly, Let the diviſion fall between E and F: Therefore becauſe A E is not parallel to C D, draw from one of the two angles D or C, the line D M within the quadrangle, and parallel to A E, then draw A M cutting E D in N; then making L M to M E as D N to N E, [this may preſently be done by drawing D L parallel to A M:] Let therefore L fall on this ſide F, ſo as that D F if it ſhould be drawn, would be parallel to A C. If that be drawn (as here it is) then draw A L. Then the triangle A E L will be equal to the quadrangle A E M D: Therefore divide B F in pro-

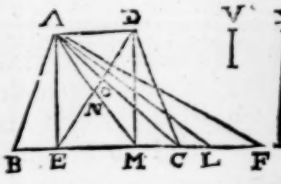
portion as V to X; and now let the diviſion fall between E and L in the point R, ſo as that B R may be to R F, as V to X; then [b] draw P ♎ parallel to A E, ſo as that the ſuperficies A E ♎ P may be equal to the triangle A E R. And becauſe the triangle A E L is greater then the triangle A E R, and the triangle A E L is equal to the quadrangle A E M D: Therefore will the quadrangle A E Q P be leſſe then the quadrangle A E M D: I ſay therefore that the line P Q doth divide the quadrangle A B C D. As was required.

b) 10. prop. of this.



Demonſtr. For the quadrangle A E C D is equal to the triangle A E F, and the quadrangle A E ♎ P is equal to the triangle A E R: Therefore the quadrangle P Q C D the reſidue, is equal to the triangle A R F the reſidue: In like manner, becauſe the quadrangle A E Q P is equal to the triangle A E R, putting the triangle A B E common to both, the quadrangle A B ♎ P will be equal to the triangle A B R: Therefore the quadrangle A B ♎ P is to the quadrangle P ♎ C D, as the triangle A B R to the triangle A R F: Therefore as B R to R; and by conſequence as V to X. Which was propoſed.

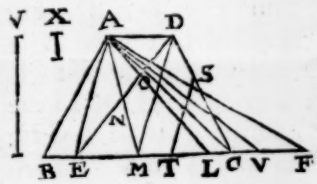
Fourthly, Let the diviſion fall in L, ſo as that B L may be to L F, as V to X: I ſay D M doth divide the quadrangle, as is required.



Demonſtr. For the triangle A E F is equal to the quadrangle A E C D, and the triangle A E L is equal to the quadrangle A E M D: Therefore the triangle A L F reſidue, is equal to the triangle D M C reſidue. In like manner, becauſe the quadrangle A E M D is equal to the triangle A E L; and putting the triangle A B E common, the quadrangle A B M D will be equal to the triangle A B L: Therefore the quadrangle A B M D is to the triangle D M C, as the triangle A B L is to the triangle A L F; and by conſequence as V to X. Which was propoſed.

Fifthly, Let the diviſion fall between L and F, in the point V, ſo as that B V may be to V F as V to X. Draw A V: Therefore becauſe the triangle D M C is equal to the triangle A L F, and A L F is greater then the triangle A V F: The triangle D M C will be greater then the triangle A V F: Therefore from the triangle D M C [c] ſeperate the triangle S T C like unto it, and equal to the triangle A V F, by the line S T, parallel to D M. I ſay therefore that that line S T doth divide the quadrangle, as is required.

c) 30. of this Book.



Demonſtr. For the triangle D M C is equal to the triangle A L F; alſo the triangle S T C is equal to the triangle A V F, the quadrangle D M T S being the reſidue, will be equal to the triangle A L V, alſo the reſidue: Therefore ſince the quadrangle A B M D is equal to the triangle A B L, the Pentagon A B T S D will be equal to the triangle A B V: Therefore the Pentagon A B T S D is to the triangle S T C, as the triangle A B V to the triangle A V F; and by conſequence as V to X. Which was to be demonſtrated.

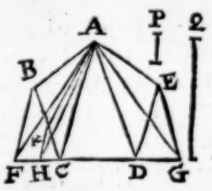
Note

¶ Note that as the quadrangle is divided by a line parallel to a line drawn from an angle thereof, and neither parallel to its ſides nor diameters; ſo may it alſo be divided by a line parallel to a line not drawn from an angle aſſigned, as by drawing a line from any angle of the quadrangle falling within the quadrangle and parallel to a line aſſigned, then the operation muſt be as we have now ſhewn.

PROP. 17. PROB. 17.

To divide a known Pentagon ABCDE *by a line* AK, *drawn from any of its angles as* A, *according to a proportion given* P *to* Q.

DRaw the lines AD and AC, and from the angle B draw BF, parallel to AC, and drawn out till it meet with DC (extended) in F. Likewiſe from the angle E draw EG parallel to AD, till it cut CD (extended) in G. Then drawing AF and AG, the triangle AFG will be [a] equal to the Pentagon ABCDE; by reaſon that the triangle ABC is equal to the triangle AFC, and the triangle AED is equal to the triangle AGD; then adding the triangle ACD common to both, what we ſaid is clear. a) 31. 1.



Demonſtr. Divide (firſt of all) the line FG in proportion as P to Q; and let the diviſion fall between F and C in the point H; ſo as that FH may be to HG as P to Q: Then draw HK parallel to BF, till it touch BC in the point K, there is then [b] the ſame proportion of BK to KC, as of FH to HC: Then drawing AK: I ſay that that line doth divide the Pentagon, as was required. b) 2. 6.

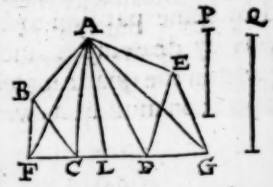
Draw AH, becauſe therefore the triangle AED is equal to the triangle AGD, adding the triangle ACD common to both, the quadrangle ACDE will be equal to the triangle ACG: Likewiſe becauſe the triangle AKC is equal to the triangle BHC, by reaſon of the Paralleliſme of KH and AC; the Pentagon AKCDE will be equal to the triangle AHG. Alſo becauſe BC is to BK, as FC to FH, there will be the ſame proportion of the triangle ABC to the triangle ABK, as of the triangle AFC to the triangle AFH: Therefore by permutation, the triangle ABC is to the triangle AFC, as the triangle ABK is to the triangle AFH. Since therefore the triangle ABC is equal to the triangle AFC, the triangle ABK will be equal to the triangle AFH: Therefore the triangle ABK will be to the Pentagon AKCDE, as the triangle AFH to the triangle AHG; and therefore as FH to HG; and by conſequence as P to Q. Which was propoſed.

Secondly, Let the diviſion fall in C, ſo as that FC may be to FG, as P to Q; then I ſay, the line AC doth divide the Pentagon, as was required.

Demonſtr. For (as is before ſhewn) the quadrangle ACDE is equal to the triangle ACG; and the triangle ABC is equal to the triangle AFC: Therefore the triangle ABC is in proportion to the quadrangle ACDE, as the triangle AFC to the triangle ACG: Therefore as FC to CG, and by conſequence, as P to Q. Which was propoſed.

Thirdly,

Thirdly, Let the diviſion fall between C and D in the point L; ſo as that the proportion of F L to L G may be as P to Q. Draw A L: I ſay the line A L doth divide the Pentagon, as was required.

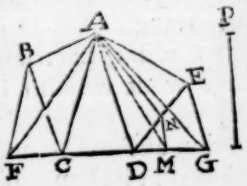


Demonſtr. For becauſe the triangle A B C is equal to the triangle A F C, putting the triangle A C L common, the quadrangle A B C L will be equal to the triangle A F L. Likewiſe putting the triangle A L D to both triangles, to wit, the triangle A E D and A G D, the quadrangle A L D E will be equal to the triangle A L G: Therefore the quadrangle A B C L is to the quadrangle A L D E, as the triangle A F L is to the triangle A L G: Therefore as F L to L G; and conſequently as P to Q. Which was propoſed.

Fourthly, If the diviſion fall in the point D: Then I ſay that the line A D divideth the Pentagon according to the Propoſition.

The Demonſtration is manifeſt, as it was made appear, when the diviſion fell in the point C, as in the ſecond Caſe.

Fifthly, Let the diviſion fall between D and G in the point M, ſo as that F M may be to M G as P to Q. Draw M N parallel to G E, till it touch D E in the point N; then draw A N, which line A N I ſay doth divide the Pentagon according to the Propoſition

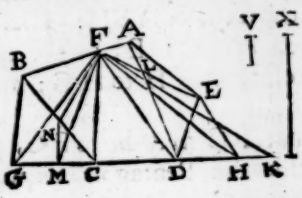


Demonſtr. For having drawn A M, it is proved as in the firſt Caſe, that the triangle A E N is equal to the triangle A G M, and that the Pentagon A B C D N is equal to the triangle A F M: Therefore the Pentagon A B C D N is to the triangle A N E, as the triangle A F M is to the triangle A M G: Therefore as F M to M G; and by conſequence as P to Q. Which was required.

PROP. 18. PROBL. 18.

By a line drawn from a point F, *in the ſide aſſigned of a known Pentagon* A B C D E, *to divide the ſaid Pentagon according to a proportion given, as* V *to* X.

DRaw F C; F D, and F E, then drawn B G parallel to F C, and E H parallel to F D, till they meet with C D prolonged on both ſides to G and H, then draw A D, cutting F E in the point L: Then extend D H to K, until the proportion of D H to H K, be as D L to L A (this may be done by imagining A K to be produced parallel to L H:) Then draw F G, F H, and F K. Divide then G K according to the proportion of V to X, and let the diviſion fall (in the firſt place) between G and C, in the point M, ſo as that the proportion of G M to M K, may be as V to X: Then divide B C in the point N, by a line parallel to B G, and there will be the ſame proportion of B N to N C, as of G M to M C: Then draw F N. which line F N, I ſay, doth divide the Pentagon according to the Propoſition.



Demonſtr. For the triangle F D E is in proportion to the triangle F A E,

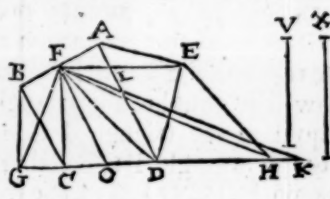
as

as D L to L A: Therefore the proportion of D H to H K is as D F H to H F K: Therefore the proportion of the triangle F D E, to the triangle F A E, is as D F H to H F K: Therefore (changing) the proportion of the triangle D F E to the triangle D F H, is as F A E to E H K; but D E H is equal to D F E (by reaſon of the Parallelisme of F D and F H;) therefore F A E is equal to F H K: Therefore the quadrangle F D E A is equal to the triangle F D K: Adding therefore F D C common to both, the Pentagon F C D E A will be equal to the triangle F C K. On the other ſide draw F M, and becauſe therefore the triangle F B C is equal to the triangle F G C; and there is the ſame proportion of B N to N C, as of G M to M C: Therefore the triangle F B N is equal to the triangle F G M, and the triangle F N C is equal to the triangle F M C: Therefore by gathering all together, it is manifeſt that the Hexagon F N C D E A is equal to the triangle M F K, and the triangle F B N is equal to the triangle F G M: There is therefore the ſame proportion of the triangle F B N to the Hexagon F N C D E A, as of the triangle F M G to the triangle F M K; and therefore as G M to M K; and by conſequence as V to X. Which was propoſed.

Secondly, Let the diviſion fall in the point C, ſo as that the proportion of G C to C K may be as V to X: I ſay then that the line F C doth divide the Pentagon according to the Propoſition.

Demonſt. For (as hath been ſhewn) the Pentagon F C D E A is equal to the triangle F C K, and F B C is equal to F G C; therefore as F B C is to F C D E A, ſo is F G C to F C K: Therefore as G C is to G K, and conſequently, as V to X. Which was propoſed.

Thirdly, Let the diviſion fall between C and D in the point O, ſo as that the proportion of G O to O K may be as V to X: I ſay then that the line F O divideth the Pentagon, as was required.



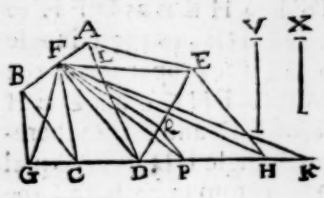
Demonſtr. For adding the common triangle F O D to the qnadrangle F D E A, and to the triangle F D K (equal to it) the Pentagon F O D E A will be equal to the triangle F O K. Likewiſe adde the triangle F C G common to the two triangles F B C and and F G C; the quadrangle F B C O will be equal to the triangle F G O: Therefore the ſame proportion is of the quadrangle F B C O to the Pentagon F O D E A, as of the triangle F G O to the triangle F O K; and therefore as G O to O K; and by conſequence as V to X. Which was propoſed.

Fourthly, Let the diviſion fall in the point D, ſo as that the proportion of G D to D K, may be as V to X: I ſay then that the line F D doth divide the Pentagon according to the Propoſition.

Demonſtr. For the triangle F C D common to both, being added to the equal triangles F B C and F G C, the Propoſition is manifeſt.

Fifthly, Let the diviſion fall between D and H in the point P, ſo as that the proportion of G P to P K, may be as V to X. Divide the line D E in the point Q, by the line P Q parallel to E H: There will be therefore the ſame proportion of D Q to Q E, as of D P to P H: Therefore the line F Q being drawn, I ſay that line F Q doth divide the Pentagon according to the Propoſition.

Demonſt. For the whole quadrangle F D E A is equal to the whole triangle F D K. But alſo the triangle F D Q is equal to the triangle F D P:

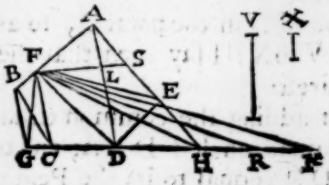


Therefore the quadrangle F Q E A remaining, is equal to the triangle F P K alſo remaining. Again, the quadrangle F B C D is equal to the triangle F G D. Adding therfore the triangle F D Q to the quadrangle F B C D, and the triangle F D P equal to the triangle F D Q being added to the triangle F G D; it is apparent that the Pentagon F B C D Q is equal to the triangle F G P; there is therefore the ſame proportion of the Pentagon F B C D Q to the quadrangle F Q E A, as of the triangle F G P to the triangle F P K; and by conſequence as V to X. Which was propoſed.

Sixthly, Let the diviſion fall in the point H: I ſay that the line F E doth divide the Pentagon, as was required.

Demonſtr. For becauſe the quadrangle F B C D is equal to the triangle F G D, and (as is afore ſaid) the triangle A F E is equal to the triangle F H K, and the triangle F D E is equal to the triangle E D H: Therefore the Pentagon F B C D E is equal to the triangle F G H: Therefore there is the ſame proportion of the Pentagon F B C D E to the triangle F A E, as of the triangle F G H to the triangle F H K; and therefore as G H to G K; and by conſequence as V to X, as was propoſed.

Seventhly, Let the diviſion fall between H and K, in the point R, ſo as that the proportion of G R to R K, may be as V to X: Then divide E A in the point S, ſo as that the proportion of E S to S A, may be as H R to R K: I ſay therefore that the line F S doth divide the Pentagon according to the Propoſition.



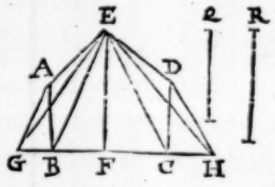
Demonſtr. Foraſmuch as the triangle A F E is equal to the triangle F H K, and the proportion of E S to S A is as the proportion of H R to R K, the triangle F E S will be equal to the triangle F H R, and the triangle F S A will be equal to the triangle F R K. But the Pentagon F B C D E is equal to the triangle F G H: Therefore the Hexagon F B C D E S is equal to the triangle F G R; there is therefore the ſame proportion of the Hexagon F B C D E S to the triangle F S A, as of the triangle F G R to the triangle F K R; and therefore as of the line G R to R K; and by conſequence as V to X. Which was propoſed.

PROP. 19. PROBL. 19.

To divide a Pentagon A B C D E, *having two ſides parallel, by a line parallel to thoſe parallel ſides, according to a proportion given, as* Q *to* R.

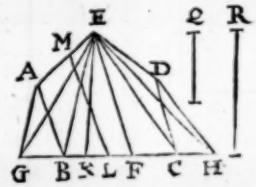
FIrſt, Let it be required to performe this by a line parallel to A B, which ſide let be parallel either to C D or D E: Let it be here parallel to C D: Then draw E F parallel to A B: Then draw E B and and E C: Then draw A G parallel to E B, and D H parallel to E C, till they meet with B C (extended both wayes) in G and H. Then divide G H in proportion as Q to R, and firſt let the diviſion fall in the point F: Then I ſay that the line E F doth divide the Pentagon, as was required.

De-



Demonſt. For becauſe A G is parallel to E B, having drawn E G, the triangle E A B will be equal to the triangle E G B, and adding the triangle E B F common to both, the triangle E G F will be equal to the quadrangle E A B F; alſo becauſe D H is parallel to E C, having drawn E H, the triangle E D C will be equal to the triangle E H C, then adding the triangle E F C common to both, the triangle E F H will be equal to the quadrangle E F C D; and before the triangle E G F was proved to be equal to the quadrangle A B F E: Therefore the quadrangle A B F E is to the quadrangle E F C D, as the triangle E G F is to the triangle E F H: Therefore as G F to F H, and conſequently, as Q to R. Which was propoſed.

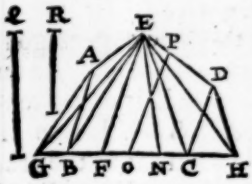
Secondly, Let the diviſion fall between G and F in the point K, ſo as that the proportion of G K to K H, may be as Q to R: Then draw E K. Becauſe therefore the triangle E G K is leſſe then the triangle E G F, and the triangle E G F is equal to the quadrangle A B F E; the triangle E G K will be leſſe than the quadrangle A B F E: Therefore [a] I joyn to A B the ſuperficies A B L M, equal to the triangle E G K, by the line L M, parallel to A B: I ſay then that the line L M doth divide the Pentagon, as was required.



a) 10. of this Book.

Demonſtr. For the triangle E G K is equal to the quadrangle A B L M, and the whole triangle E G H is equal to the whole Pentagon A B C D E: Therefore the remaining triangle E K H is equal to the remaining Pentagon M L C D E: Therefore the quadrangle A B L M is in proportion to the Pentagon M L C D E, as the triangle E G K is to the triangle E H K; and by conſequence, as Q to R. Which was propoſed.

Thirdly, Let the diviſion fall between F and H in the point N; and let E N be drawn, then the triangle E H N will be leſſe then the quadrangle E F C D; becauſe it is in proportion leſſe then the triangle E H F, which is equal to the ſaid quadrangle: Therefore [b] adjoyn to D C the ſuperficies P O C D, equal to the triangle E H N, by the line O P, parallel to C D: I ſay then that the line O P doth divide the Pentagon, as was required.



b) 10. prop. of this.

Demonſtr. For becauſe the quadrangle P O C D, is equal to the triangle E N H, and the whole triangle E G H is equal to the whole Pentagon A B C D E, the reſidual Pentagon A B O P E is equal to the reſidual triangle E G N: Therefore the proportion of the Pentagon A B O P E is to the quadrangle P O C D, as the triangle E G N is to the triangle E N H; and by conſequence as Q to R. Which was propoſed.

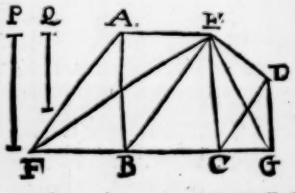
In like manner, alſo as the Pentagon A B C D E, having two ſides parallel, is divided by ratiocination upon B C, oppoſite to the angle E, intercepted within the parallel ſides, ſo making two ſides A B and D E parallel, the Pentagon will be divided by a line parallel to A B, made by ratiocination upon the ſide E A, oppoſite to the angle C, intercepted between its two parallel ſides A B and D E. And the Propoſition is manifeſt both wayes.

PROP.

PROP. 20. PROBL. 20.

To divide a Pentagon A B C D, *one of whose ſides* A B, *is parallel to one of its diameters* C E, *by a line parallel to that ſide, and alſo to that diameter, according to a proportion given,* P *to* Q.

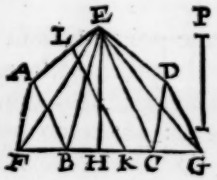
DRaw E B, and make A F parallel to E B, and D G parallel to E C; produce them to cut B C (produced on either ſide) in F and G: Then drawing E F and E G, the triangle E F G is equal to the Pentagon A B C D E, propoſed; as is manifeſt. Then divide F G in proportion as P to Q, and let the diviſion fall either in the point C, or before or after C; and firſt let it fall in the point C: I ſay then that the line E C divideth the Pentagon, as was required.



Demonſtr. For the quadrangle A B C E is equal to the triangle E F C, becauſe the triangle E C D remaining, is equal to the triangle E C G remaining; and the whole pentagon is equal to the whole triangle: Therefore the quadrangle A B C F is in proportion to E C D, as the triangle E F C to the triangle E C G: Therefore as F C to C G, and by conſequence, as P to Q. Which was propoſed.

Secondly, Let the diviſion fall between F and C in the point H, ſo as that the proportion of F H to H G, may be as P to Q. Becauſe therefore the quadrangle A B C E is equal to the triangle E F C; and the triangle E F H leſſe then the triangle E F C, the triangle E F H will be leſſe then the quadrangle A B C E: Adjoyn to the line A B [a] the quadrangle A B L K, equal to the triangle E F H, by the line K L, parallel to A B: I ſay the line K L doth divide the Pentagon as is required.

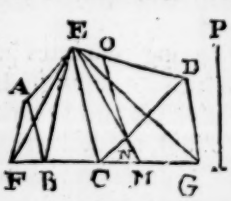
a) 10. of this Book.



Demonſtr. For becauſe the whole pentagon is equal to the whole triangle F E G, and the quadrangle A B K L is equal to the triangle E F H, the remaining pentagon L K C D E is equal to the remaining triangle E H G: Therefore the quadrangle A B K L is in proportion to the pentagon L K C D E, as the triangle E F H is to the triangle E H G; and therefore as F H to H G; and conſequently as P to Q. Which was propoſed.

Thirdly, Let the diviſion fall between C and G, in the point M, ſo as that F M may be in proportion to M G, as P to Q. Becauſe the triangle E D C is equal to the triangle E G C, and the the triangle E M C is leſſe then the triangle E G C: Therefore the triangle E M C is leſſe then the triangle E D C. Adjoyne therefore to the line E C the quadrangle E C N O equal to the triangle E M C, by the line N O, parallel to E C [b]. Separate the triangle D O N from the triangle D E C like unto it, and equal to the triangle E G M: I ſay that the line N O doth divide the pentagon according to the Propoſition.

b) 3, 10. of this.



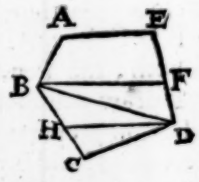
De-

Demonſtr. For becauſe the whole Pentagon ABCDE is equal to the whole triangle FEG, and the triangle OND equal to the triangle EMG, the Hexagon ABCNOE the reſidue, is equal to the triangle EFM, the reſidue: Therefore the Hexagon ABCNOE is in proportion to the triangle OND, as the triangle EFM is to the triangle EMG. Therefore as FM to MG, and by conſequence as P to Q. Which was propoſed.

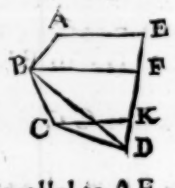
PROP. 21. THEOR. 1.

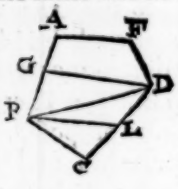
To any ſide AE, *of a Pentagon* ABCDE, *aſſigned, which is neither parallel to any of his ſides, nor any of his diameters. It is poſſible to draw two right lines within the Pentagon, from (any) two of the three angles not joyned to that ſide, which two lines ſhall be parallel to the ſide before limited.*

FOr Example, Let it be that in the Pentagon ABCDE, the ſide AE be neither parallel to any ſide thereof, nor to the diameter BD: Then I ſay that from any two of the three angles B, C, or D, two lines may be drawn within the Pentagon, both which ſhall be parallel to the ſide AE; for ſince AE and BD are not parallel, they being extended, would meet either on the part next AB, or elſe on that of ED, if they meet on the part of AB, then the line BF drawn from the point B, parallel to AE, would neceſſarily fall on the ſide ED, as in both the uppermoſt figures: But if they (*viz.* AE and BD) meet on the part of ED: Then the line DG, drawn from D, parallel to AE, muſt needs fall on the ſide AB, as in both the lower-moſt figures of this Propoſition.

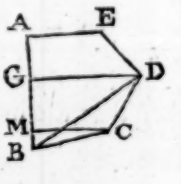


Alſo if AE and BD ſhould meet on the part next AB, as in both the firſt figures of this Propoſition; then the line BF being not parallel to CD, ſhould either meet with it on the part of FD, or on the part of BC: If on the part FD, as in the firſt of the former figures; then from D may be drawn DH, parallel to AE; falling in the ſide BC. But if BF and CD ſhould meet on the part of BC, as in the ſecond figure; then from C may be drawn a line parallel to AE, falling in the ſide ED. We have therefore BF and DH parallel to AE in the firſt figure, and we have BF and CK parallel to the ſame line in the ſecond figure of this Propoſition.





But if AF and BD ſhould meet on the part of BD, as in the two later figures of this Propoſition. Then the line DG (it being not parallel to BC) would meet with it either on the part of GB, or on the part of DC: If on the part of GB, as in the third figure, then may a line be drawn from B, *viz.* BL parallel to AE, and will fall in the ſide CD.



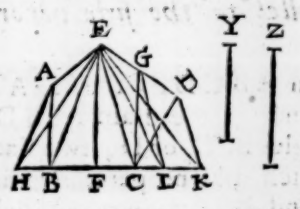
But if GD and BC meet on the part of CD, as in the third figure, then from C may be drawn CM, parallel to the line AE; and will fall in the ſide AB: We have therefore DG and BL in the third figure, and DG and CM in the fourth figure, parallels to the line

AE, and falling within the Pentagon: Therefore the whole intended to be declared, is manifeſt.

PROP. 22. PROBL. 21.

To divide a Pentagon ABCDE, *by a line parallel to one of the ſides aſſigned* AB, *which ſide is neither parallel to any of the other ſides* (ED or CD) *nor yet to the diameter* EC, *according to a proportion given*, Y *to* Z.

FRom two of the three angles C, D, and E, I draw two lines within the Pentagon, parallel to the ſide AB: Theſe two lines deſcending ſo from the angles, will fall upon the ſame ſide, or on the oppoſite ſides. Let them in the firſt place fall on the oppoſite ſides, and let them be EF and CG, ſo as that F may be in the ſide BC; and let the point G be in the ſide ED. I will reaſon alſo upon the ſide whereupon the neereſt parallel to AB falleth, to wit, upon BC. Draw EB and EC, then draw AH parallel to EB, and DK parallel to EC, till they meet with BC extended on both ſides, in the points H and K; and draw EH and EK; becauſe therefore the triangle EAB is equal to the triangle EHB, and the triangle EDC is equal to triangle EKC, adding the triangle EBC common to both, the Pentagon ABCDE will be equal to the triangle EHK: I draw therefore GL parallel to EC, then protract EL, then divide HK in proportion as Y to Z. The diviſion will fall either in F or in L, or between H and F, or between F and L: Let it fall firſt in F, ſo as that the proportion of FH to HK, may be as Y to Z: I ſay that the line EF doth divide the Pentagon, as is required.



Demonſtr. For the quadrangle EABF is equal to the triangle EFH, and the quadrangle EDCF is equal to the triangle EKF: Therefore the quadrangle EABF is in proportion to the quadrangle EDCF, as the triangle EHF is to the triangle EKF; and therefore as HF to FK; and conſequently as Y to Z. Which was propoſed.

Secondly, If the diviſion fall in L, I ſay then that the line CG doth divide the Pentagon, as is required.

Demonſtr. For becauſe EC and GL are parallels, the triangle EGC is equal to the triangle ELC. But the whole triangle EDC is equal to the whole triangle EKC: Therefore the triangle GCD is equal to the triangle ELK, alſo the quadrangle ABCE is equal to the triangle EHC: Therefore the Pentagon ABCGE is equal to the triangle EHL: Therefore the Pentagon ABCGE hath the ſame proportion to the triangle GCD, as the triangle EHL hath to the triangle ELK: Therefore as HL to LK, and conſequently as Y to Z. Which was propoſed.

Thirdly, Let the diviſion fall between H and F in the point M; and let EM be drawn. Becauſe therefore the triangle EHF is equal to the quadrangle EABF, and the triangle EHM is leſſe then the triangle EHF: Therefore will the triangle EHM be leſſe then the quadrangle EABF. Joyn therefore [a] to the line AB a ſuperficies ABNO, equal to the triangle EHM, by the line NO, parallel to AB: I ſay then that the line NO doth divide the Pentagon, as is required.

a) 10. of this.

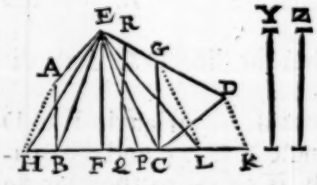
Demonſtr.



Demonſtr. For the Pentagon A B C D E is equal to the triangle E K, and the quadrangle A B N O is equal to the triangle E H M: Therefore the Pentagon O N C D E the remainder, is equal to the triangle E M K remainder: Therefore the quadrangle A B N O is in proportion to the Pentagon O N C D E, as the triangle E H M is to the triangle E M K: Therefore as H M to M K, and by conſequence as Y to Z. Which was propoſed.

Fourthly; Let the diviſion fall between F and L, in the point P, and let E P be drawn. Becauſe therefore the triangle E F L is equal to the quadrangle E F C G, and the triangle E F P leſſe then the triangle E F L; the triangle E F P will be leſſe then the quadrangle E F C G. Adjoyne therefore [b] to the line E F, the quadrangle E F Q R equal to the triangle E F P, by the line Q R, parallel to E F: I ſay then that the line Q R doth divide the Pentagon, as was propoſed.

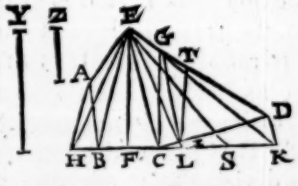
b) 10. of this.



Demonſtr. For the triangle E H P is equal to the Pentagon A B Q R E, and the whole Pentagon A B C D E is equal to the whole triangle E H K: Therefore the quadrangle R Q C D remaining, is equal to the triangle E P K: Therefore the Pentagon A B Q R E is in proportion to the quadrangle R Q C D, as the triangle E H P is to the triangle E P K: Therefore as H P to K P, and by conſequence as Y to Z. Which was propoſed.

Fifthly, Let the diviſion fall between L and K, in the point S. Becauſe the parallelilme of the lines E C and G L, doth conſtitute the triangle E G C, equal to the triangle E L C; alſo the whole triangle E D C to the whole triangle E K C: Therefore the remaining triangle G D C will be equal to the remaining triangle E K L. But having drawn the line E S, the triangle E K S is leſſe then the triangle E K L: Therefore the triangle E K S is alſo leſſe then the triangle G D C: Therefore [c] cut off from the triangle G D C, the triangle T D V like unto it, and equal to the triangle E K S, by the line T V, parallel to G C: I ſay the line T V doth divide the Pentagon as was propoſed.

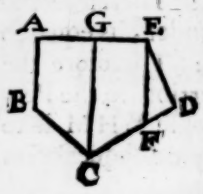
c) 3. of this.



Demonſtr. For the whole Pentagon A B C D E is equal to the whole triangle E H K, and the triangle T D V is equal to the triangle E K S: Therefore the Hexagon A B C V T E remaining, is to the triangle T D V, as the triangle E H S is to the triangle E K S: Therefore as H S to S K, and conſequently as Y to Z. Which was propoſed.

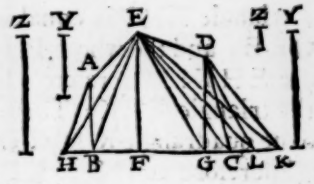
But if the two lines E F and C G, which are parallel to A B, ſhould fall ſo as that the line E F fall on the ſide C D, and the line C G on the ſide A E: Then make the angle C, and you muſt reaſon upon the line A E, as was done before upon B C, and you may atteine your deſire. But if the two lines which were protracted parallel to A B, happen to fall on one and the ſame ſide, then muſt you reaſon upon that ſide: As for Example, Let it be that in the Pentagon A B C D E, the two lines E F and D G protracted parallel to the line A B, fall upon the ſide B C: Then draw A H parallel to E B, and D K parallel to E C: Draw alſo E G,

E G, and a line D L parallel thereto: It is manifeſt by what hath been premiſed, that the triangle E H K is equal to the Pentagon A B C D E, and the triangle E H L is equal to the ſame Pentagon A B C D E, and ſo is left the triangle D G C equal to the triangle E L K. Divide then H K in proportion as Y to Z; and let the diviſion fall either in F or in L, or betwixt them and the extreames: Therefore in the firſt place, let the diviſion fall in F, ſo as that H F may be to F K as Y to Z: I ſay the line E F doth divide the Pentagon, as was propoſed.

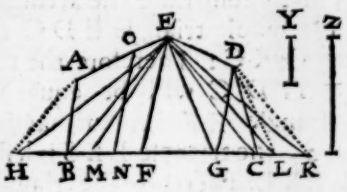


Demonſtr. For the quadrangle A B F E is equal to the triangle E H F, and the quadrangle E F C D is equal to the triangle E F R: Therefore the quadrangle A B F E is to the quadrangle E F C D, as the triangle E H F is to the triangle E F K, and by conſequence as Y to Z. Which was propoſed.

Secondly, Let the diviſion fall in L: I ſay that the line D G doth divide the Pentagon, as was propoſed.

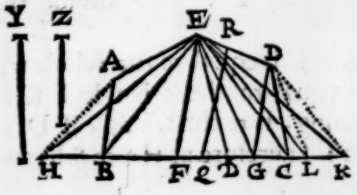


Demonſtr. For becauſe the triangle E G D is equal to the triangle E G L; and the quadrangle A B G E is equal to the triangle E H C; the Pentagon A B G D L will be equal to the triangle E H L. But the triangle D G C is equal to the triangle E L K: Therefore the Pentagon A B G D L is to the triangle D G C, as the triangle E H L is to the triangle F L K: Therefore as H L to L K, and conſequently, as Y to Z. Which was propoſed.



d) 10. of this Book.

Thirdly, Let the diviſion fall in M, between H and F; having drawn the line E M, Let [d] the quadrangle A B N O be made equal to the triangle E H M, by the line N O, parallel to A B: It is manifeſt therefore as before that the quadrangle A B N O is to the Pentagon O N C D E, as the triangle E H M is to the triangle E M K; and by conſequence as Y to Z: Therefore the line O N doth divide the Pentagon, as was required.



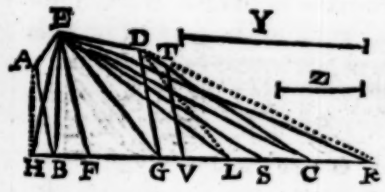
e) 10. of this.

Fourthly, Let the diviſion fall between F and L, in the point P: Then having drawn the line E P, Let there be made [e] the quadrangle E F Q R equal to the triangle E F P: Therefore the Pentagon A B Q R E is equal to the triangle E H P: Therefore the Pentagon A B Q R E is to the quadrangle R Q C D, as the triangle E H P is to the triangle E P K: Therefore as A P to P K, and by conſequence as Y to Z. Which was propoſed.

Fifthly, Let the diviſion fall in S, between L and K, ſo as that H S may be to S K as Y to Z. Becauſe therefore (as aforeſaid) the triangle D G C is equal to the triangle E L K, the triangle E S K will be leſſe then the triangle D G C cut off: Therefore [f] from the triangle D G C another triangle, to wit, T V C like unto it, and equal to the triangle E S K, by the line T V, parallel to D G: I ſay then that the line T V

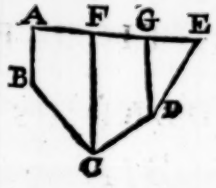
f) 3. of this.

doth



doth divide the Pentagon, as was required.

Demonſtr. For becauſe the triangle T V C is equal to the triangle E S K, and the whole Pentagon A B C D E is equal to the whole triangle E H K: Therefore the Hexagon A B V T D E is equal to the whole triangle E H S: Therefore the Hexagon A B V T D E, is to the triangle T V C, as the triangle E H S is to the triangle E S K; and by conſequence as Y to Z. Which was propoſed.



But if the two lines which ſhall be drawn parallel to A B, fall upon the ſide A E, according to which the lines C F and D G doe fall, conſtitute the angle C, and reaſon on the line A E, as we have done on the line B C, and (as before) we ſhall arrive to our deſire.

The End of MACHOMETUS BAGDEDINUS,

OF THE DIVISION OF SUPERFICIES.

A LITTLE BOOK CONCERNING THE DIVISION OF SUPERFICIAL FIGURES.

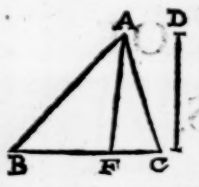
Written in Latine by FREDERICUS COMMANDINUS of *URBIN.*

PROBLEME I.

To divide a right lined figure according to a proportion given, from a point given in any part of the Ambitus or Circuit thereof, whether the ſaid point be taken in any angle or ſide of the figure.

A Right lined figure is here taken to be ſuch as is conteined by (an equal number) a like number of ſides and angles.

A triangle divided from a point in an angle.



LEt the triangle A B C be divided in proportion as D to E, from a point A in an angle: Divide B C [a] in F, ſo as that B F may be to B C as D to E; and draw A F, which doth divide the triangle as is required.

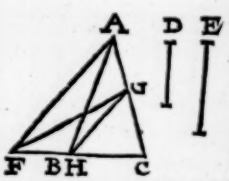
a) 10. 6.

Demonſtr. For the triangle A B F is to the triangle A F C, [b] as B F is to F C, that is, as D to E.

b) 1. 6.

A triangle divided from a point in a ſide.

LEt G be a point given in the ſide A C, from whence the triangle is to be divided as D to E, by a right line. Draw G B, and from A draw a line parallel to G B, as the line A F: Then draw G F, and divide F C in the point H, ſo as that F H may be to H C, as D to E; the point H will then fall either in B or between F and B, or between B and C. If it fall in B (as here it doth) the line G B performeth the Probleme.



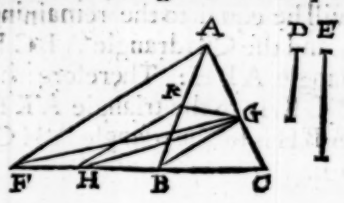
Demonſtr. For the triangle G F B is to the triangle G B C, as F B is to B C; that is, as D to E. But the triangle A B G is equal to the triangle G F B (being on the ſame baſe) and having the ſame altitude [c]: Therefore the triangle A B G is

c) 37. 1.

to

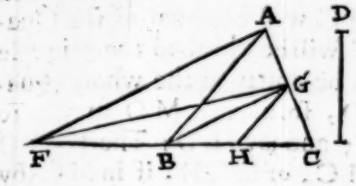
to the triangle G B C, as the triangle G F B is to the ſame triangle G B C, that is, as D is to E.

But if H fall between F and B: Draw H K parallel to G B, which will cut A B in K: Then draw G H and G K. I ſay then the line G K doth divide the triangle as was required.



Demonſtr. For again, the triangle A B G is equal to the triangle G F B; and adding the triangle G B C common to both, the triangle A B C is equal to the triangle G F C. But the triangle G K B is equal to the triangle G H B: Wherefore the remainder is equal to the remainder, to wit, the triangle A K G to the triangle G F H; and therefore the quadrilateral figure G K B C is equal to the triangle G H C: Therefore the triangle A K G, is to the quadrilateral figure G K B C, as the triangle G F H is to the triangle G H C, that is, as D to E.

But if H fall between B and C, draw G H, which again performeth the Probleme.

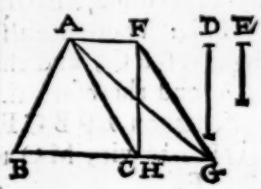


Demonſtr. For ſeeing that the triangle G F B is equal to the triangle A B G. Adding then the triangle G B H common to both, the triangle G F H is equal to the quadrilateral figure A B H G: Therefore the triangle G F H is to the triangle G H C, to wit, D to E, as the quadrilateral figure A B H G, is to the triangle G H C.

If the ſame point be taken in another, either ſide or angle the ſame reaſoning is to be uſed.

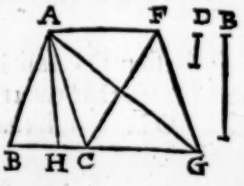
To divide a quadrilateral figure from an angle.

LEt the quadrilateral figure (or quadrangle) A B C F, be divided by a right line from the angle A, in proportion as D to E: Joyne A C, and from F draw a parallel to it, as F G, which will meet with B C produced in G. Joyn A G, then the triangle A C G is equal to the triangle A C F, and adding the triangle A B C common to both, the triangle A B G will be equal to the quadrangle A B C F. Let B C be divided as D to E, now in H, ſo as that B H may be to H G as D to E. And for that the point H falls in C, the Problem is already performed.



Demonſtr. For the triangle A B C is to the triangle A C F, as to the triangle A C G, that is, as D to E.

If the point of diviſion H, fall between B and C, the only joyning of A H performeth the Probleme.



Demonſt. For the quadrangle A H C F is equal to the triangle A H G: Wherefore the triangle A B H is to the quadrangle A H C F, as the triangle A B H, is to the triangle A H G, to wit, as D to E.

But

But if H fall between C and G. Again to F C, having drawn H K parallel to A C, and joyned A H and A K, the line A K will divide the Quadrangle as was required.

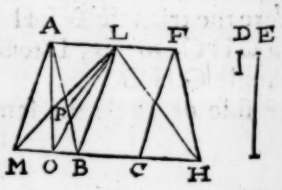


Demonſtr. For the triangle A C K is equal to the triangle A C H; therefore the remaining triangle A K F will be equal to the remaining triangle A H G, and the Quadrangle A B C K equal to the triangle A B H: Therefore the Quadrangle A B C K, is to the triangle A K F, as the triangle A B H is to the triangle A H G, that is, as D to E.

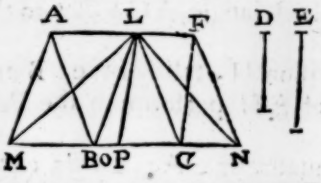
A Quadrangle divided from a point in a ſide.

Let any point be taken in the ſide A F, as L, from whence it is required to divide the Quadrangle in proportion as D to E.

Draw the lines L B and L C, and let B C be produced on both ſides, and to it from A, let there be drawn the line A M parallel to L B; and from F let F N be drawn parallel to L C; and joyn L M and L H: Then by what hath been ſhewn, the triangle L M C will be equal to the Quadrangle A B C L; alſo the triangle L H C will be equal to the triangle L C F, and the whole triangle L M H will be equal to the whole Quadrangle A B C F. Let M H be divided in O, ſo as that M O may be to O H, as D to E; and joyn L O: The point O then will fall in M C, or in C H, if in M C (by the doctrine before taught) divide the Quadrangle A B C L by a right line drawn from the angle L, which let be L P, ſo that the parts (*viz.* the triangle L P A) may be to the Quadrangle L P B C, as M O to O C: I ſay that the line L P doth divide the Quadrangle, as is required.



Demonſtr. For the point P will be either in A B or B C: Let it be firſt in A B: And becauſe the triangle L P A is to the Quadrangle L P B C, as M O is to O C, to wit, the triangle L M O to the triangle L O C; in compounding, the Quadrangle A B C L is to the Quadrangle L P B C, as the triangle L M C is to the triangle L O C, and by permutation. But the triangle L M C, is equal to the Quadrangle A B C L: Therefore alſo the triangle L O C will be equal to the Quadrangle L B P C, and the triangle L M O will be equal to the triangle L P A; and therefore the remaining triangle L O H will be equal to the Pentagon L P B C F: Therefore the triangle L M O, is to the triangle L O H, to wit, M O to O H, as the triangle L P A is to the Pentagon L P B C F.



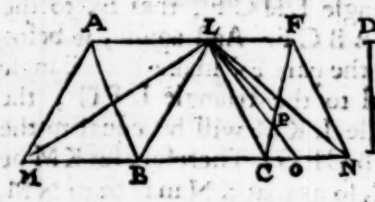
Then let P be in B C as (in the other figure) we will ſhew in the ſame manner that M O is to O N, as the Quadrangle A B P L is to the Quadrangle L P C F required.

But if the point of diviſion O, fall in C N: Let the triangle L C F be divided by the line L P, ſo that the triangle L C P may be to the triangle L P E, as C O to O N, and that which was required is done.

Demonſtr. For becauſe the triangle L C P, is to the triangle L P F, as

C O

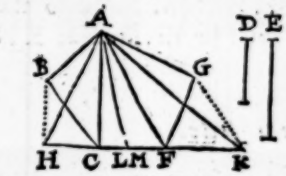
CO to ON; that is, as the triangle LOC to the triangle LON; in compounding, the triangle LCF is to the triangle LPF, as the triangle LCN is to the triangle LON; and by permutation, the triangle LCN is equal to the triangle LCF: Therefore also the triangle LON is equal to the triangle LPF; and the remaining triangle LMO is equal to the pentagon ABCPL. Wherefore as the triangle LMO is to the triangle LON; that is, as MO to ON, that is, as D to E; so is the pentagon ABCPL to the triangle LPE: Therefore the Quadrangle ABCF, &c. Which was to be done.



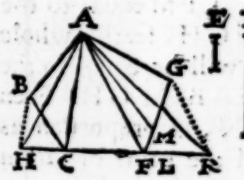
¶ But if the point be given in any other either, angle or side of the Quadrangle, the Proposition is wrought after the same manner.

A Pentagon divided from an angle.

Let the pentagon ABCFG be to be divided by a right line drawn from the angle A, in proportion as D to E: Let there be drawn the lines AC and AF, and from B and G, upon CF, extended on both ends; let there be drawn BH parallel to AC, and GK parallel to AF; and having drawn AH and AK, the triangle AHF will be equal to the Quadrangle ABCF, and the triangle AFK will be equal to the triangle AFG, and the whole triangle AHK is equal to the whole pentagon ABCFG: Let HK be cut in L, so as that HL may be to LK, as D to E; the point L then will either be in HF or FK; and if in HF, let the Quadrangle ABCF (by the Rules aforesaid) be divided by AM, issuing from the angle A, so as that the parts thereof may be in proportion as HL to LF: I say the line AM doth divide the Pentagon, as is required.



Demonstr. For (by the reasons aforesaid) the triangle ABM is to the pentagon AMCFG, or as in the other Scheme, the Quadrangle ABCM is to the Quadrangle AMFG, as HL is to LK, required.

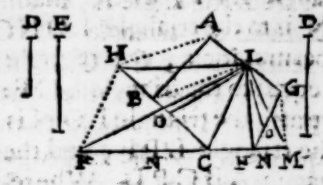


But if L doth fall in FK, in like manner, AM drawn from the angle A, doth divide the triangle AFG in proportion as FL to LK, and sheweth that the pentagon ABCFM is to the triangle AMG, as HL is to LK, that is, as D to E.

To divide a Pentagon from a side.

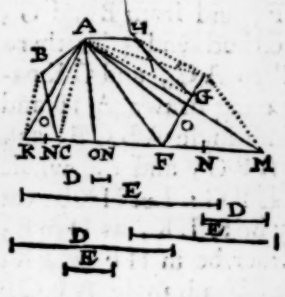
Let the point L be taken in the side AG, from which point a line drawn, doth divide the pentagon in the proportion given, to wit, of D to E. Draw LC and LF, and CB being produced, on the side B, let a triangle (by the Rules before) be made, to wit, the triangle LHC, equal

to the Quadrangle L H C F. Then producing C F on the part C, make the triangle L K F equal to the Quadrangle L H C F, that is, to the pentagon L A B C F. And again, it being produced on the part F, make the triangle L F M equal to the triangle L F G; the whole triangle L K M will be equal to the pentagon A B C F G: Therefore let K M be divided in N, so as that K N may be to N M, as D to E. And if the point N fall in K F, divide the pentagon L A B C F by L O, so as that the Quadrangle L A B O may be to the Quadrangle O C F L, as K N to N F: The Quadrangle L A B O will be to the pentagon O C F G L, as K N to N M: In the same manner, it may be demonstrated that the Hexagon L A B C F O, is to the triangle L O G, as K N to N M; that is, as D to E. Which ought to be done.



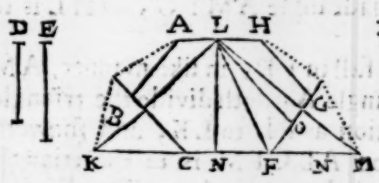
To divide a Hexagon from an angle.

Let the Hexagon be A B C F G H, and the angle A from whence it ought to be divided in proportion as D to E, draw A F, being extended both wayes, make the angle A K F equal to the Quadrangle A B C F, and the triangle A F M equal to the Quadrangle A F G H: Then the whole triangle A K M will be equal to the Hexagon A B C F G H: Then divide K M in the point N, so as that K N may be to N M, as D to E. And if the point N fall in K F, divide the Quadrangle A B C F by a line drawn from the angle A, so that its parts be in proportion as K N to N F. But if the point N fall in F M, divide the Quadrangle A F G H, as F N to N M, and so the Hexagon A B C F G H will be divided as K N to N M; that is, as D to E, required.



To divide a Hexagon from a side.

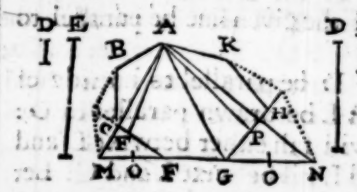
Let there be taken in the side A H the point L, from whence a line drawn, may divide the Hexagon given, according to the proportion of D to E. Draw L F, and having produced C F, make the triangle L K F equal to the pentagon L A B C F, and the triangle L F M equal to the quadrangle L F G H, so the whole triangle L K M will be equal to the whole Hexagon A B C F G H. Then divide K M in N, in proportion as D to E, and if the point N fall in K F. Let the pentagon L A B C F be divided from the angle L in proportion as K N to N F. And if it fall in F M, let the Quadrangle L F G H be divided according to the proportion of F N to N M, so will the whole Hexagon be divided by a line drawn from L, in proportion as K N to N M, that is, as D to E, required.



To

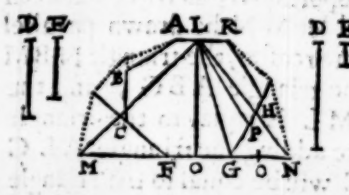
To divide an Heptagon from an angle.

Let the Heptagon A B C F G H K, be divided from the angle A, in proportion as D to E: Let A G be drawn, and make the triangle A M G equal to the pentagon A B C F G, and the triangle A G N equal to the quadrangle A G H K; and the whole triangle A M N equal to the whole Heptagon A B C F G H K: Let M N be cut in the point O, in proportion as D to E. And if O fall in M G: Let the pentagon A B C F G be divided as M O is to O G, with the line A P; and if it fall in G N, divide the quadrangle A G H K as G O to O N, and so the Heptagon will be divided in proportion as M O to O N.



To divide an Heptagon from a point in a side.

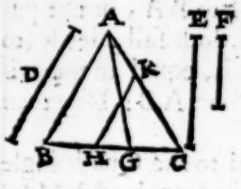
In the last place, Let the point L be in the side A K, and from L a line be drawn to divide the Heptagon according to the proportion of D to E. Let there be drawn the line L G, and constitute the triangle L M G, equal to the Hexagon L A B C F G; and let the triangle L G N be made equal to the quadrangle L G H K, and so as that the whole triangle L M N will be equal to the Heptagon A E C F G H K. Again, Let M N be cut according to the given proportion in O, and if O fall in M G, divide the Hexagon in proportion as M O to M G. But if it fall in G N (as on the left hand) divide the quadrangle according to the proportion of G O to O N. And the whole Heptagon will be divided as M O is to O N, that is, as D to E given. In like manner one may proceed to other Figures, of how many angles or sides soever. Which was to be done.



PROBLEME II.

To divide a right lined figure G A B C, *according to a proportion given* E *to* F, *by a right line parallel to another line given* D.

Divide B C in G, so as that B G may be to G C, as E to F; then either the line D is parallel to one side of the triangle, or to none of them. First, suppose it parallel to the side A B, and find a mean proportional C H, and by H let H K be drawn parallel to B A: I say H K divides the triangle, as is required.



Demonstr. For having drawn A G, the triangle A B G will be to the triangle A G C, as B G to G C, that is, as E to F; and in compounding, the triangle A B C is to the triangle A G C, as B C to C G. But B C is to C G, as the triangle A B C is to the triangle K H C; [a] for the triangles A B C and K H C are like, and B C to C G hath dupli-

a) 19. 6.

duplicate proportion of BC to EH: Wherefore the triangle KHC is equal to the triangle AGC, and the remaining Quadrangle ABHK is equal to the triangle ABG: Therefore the Quadrangle ABHK is to the triangle KHC, as the triangle ABG is to the triangle AGC; to wit, as E to F.

In like manner, it will be demonstrated, if the given line be parallel to the side BC or CA required.



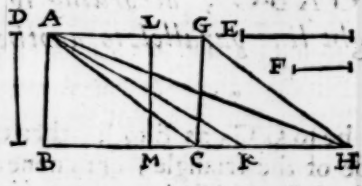
If the given line D be parallel to no side of the triangle, Let AL be drawn parallel to D: Then the point G will fall either betwixt L and C, or betwixt B and L; if betwixt L and C: Let there be taken between LC and CG, a mean proportional; and draw MN parallel to AL: Then the triangle NMC (by what is before demonstrated) will be equal to the triangle AGC; and the Quadrangle ALMN will be equal to the triangle ALG. Wherefore adding the triangle common to both, to wit, ABL, the Quadrangle ABMN will be equal to the triangle ABG: Therefore the Quadrangle ABMN is to the triangle NMC, as E to F.



But if the point G fall betwixt B and L, then also find a mean proportional, as BM, between LB and BG; and let MN be drawn parallel to AL. By the same reason, the triangle NBM will be equal to the triangle ABG, and the Quadrangle ANML is equal to the triangle AGL: Therefore adding the triangle ALC common to both, the Quadrangle ANMC will be equal to the triangle AGC: Therefore the triangle ABC is divided in the proportion given, by a parallel to D, as was required.

A Quadrangle ABCG, *divided by a line parallel to a line given* D, *in proportion as* E *to* F.

Where D is parallel to a side of a Quadrangle, as here to AB: Joyn AC, and from the point G draw GH parallel to AC, which will meet with BC extended in H; then let AH be drawn: Therefore (by what is before said) the triangle ACH is equal to the triangle ACG; and adding to both the common triangle ABC, the triangle ABH will be equal to the Quadrangle ABCG: Let BH be divided in K, so as that BK may be to KH, as E is to F; and let AK be joyned, the side then of the Quadrangle is parallel to BA, or not parallel, and if it be parallel, howsoever the point K doth fall, let [b] (to the line AB) there be applyed the superficies ABML, equal to the triangle ABK, so as that LM be parallel to the said line AB: I say then that LM performes the Proposition.

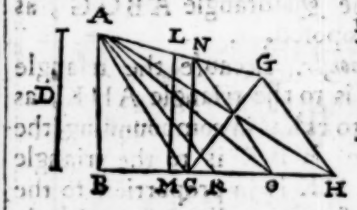


b) 10. of the former book.

Demonstr. For because the triangle ABH is equal to the Quadrangle ABCG, and the triangle ABK is equal to the Quadrangle ABML: The triangle AKH remaining, will be equal to the remaining Quadrangle LMCG: Therefore as the Quadrangle ABML is to the Quadrangle

drangle LMCG; so is the triangle ABK to the triangle AKH. But the triangle ABK is to the triangle AKH, as BK is to KH, that is, as E is to F: Therefore the Quadrangle ABML is to the Quadrangle LMCG, as E to F.

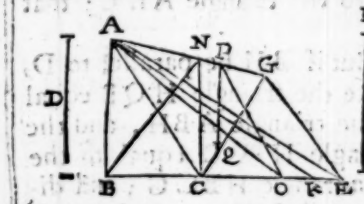
But if CG be not parallel to the side BA: Let there be drawn from one of the points, as CG, within the Quadrangle, a right line, parallel to BA; let that be now CN, and from N, draw NO parallel to AC, and joyn AO: Then the triangle ABO will be equal to the Quadrangle ABCN: Therefore if K fall in O, the line CN performes the Prposition.



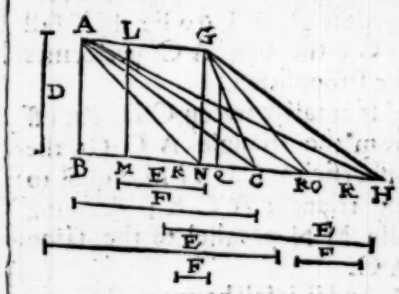
Demonstr. For the Quadrangle ABCN is to the triangle CNG, as the triangle ABO is to the triangle AOH; that is, as BK is to KH, and as E is to F.

But if K doth fall between BO, apply [c] to AB a Superficies, equal to the triangle ABK, which let be ABML; so as that ML may be parallel to AB; which may in like manner be demonstrated, to divide the Quadrangle ABCG, as was proposed. c) 10. of the former book.

Lastly, if K fall between OH, divide the triangle NCG by the line PQ, parallel to NC, in proportion as OK to KH; that is, as the triangle AOK to the triangle AKH; and since the triangle NCG is equal to the triangle AOH, the Superficies NCZP will be equal to the triangle AOK, and the triangle PQG equal to the triangle AKH: Therefore the pentagon ABCQP is equal to the triangle ABK, and the same pentagon ABCQP, is to the triangle PQG, as BK is to KH, that is, E to F.

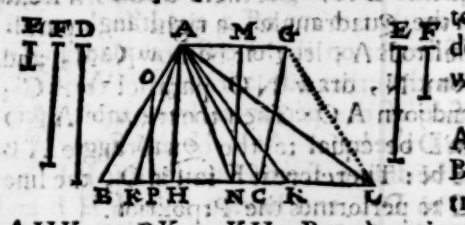


In like manner proceed, if from G there be drawn within the Quadrangle the line GN, parallel to AB, (as in the other figure it doth appear. For having joyned AN and AC, and from G drawn GO, which is parallel to the said AN; and having drawn GH which is parallel to AC. Lastly, Let there be joyned AO and AH: Then the triangle ABO will be equal to the Quadrangle ABNG; and the triangle ABH equal to the Quadrangle ABCG. And if the point K fall in O; then NG performes the Propositon. But if it fall between BO, do as in the former rules. And if it fall between OH, then from the triangle GNC cut off the Superficies GNQP, equal to the triangle AOK; draw PQ parallel to GN, and it is done as was required. But if D be not parallel to any of the sides of the Quadrangle ABCG; draw from one of the points AB, within the Quadrangle, a right line, parallel to D. Let it first be AH, and joyning AC, let GL be drawn parallel (from the point L) to AC, which produced, may meet with BC in L; and joyning AL, the triangle ABL will be equal to the Quadrangle ABCG. Let BL be divided in the point K, so as that BK may be to KL, as E



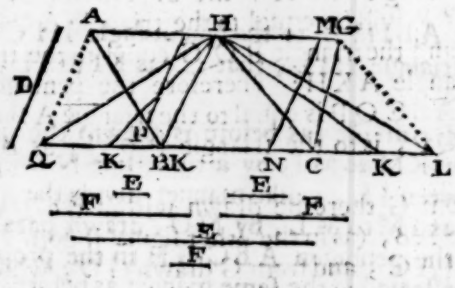
to F. Then the point K will either fall in H, or between H L, or between B H, if it fall in H, the right line A H performeth the Propoſition.

But if it fall between H L (by the former Rules divide the Quadrangle A H C G in proportion as H K is to K L, by the right line M N, parallel to A H, that is to D, which divideth the Quadrangle A B C G, as was propoſed.



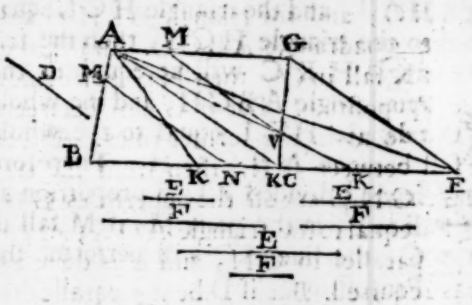
Demonſtr. Becauſe the triangle A B H is to the triangle A H K, as B H is to H K; in compounding, the triangle A B K is to the triangle A H K, as B K to K H. But the triangle A H K is in proportion to the triangle A K L, as K H to K L: Therefore from equality, the triangle A B K is to the triangle A K L as H K to K L. But the Quadrangle M N C G is equal to the triangle A K L: Therefore the Quadrangle A B N M is to the Quadrangle M N C G, as B K to K L, that is, as E to F.

Laſtly, if K fall between B H, having drawn A K, cut from the triangle A B H the Superficies A O P H, equal to the triangle A H K, by O P, parallel to A H; the remaining triangle O B P will be equal to the remaining triangle A B K: Therefore the triangle O B P is to the pentagon A O P C G, as the triangle A B K is to the triangle A K L; that is, as B K to K L, to wit, as E to F.



But if B H be parallel to D, make the triangle H Q B equal to the triangle A B H, and the triangle H B L equal to the Quadrangle H B C G; and dividing Q L in proportion as E to F, in the point K: If K fall in B, the line B H will performe the Probleme. If it fall between B L or Q B, then proceed as before newly is ſhewn.

But if the joyned line A C be parallel to D, make to the triangle A C G an equal triangle, as A C L; and dividing B L in K, according to the proportion given E to F: If K fall in C, the line A C performes the Propoſition.



If it fall between C L, cut off from the triangle A C G, the Superficies A C N M, equal to the triangle A C K; drawing alſo M N parallel to the ſame A C.

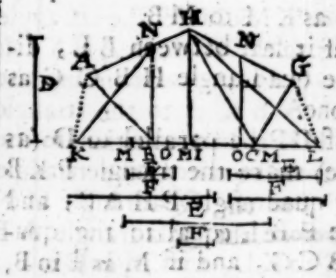
And if it fall between B C, cut from the triangle A B C a Superficies A C N M equal to the triangle A K C, by a right line M N, parallel to A C; and in like manner, it may be demonſtrated that the Quadrangle A B C G is divided according to the proportion of E to F, as was required.

No otherwiſe do we proceed if the joyned line B G were parallel to D.

A

A Pentagon divided in proportion as E *to* F, *by a line parallel to* D.

Let there be drawn from ſome point either in the angle or ſide, a right line to the baſe, parallel to D, ſo as that it cut off a quadrangle on either ſide, or on the one ſide a quadrangle, and on the other ſide a triangle: We make any ſide of the pentagon lying moſt oppoſite to D the baſe. As in the firſt figure, Let there be drawn from H a right line HI, parallel to D, and having joyned H B and HC; let there be drawn from A the line AK, parallel to HB, which being produced, may meet with CB in K; and from G, let GL be drawn, parallel to HC, and meeting with BC produced in L; and let HK and HL be joyned: Then the triangle HKI will be equal to the quadrangle ABIH, and the triangle HIL will be equal to the quadrangle HICG, and the whole triangle HKL will be equal to the whole pentagon.

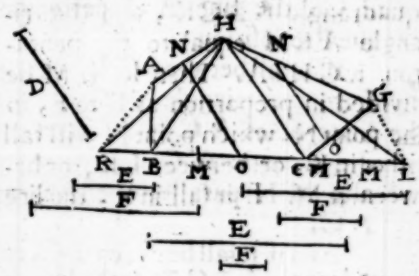


Divide KL in the point M, in proportion as E to F: Therefore M will fall either in I, or between KI: If M fall in I, the right line HI performeth the Propoſition.

Demonſtr. For the quadrangle ABIH is to the quadrangle HICG, as the triangle HKI is to the triangle HIL; that is, as KI to IL, to wit, as E to F.

If the point fall between KI, divide (as before is ſhewn) the quadrangle ABIH in proportion as KM to MI, by a right line NO, parallel to HI. And if it fall between IL, in like manner divide the quadrangle HICG in proportion as IM to ML, by NO, drawn parallel to HI, and NO will divide the pentagon ABCGH in the proportion given, which will be demonſtrated in the ſame manner as before.

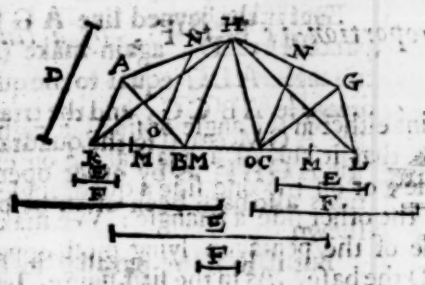
Again, in the following figure, in which HC is parallel to D, by joyning HB, the triangle HKB may be made equal to the triangle HAB, and the triangle HCL equal to the triangle HCG, then the triangle HKC will be equal to the quadrangle ABCH, and the whole triangle HKL equal to the whole pentagon ABCGH: Therefore having divided KL in proportion as E to F, in the point M, if M fall in C, the line HC will performe the Propoſition.



If it fall between KC, divide the quadrangle ABCH in proportion as KM is to MC.

But if it fall between CL, divide the triangle HCG in proportion as CM to ML, and ſo the pentagon will be divided as is required.

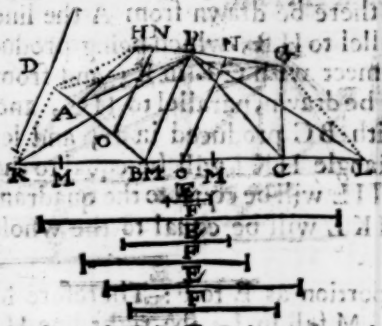
No otherwiſe ſhall it be done, if HB be parallel to D, for then will be



be made the triangle H K B, equal to the triangle H A B; and the triangle H B L equal to the quadrangle H B C G: Wherefore if M fall in the point B, the line H B performeth the propoſition.

If it fall between K B, Let the triangle H A B be divided in proportion as K M to M B.

But if it fall between B L, divide the quadrangle H B C G as B M to M L, and what was required is done.



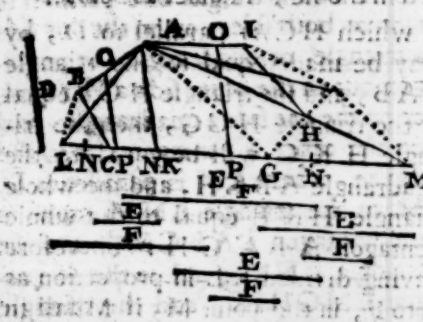
Laſtly, if B P be parallel to D (as in this figure) make the triangle P K B equal to the quadrangle P H A B; and the triangle P B L equal to the quadrangle P B C G, and if M fall in B, the ſame line B P will performe the Propoſition.

If it fall between K B, divide the quadrangle P H A B in proportion as K M to M B.

But if it fall between B L, divide the quadrangle P B C G in proportion as B M to B I, and do the like in all other pentagons, and that which was required will be done.

A Hexagon A B C G H I, *divided in proportion as* E *to* F, *by a line drawn parallel to* D.

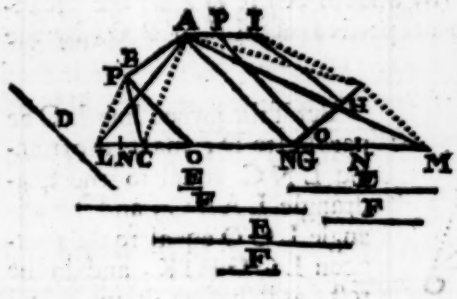
Let there be drawn from any point to the baſe a right line parallel to D, ſo as that it cut off either a quadrangle or pentagon on both ſides, or on the one ſide a triangle or quadrangle, and on the other ſide a pentagon, (as in the figure.) And let there be drawn from A, a right line A K, parallel to D, and let the triangle A L K be made equal to the quadrangle A B C K, and the triangle A K M equal to the pentagon K G H I A. Then let L M be divided in proportion as E to F, in the point N, which point N will fall either in K, or between L K, or between K M. If it fall in K, the line A K performeth the Propoſition.



If it fall between L K, divide the quadrangle A B C K according to the proportion of L N to N K, by a line O P, parallel to A K.

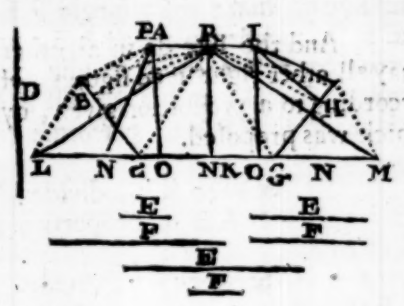
If it fall between K M (by what hath been demonſtrated) divide the pentagon A K G H I, in proportion as K N to N M, by the right line O P, parallel to A K.

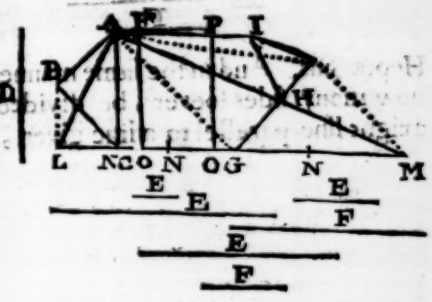
But



But if the joyned line A G be parallel to D, again make the triangle A L G equal to the quadrangle A B C G, and the triangle A G M equal to the quadrangle A G H I, as by other operations alſo hath been often before ſhewn.

But if R K be parallel to D, make the triangle R L K equal to the Pentagon R A B C K, and the triangle R K M equal to the pentagon K G H I R.

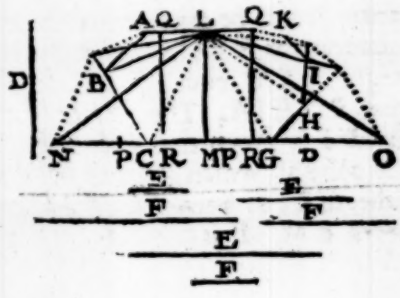




Laſtly, if the joyned line A C be parallel to D, make the triangle A L C equal to the triangle A B C, and the triangle A C M equal to the pentagon A C G H I, do the reſt as hath been ſhewn by the former rules, and the Hexagon will be divided as was required.

A Heptagon A B C G H L K, *divided in the proportion of* E *to* F, *by a line parallel to a line given, as* D.

Let there be drawn from any point to the baſe, a right line parallel to D, which ſhall either cut off a pentagon on both ſides, or on the one ſide a triangle, or quadrangle, or pentagon, but on the other ſide a Hexagon, or on the one ſide a quadrangle, and on the other a pentagon, as in the figure, where L M is parallel to D. Make the triangle L N M equal to the pentagon L A B C M, and the triangle L N M q ual to the Hexagon L M G H I K, and the triangle L M O equal to the Hexagon L M G H I K, and having cut N O according to the proportion of E to F in the point P. If P fall in M, the right line L M performeth the Probleme.



If it fall between N M, divide the pentagon L A B C M according to the proportion of N P to P M, by the line Q R, parallel to L M.

If it fall between M O, divide (by what is before taught) the Hexagon L M G H I K according to the proportion of M P to P O, by the right line L M, parallel to D.



But if the joyned line L C be parallel to D, make the triangle L N C equal to the quadrangle L A B C, and the triangle L C O equal to the Hexagon L C G H I K, and do the reſt, as is before ſhewn. And the Heptagon will be divided, as was required.

And the like do in all other Heptagons. And in the ſame manner may all other right lined figures, of how many ſides ſoever, be divided according to a proportion given, by a right line parallel to a line given, which was propoſed.

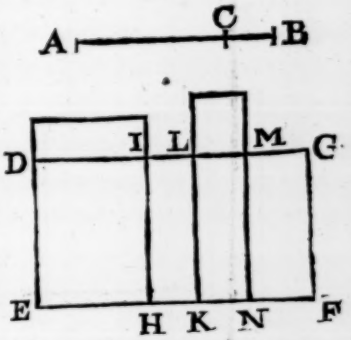
FINIS.

ERRATA.

Pag	marg line	Page line	Read
11		20	*dele* not
19	7		23 d. 1.
20		39	lines
22		33	*dele* alſo
22	8		3 Com. ſent.
28		40	A C F
29		4	will not cut
30	6		15 prop.
30	7		16 prop.
30	9		13 prop
35		16	E C D
35		26	*del* equal
36		26	*dele* equal
36		27	*dele* equal
37		28	G E
37		31	A and D
40	2		13 prop.
40		9	D H G
40		29	B G H
46	6		31. 1.
47	3		34. 1.
58		24	28
59		9	E G H
59		25	C B
60	5		31. 1.
61		37	K G
64		1	R H T
64		35	be a right angle
64		40	E F G
65	4		47. 1.
70	2		47. 1.
71		24	E B
71		30	D E
72			E ..... 24 F
72			F ..... G 13
76		13	D B & the ſquare
84		18	*dele* let (of C B
84		21	as
89	3		5. 1
90		35	within at B
90		40	*dele* it
91		25	*dele* but G F & F C are greater than the other ſide G C.
95		40	C E
96		9	F C E G
96		18	F C and G E
98		9	B E
101		4	D A B
102	1		20. 3.
106	3		27. 3.
108	1		18. 3.
108	2		22. 3.

Page	marg line	Page line	Read.
110	3		5. 2.
113		10	A G
118	3		16. 3.
120		1	prop. of the third
120		32	*de.* the equal lines
124	4		26. 3.
124	5		29. 3.
124	6		27. 3.
124		40	½ of 2 right angls
126		17	C B F
126	4		Cor. 16. 3.
126		40	A F and B F
127		32	of 2 right angles
128		27	equall to B H
134		41	divers
135		41	greater inequa-
138		46	$\frac{1}{33}$ (lity
139		25	one of the which
143		7	of the other
143		20	as of D to E
143		24	*dele* and
151		17	and E H be added
153	1		3. 5.
159	2		6. 5.
164	7		15. 5.
165	6		10. 5.
166	4		11. 5.
197	3		19. 5.
177		14	*dele* of
178		24	D E F
178		25	E H
179	3		38. 1.
180	7	33	39. 1.
181		36	as B D
185			*de.* A D B are right
188	2	27	31. 3. (angles
196		24	B C to C G
198			G B C
200	2		43. 1.
202	2		25. 6.
204	7		19. or 20. 6.
206	2		28. 3.
206	3		27. 3.
182		*ult*	in
186		10	there
191	9		12. c. f.
193		26	to A and to D
202		5	by 2 and 2
205	6		26. 7.
206		30	any Prime num.
207		6	to E the fourth

Page	marg line	Page line	Read
208		18	A B and C
212	2		29. 7.
216		29	I of B in E
219		36	of D in E
220		11	E and H
220		32	*ada* [to the ſide D
226		31	A and C
231		16	D, E and G
234		27	proportional
236		27	as C meaſures G
241		5	A, B and D
242	3		12. c. f.
248		8	A, BC, D, & E F
248		17	D is to B C
259		3	*dele* the
259		10	and therefore
261		8	C to a number D
271		34	incomenſrable
276		43	*de* Being equal to the medial A B. A C & H I rational.
282		9	ſquare rational
287	10		Cor. 24. 10.
289		3	the rectangle
295		17	to the Rational
297		33	no ſquare greater
303	3		22. 10.
310		10	D L and L G
313		33	as the rectangle
324		28	*dele* a compound to the ſaid
334		*pe.*	which with
336	8		10. 10.
337	3		23, 10.
344			line 2 & 3, *dele* [and to G H the rectangle H E equal to A B



This Scheme belongs to the 61, 62, and 63, Propoſitions of the Tenth Book.

Errors in the Diagrams.

First Book, Prop. 16. B F C for B E C. Third Book prop. 36 E omitted in the ſecond ſcheme. Prop. 27. C ſtands in place of I. Fourth Book Def. 1. a line omitted from K to L. Prop. 5, a line omitted from A to F. Prop 12, F is in place of L, and F muſt be in center. Fifth Book, Prop. 3, line C is not to be divided. Sixth Book, prop. 1, the line A L omitted. prop, 16, E is put for F in the rectangle. Seventh Book, prop. 14, for F...2, F...3. Eighth Book, prop, 5, for 19, 16. prop 6, for G 9, G 6. prop. 12, for E 6, E 9. prop. 15, for I 22, I 72. prop. 18: for A 13, A 12. Ninth Book, prop. 23 E omitted. Tenth Book, prop. 45, V in the firſt, and M in the ſecond ſcheme omited. The Scheme in prop. 49 relates to prop. 50. prop. 51. place 6 between the third and fourth points.